Report of the Committee on Waves, appointed by the British Association at Bristol in 1836, *and consisting of* Sir John Robison, K.H., *Secretary of the Royal Society of Edinburgh, and* John Scott Russell, Esq., M.A. F.R.S. *Edin.* (*Reporter*).

Since the period of their appointment, the Committee have been almost incessantly occupied in carrying on the researches committed to them. The extent and multifarious nature of the subjects of inquiry have rendered it impossible to terminate the examination of all of them in so short a time; but it is their duty to report the progress which they have made, and the partial results they have already obtained, leaving to the reports of future years such portions of the inquiries as they have not yet undertaken. As far as they can judge from present indications, there are wide fields of novel and important science opening up in this direction, which will furnish an ample harvest of rich knowledge for the labour of several succeeding seasons.

The *Subjects of Inquiry* with which the Committee were charged are the following:—

What is a Wave?—What are the varieties, phenomena, and laws of waves in regard to generation and propagation in various circumstances?

Of what nature are the *Waves of the Sea?*

Is the *Tidal Elevation* a wave obeying the same laws with any other order of wave?

Is the propagation of the tide-wave affected by *Local Winds?* and if so, in what manner?

These were questions to which, in the existing state of our knowledge of hydrodynamics, we had no grounds either dogmatical or empirical to form a reply, and it was therefore of importance to the advancement of the science of hydrodynamics that we should be able to fill up this *hiatus valde deflendus*. The question of the propagation possessed interest not only in a scientific view, but also from its practical importance; for it had been found in the earlier proceedings of this Association that the beautiful physical phenomena of *waves* were not only employed as agents to convey through the air the intimations of distant events to the sense of hearing, and to waft to the eye the exquisite sensations of light and colour, but were likewise employed in the practical uses of every-day life, and took an important part in that commercial intercourse by means of which the comforts of life and the advancement of civilization are immediately promoted. It had been ascertained by former researches that the resistance of fluids to bodies moving through them is affected by an element which had not been formerly recognised; that the new element which had given rise to contradictory and apparently anomalous phenomena was a wave produced in the fluid by the moving body; and that this wave affected the amount of resistance, either positively or negatively, according as the velocity of the wave was greater or less than that of the moving body*. It became, therefore, an inquiry of theoretical and general interest, as well as of special and practical importance to the art of navigation, to determine with great accuracy the laws of this wave. It had already been satisfactorily established that the velocity of propagation of this wave was nearly that due to half the depth of the fluid, that this velocity was independent of the form of the generating solid, and of the generating velocity of the solid. But this law had not been extended to channels of different forms; neither had the conditions necessary to the existence of this wave, nor the nature of the mechanism by which its propagation takes place, been described and ascertained. This wave had been called the great solitary wave of the fluid, but its relation to other waves, and its identity or diversity, had not been determined.

It was also necessary to determine the nature and class of the waves with which we are most familiar, and which we see at the surface of water agitated by the wind, and which break on the shores of the sea. Do these belong to the previous class of waves, or do they not? their form and velocity have been thought to depend in some measure on the depth. Do they belong to the first class of waves, or are they a different class?

But the most important of all these investigations, both in relation to the advancement of physical science and to the practical value of their results, are probably those which refer to the propagation of the tide. The recent researches of Mr. Lubbock and Mr. Whewell, carried on in connexion with this Association and by its assistance, have conferred on the subject of the tides the interest of novelty as well as scientific value. Their researches have gone far towards removing the stigma

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* *Researches in Hydrodynamics*, by John Scott Russell, Esq., M.A. F.R.S. *Edinburgh, Phil. Trans. R.S.E.*, 1836.

cast upon science by the imperfect state of this branch of knowledge. That the solar and lunar attractions produced some effect upon the tides, every one knew; but the problem was far from having been reduced into that condition in which it could be said that the phenomena of the heavens being given, the tides could be determined in magnitude and in time. So perfect, however, has this prediction lately become, that Mr. Lubbock has said that, considering how well theory agrees with observation, he is not sanguine that any material improvements in prediction will hereafter be made. And, indeed, this assurance appears to rest on valid grounds when it is considered that the tide tables which have resulted from his researches, and those of Mr. Whewell, give predictions whose errors are within the limits of the errors of observation.

But although the Celestial Mechanism of the tides has been thus perfectly analysed and explained, there remain a great variety of considerations relating to the propagation of tides along the surface of the globe which are as yet unexplained; these constitute the Terrestrial Mechanism of the tides. It is in the *generation* only of the tide that the solar and lunar attraction produce their effects: over the subsequent *propagation* of them, they exercise little or no influence. It is not until 50 or 60 hours after their creation that the tides reach our shores, having moved in the interval in every possible direction, and with every velocity from 100 to 10 miles an hour. This moving elevation of fluid may be conveniently designated a *wave*, and its history will be the history of the *tidal wave*; but to confer upon it the name of wave does not imply that its laws are those which belong to any other similar elevation with which we are acquainted. It was necessary to investigate the nature of this tide wave—to examine the hydrodynamical mechanism by which it is transferred from one place to another,—to determine the laws which regulate its form and its velocity—to ascertain if any relations exist between the form and dimensions of its bed, and its own form and rate of transference. These and many similar points were still unknown. Laplace has said, in speaking of these points, "les circonstances dont elles dependent, ne sont pas connues." Mr. Lubbock, in reference to the fluctuation of the establishment, says, "this perplexing fluctuation presents an insuperable obstacle to extreme accuracy in tide predictions until it can be explained; at present we are only left to conjecture respecting the cause." And similar sentiments are expressed by Mr. Whewell in the seventh series of his researches on the tides, read on the 7th March, 1837. He observes, "I cannot conclude this paper without again pointing out that a great number of curious facts in fluid motion are established by these tide researches, of which it may be hoped that the theory of hydrodynamics will one day be able to render a reason." It was, therefore, necessary to investigate the subject of the terrestrial mechanism of the tides, that is, to determine the nature of the mechanism by which this tide wave is transferred from one part of the waters of a given channel to another. At the meeting of the Association at Bristol, Mr. Whewell had expressed his opinion that the great primary wave of Mr. Russell and the tidal wave would be identified.

The effect of wind upon the propagation of the tide wave was also a subject of importance. The magnitude of the tide is admitted generally to be affected by it in some way, but it is matter of doubt, whether the time of the tide, or rather the velocity of the tide wave, is at all affected. M. Daussy denies the existence of such an effect in the French observations, while it has been found by Mr. Lubbock in the London tides. It was necessary to determine this point with great accuracy.

Besides their direct and theoretical use, there was another point of some importance in these researches concerning the tide wave, viz., that if the tide wave should be found to obey the law of the great primary wave of fluid, we should be put in possession of the principles on which the improvement of tidal rivers might be effected.

Method of Inquiry.—The following order was adopted by the Committee in the means by which they endeavoured to carry on the inquiry with which they were entrusted:

The observations on the nature of the tide wave were those which it was important to obtain in the first place, as they required peculiar facilities which were not likely to be readily found.

Fortunately it occurred to one of the Committee that the river Dee in Cheshire was peculiarly suitable to their purpose. It was their object to determine whether the same law which regulated the propagation of the wave previously examined by Mr. Russell in experimental canals, was followed by the tide wave in its propagation, or whether the velocity of the tide wave were proportional to a certain depth in a certain form of channel. It was necessary for this purpose that a channel of uniform dimensions should be obtained which could be easily measured, and which should possess a tidal wave capable of being easily observed. Now it happened that the river Dee is, in part of its channel, tolerably regular, having been formed artificially through a considerable part of its length; it was thought likely to answer the purpose.

In the month of September Mr. Russell visited Cheshire for the purpose of instituting the observations. He found the river more perfectly suitable than could have been anticipated. For more than five miles the channel of the river is perfectly straight, of a depth and width nearly uniform, inclosed between banks that are even and well kept, and that have everywhere the same slope, while the bottom has the slight declivity of 10 inches per mile. Along this channel the tide rolls with a moderate velocity, sometimes marked by a crested surge, and sometimes commencing by a motion hardly perceptible, and here it is inclosed by banks so high as to protect the wave most perfectly from the action of the wind from every point except two.

The channel of the river was measured and sounded with great care, and observations of its tidal wave will be found in this report. The form of the tide wave is given in plate (VI.).

The observations on the Dee having furnished data for the determination of the law of the propagation of the tidal wave in a given regular channel, it was only necessary further to ascertain the nature of its motion in a channel of a less regular form, and to determine the effect of the wind upon it. But the difficulty in this case was enhanced by the circumstance that a most minute and expensive survey would be required to determine the figure of such a channel with the accuracy necessary to furnish data for calculation. In this however the Committee were again fortunate. The River and Frith of Clyde on the West of Scotland presents a long and varied tidal channel which has all the variety of form necessary for such an investigation. The navigation of this river is under the management of a Board of Trustees, under whose superintendence it has been greatly improved, and who have been at great pains to determine its condition by very careful surveys. To that Board your Committee made application, and having the kind assistance of Sir Thomas Brisbane, who, as a former President of the Association, took a deep interest in forwarding its views, they succeeded in obtaining the effective cooperation of the Board of Trustees of the Clyde in carrying on an investigation which they considered of much importance to the navigation and future improvement of their own river.[*] Their excellent engineer, Mr. Logan, was immediately placed in communication with Mr. Russell, and

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[*] The thanks of the British Association were afterwards tendered to the Trustees of the Clyde for their liberality.

instructed to afford every facility and assistance in his power; a most accurate survey of the river, with a longitudinal section and accurate transverse sections at every half mile were obtained, and a geometrical level of 18 miles was laid down with great precision. On this line were erected tide gauges of a peculiar construction, on which a small fraction of an inch could be read with ease even in a rough sea, and at a considerable distance from the instrument. These were placed at nine stations, and were simultaneously observed by careful observers every five minutes during at least one tide each day. The form and velocity of each tide wave were thus ascertained with the desired accuracy. Application was at the same time made to Captain Denham, a well-known member of this Association, who was kind enough to cause such observations of the corresponding high waters at the Liverpool Docks to be made as the nature of the situation would afford; and these, although less perfect than they would have been had the new arrangements for that purpose been completed which the interest taken by the British Association has been the means of originating, were yet sufficient to enable us to determine the tidal interval of the ports in the Clyde with Liverpool more accurately than hitherto. The observations after laborious corrections and reductions were all referred to mean solar line on the meridian of the observatory of the University of Glasgow, kindly granted by Professor Nicol for the purpose of regulating the chronometers.

The waves of the sea formed the subject of careful attention to your Committee. For this purpose one of them obtained the use of the Mermaid yacht, of Mr. Bogle, of Glasgow, kindly granted at the request of Mr. Allan, the secretary of the Northern yacht squadron, for the purpose of making the necessary observations at sea. The weather was rather unfavourable. The vessel encountered alternately severe gales and dead calms, which first drove her to seek shelter and then prevented her from leaving her asylum. By means however of these observations, and of others made in steam vessels crossing the Irish Channel, the results aimed at were obtained. This series was afterwards completed by observations made on the sea shore, by which the phenomena of surges have been perfectly explained.

The series was concluded by observations made in experimental reservoirs and channels. These were constructed of a variety of forms. The waves were generated in different ways and of very different species. An apparatus was contrived by which very great accuracy was obtained in the determination of velocity. A considerable series of these observations are given at the end of this report exactly as they were made, and

in such an extent of detail as to furnish any future theorist with data as minute as those he might obtain by individual observation. This branch of inquiry is however so extensive, that this report only gives the commencement of the series, the powers of the Committee having been extended during another year for continuing the inquiry.

General Results.—The following are nearly the general results of these inquiries in so far as they have hitherto been obtained.

1. The existence of a GREAT PRIMARY WAVE of fluid, differing in its origin, its phenomena, and its laws from the undulatory and oscillatory waves which alone had been investigated previous to the researches of Mr. Russell, has been confirmed and established.

2. The velocity of this wave in channels of uniform depth is independent of the breadth of the fluid, and equal to the velocity acquired by a heavy body falling freely by gravity through a height equal to half the depth of the fluid, reckoned from the top of the wave to the bottom of the channel.

3. The velocity of this primary wave is not affected by the velocity of impulse with which the wave has been originally generated, neither do its form or velocity appear to be derived in any way from the form of the generating body.

4. This wave has been found to differ from every other species of wave in the motion which is given to the individual particles of the fluid through which the wave is propagated. By the transit of the wave the particles of the fluid are raised from their places, transferred forwards in the direction of the motion of the wave, and permanently deposited at rest in a new place at a considerable distance from their original position. There is no retrogradation, no oscillation; the motion is all in the same direction, and the extent of the transference is equal throughout the whole depth. Hence this wave may be descriptively designated THE GREAT PRIMARY WAVE OF TRANSLATION. The motion of translation commences when the anterior surface of the wave is vertically over a given series of particles, it increases in velocity until the crest of the wave has come to be vertically above them, and from this moment the motion of translation is retarded, and the particles are left in a condition of perfect rest at the instant when the posterior surface of the wave has terminated its transit through the vertical plane in which they lie. This phenomenon has been verified up to depths of five feet.

5. The elementary form of the wave is cycloidal; when the height of the wave is small in proportion to its length the curve

is the prolate cycloid, and as the height of the wave increases the form approaches that of the common cycloid, becoming more and more cusped until at last it becomes exactly that of the common cycloid with a cusped summit; and if by any means the height be increased beyond this, the curve becomes the curtate cycloid, the summit assumes a form of unstable equilibrium, the summit totters, and falling over on one side forms a crested wave or breaking surge.

6. A wave is possible in forms of channel where the depth is not uniform throughout the whole depth. The full consideration of this subject is reserved for next report. It appears however that where the difference between the depth of the sides is considerable, one part of the wave will continue during the whole period of propagation in the act of breaking, so as to show that in these circumstances a continuous wave is impossible. In other cases the ridge of the wave rises so much higher on the shallower part of the fluid as to produce a given velocity without exceeding the limits of equilibrium, and in those cases the wave becomes possible, and the velocity appears to coincide closely with that which we obtain by supposing the wave resolved into vertical elements, each having the velocity due to the depth and then integrating.

For example, let the form of the channel be

$$y = m\,x^n$$

$x\,\delta y$ = vertical element of area

$\therefore \frac{1}{2} a^2 x$ = the square of the velocity of the element,

and

$\frac{1}{2}\,a^2\,x^2\,\delta y$ = the square of the velocity multiplied by wave,

whence,

$$\int \frac{1}{2} a^2 x^2 \delta y = \int \frac{1}{2}\,a^2\,x^2\,m\,n\,x^{n-1}\,\delta x$$

$$= \int \frac{1}{2}\,a^2\,m\,n\,x^{n+1}\,\delta x$$

$$= \frac{1}{2}\,a^2\,\frac{m\,n}{n+2},\,x^{n+2} + \text{C}.$$

But since

$$\int x\,\delta y = \frac{m\,n}{m+n}\,x^{n+1}$$

$$\therefore v^2 = \frac{1}{2}\,a^2\,\frac{n+1}{n+2}\,x$$

$$\text{and } v = a\left(\frac{1}{2}\,\frac{n+1}{n+2}\,x\right)^{\frac{1}{2}}$$

If $y = m\,x$

$$v = a\sqrt{\frac{1}{3}\,x}$$

If $y = m\,x^2$

$$v = a\sqrt{\frac{3}{8}\,x}, \text{ \&c.}$$

Hence in the rectangular channel the velocity being that of gravity due to half the depth.

In the sloping or triangular channel the velocity is that due to one-third of the greatest depth. In a parabolic channel the velocity is that due to three-eighths or three-tenths of the greatest depth according as the channel is convex or concave.

From the identity of this formula with that for the centre of gravity, it appears that the velocity of the great primary wave of translation of a fluid is that due to gravity acting through a height equal to the depth of the centre of gravity of the transverse section of the channel below the surface of the fluid.

7. The height of a wave may be indefinitely increased by propagation into a channel which becomes narrower in the form of a wedge, the increased height being nearly in the inverse ratio of the square root of the breadth.

8. If waves be propagated in a channel whose depth diminishes uniformly, the waves will break when their height above the surface of the level fluid becomes equal to the depth at the bottom below the surface.

9. The great waves of translation are reflected from surfaces at right angles to the direction of their motion without suffering any change but that of direction.

10. The great primary waves of translation cross each other without change of any kind in the same manner as the small oscillations produced on the surface of a pool by a falling stone.

11. The WAVES OF THE SEA are not of the first order—they belong to the *second or oscillatory order* of waves—they are partial displacements at the surface which do not extend to considerable depths, and are therefore totally different in character from the great waves of translation, in which the motion of displacement of the particles is uniform to the greatest depth. The displacement of the particles of the fluid in the waves of the sea is greatest at the surface and diminishes rapidly. There

are generally on the surface of the sea several coexistent classes of oscillations of varying direction and magnitude, which by their union give the surface an appearance of irregularity which does not exist in nature.

12. When waves of the sea approach a shore or come into shallow water, they become waves of translation, and obeying the laws already mentioned, always break when the depth of the water is not greater than their height above the level.

13. Waves at the surface of the sea do not move with the velocity due to the whole depth of the fluid: may they not move with the velocity due to that part which they do agitate, or to some given part of it?

14. A circumstance frequently observed when the waves break on the shore, has been satisfactorily accounted for by the examination of the constitution of the waves of the sea. It has been frequently observed that a certain wave is the largest of a series, and that these large waves occur periodically at equal intervals, so that sometimes every 3rd wave, every 7th, or every 9th wave is the largest. Now as there are almost always several coexistent series of waves, and as one of these is a long gentle "under swell," propagated to the shore from the deep sea in the distance, while the others are short and more superficial waves generated by a temporary breeze of reflections from a neighbouring shore; so it will follow that when the smaller waves are $\frac{1}{3}$, or $\frac{1}{7}$, or $\frac{1}{9}$th, or in any other given ratio to the length of longer ones, those waves in which the ridges of the two series are coincident, will be the periodical large waves; and if there be three systems of coexistent waves, or any greater number, their coincidences will give periodical large recurring waves, having maxima and minima of various orders.

15. The TIDE WAVE appears to be the only wave of the ocean which belongs to the first order, and appears to be identical with the great primary wave of translation; its velocity diminishes and increases with the depth of the fluid, and appears to approximate closely to the velocity due to half the depth of the fluid in the rectangular channel, and to a certain mean depth which is that of the centre of gravity of the section of the channel. It is, however, difficult to determine the limits within which the tide wave retains its unity; where portions of the same channel differ much in depth at points remote from each other, the tide waves appear to separate.

16. The tide appears to be a compound wave, one elementary wave bringing the first part of flood tide, another the high water, and so on; these move with different velocities according to the depth. On approaching shallow shores the anterior tide waves move more slowly in the shallow water, while the posterior waves moving more rapidly, diminish the distance between successive waves. The tide wave becomes thus dislocated, its anterior surface rising more rapidly, and its posterior surface descending more slowly than in deep water.

17. A tidal bore is formed when the water is so shallow at low water that the first waves of flood tide move with a velocity so much less than that due to the succeeding part of the tidal wave, as to be overtaken by the subsequent waves, or wherever the tide rises so rapidly, and the water on the shore or in the river is so shallow that the height of the first wave of the tide is greater than the depth of the fluid at that place. Hence in deep water vessels are safe from the waves of rivers which injure those on the shore.

18. The identity of the tide wave, and of the great wave of translation, show the nature of certain variations in the establishment of ports situated on tidal rivers. Any change in the depth of the rivers produces a corresponding change on the interval between the moon's transit and the high water immediately succeeding. It appears from the observations in this report, that the mean time of high water has been rendered 37 minutes earlier than formerly by deepening a portion of about 12 miles in the channel of a tidal river, so that a tide wave which formerly travelled at the rate of 10 miles an hour, now travels at the rate of nearly 15 miles an hour.

19. It also appears that a large wave or a wave of high water of spring tides travels faster than a wave of high water of neap tides, showing that there is a variation on the establishment, or on the interval between the moon's transit and the succeeding high water, due to the depth of the fluid at high water, and which should, of course, enter as an element into the calculation of tide tables for an inland port derived from those of a port on the sea shore. The variation of the interval will vary with the square root of mean depth of the channel at high water.

These results give us principles, 1st, for the construction of canals; 2nd, for the navigation of canals; 3rd, for the improvement of tidal rivers; 4th, for the navigation of tidal rivers; 5th, for the improvement of tide tables.—See the Transactions of the Sections at the end of the volume.

First Series of Observations.

Experiments on Waves in Artificial Reservoirs.—As this portion of the experiments was made in continuation of a series of experiments in which Mr. Russell had been previously engaged, and of which he from time to time announced the results to the British Association at Dublin and at Bristol, and as these notices were omitted in the last volume of the Report, but promised by the Secretary to be included in the present one, it will be proper to state what had been brought to light in those experiments on waves previous to the appointment of this Committee.

At the Dublin meeting of the Association Mr. Russell stated that he had been induced to make a series of experiments on waves in certain circumstances, from having found that the resistance of fluids to the motion of floating bodies was very much affected by the phenomena of the waves generated in the fluid by the motion of these bodies; and that many of the imperfections of that part of hydrodynamical science which treats of the resistance of fluids, would be removed by an acquaintance with the laws of the motion of waves. One of the great instances of deficiency in our theoretical knowledge, when applied to practical uses, occurred in the question of the force required to give motion to a vessel in a confined channel, a canal, or a small river; in these cases a vessel at certain points of her progress encountered extreme resistance, and at other, still higher velocities, experienced diminutions of resistance equally extraordinary and anomalous. These facts had set at defiance all previous theory; but it was found that a knowledge of the laws of the generation and propagation of waves in a fluid was all that was required to solve these difficulties and to remove these anomalies. For this purpose he had undertaken a series of experiments on waves carried on during the years 1834 and 1835.

The Wave which had been thus found to form so important an element in the resistance of fluids, was found to be a phenomenon of a very different nature from those waves which had previously occupied the attention of the physical investigator. This phenomenon presents itself as a Solitary Progressive Elevation of the surface of a quiescent fluid, neither preceded nor followed by any secondary or successive phenomena, totally distinct from the *oscillatory waves*, and from such waves as the ripple on the surface of a lake agitated by the wind, and the concentric circular oscillations of a calm sheet of water into which a stone has been dropped, and from the waves which are presented on the surface of an agitated sea. This wave presents simply the phenomenon of an elevation of fluid transferred from place to place of the fluid, finding the fluid perfectly at rest, and leaving it in an equally perfect state of equilibrium. Many philosophers have examined the theory of waves, but they all appear to have considered only the oscillatory, successive, and gregarious waves. Newton considered them as represented by the oscillations of a column of fluid in a bent tube, and assigned to them laws analogous to those of the pendulum; Granesaude followed the theory of Newton; D'Alembert adopted Newton's theory, and pursued this investigation considerably further; and Lagrange improved it by removing some former limitations inconsistent with the phenomena; Laplace formed a new theory, in which the oscillatory waves are supposed to be formed by immersing a solid of a given form in the fluid and suddenly withdrawing it; Gerstner gives a very beautiful theory of waves, in which the observed phenomena of oscillatory waves of the larger class are very accurately represented; Poisson, Cauchy, and Fourier have discussed the mathematico-physical question of very minute oscillatory waves with so much success, as to represent some of the phenomena with considerable accuracy; and the results of these theoretical views have been examined very carefully in the experiments of Bremontier, Flaugergues, Bidone, and the Webers. But in none of these inquiries has the phenomenon of the solitary wave attracted any attention; and, indeed, so far from having been satisfactorily examined, its very existence does not appear ever to have been distinctly recognised.

This *solitary progressive elevation* appears to be the *wave of the first order*, and has been called by Mr. Russell the Great Primary Wave of the fluid. And its phenomena are of that invariable and decided character, which claim for it such a distinction.

The great primary wave was first observed by Mr. Russell in 1834. By the impulse of a vessel drawn by horses a considerable portion of fluid was raised above the level of the rest of the fluid in a channel of limited breadth and depth. The elevation thus formed was observed to assume a peculiar and regular shape extending across the whole breadth of the channel, and to propagate itself along the surface of the quiescent fluid with a velocity of nearly eight miles an hour; which velocity and form appeared to continue unchanged, although followed for about the distance of a mile.

The following experiments were made for the purpose of determining whether the velocity of this wave were not affected by the initial velocity given to the fluid at its generation by the moving body. The velocity of genesis, or of the vessel by whose displacement the elevation of fluid was produced, is given in miles per hour, and the time occupied by the wave in describing 700 feet is given in seconds.

	Velocity of genesis.	Space described by the wave.	Interval of time.
(1.)	5 miles an hour	700 feet	62· seconds
(2.)	3 ——	700 —	61· —
(3.)	10 ——	700 —	61· —
(4.)	7 ——	700 —	62· —
(5.)	7 ——	700 —	62· —
(6.)	4 ——	700 —	61·5 —

From this it is manifest that the velocity of the propagation of the wave does not vary with the velocity of its genesis.

To determine whether the height of the wave produced any variation in its velocity, the following experiments were made:

	Height of the wave above the level.	Space described.	Interval.
(7.)	6·0 inches	700 feet	61·50 seconds
(8.)	5·0 —	700 —	61·75 —
(9.)	3·5 —	700 —	62·50 —
(10.)	2·0 —	700 —	63·50 —

It appears from these examples that, in a given reservoir of fluid, the higher wave moves more rapidly than the lower; and it was afterwards found that the increase of height was equivalent in its effect on the velocity to an equal addition to the depth of fluid in the reservoir.

To determine whether the depth of the fluid affected the velocity of the wave, the following experiments were made in the same channel filled to different depths:

	Depth of fluid.	Space described.	Velocity of wave.
(11.)	5·6 feet	486· feet	9·594 miles an hour
(12.)	3·4 —	150· —	7·086 ——

The former of these observations is exclusive of the height of the wave, and adding six inches to the depth of the fluid in this case, the height of the wave being already added to the depth in (12.), we find that the velocities are nearly proportional to the square roots of the depths, and are nearly equal to the velocities that would be acquired by a heavy body in falling through heights equal to half the depth of the fluid.

In the last case the channel was rectangular, and consequently the depth of the fluid was uniform across the whole depth of the channel; it was next of importance to ascertain what law held in those cases where the depth diminished towards the edges of the channel. For this purpose two channels were selected having the greatest depths in their middle and diminishing towards the sides. The following are the results:

	Greatest depth in the middle of the channel.	Space described.	Velocity of wave.
(13.)	5·5 feet	1000 feet	7·84 miles an hour
(14.)	4·0 —	820 —	6·09 ——

In these instances the diminished depth at the sides has diminished the velocity of the wave below that due to the greatest depth in a ratio in the first example nearly of 9·5 to 7·8, and in the second of 7· to 6·. See Experiments (11) and (12).

The following three experiments are instructive as having been made on channels in which the maximum depth was nearly the same in all; but in (15) the depth remained constant to the side which was vertical. In (16) the sides had a slope of nearly 20°, and in (17) a slope of nearly 40°, so as to diminish the depth towards the sides.

	Maximum depth.	Form of channel.	Space described.	Velocity.
(15.)	5·6 feet	Rectangular	486 feet	9·59 miles
(16.)	5·5 —	Slope of 20°	2038 —	8·83 —
(17.)	5·5 —	Slope of 40°	1000 —	7·84 —

From these it is manifest that the depth of the channel, while it modifies the depth of the fluid, affects the velocity of the wave. It was not found that the breadth of the channel produced any similar effect.

The results obtained from the experiments of 1834 and 1835 were considered by the Association of sufficient novelty and importance to point out the propriety and advantage of instituting a fuller and more minute series of experiments concerning the nature of the wave, in which all its phenomena and laws should be determined with as much precision as possible.

The subjects of inquiry which immediately presented themselves were the following:

1. To determine whether different methods of generating the wave influence its subsequent phenomena.
2. To determine with accuracy the velocity of the wave in given circumstances.
3. To ascertain the form or forms of the wave.

4. To determine the manner in which the depth and breadth of the channel affect the velocity and form of the wave.

5. To determine the influence of form in the channel on the form and velocity of the wave.

6. To ascertain the nature of the mechanism by which the wave is propagated from one place to another; or to answer the question, What is the wave?

7. To ascertain the difference between *the primary wave* and waves of other descriptions.

8. To determine the effects of solid bodies or obstacles on the motion of waves, and the effect of waves on one another, and conversely—the effect of waves on solid bodies, either at rest or moving through them, immersed in them, or floating upon their surface.

9. To determine the effects of waves on one another.

For the purpose of obtaining some of these results with the requisite precision, there was provided the following

EXPERIMENTAL APPARATUS.

Experimental reservoir.—A rectangular reservoir, formed with much precision, was provided for the purpose of containing the fluid to be made the subject of experiment. Its sides were supported by strong brackets, and the whole was raised on a strong frame to a height convenient for experiment; the whole length of the reservoir was 20 feet precisely, an additional length of 7·3 inches having been reserved to form a generating chamber in connexion with the reservoir. The dimensions of the reservoir are,

Length of experimental reservoir . . .	20 feet
Breadth of experimental reservoir . . .	1 foot.

The bottom of the reservoir was placed with care in the horizontal plane, so that it could be filled and emptied conveniently. The reservoir is represented in Plate I., fig. 1. A is the transverse section, B and D are longitudinal sections of the levels of the reservoir.

Method of determining the velocity.—A channel of great length may appear at first sight more suitable to the determination of velocity than the comparatively short one here employed, whose whole length was traversed by some of the waves in less than five seconds; and it would have been preferable for that purpose had not the method of reflection been employed, by which all the advantages of that method when employed in the repeating circle and other instruments are obtained for the diminution of errors of observation, and by which also the probability in favour of accuracy in the result is elevated to the region of certainty. It was found that when a smooth plane surface, of sufficient rigidity, was immoveably fixed at the end of the channel, at right angles to the direction of the wave's transmission, the wave was thereby reflected without sensible change in its form, magnitude, or velocity. Two such reflecting surfaces being placed at opposite ends of the reservoir, it was found that the wave might be reflected from one end to the other over successive spaces of 20 feet, and thus brought repeatedly to the same points of observation. In this way the same wave was observed during so many as 60 successive transits after 60 successive reflections, having thus passed over a course equal in length to 1200 feet, and occupying an interval of 320 seconds, giving the power of observing it 60 times in its transit past a given point. It was thus brought under the eye of three observers at three different parts of the reservoir during a single transit. The whole internal surface of the reservoir was accurately divided into feet, inches, and minuter divisions.

Means of observing the transit.—To observe the instant of the transit of a wave past a given point is a matter of some difficulty, especially when the wave is long and flat. A wave one-tenth of an inch high and three feet long is scarcely sensible to the eye until its vertex has passed; its commencement and end are perfectly insensible, and its summit so flat that it is impossible directly to observe its place with precision. To obviate these difficulties, the following apparatus was provided. A plane mirror, M, (Fig. 2. Plate 1.) was raised on a frame to a height of four feet above the surface of the water. On this mirror the image, I, of a bright flame was thrown, and the mirror was adjusted so as to reflect this image upon the surface of the water (at W). A second mirror (m) was placed over this second image, so as to intercept the rays reflected from the surface of the water, and to return them finally through an eye-piece to the observer. The path of the ray was preserved during the whole of its extent in a plane at right angles to the direction of the motion of the wave. Parallax in observation was avoided by a micrometer wire in the eye-piece, which was kept in coincidence with an opaque line passed through the image at M and so reflected in m, and with a line of division, D, seen directly without reflection past the edge of the mirror m. The observer was thus enabled to compare the place of the centre of the reflected image by coincidence with fixed lines. When perfectly at rest the coincidence was perfect. When the centre of the wave was at W^{II}., figs. 2 and 3, the rays of light also reflected from a plane surface, perfectly horizontal, presented the same coincidence; but when the anterior part of the wave W^{I}, figs. 2 and 3, was that on which the rays fell, the image was carried in the direction of the motion; and, on the other hand, when the posterior surface of the wave reflected the image, it was transferred to the other side, as in the point W^{III}. When, therefore, the transit of a wave took place, the following phenomena presented themselves to the observer. The image continued at rest, as seen in fig. 3, until the approach of the wave; from the instant at which the transit began until the instant of the passage of the crest of the wave, the image appeared on the anterior side of the wire, as in fig. 4; but during the remainder of the transit, the image was found on the posterior side of the wire, as in fig. 5; and therefore the instant of the transit of the crest of the wave across the line was also the instant of the passage of the image from one side to the other across the wire: now, as the whole time of the transit did not amount to a second, this instant was given with the required precision, and although the elevation of the surface was not in many cases perceptible to the eye, the transit of the image was perfectly satisfactory.

For obtaining the dimensions of the wave with precision, various expedients were resorted to; there were provided glass tubes (gauges or *indices*) communicating with the channel at different depths; they are represented in fig. 6. The centre of each tube opens into the side of the reservoir at successive inches of its height, and after continuing horizontally for a certain space, is turned up vertically, and rises above the level of the water; the tubes thus become filled, and the water in each tube being tinged with colouring matter becomes distinctly visible, so that the variations of height are read with ease and precision on the graduated scale behind the tubes to hundredths of an inch. For a very elegant method of ascertaining the length of the wave with precision, Mr. Russell is indebted to Professor STEVELLY of Belfast, who suggested that fine points, similar to those used in the standard cistern barometers, should be applied to the surface of the water, so as to show by the instant of their submersion in the fluid, or emergence from it, the origin and end of the wave. This method was found to possess much precision; the phenomena of capillary attraction mark the instants of contact and separation with vividness, by the reflection of rays of light from the concave surface of the fluid raised around the point, and their disappearance on separation. The contact of this point with its image in the water was also a phenomenon marking the place of the surface of the fluid with minute accuracy. When the two points, placed at the beginning and end of the wave, showed the phenomena of immersion and emergence at the same instant, their distance was equal to the length of the wave. It was, however, necessary to have some means of bringing both points under the eye at the same instant, in order to determine with accuracy the coincidence of contact in both cases; the arrangements are given in fig. 7. P and P are points in contact with the surface of the fluid at the extremities of a wave; rays of light from them are reflected by the mirrors p and p to the eye at O, and are thus observed simultaneously. By these means, the points being removed further apart, or brought nearer, until the contact became simultaneous, and the distance of the points equal to the length of the waves, the height of the wave was determined by the glass indices in fig. 6.

Apparatus for generating the Waves.—Generating reservoir A. fig. 8, consisted of a continuation of the experimental reservoir A, B, D, of fig. 1, which was separated from it or connected with it by means of a sluice; so that by filling the generating reservoir with water to a higher level than the experimental reservoir while the sluice was closed, on raising it the water descended, producing a wave, of which the volume was known. The area of the horizontal section of the generating reservoir is 76·27 square inches, its length being 6·33 inches in the direction of the motion of the wave, and 12·05 inches its breadth at right angles to this; the detached generating chamber B, fig. 9, was a rectangular parallelopipedon, open at top and bottom, and so accurately fitted to the bottom of the reservoir as, when resting on it, to be capable of containing water to any height, but on raising it from the bottom by which it had been thus temporarily closed, the fluid descended, producing a wave of given volume. The area of the horizontal section of the chamber is 68·32 inches, being 6·1 inches long and 11·2 inches wide. A solid parallelopipedon, C. fig. 10, was used to generate waves, by protruding it to a given depth in the fluid; the area of its horizontal section being 88·32 inches, and its dimensions 24·0, 12·05, and 7·33 inches. Another detached generating chamber, D., was 2·98 inches, being 11·92 inches broad and 24· inches deep, being an area of 35·52 square inches in its horizontal section. In those cases where volume of the wave was not of importance, the wave was produced by the impulse of a flat surface pressed horizontally on the fluid.

Analysis of Experiments.—The original experiments are themselves given at the end of this paper, for the purpose of enabling any one who may be disposed to make use of them for any future purpose, either of framing or testing a theory, to make use of them much in the same way as if he had himself made the experiments. The wave having been generated was first observed in the glass index, fig. 6, placed near to the generating reservoir; then it passed under the transit station where its transit was observed, and the time registered either by one or two observers, and then its height was cleared in another glass index near the other reservoir; the wave having undergone the first reflection was returned, and the same observations were repeated during a number of successive reflections. See Experiments page 465—491.

The collection of tables at the end of this report gives the history of a series of waves in which these phenomena are carefully recorded.

Explanation of Tables.—For the sake of ready reference, there is given at the beginning of each table (see Wave I.) the approximate depth of the fluid, and the date of experiment, thus:

2d Aug. 1837. Wave I. Depth, 4 inches.

The next line contains the mode of generation, written thus:

Created by reservoir A. Volume of added fluid = 153·5 inches.

The reservoir A, fig. 1, Plate I., the detached chamber B, fig. 9, the solid parallelopipedon C, fig. 10, and chamber D, have already been described, and are successively referred to in the manner now stated; and in Wave IX. for example, the means of generation was the flat sluice in fig. 8, held in the hand, passed down to the bottom of the fluid, and moved horizontally so as to displace the fluid from the reservoir A.

The method of observing is next given, as for example in Wave I.

Transits observed directly at index, and without reflection—

when the unassisted eye of the observer detected by inspection the transit of the ridge of the wave passing the place of the indices at γ. fig. 6; but in other cases the eye was arrested by the reflected image in the transit apparatus already described, figs. 2, 3, 4, and 5, as for example in Wave V., where we have

Transits observed by the reflected image at the central station.

The next line gives the depth of the fluid in the channel, previous to the commencement of the experiment, first of all as directly observed in the glass indices, figure 6, on the scale of which the deviation from approximate depth, already given at the head, (Depth, 4 inches,) is read off with the appropriate sign + or −; and the mean depth of the fluid having been already compared by direct experiment with the scale of the index, and a correction for error of scale applied, the true result is given at the end as the mean depth of the fluid when at rest, freed from instrumental error, thus:

Statical level observed at $\left\{ \begin{array}{l} \gamma = -0·05 \\ \delta = -0·01 \end{array} \right\}$ corrected statical depth=3·942 inches.

In the table of the observations, *column* A gives the number of feet passed over by the wave, reckoning from the instant at which the first observation of time in *column* B was made on either or both of the chronometers α and β. In *column* C are given the readings of the index γ at that end of the reservoir where the wave was generated, and from which the observations are begun, and of the index δ placed towards the other end of the reservoir. In *column* D the observations of column C have been freed from the error of the index scale, so as to represent the true height of the ridge of the wave above the statical level of the fluid; and in *column* E the true height of the wave has been added to the statical depth of the fluid, so as to give the whole depth reckoned from the ridge of the wave to the bottom of the reservoir.

The observations were made in the following manner. The wave having been generated, was generally allowed to traverse the whole length of the reservoir, and return to γ before commencing the observations of time and space; this was done for the purpose of allowing the wave to assume its determinate form, which it did not generally acquire until it had remained for some time unaffected by external impulse; and this delay also allowed the secondary oscillations of the fluid to disappear. On the return of the wave to γ its height was carefully observed; after passing γ its transit past the central station was assumed as the zero for time, its height was observed at δ, and once more on its return to γ, so that the interval between the observations was an interval due to 20 feet or 40 feet, according as the observations were made on successive or alternate transits; the successive transits being used when the velocity was small, and the alternate ones when the velocity was such as not to afford sufficient intervals for observing and noting with composure.

The intervals between the transits were obtained with considerable precision, as may be gathered from the following observations made by independent observers.—See Wave XLV.

Chronometer α.	Chronometer β.	Difference of interval.	Chronometer α.	Chronometer β.	Difference of interval.
0·0	0·0	0·0	89·00	89·5	0·00
9·75	9·5	− 0·25	99·50	100·0	0·00
19·50	19·0	− 0·25	110·00	110·5	0·00
28·50	29·0	0·00	120·50	121·0	0·00
38·50	39·0	0·00	131·00	131·5	0·00
48·50	49·0	0·00	141·50	142·0	0·00
58·50	59·0	0·00	151·50	152·5	− 0·50
68·50	69·0	0·00	162·50	163·0	+ 0·50
79·00	79·5	0·00	173·00	173·5	0·00

One of the first objects of inquiry was, to determine whether there existed any important difference in the phenomena of waves generated by different methods and by bodies of different forms, or to ascertain whether a wave being given in height and depth, the phenomena were the same and independent of the source from which it had been originally derived. To give the value of the comparison, we shall collate the history of four waves generated by four different methods, and very nearly of the same magnitude and in the same depth of fluid.

WAVE XIX. Generated by protrusion of solid C. Depth = 3·95 in.		WAVE XV. Generated from chamber B. Depth = 3·87 in.		WAVE VIII. Generated by simple impulsion. Depth = 4·15 in.		WAVE VII. Generated from reservoir A. Depth = 4·07 in.	
Sec.	In.	Sec.	In.	Sec.	In.	Sec.	In.
10·5	5·40	...	...	...	...	...	...
10·5	5·22	...	5·30	...	5·10	...	..
10·5	5·15	10·0	5·32	11·0	5·02	...	...
10·5	5·02	10·5	5·20	11·0	4·95	...	...
10·5	4·83	11·0	5·03	11·5	4·85	...	...
12·0	4·76	11·0	4·96	11·5	4·75	...	...
12·0	4·67	11·5	4·68	11·0	4·69	...	...
11·5	4·58	11·0	4·60	11·5	4·61	...	4·62
11·5	4·55	12·0	4·55	12·0	4·55	...	4·58
11·5	4·50	12·0	4·43	11·0	4·48	11·5	4·52
11·5	4·42	11·0	4·36	12·5	4·43	11·5	4·46
				11·5	4·40	11·5	4·40
				11·5	4·37	11·5	4·35
				12·5	4·36	12·0	4·27
				12·0	4·33	12·0	4·26
				12·0	4·29	12·0	4·26
11·13	4·82	11·11	4·84	11·60	4·66	11·70	4·41

These columns contain the intervals of description of successive spaces of 40 feet each, with the mean depth reckoned from

the top of the wave, ascertained from the mean of three observations in each distance of 40 feet. The waves were generated by four different methods, the depth of the fluid and the height of the wave are different in each; so that on comparing them together, we have to take into consideration the variations of the conditions. Now between the mean interval of the successive transits in XIX. and XV., the difference is only two-hundredth parts of a second, and between the mean height of the wave in the former case, and in the latter, there is a corresponding difference with the same sign, amounting to two-hundredth parts of an inch—between VIII. and VII. the same coincidence exists. The same harmony runs through that whole series of observations from Wave I. to Wave XXVI., and appears to warrant the conclusion, that *between waves of this order, generated in very different methods, no sensible difference in the law of propagation can be distinguished.* In the remaining series of observations, the protrusion of solid C was the method generally adopted for generating the waves, as it was found convenient and precise. Various other methods, such as suspending the fluid by atmospheric pressure and the immergence of bodies of different forms, were tried, without sensible difference on the result.

Waves were then generated in different depths of the fluid, and having different heights, for the purpose of determining the velocity due to them with all the precision which the method was capable of affording. The three columns of figures which follow, are a short table of results, and in a fourth column are given a few theoretical numbers, representing the height due to half the depth of the fluid, reckoning from the ridge of the wave. The first of these columns gives the total depth reckoned from the top of the wave, the second column is the height of the wave itself above the quiescent fluid, and the third the observed velocity.

Total depth.	Height of the wave.	Velocity observed.	Velocity due to half the depth.
1·00	...	...	1·636
1·05	0·05	1·64	...
1·30	0·15	1·84	...
2·00	...	...	2·314
2·19	0·29	2·30	...
3·00	...	...	2·834
3·10	0·16	2·87	...
3·23	0·15	2·99	...
4·00	...	...	3·273
4·00	0·19	3·33	...
4·08	0·13	3·24	...
4·20	0·13	3·33	...
4·31	0·24	3·40	...
5·00	...	...	3·701
5·20	0·10	3·73	...
5·25	0·15	3·72	...
6·00	...	...	4·008
6·40	0·15	4·04	...
6·47	0·27	4·14	...
6·74	0·54	4·32	...
7·00	...	...	4·333
7·33	0·29	4·39	...
7·44	0·40	4·44	...
8·00	...	...	4·628

Table of Experiments in Rectangular Channel.

Reference to original observations.	Total depth from the ridge of the waves.	Height of the wave.	Time occupied in describing space in next column.	Space described.	Velocity of wave in feet per sec.
	Inches.	Inches.	Seconds.	Feet.	
XXIX. ...	1·05	·05	36·5	60·0	1·64
XXVII....	1·10	1·10	23·5	40·0	1·70
XXVIII...	1·20	·20	22·7	40·0	1·76
XXXIII. .	1·30	·15	22·0	40·0	1·81
XXXV....	1·62	·32	29·0	60·0	2·06
XXXVI...	2·19	·29	34·7	80·0	2·30
XLI.	3·09	·15	27·5	80·0	2·90
XL.	3·11	·17	14·0	40·0	2·85
XLI.	3·16	·22	21·0	60·0	2·71
XL.	3·20	·26	22·0	80·0	2·72
XLI.	3·23	·29	27·0	80·0	2·96
XXVI. ...	3·23	·15	69·5	200·0	2·99
XXVI. ...	3·32	·24	27·0	80·0	2·96
XXXVIII.	3·35	·35	27·0	80·0	2·96
XL.	3·38	·44	19·5	60·0	3·07
XLI.	3·41	·47	20·0	60·0	3·00
XV........	3·40	·32	27·0	80·0	2·96
XXVI. ...	3·50	·44	26·0	80·0	3·08

Reference to original observations.	Total depth from the ridge of the wave.	Height of the wave.	Time occupied in describing space in next column.	Space described.	Velocity of wave in feet per sec.
	Inches.	Inches.	Seconds.	Feet.	
XXXVII..	3·50	·50	19·0	60·0	3·15
XXXIX. .	3·60	·66	13·0	40·0	3·07
XXV.......	3·61	·53	26·5	80·0	3 02
XL.	3·69	·75	18·5	60·0	3·24
XXVI. ...	3·81	·73	25·0	80·0	3·20
XXXVIII.	3·81	·81	18·5	60·0	3·24
XXXIX. .	3·84	·92	18·5	60·0	3·24
XL.	3·90	·96	12·0	40·0	3·33
XXV.......	3·97	·81	24·5	80·0	3·22
II.	4·00	·19	36·0	120·0	3·33
IV..........	4·08	·13	74·0	240·0	3·24
II.	4·12	·31	24·2	80·0	3·30
IV..........	4·15	·34	25 0	80·0	3·20
VII.	4·20	·13	36·0	120·0	3·33
IV..........	4·25	·45	47·7	160 0	3·35
VII.	4·31	·24	46·75	160·0	3·40
II.	4·40	·59	23·5	80·0	3·40
IV..........	4·45	·64	35·5	120·0	3·38
VII.	4·49	·42	34·75	120·0	3·46
XIX.	4·51	·56	42·5	160·0	3·76
XV.	4·61	·74	22·5	80·0 }	3·52
XIX.	4·75	·80	23·0	80·0 }	
XLV.......	5·20	·10	32·0	120·0	{ 3·73 3·77 3·80
XV.	5·21	1·34	31·5	120·0	
XLV.......	5·25	·15	43 0	160·0	3·72
XLV.......	5·35	·25	21·2	80·0	3·77
XLIII. ...	5·40	·36	32·0	120·0	3·75
XLV.......	5·50	·40	21·0	80·0	3·80
XLIII. ...	5·61	·57	39·5	160·0	4·05
XLVI. ...	5·80	·70	20·0	80·0	4·00
XLIII. ...	5·82	·78	30·5	120·0	3 93
XLV.......	5.82	·72	20·5	80·0	3·90
XLIII. ...	6·15	1·05	19·0	80·0	4·21
XLVI. ...	6·15	1·13	19·0	80·0	4·21
XLV.......	6·26	1·16	29·5	120·0	4·08
XLV.......	6·40	1·30	28·7	120·0	4·18
L.	6·40	·15	49·5	200·0	4·04
XLIX. ...	6·47	·27	29·0	120·0	4·14
XLIX. ...	6·54	·34	39·5	160·0	4·05
L.	6·56	·31	39·0	160·0	4·10
XLIX. ...	6·65	·45	29·0	120·0	4·14
XLVI. ...	6·69	1·59	18·5	80·0	4·32
XLVIII....	6·74	0·54	18·5	80·0	4·32
L.	6·75	0·50	48·5	200·0	4·13
L.	6·86	·61	38·0	160·0	4·21
XLVIII....	6·90	·70	37·5	160·0	4·29
XLIX. ...	7·20	1·0	37·0	160·0	4·32
LIII.	7·42	·38	45·5	200·0	4·40

Reference to original observations.	Total depth from the ridge of the wave.	Height of the wave.	Time occupied in describing space in next column.	Space described.	Velocity of wave in feet per sec.
	Inches.	Inches.	Seconds.	Feet.	
LV.	7·33	·29	73·0	320·0	4·39
LV.	7·44	·40	36·0	160·0	4·44
LI.	7·68	·64	28·0	120·0	4·37
LIII.	7·70	·66	27·0	120·0	4·43
XLVIII....	7·74	1·54	26·5	120·0	4·44
LV..........	7·75	·71	35·5	160·0	
LIII.	7·79	·75	27·0	120·0	4·43
LII.	7·82	·78	26·5	120·0	4·53
LI.	7·84	·80	27·0	120·0	4·43
LV..........	7·87	·83	26·5	120·0	4·53
LII.	8·00	·78	26·5	120·0	4·53

Observations on the influence of the form of the channel on the propagation of the wave extend from Wave LVI. to Wave CXLIX., at the end of the report.

The triangular channel H was of the form given in Plate III., fig. 2, its depth having varied by the quantity of water poured in, its vertex undermost, one side vertical and the other inclined to the horizon at an angle whose radius is to its tangent as 3 to 2. In all these experiments the wave was observed to be low and flat on the deep side of the channel, while it remained high and cusped on the shallow side; it was also long on the deep side, and diminished in length uniformly with the diminution in depth. The following table contains an analysis of the experiments in the channel H. The first column refers to the individual wave made the subject of experiment, so that it may be referred to in its place at the end of the report. The second column contains the total depth reckoned from the top of the wave on the deep side. The third column gives the height of the wave. The fourth column contains the number of seconds employed in describing the number of feet given in the fifth column; and the last column is the resulting velocity.

It should be recollected, before proceeding to compare these observations with any formula, that the attraction of the sides at the bottom of the channel in the acute angle of the channel must be considered as having fixed a portion of the fluid which was not affected by the motion of the wave, and which should therefore be subtracted from the effective depth.

Analysis of Observations of Waves in the Triangular Channel H., Plate III., fig. 2.

Reference to original observations.	Total depth from the ridge of the wave.	Height of the wave.	Time occupied in describing space in next column.	Space described.	Velocity of wave in feet per sec.
	Inches.	Inches.	Seconds.	Feet.	
LVIII. ...	4·15	·15	36·5	80·	2·19
LIII. ...... LIX.	4·23	·22	33·0	80·	2·42
LVI. ...... LX.	4·32	·31	31·0	75·5	2·43
LVII. ...... LVIII. ... LIX.	4·38	·37	47·0	115·5	2·46
LVIII. ...	4·71	·70	13·5	35·5	2·62
LX. ...... LXI.	4·81	·80	29·5	75·5	2·57
LXIX. ...	4·86	·85	14·0	35·5	2·53
LXII.......	5·29	·18	31·0	80·0	2·58
LXV.......	5·44	·33	45·5	120·0	2·63
LXIII. ...	5·55	·44	58·0	160·0	2·75
LXII.......	5·59	·48	30·0	80·0	2·66
LXV.......	5·99	·88	12·0	35·5	2·95
LXIII. ... LXIV. ...	6·01	·90	24·5	71·0	2·89
LXVI. ...	6·18	·14	28·0	80·0	2·85
LXVII. ... LXVIII....	6·26	·21	55·5	160·0	2·88
LXVI. ...	6·38	·34	14·0	40·0	2·85
LXII.......	6·44	1·33	12·0	35·5	2·95
LXVII. ...	6·52	·48	26·5	80·0	3·02
LXVI. ... LXVIII...	6·78	·74	35·0	111·0	3·17
LXX.......	7·10	·60	26·5	80·0	3·02
LXXV. ...	7·12	·08	39·5	120·0	3·03
LXXII. ... LXXI. ...	7·15	·11	78·5	240·0	3·05
LXXIII... LXXIV....	7·16	·12	52·5	160·0	3·04
LXXI. ...	7·21	·17	26·5	80·0	3·02
LXXIII...	7·36	·32	26·5	80·0	3·02
LXXV. ...	7·51	·47	25·0	80·0	3·20
LXXIV....	7·53	·47	24·0	80·0	3·33

The triangular channel K was of the form given in Plate III., fig. 3, the breadth at the surface of the water being 12 inches, the depth 4 inches to 0. It was observed that during the whole of the experiments the wave was long and low on the deep side; short and pointed, and considerably higher and continually breaking, on the shallow side, so as to leave behind a long train of secondary waves.

The trapezoidal channel L was formed by the addition of a rectangular portion, 1 inch deep, to channel K. See Plate III. fig. 4.

The trapezoidal channel M was formed by the addition of a rectangular portion, 1 inch deep, to channel L.

Analysis of Observations of Waves in the Channels K, L, M.

Reference to original observations.	Total depth from the ridge of the wave.	Height of the wave.	Time occupied in describing space in next column.	Space described.	Velocity of wave in feet per sec.
K					
	Inches.	Inches.	Seconds.	Feet.	
LXXIX...	4·14	·10	19·5	40·0	2·05
LXXVIII.	4·21	·17	17·5	40·0	2·28
LXXVI.... LXXVII..	4·42	·37	40·75	102·2	2·50
LXXVIII. LXXIX...	4·46	·41	31·7	82·2	2·60
LXXVIII.	5·31	1·27	5·0	14·6	2·92
L					
LXXXV..	5·24	·24	12·5	40·0	3·20
LXXXII..	5·42	·42	13·5	40·0	3·00
LXXXI... LXXXIV. LXXXV..	5·53	·53	42·0	120·0	2·90
LXXXIV.	5·68	·68	13·5	41·1	3·04
LXXXIII.	5·70	·70	12·7	41·1	3·23
LXXXII..	5·77	·77	20·0	61·1	3·05
LXXXI... LXXXIV.	6·41	1·41	8·5	29·2	3·43
LXXXII..	6 47	1·47	4·5	14·6	3·24
LXXXIII.	6·67	1·67	4·0	14·6	3·65
LXXXV..	6·92	1·92	4·0	14·6	3·65
M					
XC. ...... XCIII. ...	6·41	·40	13·0	40·0	3·08
XC. ...... XCIII. ...	6·87	·86	11·4	40·0	3·50
XC. ...... XCIII. ...	7·43	1·42	9·25	35·7	3·86

The wedge-formed channel was of uniform depth, twelve inches wide at the broad end, and tapering to an edge at the other; the wave on entering the channel at A was observed; its height was again taken at B, when it had advanced half the length of the channel, and had been diminished one half in breadth; and at C, after having passed along three-fourths of the length of the channel, the height was again observed. The wave was observed breaking invariably at the height of about 3·6 inches above the level of the fluid; and the distance from D, the end of the channel, when it broke, is given with the sign minus prefixed. On entering the channel the wave was low, but gradually increased as it reached the narrower parts of the channel, becoming acuminated; and at last having gained the cusped cycloidal form, broke at the crest, and passed into the centre angle of the wedge, when it rose suddenly over the sides of the channel in a sharp vertical *jet d'eau.* A table of these experiments is given at the end, comprehending Waves XCIV.—CVI.

The sloping channel, Plate II. fig. 6, was formed to imitate a sloping sea beach; its slope rose 1 in 51. The wave entered the deep end at a given height, then gradually became more acuminated, formed a cycloidal cusp, and broke. Its height on entering, its height when breaking, and the place at which it broke were observed and are given in the observations at the end from Wave CVII. to CXXXII. The numbers in the last column are the depths corresponding to the place of breaking observed in the preceding column, and this table shows that the depth at breaking corresponds with remarkable accuracy to the height of the wave.

A considerable number of observations were made upon the translation of the particles of the fluid during the transit of a wave, but the results are not of a numerical character, being all comprehended in the general expression that the translation of the particles takes place wholly in the direction of the motion of the wave; that it is of equal extent from the surface to the bottom of the channel, that it is permanent, that the particles which were in the same vertical plane previous to translation are still so after translation. This is not the case in other species of waves; the particles oscillate in opposite directions with an alternating motion.

Experiments were also made on waves formed by the removal of a solid body from a quiescent fluid; these are called negative waves, but the investigation of them has not yet been completed.

Second Series of Observations.

On the Waves of the Sea.—Are the waves on the surface of the sea, when it is agitated by the wind, of the same nature with the waves which have already been examined by experiment?

Does their velocity depend on the depth of the fluid? Is their form cycloidal? What is the cause of their breaking on the shore? And what law is observed in their breaking? Why do waves in any circumstances break? What is a breaker? These are some of the questions which the Committee have examined, and their results are of importance to theory and to navigation.

The Committee obtained for the purpose of their observations on the waves of the sea the use of one of the yachts of the Royal Northern Yacht Squadron, which was kindly granted by her proprietor, James Bogle, Jun., Esq., at the request of the secretary. The Mermaid was an excellent sea vessel, but the weather was unfortunate; she was alternately becalmed and bestormed; one day driven into harbour for refuge and the next day prevented by calms from leaving harbour. Out of eight days occupied in this way not more than one was favourable to observation. By subsequently crossing the Irish Channel in steam-vessels one or two observations of a sufficiently accurate nature were obtained.

From these observations it appears to be established that the velocity of the waves at the surface of the deep water is not a direct function of the depth.

In a depth of 50 to 60 fathoms the velocity was 13·5 miles an hour.
In a depth of 53 fathoms the velocity observed was 20 miles an hour.
In a depth of 60 to 70 fathoms the velocity was 17 miles an hour.
In a depth of 34 to 40 fathoms the velocity was 17½ miles an hour.

In a depth of 51 fathoms the waves produced by a steam vessel passing at the distance of about a mile, moved at the rate of only 4·3 feet in a second.

It thus appears that the waves produced by the wind on the surface of the deep sea do not follow the same law with the great wave of the fluid. In other words they are not primary but secondary waves, or waves of some inferior order. They do not move with the velocity due to half the depth of the fluid in which they are generated.

The following are the most important and accurate observations made on this subject.

Observations.—The observations were made by bringing the vessel nearly to rest in a direction at right angles to the ridge of the wave. The cork fenders of the vessel were then attached at equal distances to the log-line, and spaces of 200 feet were marked off upon it. The time was taken by a common chronometer; the observations made were upon the transits of the top of the wave under the floating buoys attached to the log-line.

1. 4th Oct. 1836, lat. 55° 38′ N., long. 4° 49′ W.
Off the Cambray Islands, 60 to 70 fathoms.
Space 200 feet, time 7 sec. to 9 sec. = 25 feet per sec. = 17 miles an hour.

2. 4th Oct., 1836, lat. 55° 32′ N., long 4° 52′ W.
Off the Isle of Arran, 50 to 60 fathoms.
Space 200 feet, time 10 sec. = 20· feet per sec. = 13·5 miles an hour.

3. 5th Oct. 1836, lat. 55° 29′ N., long. 4° 54′ W.
Of Pladda Lights, 20 to 16 fathoms.
Space 200 feet, time 11 sec. to 12 sec. = 17·3 feet per sec. = 11¼ miles per hour.

4. 12th Oct. 1836, lat. 54° 5′ N., long. 5° 31′ W.
Off Ardglass Light, in 34 to 40 fathoms.

$$\text{Space} = 345 \text{ feet} \left\{ \begin{array}{l} \text{Time.} \\ 9{\cdot}3 \text{ sec.} \\ 10{\cdot}0 \\ 9{\cdot}3 \\ 8{\cdot}6 \\ 10{\cdot}0 \end{array} \right\} = 35 \text{ feet} - 9 = 17\tfrac{1}{2} \text{ miles.}$$

5. 12th Oct. 1836, lat. 54° 1′ N., long. 5° 37′ W.
in 53 fathoms.

$$\text{Space} = 345 \text{ feet} \left\{ \begin{array}{l} 9{\cdot}3 \text{ sec.} \\ 8{\cdot}6 \\ 8{\cdot}6 \end{array} \right\} = 39 \text{ feet} - 9 = 20 \text{ miles per hour.}$$

6. 12th Oct. 1836, lat. 53° 58′ N., long. 5° 39′ W.
in 46 to 44 fathoms.
Space = 345 feet, time = 9·3 sec. = 37 feet —9 = 19 miles.

The observations (4-6) were made against a very strong breeze and very high waves, about 8 or 9 feet high, and the vessel was going in the opposite direction at about the rate of six miles an hour.

7. In 51 fathoms water the City of Glasgow steam packet passed; her waves were about 20 inches high, about 12 feet apart, and passed over a space = 150 feet in 35 sec. = 4·3 feet per sec.

It became of importance to determine whether the waves of the sea produce an agitation which extends to the deep parts of the water. It was found that even in moderate depths they do not. Thus in a depth of 12 feet—short quick waves, 9 inches high and 4 or 5 feet long, do not sensibly affect the water at the bottom, while waves thirty or forty feet long, oscillating at intervals of 6 or 8 seconds, produce a sensible effect, although much less than at a point nearer the surface. The circumstances of these partial oscillations opens up a field of future research. The observations made on this subject were obtained by plunging a glass tube to a considerable depth, so that the column of water contained in it should only be affected by the forces acting upon the particles of the fluid at the depth of its orifice below the surface. In this way it was ascertained that neither in velocity of the wave-surface, nor in the motion of transference of the particles, do the waves of the sea resemble the great primary wave of translation of the previous experiments.

It is difficult to ascertain with precision the form of the waves of the sea; they appear to belong to the family of the cycloid, The summit of the wave is round and flat so long as its height bears only a small ratio to its length in the direction of its motion; but as the height increases the summit of the wave becomes more and more acuminated, and the limit to which the height of a wave approaches, but which it never appears to exceed, is nearly a third part of its length. If the wave belong to the cycloidal family, and if its length being constant the height vary with the generating radius, the rolling circle continuing the same, we shall have a series of lines accurately representing the form of the waves. See Plate II. fig. 1. Now it is manifest that when the describing radius of the wave becomes greater than the radius of the rolling circle, the curve ceases to have a form of possible equilibrium, and that portion which falls down from the top of the wave constitutes the white crest which we observe on the summits of the largest waves, when they are said to break.

There is generally much confusion in the appearance of an agitated sea. The waves do not appear regular in their forms, their intervals, or their velocities. Sometimes a wave seems to stand still or even to retrograde, and frequently after the eye has traced a wave for a considerable time it suddenly disappears altogether. Close attention will however discover some method in this irregularity.

The surface of the sea is seldom covered with only one series of successive waves. Every breeze that ruffles the surface of the sea generates a series of waves that move in the direction of the motion of the wind. These waves do not subside with the breeze which raised them, but continue their oscillations until the adhesion of the water or the resistance of the shore has diffused the elevated fluid uniformly over the surface. In the mean time a second breeze springs up in another direction, and new waves rise to its pressure and follow its direction; they mingle with those of the former wind without becoming mixed with them. Two distinct series of waves are now coexistent, and give rise to more complex phenomena. A third gale arises, and a new class of waves intersect and overlap the two former, while the long low swell—the residue and telegraph of some distant storm—rolls across the whole, and to the untutored eye leaves nothing to be looked on but a chaos of tumultuous, troubled waters. The seeming chaos is however to be analyzed by patient attention: by ascending the mast of the ship, or standing on an elevated rock on the shore, much of this apparent confusion may be dispelled; and by attention to the phenomena of coexistent oscillations every thing may be understood.

When a breeze has been blowing for some time in one direction, and the wind has shifted round into the opposite one and blown with nearly equal force, the two sets of waves may be distinctly seen moving in opposite directions; if they be of nearly equal dimensions a very singular appearance results. When the crests coincide, the ordinates of the compound wave surface become the same ordinates of the elementary waves, and their difference when the crest of the one is in the cavity of the other; so that the sea is alternately in the forms represented in *c* and *d*, fig. 2, Plate II.

When these two systems of waves are compounded with a third system arising from some other breeze, or by a third system resulting from the reflection of a bold coast, the third series combines with the two former in the manner represented in fig. 3, with an appearance of still less regularity, and so on for any number of parallel systems.

It is manifest that if these parallel systems be compounded with transverse systems, making any angle with the first, we shall have a compound system of surfaces of double curvature so complex in its structure as to represent the phenomena of the most troubled sea. On all occasions where the sea was observed, there were found two or more such systems of coexistent waves.

The phenomena of the waves at the surface of the sea appear to coincide very well with the hypothesis, that when a wave agitates the fluid only to a small depth it may be considered as formed in a shallow canal of that depth; for it may be observed that a short wave of a given height is always more pointed than a longer wave of the same height, and also that whenever a wave reaches the limit of the cycloidal form it breaks.

Whenever the height of a wave exceeds the limit of the cycloidal form due to its depth, the wave, after having become cusped or pointed, passes into the nodated form of unstable equilibrium and is broken. See figs. 4 and 5.

Whenever a wave of a higher order coincides with the ridge of one of an inferior order, its curvature at the crest will be a maximum, and it may break, although it would not have broken on any other part of the wave. See figs. 4 and 5, Pl. II. From this cause a large wave frequently exhibits the appearance of a breaking wave, although its own figure has not approached the limits of equilibrium; but in that case it is not the large wave which is breaking, but the smaller one on its summit, whose curvature is then increased by the amount of the curvature of the greater wave at the crest.

Waves break on the shore when they reach the point where the depth of the fluid becomes nearly equal to the height of the wave above the fluid. When at a distance from the shore they may be observed long and low, see fig. 6; as they approach the shallow part of the shore they gradually assume the greater curvature due to the increased ratio of height to depth; the form at last becomes cusped and perfectly cycloidal, the equilibrium of the summit ceases, and the particles of water on the extreme ridge of the wave, abandoned to the force of gravity, and aggregated in spherical drops by this cohesion, present to the eye the white foaming crest by which breakers are distinguished. Waves of great height are thus broken on the beach at a greater distance from the shore than such as are smaller.

The depth of water may be judged of by the form and height of the waves. See fig. 7, Plate II. Where a wave of a given height can exist, suppose a wave of five feet, the water must have a depth below the surface of at least five feet, and wherever in a calm day waves are broken, the depth of the water is equal to their height above its surface.

It must be observed that the existence of a strong *wind* will often destroy the equilibrium of the ridge of a wave, independent of depth or of the equilibrium of its proper form. When the curvature of the ridge of the wave becomes considerable, and it approaches the cusped form, the direct incidence of the wind upon the surface of the ridge will derange the equilibrium of the thin and slender column presented by the top of it before it reaches the limits of undisturbed equilibrium. Hence the phenomenon well known to sailors, that a very strong wind will blow the sea down, in other words, that it will blow off the ridges of the highest waves, and keep them from attaining the height they afterwards reach when the gale has subsided. The highest seas are thus generated by the continuance of a strong gale in one direction rather than by the sudden and short impulse of a hurricane; for in the former case the wind only breaks the summits of the smaller waves as they rise to the top of the larger ones, so as to add the mass of the smaller to the crest of the larger waves, without injuring the equilibrium of the latter; these continual additions increase the magnitude of these great waves, while the force of the gale is not sufficiently great to derange their equilibrium. The waves in these circumstances go on increasing in magnitude.

The phenomena of waves breaking on the shore were observed principally on a very fine smooth beach of sand, having a slope towards the sea of 1· in 50·; so perfectly plane and level was it at the time when the observations were made, that a single wave a mile in breadth might be observed advancing to the shore, so perfectly parallel to the edge of the water that the whole wave rose, became cusped, and broke at the same instant; a line of graduated rods was fixed in the water at different depths from 6 inches to 6 feet in length, and it was observed that every wave broke exactly when its height above the antecedent hollow was equal to the depth of the water. At another time when the direction of the waves was oblique to the edge of the water, the breaking crest moved along from one end of the shore towards the other, uniformly and gradually as the wave advanced to the point of breaking depth, resembling the *feu de joie* of a file of soldiers.

When a wave that has been breaking on a shallow part of the water comes suddenly into deeper water, the form ceases to be crested, see Plate II.; and the wave subsides into the figure due to the depth.

The phenomena of waves breaking on the shore were accurately obtained in the experiments No. 107—132, page 492. Plate II. figs. 6 and 7.

Third Series of Observations.

On the Tide Wave of the River Dee in Cheshire.—The object of this series of observations was the comparison of the *tidal wave* moving in a given channel with the great *primary wave of translation* previously examined by Mr. Russell.

To this object the river Dee is peculiarly suitable. Plate VI. fig. 1. gives a plan of that river at low water. The upper portion of the channel of the river is artificial. The waters of the river were turned into a new course about the middle of the last century. Of this course about 5¼ miles forms a perfectly straight canal, along which a large and rapid tidal wave is transferred with great velocity. The two points A and B on the plan were selected as stations of observation. The distance between A and B was carefully measured; transverse sections of the river were made, and soundings were taken throughout the whole length of the channel.

The distance between A and B . . . . = 5·275 miles.
The mean depth of the channel at low water = 3·0 feet.
The bed of the river has a slope nearly . = 3·8 feet.

The opposite sides of the river are parallel embankments about 500 feet apart at high water mark, but nearly half that breadth is occupied by groins, as shown in the sections of the river, figs. 2, 3, and 4, Plate VII., and the intervals between them are filled up with high banks of sand.

The tides selected to be observed were those which differed most in magnitude, and which were least affected by disturbing influences. They were made when the weather was settled, when there was no sensible wind, and when the river was as nearly as possible in its natural state. One entire tide wave was obtained on the 7th of September, and two others on the 9th and 13th of that month. In the latter two cases the river was a few inches fuller than in the former, as will appear by inspecting the table of observations which follows.

From these observations it may be useful to make the following extracts.

First wave of flood tide, 7th Sept. reached	
Station A at	6^h 50^m
Station B at	7 50·
Time of describing 5·275 miles	1 0·
First wave of flood tide, 9 Sept. reached	
Station A at	8 5·
Station B at	8 50·
Time of describing 5·275 miles	0 45·
First wave of flood tide, 13th Sept. reached	
Station A at	10 15·
Station B at	11 0·
Time of describing 5·275 miles	0 45·
The wave of high water of 7th Sept. reached	
Station A at	9 21·
Station B at	9 50·75
Time of describing 5·275 miles	0 29·75
The wave of high water of 9th Sept. reached	
Station A at	10 35·
Station B at	10 54·5
Time of describing 5·275 miles	0 19·5
The wave of high water of 13th Sept. reached	
Station A at	12 35·
Station B at	12 53·5
Time of describing 5·275 miles	0 18·5

The following table contains the corresponding velocities of the waves.

WAVE.	Velocity in miles.	Height of Wave at A.	Height of Wave at B.	Mean Depth.	Height due to the Velocity.
I.	5·2	0·ft. 8·in.	0·ft. 6·in.	3·ft. 7·0in.	2·0ft.
II.	7·0	1· 6·	0· 7·	4· 10·5	3·5
III.	7·0	2· 8·	1· 1·	5· 0·5	3·5
IV.	10·5	9· 2·1	6· 4·	10· 9·0	7·0
V.	16·2	13· 5·7	10· 6·	15· 11·8	16·0
VI.	17·1	15· 8·	13· 0·	17· 4·0	17·0

In order to make these observations the foundation of any conclusions, it will be necessary to observe that it is scarcely possible to determine whether the wave which brings flood tide to the lower station be the same with that which afterwards brings flood tide to the higher station; on the other hand it seems more likely that the wave which passed the lower station was diffused over the intermediate space in the channel, and was overtaken by a subsequent part of the tide, which had not reached the lower station till a considerable time after the first wave had passed it. This is not a conjecture, but has frequently been observed in similar cases where the first wave being become diffused in the channel ceased to pass onwards and was overtaken by a subsequent wave. The result obtained in the case of waves I., II., and III. of flood tide is consistent with this view, and shows that in these cases the progress of flood tide is slower than the velocity due by gravity to the wave of the fluid. It is also consistent with the experiments of the previous part of this paper, that a breaking wave or bore, as this was, has a slower velocity than one which does not break.

Waves IV., V., and VI., the waves of high water, have almost exactly the velocities of great waves of translation of the fluid. It will be seen at once by examining the transverse sections of the river, that wave IV. must suffer great retardation from the circumstance that its progress is continually intercepted by the groins to which it is almost exactly equal in height, while waves V. and VI. rise above them and accordingly approximate more closely to the velocity due to the depth. The form of these waves and their antecedent bores are given in Plate VI. figs. 1 and 2. and the observations from which they are deduced are given in the following table.

Tide Wave of the 7th Sept., River Dee, 1836.

Station A, Jarvis Obs., Chron. No. 4 stand. H. W. 9^h 21^m = 9ft. 2·1in.								Station B, Jones Obs., Chronom. No. 2, cor. = + $0·75^m$. H. W. $9^h 50^m·75$ = 6ft. 4in.							
Flood.				Ebb.				Flood.				Ebb.			
h.	m.	ft.	in.	h.	m.	ft.	in.	h.	m.	ft.	in.	h.	m.	ft.	in.
9	20	9	2·1	9	20	9	2·1	9	50	6	4·	9	50	6	4·
	15	9	1·7		Not	observed.			45	6	3·7		55	6	3·7
	10	9	1·7						40	6	3·0	10	0	6	2·5
	5	9	1·7						35	6	2·0		5	6	2·0
	0	9	1·5						30	6	0·7		10	6	0·5
3	55	9	1·5						25	5	11·		15	5	11·0
	50	9	0·5						20	5	8·7		20	5	9·7
	45	8	10·5						15	5	6·3		25	5	8·0
	40	8	..						10	5	3·7		30	5	6·7
	35	8	6·5						5	5	0·7		35	5	5·0
	30	8	4·						0	4	10·0		40	5	3·0
	25	8	1·					8	55	4	7·0		45	5	1·5
	20	7	10·5						50	4	4·		50	4	11·7
	15	7	7·						45	4	0·7		55	4	10·5
	10	7	4·						40	3	10·	11	0	4	9·0
	5	7	1·						35	3	6·5		5	4	7·5
	0	6	8·3						30	3	3·5		10	4	6·0
7	55	6	6·5						25	3	0·3		15	4	4·
	50	6	5·0						20	2	9·2		20	4	3·
	45	5	11·5						15	2	6·5		25	4	1·5
	40	5	8·5						10	2	3·3		30	4	0·
	35	5	6·0						5	2	0·		35	3	10·5
	30	5	1·0						0	1	9·		40	3	9·
	25	4	9·					7	55	1	6·		45	3	7·5
	20	4	4·						50	1	2·		50	3	6·
	15	4	0·						45	0	8·3		55	3	4·7
	10	3	7·						40	0	7·	12	0	3	3·0
	5	3	1·						35	0	5·		5	3	3·1
	0	2	8·						30	0	4·		10	3	0·5
6	55	2	1·						25	0	3·7		15	2	11·2
	50	1	9·						20	0	3·5		20	2	10·5
	45	1	1·						15	0	3·		25	2	9·3
	40	0	11·						10	0	2·7		30	2	8·0
	35	0	8·						5	0	2·5		35	2	7·0
	30	0	6·						0	0	2·0		40	2	6·0
	25	0	..					6	55				45	2	5·0
	20	0	2·						50				50	2	4·2

Tide Wave of the 9th Sept., River Dee, 1836.

Station A, Jarvis Obs., Chron. No. 4 stand. H. W. 10^h 35^m = 13 ft. 5·75 in.								Station B, Jones Obs., Chron. No. 2 cor. = − $0·5^m$. H. W. 10^h $54·^m5$ = 10 ft. 6 in.							
Flood.				Ebb.				Flood.				Ebb.			
h.	m.	ft.	in.	h.	m.	ft.	in.	h.	m.	ft.	in.	h.	m.	ft.	in.
10	35	13	5·7	10	35	13	5·7	10	55	10	6·	10	55	10	6·
	30	13	5·5		40	13	5·7		50	10	5·5	11	0	10	5·5
	25	13	4·5		45	13	5·		45	10	4·		5	10	5·
	20	13	3·		50	13	3·		40	10	2·5		10	10	3·5
	15	13	1·		55	13	1·5		35	10	0·5		15	10	2·0
	10	12	1·1	11	0	12	11·		30	9	10·		20	10	0·
	5	12	8·5		5	12	9·		25	9	7·5		25	9	9·5
	0	12	6·5		10	12	7·5		20	9	5·		30	9	7·0
9	55	12	4·0		15	12	5·0		15	8	11·?		35	9	3·5
	50	12	0·5		20	12	2·5		10	8	9·5		40	9	1·0
	45	11	10·0		25	12	0·		5	8	5·		45	8	9·5
	40	11	7·5		30	11	9·		0	8	1·5		50	8	7·0
	35	11	4·		35	11	6·	9	55	7	8·5		55	8	3·5
	30	11	1·5		40	11	3·		50	7	3·	12	0	8	0·0
	25	10	11·		45	11	0·		45	6	10·5		5	7	9·0
	20	10	7·5		50	10	9·		40	6	6·		10	7	6·5
	15	10	4·		55	10	5·		35	6	1·		15	7	3·5
	10	9	11·	12	0	10	2·		30	5	8·		20	7	2·0
	5	9	6·		5	9	11·		25	5	3·		25	6	11·
	0	9	1·		10	9	8·		20	4	10·		30	6	8·
8	55	8	8·		15	9	4·		15	4	4·5		35	6	6·
	50	8	2·		20	9	1·		10	3	10·5		40	6	3·
	45	7	8·		25	8	10·		5	3	5·		45	6	1·
	40	7	1·5		30	8	7·5		0	2	10·		50	5	10·5
	35	6	7·5		35	8	4·	8	55	2	3·		55	5	8·5
	30	6	0·		40	8	1·		50	1	5·	1	0	5	5·0
	25	5	5·		45	7	11·		45	0	10·		5	5	4·
	20	4	9·		50	7	8·		40	0	4·		10	5	1·
	15	4	0·		55	7	5.		35	0	4·		15	4	11·5
	10	3	2·	13	0	7	2·		30	0	4·		20	4	9¼·
	5	2	4·		5	6	11·						25	4	7·
	0	0	10·		10	6	6·7						30	4	5·5
					15	6	5·						35	4	3·
					20	6	2·5						40	4	1·5
					25	5	11·5						45	3	11·5
					30	5	9·0						50	3	10·
					35	5	5·2						55	3	8·

The observations under the words flood and ebb are uncorrected for the error of the chronometer: the correction is given at the head of each column.

The time and magnitude of high water are correctly given at the head of each column.

The time and magnitude of the same tides as given in the almanack for Liverpool are
Sept. 7, 8^h 57^m 10 ft. 10 in.
Sept. 9, 10 26 13 7
Sept. 13, 0 37 18 0

Tide Wave of the 13th Sept., River Dee, 1836.

Station A, Jarvis Obs., Chron. No. 4 stand. H. W. 12h 35m = 15 ft. 8 in.								Station B, Jones Obs., Chron. No. 2 cor. = – 1·m5. H. W. 12h 53·m5 = 13 ft. 0·in.							
Flood.				Ebb.				Flood.				Ebb.			
h.	m.	ft.	in.	h.	m.	ft.	in.	h.	m.	ft.	in.	h.	m.	ft.	in.
12	35	15	8·	12	35	15	8·	12	55	13	0·	12	55	13	0·
	30	15	7·5		40	15	7·		50	12	11·7	1	0	12	11·7
	25	15	6·5		45	15	6·		45	12	10·7		5	12	9·5
	20	15	5·		50	15	5·		40	12	10·0		10	12	6·5
	15	15	4·		55	15	4·		35	12	8·0		15	12	3·7
	10	15	3·	1	0	15	3·		30	12	5·0		20	12	0·7
	5	15	1·5		5	15	0·		25	12	0·		25	11	10·
	0	15	0·		10	14	9·		20	11	7·		30	11	7·
11	55	14	9·		15	14	5·5		15	11	2·		35	11	4·
	50	14	5·5		20	14	2·7		10	10	7·5		40	11	1·
	45	14	2·		25	13	11·		5	9	11·5		45	10	10·5
	40	13	10·		30	13	9·		0	9	4·0		50	10	7·0
	35	13	6·		35	13	5·	11	55	8	9·0		55	10	3·0
	30	13	2·		40	13	2·		50	8	1·0	2	0	10	0·
	25	12	9·		45	12	10·		45	7	7·0		5	9	8·
	20	12	5·		50	12	7·		40	7	3·5		10	9	4·
	15	12	3·		55	12	3·		35	6	9·		15	9	0·
	10	11	7·	2	0	12	0·		30	5	11·5		20	8	9·
	5	10	10·5		5	11	7·5		25	5	4·0		25	8	6·
	0	10	4·		10	11	4·		20	4	8·0		30	8	3·
10	55	9	6·		15	11	0·		15	3	11·0		35	7	11·5
	50	8	10·		20	10	6·5		10	3	0·0		40	7	8·
	45	8	0·		25	10	3·		5	2	3·0		45	7	5·
	40	7	3·		30	10	0·		0	1	3·0		50	7	3·
	35	6	5·5		35	9	6·	10	55	0	2·		55	7	0·
	30	5	9·		40	9	1·5					3	0	6	9·
	25	4	10·		45	8	11·						5	6	5·
	20	3	11·		50	8	6·						10	6	3·
	15	2	10·		55	8	2·						15	6	1·
	10	0	..	3	0	7	10·						20	5	9·
	5	..	..		5	7	6·5						25	5	7·
	0	..	..		10	7	2·						30	5	4·
					15	6	10·7						35	5	0·
					20	6	7·3						40	4	9·
					25	6	3·7						45	4	6·
					30	6	0·5						50	4	3·7
					35	5	9·						55	4	1·
					40	5	5·5						60	3	10·5

It was observed that the flood tide of the 13th was attended in passing the lower station, A, with a very considerable breaking bore or surge on the sides.

Both of the tide gauges were placed in deep water at some distance removed from the banks of the river.

Fourth Series of Observations.

On the Tide Wave of the River and Frith of Clyde in Scotland.—The observations on the river Dee having been necessarily very limited in number, and in the means as well as objects of inquiry, suggested the nature and indicated the necessity of a more extensive series of observations of the tide wave in its progress along some limited channel whose dimensions might be determined with the requisite precision. The river and frith of Clyde on the west coast of Scotland were at once suggested to the Committee, as in every way suitable to the objects of their inquiry. That river is, like the Dee, contained in a channel, which is a work of art rather than of nature, having been rendered one of the finest rivers in Britain by the perseverance, enterprise, and wealth of the citizens of the important manufacturing and commercial city, whose merchandise it transports from all quarters of the world. Your Committee made application, with the assistance of Sir Thomas Brisbane, the President of the Royal Society of Edinburgh, and one of the former presidents of the British Association, to the board of Commissioners or Trustees of the navigation of the Clyde, and were fortunate in obtaining their cordial and effectual co-operation in conducting all the observations and obtaining all the information they required. The willing assistance of Mr. Logan the engineer of the river, was given in conducting the observations; at his request simultaneous observations were made at several other ports in the vicinity; Captain Denham, R.N., of Liverpool, was good enough to order similar observations at that port; Professor Nicol kindly placed the instruments in the college observatory at their disposal, for regulating the time-keepers of the observers, and nothing was omitted that could give the observations value and general interest. Moreover, it was fortunate that the board of Trustees had just obtained very accurate surveys of the river with transverse sections, at distances of each quarter of a mile; and they further ordered that the stations of observation should be connected by a system of levelling. These were all placed at the disposal of the Committee by the Trustees and their engineer, who were of opinion that observations of that nature were of equal value to the practical navigation and improvement of their river, as to the theoretical speculations of the British Association.

Throughout the greater part of 18 miles, the distance between Glasgow and Port-Glasgow, the river Clyde is little more than an inland tidal canal, excavated and embanked by artificial means; it then expands into a frith of considerable breadth, extending about 25 miles down to the outer Cumbray Island, where it terminates. The whole of this space was embraced by the observations.

Plate VII. contains a plan of the river Clyde; the stations at which tide gauges were erected and observers placed are marked in the plan. Nine different stations were occupied; the first of these was at the harbour at Glasgow, immediately below which, the river is for about 5 miles of nearly uniform width and depth, and in this division were three stations, No. I., II., and III. The next division of the river is wide, irregular, and of variable depth, comprehending stations III., IV., and V. The third division of the river is deep and broad from station V. to station VII. The river then opens out into a wide and deep frith, and at a distance of about 5 miles further down, on opposite sides of the frith, were placed stations VIII. and X. Station IX. was at the light-house on the outer Cumbray Island, which stands at the mouth of the frith. A great variety of channel was thus included in the observations.

The observations were made with a tide-gauge, constructed for the purpose of preventing the oscillations of the waves of the surface; a glass tube traversed the scale; the tube open above terminated below in a stop-cock, by which the aperture was regulated, and which communicated with a long narrow pipe descending into deep water; the indications of this gauge were free from inconvenient oscillation, even in a rough sea, to which it was exposed. The scale of the gauge was so constructed as to be read with ease at a considerable distance. This gauge is recommended as one that can be used with ease and perfect accuracy by a telescope from a great distance, and which might therefore afford the means of making observations in situations where otherwise it would be impracticable. The indications of the gauge were written down every five minutes during the entire progress of one tide wave each day, and of two successive tide waves on the evening of each Friday. The following table contains the observations of heights, all referred to the same level, as accurately determined by Mr. Kyle for the first seven stations, and as interpolated for eight and nine.

TABLE of the Observations on the Height of the Tides in the River and Frith of Clyde during April and May, 1837.

STRENGTH & DIRECTION OF WIND:		April 20. Gentle, S.W.	April 21. Gentle, S.E. & E.	April 22. Fresh, E. & S.E.	April 24. Gentle, E. & W.	April 25. Moderate, S. & S.E.	April 26. Strong, W.&S.W.	April 27. Gentle, W.&S.W.	April 28. Gentle, W.&S.W.	April 29. Fresh, E. & S.E.	May 1. Strong, W.&S.W.
		Ft In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.
STATION I. BROOMIELAW. —To find the Height on the Gauge, subtract 3·44 Feet.	High W. M.	...	11 10·25	11 4·2	11 9·2	12 1·2	11 11·2	10 11·2	10 2·3	9 9·2	13 0·2
	Low W. M.	...	...	...	4 0·3	4 6·3	4 7·0	4 3·7	3 10·5	4 2·3	...
	Rise and Fall,	...	...	...	7 8·9	7 6·9	7 4·2	6 7·5	6 3·8	5 6·9	...
STATION II. CRAWFORD'S QUAY.—To find the Height on the Gauge, subtract 3·01 Feet.	High W. M.	11 6	11 9·5	11 3·2	11 8·0	12 0·2	11 10·7	10 11·2	10 1·7	9 10·2	12 10·7
	Low W. M.	3 9	3 9·2	3 8·7	3 11·2	4 2·2	4 4·7	4 1·0	3 8·2	...	6 4·7
	Rise and Fall,	7 9	7 11·3	7 6·5	7 8·8	7 10·0	7 6·0	6 10·2	6 4·5	...	6 6·0
STATION III. CLYDE BANK. —To find the Height on the Gauge, subtract 3·01 Feet.	High W. M.	...	11 6·1	11 1·7	11 5·1	11 9·9	11 7·6	10 8·1	9 11·6	9 9·1	12 7·1
	Low W. M.	...	3 7·6	3 9·1	3 10·1	4 1·1	4 3·1	3 11·1	3 6·3	3 7·1	6 1·2
	Rise and Fall,	...	7 10·5	7 4·6	7 7·0	7 8·8	7 4·5	6 9·0	6 5·3	6 2·0	6 6·9
STATION IV. RASHILEE PIER.—To find the Height on the Gauge, subtract 3·3 Feet.	High W. M.	...	...	...	...	...	11 6·0	10 7·2	9 10·5	9 10·0	...
	Low W. M.	...	...	...	...	...	4 0·2	3 8·7	3 4·2	3 11·7	...
	Rise and Fall,	...	...	...	...	...	7 5·8	6 0·5	6 6·3	5 11·3	...
STATION V. BOWLING BAY. —To find the Height on the Gauge, subtract 2·6 Feet.	High W. M.	...	...	...	11 6·3	11 10·3	11 4·8	10 6·3	9 8·3	9 8·3	12 4·5
	Low W. M.	...	...	...	...	2 11·3	3 4·3	3 0·3	2 8·8	2 8·3	4 3·3
	Rise and Fall,	...	...	...	...	9 11·0	8 0·5	7 6·0	6 11·7	7 0·0	6 1·2
STATION VI. GARMOYLE.— To find the Height on the Gauge, add 7 Inches.	High W. M.	...	...	...	11 3·7	11 8·5	11 1·0	10 2·0	9 5·5	9 8·7	11 8·0
	Low W. M.	...	...	...	...	1 0·0	1 9·0	1 0·2	1 8·2	1 7·0	1 10·0
	Rise and Fall,	...	...	...	...	10 8·5	9 4·0	9 1·8	7 9·3	8 1·7	9 10·0
STATION VII. PORT GLASGOW.—To find the Height on the Gauge, subtract 1·03 Feet.	High W. M.	...	11 3·8	10 9·4	11 0·8	11 6·9	10 10·6	10 1·1	9 4·1	9 5·8	11 3·4
	Low W. M.	...	...	...	...	0 10·2	...	1 11·8	1 10·1	1 9·8	1 6·3
	Rise and Fall,	...	...	...	...	10 8·7	...	8 1·3	7 5·0	7 8·0	9 9·1
STATION VIII. CLOCH LIGHT.—To find the Height on the Gauge, subtract 6 inches.	High W. M.	...	...	...	10 9·5	11 3·5	10 7·0	9 9·5	9 1·5	9 4·5	...
	Low W. M.	...	...	...	...	...	1 9·0	1 10·	1 9·7	1 10·5	1 3·2
	Rise and Fall,	...	...	...	...	...	8 10·0	7 11·5	7 3·8	7 6·0	...
STATION IX. CUMBRAY LIGHT.—To find the Height on the Gauge, subtract 6 inches.	High W. M.	...	...	...	...	...	...	...	...	...	...
	Low W. M.	...	...	...	...	...	...	...	...	...	...
	Rise and Fall,	...	...	...	...	...	...	...	...	...	...

TABLE of Observations continued.

STRENGTH & DIRECTION OF WIND		May 2. Strong, W.	May 3. Gentle, E.	May 4. Strong, N.W.	May 5. Moderate, E. & W.	May 6. Gentle, E. & W.	May 8. Moderate, E.	May 9. Smart, N.E.	May 10. Gentle, S.	May 11. Gentle, E. by S.	May 12. Gentle, S.	May 13. Gentle, S.
		Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.	Ft. In.
STATION I. BROOMIELAW.—To find the Height on the Gauge, subtract 3·44 Feet.	High W. M.	12 2·7	11 10·7	12 5·5	11 9·7	11 4·5	10 11·2	10 4·2	10 2·2	9 6·5	9 6·0	...
	Low W. M.	5 8·7	4 2·3	4 10·0	4 1·7	3 7·0	3 10·2	3 8·8	3 8·8	3 5·8	3 10·2	...
	Rise and Fall,	6 6·0	7 8·4	7 7·5	7 8·0	7 9·5	7 1·0	6 7·4	6 5·4	6 0·7	5 7·8	...
STATION II. CRAWFORD'S QUAY.—To find the Height on the Gauge, subtract 3·01 Feet.	High W. M.	12 1·7	11 9·2	12 4·2	11 8·5	11 3·2	...	...	...	...	...	...
	Low W. M.	5 4·5	4 2·2	4 8·2	4 0·2	3 7·7	...	...	...	...	...	...
	Rise and Fall,	6 9·2	7 7·0	7 8·0	7 8·3	7 5·5	...	...	...	...	...	...
STATION III. CLYDE BANK.—To find the Height on the Gauge, subtract 3·01 Ft.	High W. M.	11 11·3	11 7·3	12 0·1	11 5·3	11 0·9	...	...	...	...	...	...
	Low W. M.	5 0·1	3 11·1	4 3·1	3 11·6	3 6·1	...	...	...	...	...	...
	Rise and Fall,	6 11·2	7 8·2	7 9·0	7 5·7	7 6·8	...	...	...	...	...	...
STATION IV. RASHILEE PIER.—To find the Height on the Gauge, subtract 3·3 Feet.	High W. M.	...	...	...	...	...	...	...	...	...	...	...
	Low W. M.	...	...	...	...	...	...	...	...	...	...	...
	Rise and Fall,	...	...	...	...	...	...	...	...	...	...	...
STATION V. BOWLING-BAY.—To find the Height on the Gauge, subtract 2·6 Feet.	High W. M.	11 8·3	11 6·6	11 10·3	11 5·6	11 2·3	...	...	...	...	...	...
	Low W. M.	4 1·8	3 1·8	3 2·3	3 0·3	2 9·8	...	...	...	...	...	...
	Rise and Fall,	7 6·5	8 4·8	8 8·0	8 5·3	8 4·5	...	...	..	...	...	...
STATION VI. GARMOYLE.—To find the Height on the Gauge, add 7 Inches.	High W. M.	11 3·2	11 3·2	11 5·0	11 2·0	10 11·	...	...	...	...	...	...
	Low W. M.	2 7·0	1 7·0	1 3·0	1 1·5	0 10·	...	...	...	...	...	...
	Rise and Fall,	8 8·2	9 8·2	10 2·0	10 0·5	10 1·	...	...	...	...	...	...
STATION VII. PORT GLASGOW.—To find the Height on the Gauge, subtract 1·03 Feet.	High W. M.	11 1·3	11 1·3	11 3·1	10 11·8	10 9·6	...	...	...	...	...	...
	Low W. M.	2 4·3	1 7·3	0 10·9	0 10·8	0 5·8	...	...	...	...	...	...
	Rise and Fall,	8 9·0	9 6·0	10 4·2	10 1·0	10 3·8	...	...	...	...	...	...
STATION VIII. CLOCH LIGHT.—To find the Height on the Gauge, subtract 6 In.	High W. M.	10 9·2	10 10·0	10 10·5	10 7·7	10 6·0	9 11·7	9 4·2	9 1·2	8 8·0	8 5·7	9 0·0
	Low W. M.	2 5·5	1 6·5	1 0·0	0 8·2	0 5·0	1 1·2	1 2·5	1 8·7	2 2·0	3 2·2	2 4·0
	Rise and Fall,	8 3·7	9 3·5	9 10·5	9 8·5	10 1·0	8 10·5	8 1·7	7 4·5	6 6·0	5 3·5	6 8·0
STATION IX. CUMBRAY LIGHT.—To find the Height on the Gauge, subtract 6 in.	High W. M.	...	10 0·2	10 1·0	9 9·5	9 8·2	9 2·5	8 7·2	8 4·5	7 11·7	7 9·7	...
	Low W. M.	...	1 4·7	0 11·0	0 8·0	0 6·0	0 11·2	1 0·7	1 6·5	1 10·7	2 10·2	...
	Rise and Fall,	...	8 7·5	9 2·0	9 1·5	9 2·2	8 3·3	7 6·5	6 10·0	6 1·0	4 11·5	...

From these observations it appears that the summit of the tide wave increases in height as it ascends the river. From station VII. to station VI. this increase amounts to about 2 inches; at station V. it amounts to 5·2 inches; at III. it has become 6·1 inches; and at Glasgow 10·1 inches is the difference between the level of the wave of high water above that at Port Glasgow, 18·5 miles below. This difference varies slightly with the state of the tides, and with the condition of the current of fresh water in the river. At low water the surface of the river is higher at Glasgow than at Port Glasgow by 33 inches; at station III. this difference is 27 inches, at IV. about 25 inches, and at V. about 12 inches.

	Difference of level at H. W.	Difference of level at L. W.
Station I.	10·1 inches.	33 inches.
Station II.	9·1 inches.	31 inches.
Station III.	7·0 inches.	27 inches.
Station IV.	6·1 inches.	25 inches.
Station V.	5·2 inches.	12 inches.
Station VI.	2·2 inches.	5 inches.
Station VII.	0·0 inches.	0 inches.

The comparison of these numbers with the channel of the rivers in Plate VIII. will be interesting, as showing the influence of the form of the channel upon the height of the tide wave and the current of the river.

Plate V. is a diagram showing the height of the tide wave as it reached the successive stations in various states of the wind. The waves are transposed so as to have a common origin, at station VIII. The effect of westerly winds in increasing the height of the wave, and of easterly winds in depressing it, is manifest. The wave of the 24th of April is curious in this respect, that whereas the wind had been westerly, and changed, during the progress of the high water, to the east, so the wave which previously was higher, afterwards becomes lower than those adjacent to it; it therefore intersects them. The wind was in no case equivalent to what is considered a gale or storm.

Plate IV. represents the form of the tide wave as it passed the successive stations on the River and Frith of Clyde. A series of stars marks the centre of the wave, and has been placed there for the purpose of showing the *dislocation* of the wave, or the transposition of its higher parts forward, or the retardation of its lower parts by the shallowness of the water through which it has advanced. There is a remarkable reversion of this process in the wave of the Cumbray, Station IX., which is probably produced by the circumstance that it is the result of two waves (one behind the other). The corresponding wave at Liverpool is also given; it is also a compound of two waves, which coincide nearly in time.

From a laborious discussion of these observations, it appears that the wave of high water travels

From IX. to VIII.	in 6 min.	14 miles.	80 miles an hour.
From VIII. to VII.	in 9 min.	6 miles.	
From VII. to VI.	in 6 min.	3·75 miles.	20 miles an hour.
From VI. to V.	in 18 min.	4·25 miles.	
From V. to IV.	in 19 min.	2·5 miles.	8·1 miles an hour.
From IV. to III.	in 18 min.	2·5 miles.	
From III. to II.	in 15 min.	2·75 miles.	15 miles an hour.
From II. to I.	in 7 min.	2·75 miles.	

These results show that in the deep water being between 40 and 60 fathoms, or between 240 and 360 feet deep, the wave travels at the enormous rate of 30 miles an hour; that on reaching water from 20 to 30 feet deep, the velocity is diminished to 20 miles an hour; and from V. to II. where the river is wide, shelving, and shallow, the velocity of the tide wave is retarded to 8 miles an hour; while on ascending further up, where the banks nearly upright, and the contracted width give an increase of mean depth, the velocity has a corresponding increase to 15 miles an hour.

By examining the plans it will be apparent that we shall not err greatly if we assume the average depth of the river, from I. to III., at 15 feet. From III. to V. the river is wide and shallow, spreading over extensive banks, where there are not 2 feet of water, for which we may be allowed to take a third part of the greatest as a mean depth, or about 5 feet. In the division from V. to VII., both depth and breadth increase very rapidly to about 35 and 37; taking 25 feet as the mean depth, we have

Velocities of the Tide-wave as observed.	Mean depth.	Velocity due to depth.
80 miles an hour.	240—360 feet.	60—80 miles.
20 miles an hour.	25 feet.	19·3 miles.
8·1 miles an hour.	5 feet.	8·6 miles.
15 miles an hour.	15 feet.	14·9 miles.

The following are the results of the observations in regard to the time of high water:—

At Cloch Light,—High Water is 9 Min. earlier than at Port Glasgow.

Lazaretto-Point	5	do.	do.
Cumbray Light-house	15	do.	do.
Port-Patrick	31	do.	do.

At Cloch Light,—High Water is 51 Min. earlier than at Port Glasgow.

Liverpool	51	do.	do.
Whitehaven	62	do.	do.
Newry	85	do.	do.
Donaghadee	127	do.	do.
Port-Rush	5 h. 35	do.	do.

At Garmoyle Light,—H. W. is 6 Min. later than at Port Glasgow.

Bowling	24	do.	do.
Rashilee	43	do.	do.
Clyde Bank	61	do.	do.
Crawford's Quay	76	do.	do.
Broomielaw	83	do.	do.

Being 1 hour 23 minutes between Port Glasgow and the Broomielaw.

It is difficult to determine whether the wind produced a decided effect on the velocity of these tides. By a discussion which was attempted, it appeared that on all the days in which the easterly wind prevailed, compared with all the days on which the westerly wind prevailed, there was a difference of one minute more and of one minute less than the mean; the tide being accelerated by the coincident wind and retarded by the opposing one.

The continuation of this series of inquiries will be given in the next Report.

Description of the Tables containing the original Observations of the Waves in Artificial Channels made in 1837.

Each of the first ninety-three tables contains the history of a single wave, including the condition of the fluid previous to generation—the method of generation—the volume of the wave at the commencement of its path—the height of the wave at every transit—the interval between its transits—the space described, and the time occupied in describing it. The methods of observing and the observers' names are given, for the sake of authenticity, except in the first four experiments, which are not sufficiently perfect to form by themselves the grounds of any important conclusions.

The *approximate depth* of the fluid is given at the head of each table in the first line for convenience of reference.

The *corrected or true depth* of the fluid at the commencement of the observations is given immediately above the columns of observations, where it is given as "corrected statical depth =".

The "*observed statical level*" is the indication of the height of the fluid on the scale of the glass indices or gauges represented in Plate I., taken from an arbitrary line and affected by an index error, from which the "corrected statical level" is derived by a correction obtained from observation.

The *modes of generating the fluid* were very numerous, but as the resulting phænomena of the waves were found to be independent of the mode of generation, a sufficient number only are given to establish the means of comparison. These extend from Wave I. to Wave XXV. Those waves "created by reservoir A" were formed by filling that portion of the channel at the end of the experimental channel of Plate I. with a given volume of water, which was added to the water in the channel by the sudden removal of the sluice S, and so formed the wave. The waves "generated by impulsion" of sluice were formed by placing the sluice at the back of reservoir A, and suddenly bringing it to the front of the reservoir, so as to communicate a horizontal impetus to the fluid forming the wave. The waves "generated by detached chamber B" were formed by placing the rectangular vessel B, Plate I., at the end of the reservoir and filling it with water to a given volume: by raising the sides of this vessel from the bottom of the reservoir, the column of water was allowed to descend by gravity and generate the wave.

Column A contains the number of feet described by the wave from the commencement of the observations.

Column B contains the interval of time given by two observers and two chronometers, α and β: these intervals of time correspond to the spaces in column A.

Column C contains the observations of height of the wave made in two sets of glass indices—index γ near the end B of the experimental channel, and index δ near the end D, Plate I.

Column D contains the heights of the waves at γ and δ, freed from error of scale.

Column E contains the sum of the corrected height of the wave and of the corrected depth of the fluid, taken from a mean of the observations.

2nd Aug., 1837. WAVE I. Depth 4 inches.

Created by Reservoir A. Volume of added fluid = 152·5 inches.
Transits observed directly at Index γ, without reflection.
Statical level observed at $\left\{\begin{matrix}\gamma = -\,0\cdot05\\ \delta = -\,0\cdot01\end{matrix}\right\}$ Corrected statical depth = 3·942 inches.

A	B		C		D		E	A	B		C		D		E
feet.	α sec.	β sec.	γ in.	δ in.	γ' in.	δ' in.	in.	feet.	α sec.	β sec.	γ in.	δ in.	γ' in.	δ' in.	in.
...	...	...	0·50	0·60	0·55	0·61	...	280·	81·5	...	0·20	0·17	0·25	0·18	4·17
0·	0·0	...	0·50	0·50	0·55	0·51	4·50	320·	84·0	...	0·15	0·17	0·20	0·18	4·15
40·	12·0	...	0·47	0·43	0·52	0·44	4·46	360·	105·0	...	0·10	0·15	0·15	0·16	4·12
80·	23·6	...	0·37	0·40	0·42	0·41	4·40	400·	117·0	...	0·17	0·13	0·22	0·14	4·12
120·	35·0	...	0·30	0·31	0·35	0·32	4·34	440·	129·0	...	0·10	0·13	0·15	0·14	4·11
160·	47·0	...	0·30	0·27	0·35	0·28	4·28	480·	141·0	...	0·10	0·10	0·15	0·11	4·09
200·	58·5	...	0·27	0·23	0·32	0·24	4·25	520·	153·5	...	0·10	0·09	0·15	0·10	4·07
240·	70·5	...	0·20	0·20	0·25	0·21	4·20	560·	165·5	...	0·07	...	0·12	...	4·06

2nd Aug., 1837. WAVE II. Depth 4 inches.

Created by Reservoir A. Volume of added fluid = 152·5 inches.
Transits observed directly at Index γ, without reflection.
Statical level observed at $\left\{\begin{matrix}\gamma = -\,0\cdot20\\ \delta = -\,0\cdot12\end{matrix}\right\}$ Corrected statical depth = 3·812 inches.

A	B		C		D		E	A	B		C		D		E
feet.	α sec.	β sec.	γ in.	δ in.	γ' in.	δ' in.	in.	feet.	α sec.	β sec.	γ in.	δ in.	γ' in.	δ' in.	in.
...	...	...	0·50	0·60	0·70	0·72	...	240·	70·3	...	0·10	0·10	0·30	0·22	4·11
00·	0·0	...	0·50	0·51	0·70	0·63	4·50	280·	82·2	...	0·07	0·07	0.27	0·19	4·07
40·	11·5	...	0·47	0·42	0·67	0·54	4·48	320·	95·0	...	0·07	0·03	0·27	0·15	4·06
80·	23·0	...	0·37	0·35	0·57	0·47	4·40	360·	107·5	...	0·00	·00	0·20	0·12	4·01
120·	35·0	...	0·30	0·27	0·50	0·39	4·32	400·	119·5	...	0·00	–·02	0·20	0·10	3·98
160·	45·0	...	0·27	0·20	0·47	0·32	4·26	440·	131·0	...	–·05	–·07	0·15	0·03	3·96
200·	58·0	...	0·17	0·14	0·37	0·29	4·18	580·	...	...	...	...	...	...	...

2nd Aug., 1837. WAVE III. Depth 4 inches.

Created by Reservoir A. Volume of added fluid = 137·3 inches.
Transits observed directly at Index γ, without reflection.
Statical level observed at $\left\{\begin{matrix}\gamma = -\,0\cdot13\\ \delta = -\,0\cdot07\end{matrix}\right\}$ Corrected statical depth = 3·872 inches.

A	B		C		D		E	A	B		C		D		E
feet.	α sec.	β sec.	γ in.	δ in.	γ' in.	δ' in.	in.	feet.	α sec.	β sec.	γ in.	δ in.	γ' in.	δ' in.	in.
...	...	...	0·47	0·52	0·60	0·59	...	200·	60·0	...	0·17	0·17	0·30	0·24	4·16
0·	0·0	0·0	0·40	0·47	0·53	0·54	4·45	240·	71·25	...	0·10	0·15	0·23	0·22	4·12
40·	12·0	...	0·37	0·39	0·50	0·46	4·40	280·	83·5	...	0·10	0·11	0·23	0·18	4·10
80·	23·75	...	0·30	0·31	0·43	0·38	4·33	320·	96·0	...	0·07	0·10	0·20	0·17	4·08
120·	35·5	...	0·22	0·22	0·35	0·29	4·26	360·	107·5	...	0·05	0·09	0·18	0·16	4·05
160·	47·75	...	0·20	0·19	0·33	0·26	4·18	400·	119·5	...	0·02	0·07	0·15	0·14	4·02

14th Aug., 1837. WAVE CVII—CXXXII., Sloping Channel (O).
Channel 17 feet long, 4 in. deep at 0·, and 0· in. at 17 feet.

Wave.	Height at 0·.	Height at Breaking.	Distance from end.	Depth of Water at Breaking Point.
	inches.	inches.	feet.	
CVII.	0·9	1·4	6·6	1·5
CVIII.	...	2·1	9·4	2·2+
CIX.	1·1	1·4	7·6	1·7
CX.	...	2·5	11·0	2·5
CXI.	...	1·95	8·6	1·92
CXII.	0·5	0·8	5·0	1·1
CXIII.	1·5	2·3	11·0	2·5
CXIV.	1·3	1·9	8·3	1·9
CXV.	1·8?	2·2	9·4	2·2
CXVI.	1·25	1·9	8·3	1·9
CXVII.	...	2·9	15·0	3·4
CXVIII.	2·5	2·7	12·2	2·7
CXIX.	...	0·8	3·0	0·7
CXX.	1·1	1·4	6·3	1·4
CXXI.	0·2	0·3	2·1	0·4
CXXII.	1·0	1·2	5·5	1·2
CXXIII.	0·5	0·5	4·0	0·9
CXXIV.	0·8	0·8	4·3	1·1
CXXV.	0·2	0·3	2·5	0·5
CXXVI.	0·5	0·7	4·0	0·9
CXXVII.	1·2	1·7	7·5	1·7
CXXVIII.	2·0	2·7	11·3	2·6
CXXIX.	2·2	2·7	11·0	2·5
CXXX.	2·0	2·4	10·3	2·4
CXXXI.	1·5	2·0	9·0	2·1
CXXXII.	...	2·5	11·0	2·5

14th Aug., 1837. WAVE CXXXIII—CXLIX. Sloping Channel (O).
Channel 17 feet long, 4 in. deep at 0·, and 0· in. at 17 feet.

Wave.	Time from 0· to Breaking.	Whole Time from 0 to D.	Place of Breaking.	Depth of Water at Breaking Point.
	sec.	sec.	feet.	
CXXXIII.	2·0	5·5	9·3	2·2
CXXXIV.	2·0	5·5	10·0	2·3
CXXXV.	2·0	5·5	10·0	2·3
CXXXVI.	3·5	6·0	6·5	1·4
CXXXVII.	4·0	6·0	5·0	1·4
CXXXVIII.	5·0	7·0	3·9	0·9
CXXXIX.	6·0	7·0	3·0	0·7
CXL.	4·0	6·5	5·0	1·1
CXLI.	5·0	6·5	4·0	0·9
CXLII.	5·0	7·0	4·3	1·0
CXLIII.	6·5	7·5	1·5	0·2
CXLIV.	2·0	5·0	9·7	2·2
CXLV.	2·0	5·5	11·0	2·5
CXLVI.	0·5	5·5	16·0	3·7
CXLVII.	0·0	5·5*	17·0	4·0
CXLVIII.	0·0	5·5†	16·0	3·7
CXLIX.	0·0	5·5‡	15.0	3·4

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* This large wave was an inch high at D, and was reflected.

† This large wave was an inch high at D, became doubled by reflection, and returned to 0 in 6·5 seconds.

‡ This large wave was 0·75 inch high at D, was reflected, and returned to O in 7·0 seconds.

Lizars sc

Description of Plates accompanying the Repo

Plate I. contains the apparatus of the experiments on Fig. 1, A is a transverse section of the experimental chan the sides of which were made smooth and as nearly plane surfaces as possible; the whole internal surface being divided into feet, inches, tenth parts of an inch, &c., for convenient observation. B and D are the two ends of the same channel, and are elevated, so as to reflect the waves from vertical surfaces. C is the generating reservoir referred to in the experiments as "Generating Reservoir A." Fig. 2 shows the apparatus for observing transits of the wave by reflexion. I is the luminous object from which the rays falling on the plane mirror M are thrown down on the surface of the fluid at W, and thence reflected on the small mirror *m*, to the eye of the observer. W^1, W^2, and W^3, show a single wave in successive positions, and figs. 3, 4, and 5, show the places of the image corresponding to those positions. Fig. 8 shows the generation of the wave from "Reservoir A," by removing the sluice S. Fig. 9, B represents the generating chamber, resting on the bottom of the experimental channel, and containing the fluid which generates the wave when the sides of the chamber are raised from the bottom. Fig. 10 represents the solid parallelopipedon C; and that part of it towards D represents the form and magnitude of the chamber and the solid D.

Plate II. gives the forms of the waves of the sea referred to in pages 445—451 of the Report. Fig. 1, the cycloidal forms. Fig. 2, *a* and *b*, elementary waves, moving in opposite directions; *c* and *d*, the result of this combination at successive instants of time. Figs. 3 and 5 are forms observed to result from the combination of three or four co-existent classes of waves moving in different directions. Figs. 4, 5, 6 and 7 show the manner in which waves break, either from the coincidence of a wave of a higher or with the crest of a lower wave, so as to give the form of unstable equilibrium, or from the excess of the height of the wave above the depth of the fluid.

Plate III. exhibits the relation of the velocity of the waves to the depth, as taken from the experiments in the rectangular channel, fig. 1, and in the channels, fig. 2, H, fig. 3, K, and fig. 4, L. The horizontal abscissæ are depths of the fluid, and the vertical ordinates the corresponding velocities.

Plate IV. represents the form of a tide-wave as it passed the successive stations referred to in the observations on the Clyde. The corresponding tide-wave of Liverpool Docks is given in the same plate. The stars in each wave mark its centre of length, and serve to show the increasing dislocation of the tide-wave during its ascent along the river.

Plate V. shows the line described by the summit of the tide-wave during its transit along the Frith of Clyde and the manner in which it was affected by the wind. The wave of the 3rd of May was nearly calm; and that of the 24th of April is remarkable as having been described partly during a westerly wind and partly during an easterly wind, and so falling partly above and partly below the 3rd of May, while none of the others present instances of intersection.

Plate VI. gives the form of the tide-wave of the river Dee.

Plate VII. contains the channel of the river Dee, with sections.

Plate VIII. is the channel of the river Clyde, with sections.

PLATE I.

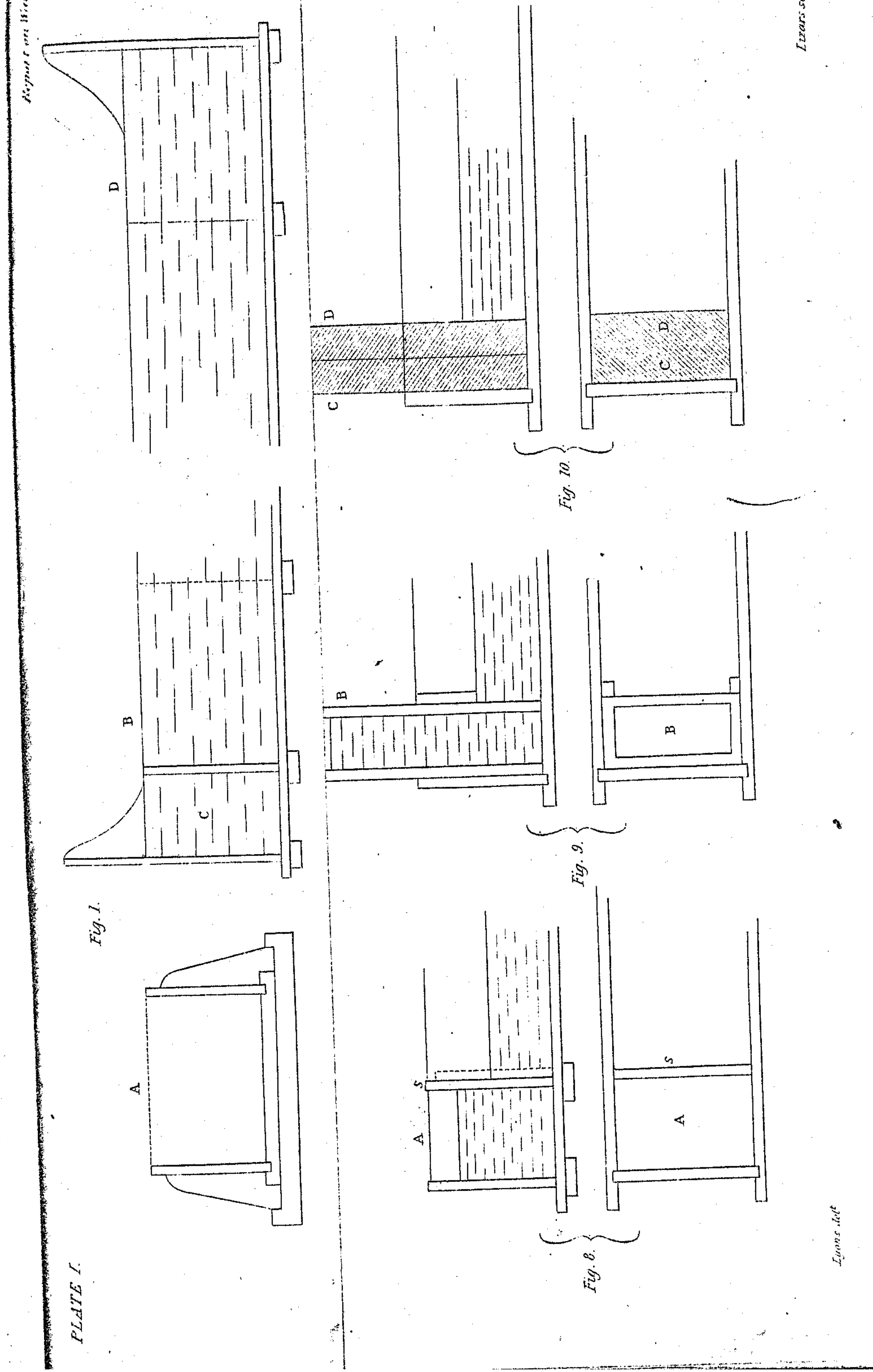

Fig. 1.

Fig. 8.

Fig. 9.

Fig. 10.

Lyons del.

Lizars sc.

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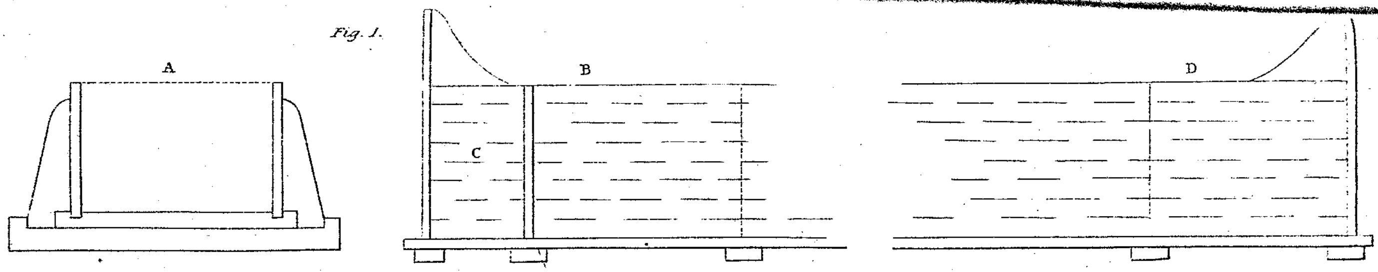
Fig. 1.



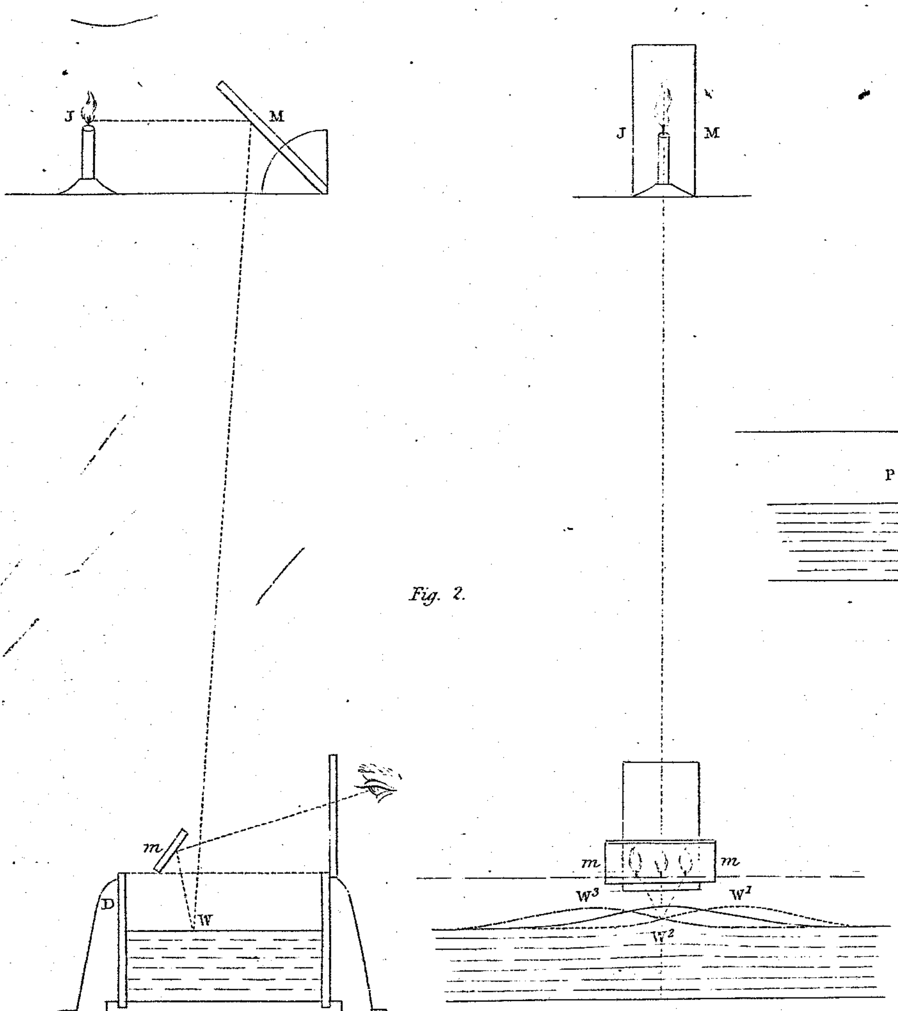


Fig. 2.

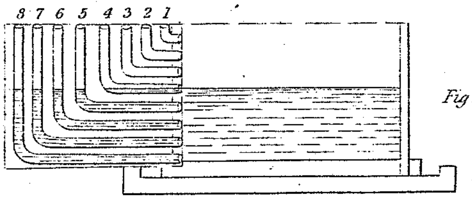


Fig. 6.

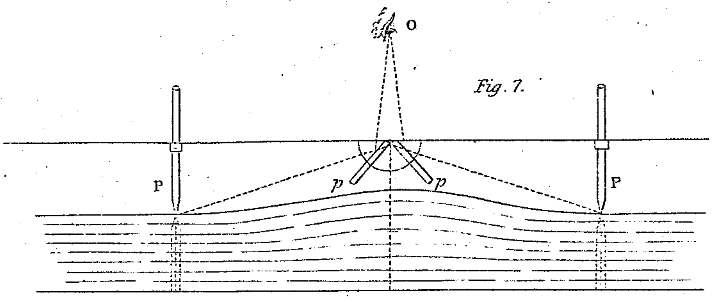


Fig. 7.

Fig. 3.

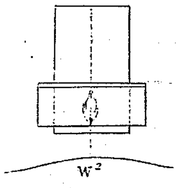


Fig. 4.

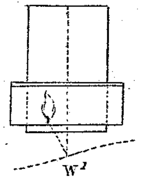
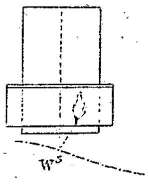


Fig. 5.



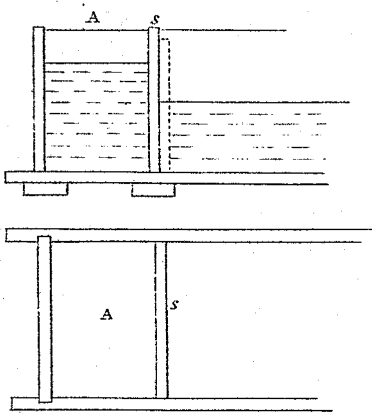


Fig. 8.

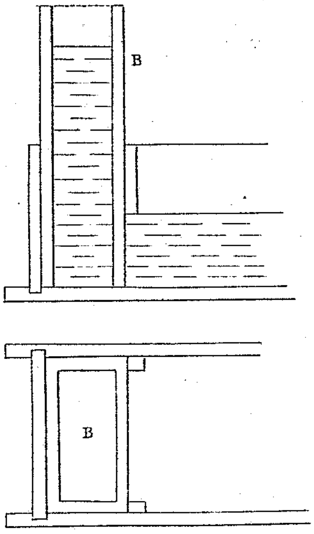


Fig. 9.

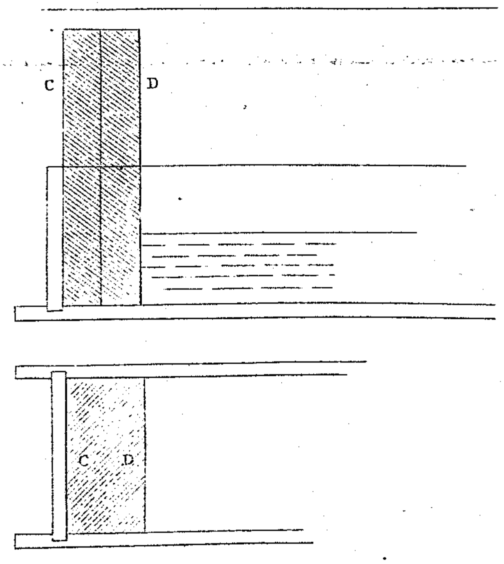


Fig. 10.

Lyons del.

Lizars sc.

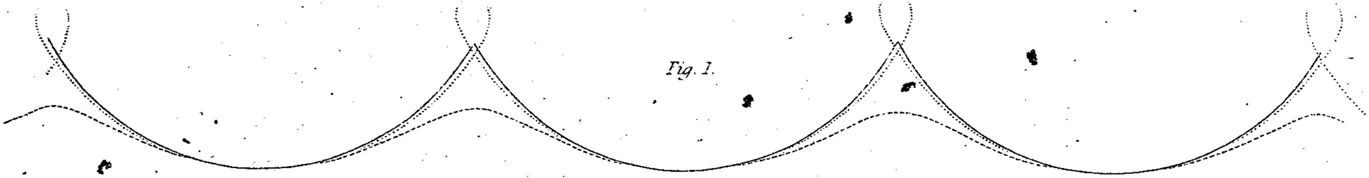
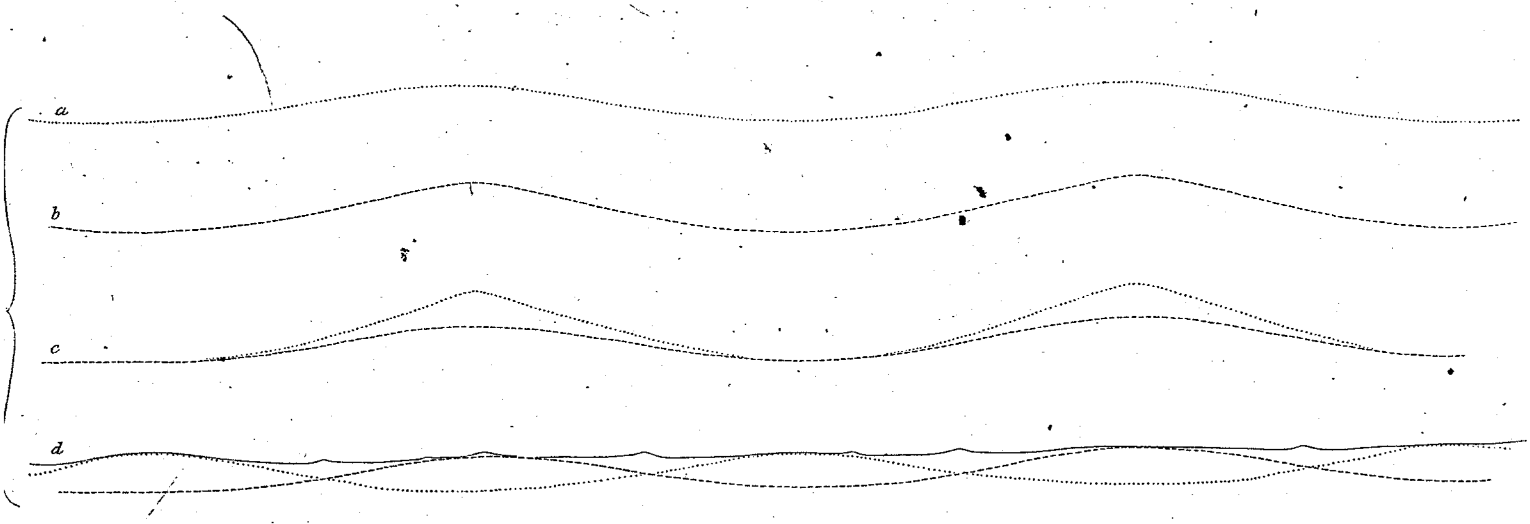


Fig. 1.



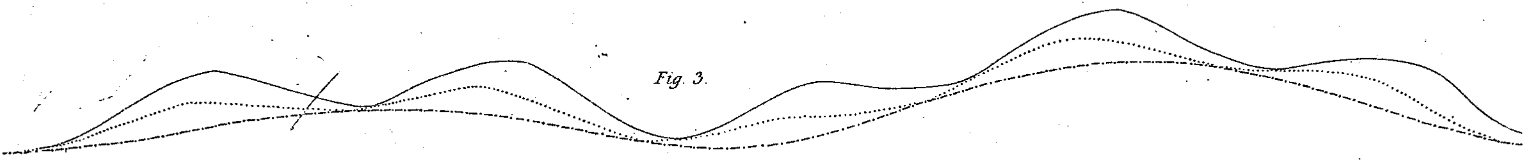


Fig. 3.

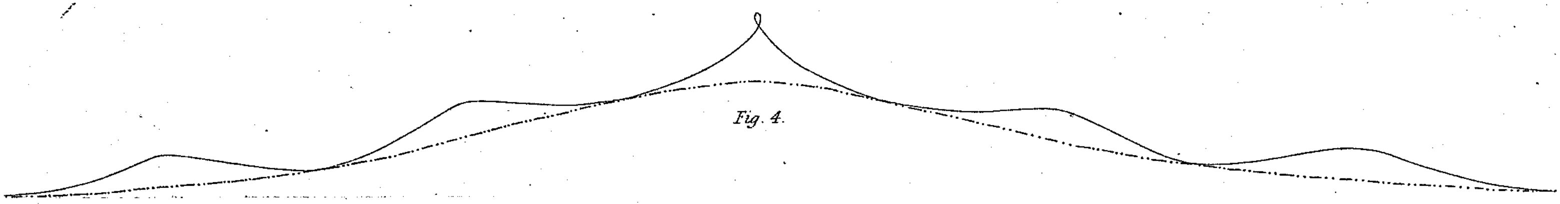


Fig. 4.

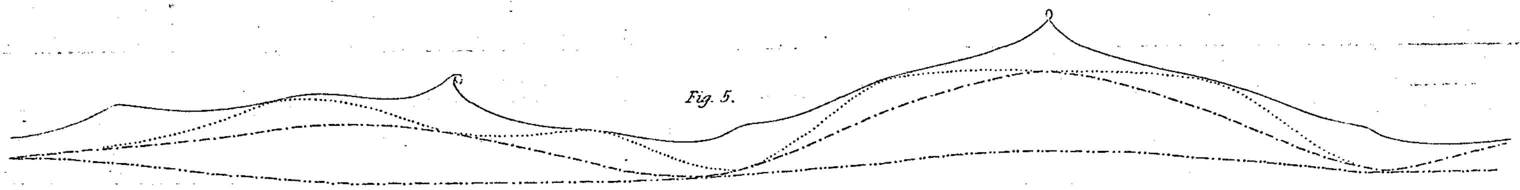
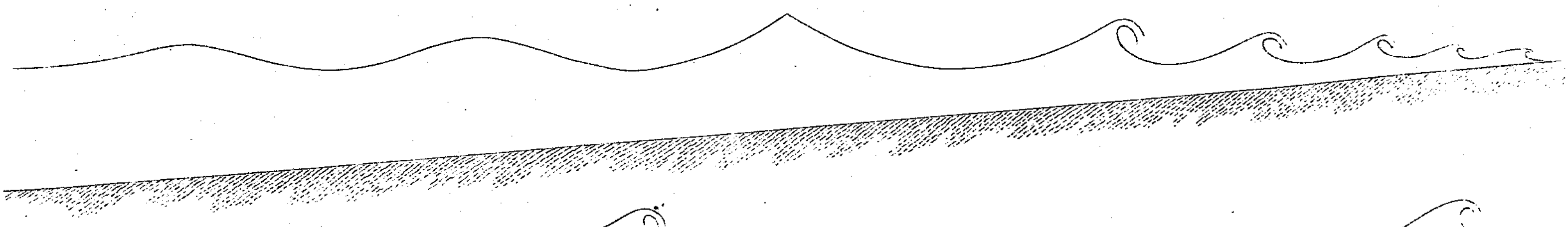
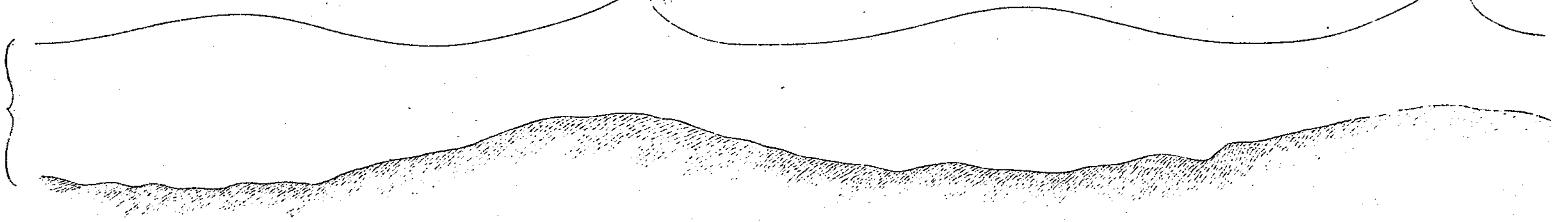


Fig. 5.





Lyons delt.

Lizars sc.

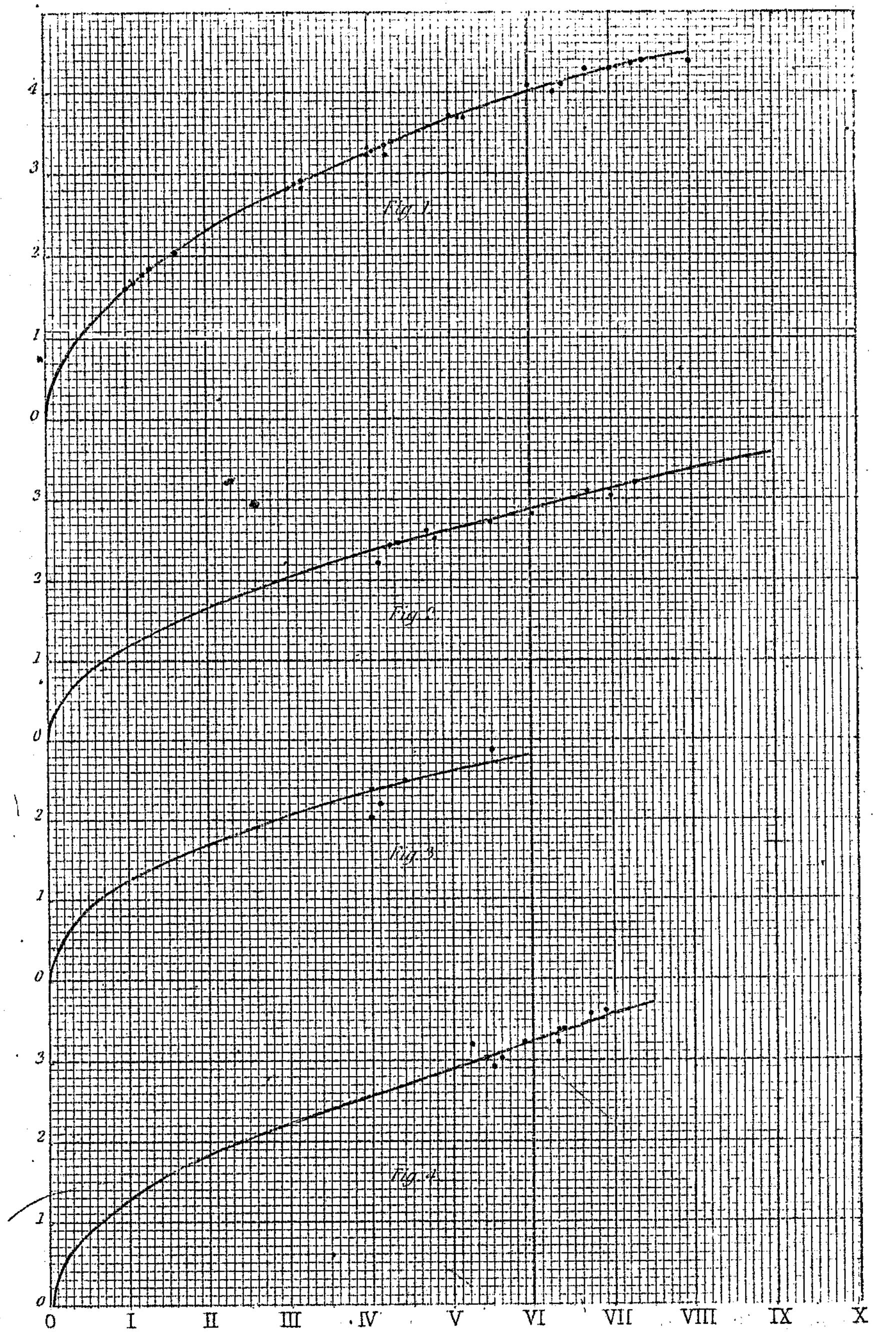


Fig. 1.

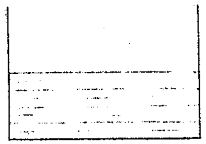


Fig. 2.

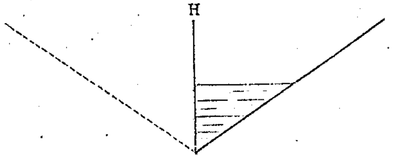


Fig. 3.

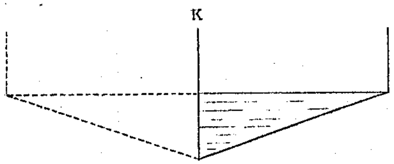
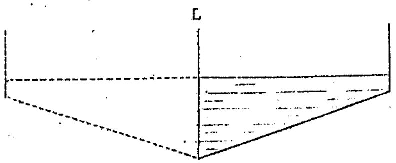


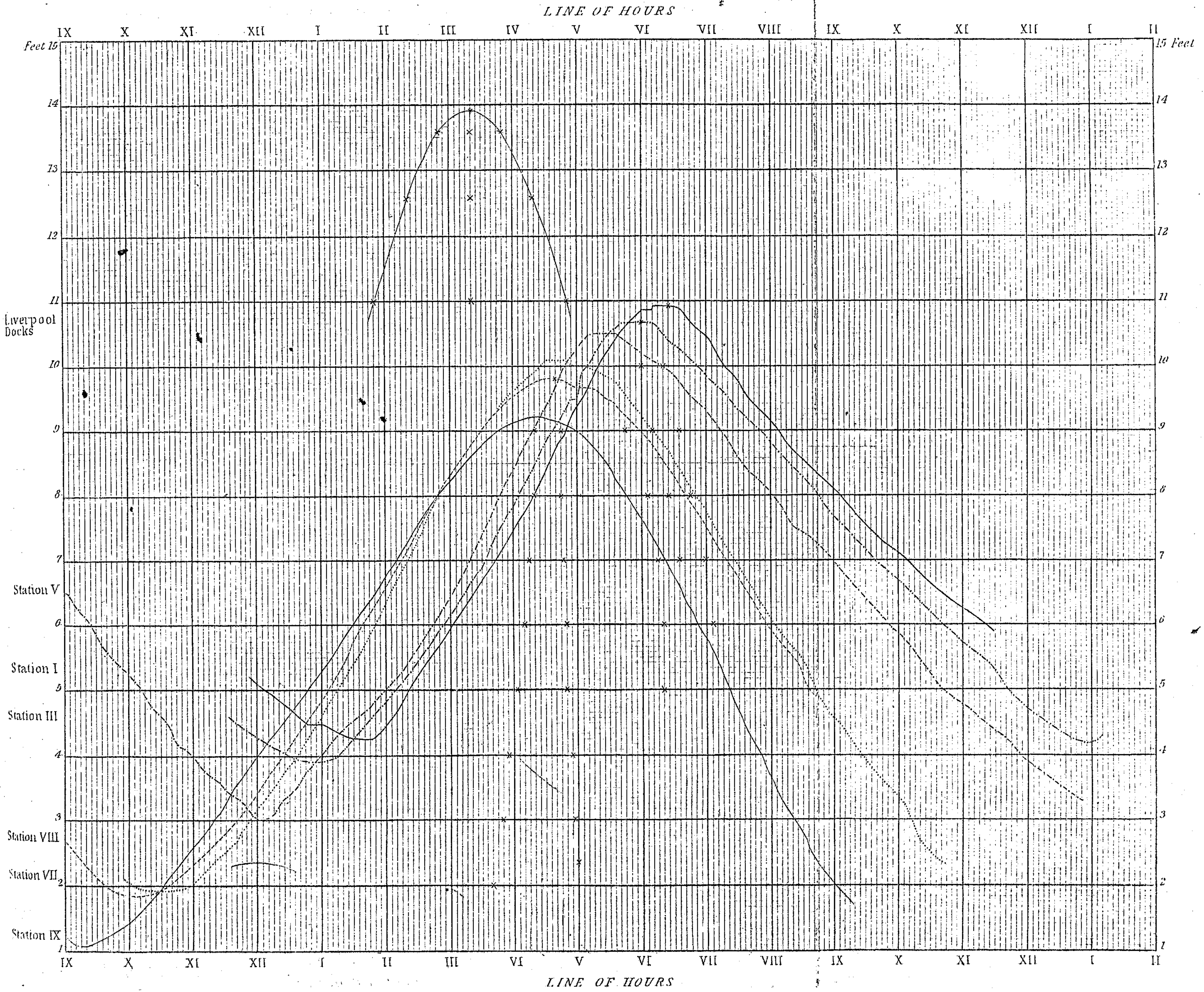
Fig. 4.



Lyons del.

Lizars sc.

PLATE IV.


LINE OF HOURS
IX
X
XI
XII
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II
III
IV
V
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VII
VIII
IX
X
XI
XII
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II
Feet 15
14
13
12
11
10
9
8
7
6
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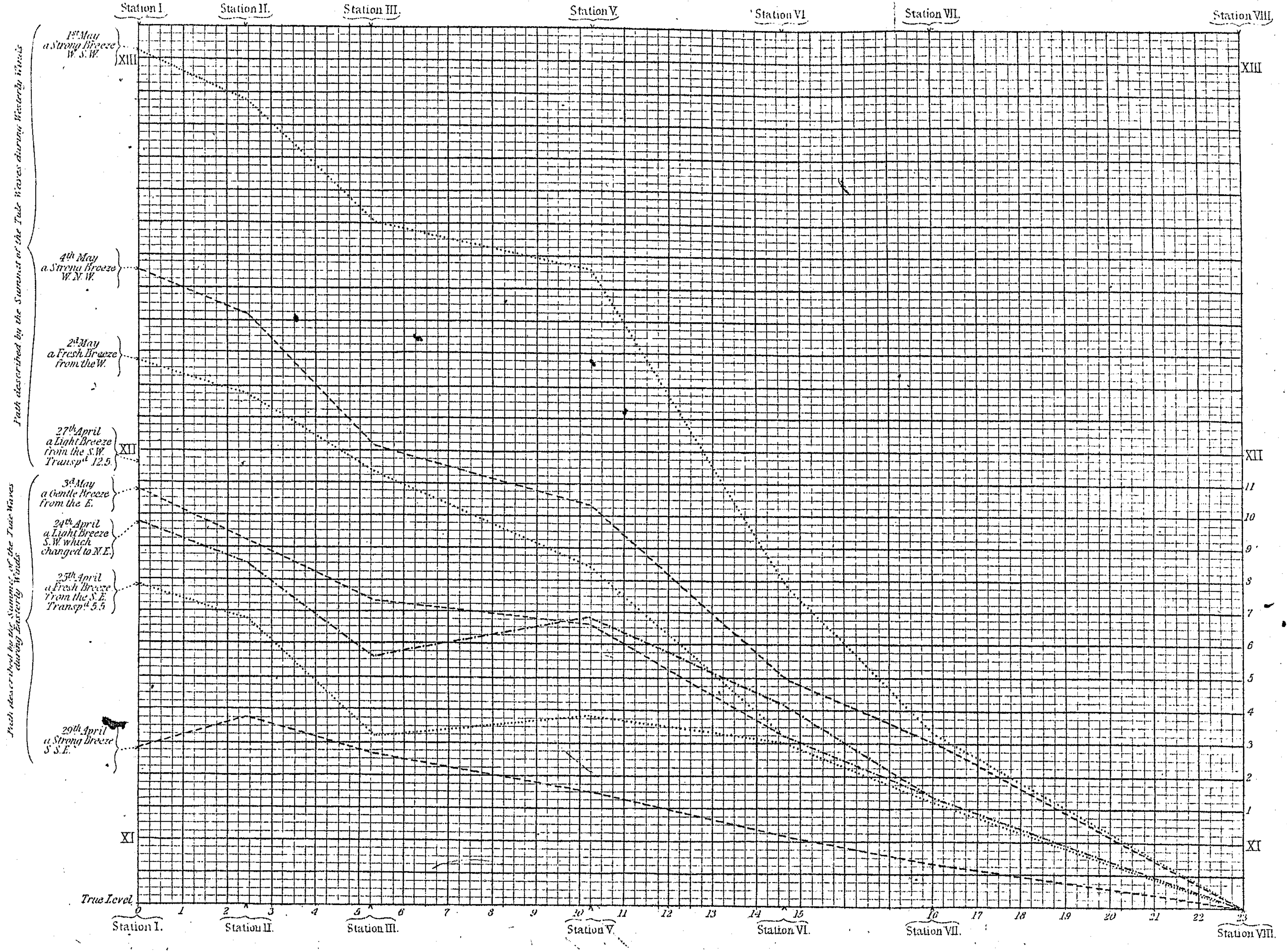
2
1
15 Feet
Liverpool Docks
Station V
Station I
Station III
Station VIII
Station VII
Station IX
LINE OF HOURS


Station I.
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Station VIII.
Path described by the Summit of the Tide Waves during Westerly Winds
1st May a Strong Breeze W. S. W.
4th May a Strong Breeze W. N. W.
2d May a Fresh Breeze from the W.
27th April a Light Breeze from the S.W. Transpd 12.5
Path described by the Summit of the Tide Waves during Easterly Winds
3d May a Gentle Breeze from the E.
24th April a Light Breeze S.W. which changed to N.E.
25th April a Fresh Breeze from the S.E. Transpd 5.5
29th April a Strong Breeze S.S.E.
XIII
XII
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True Level
0
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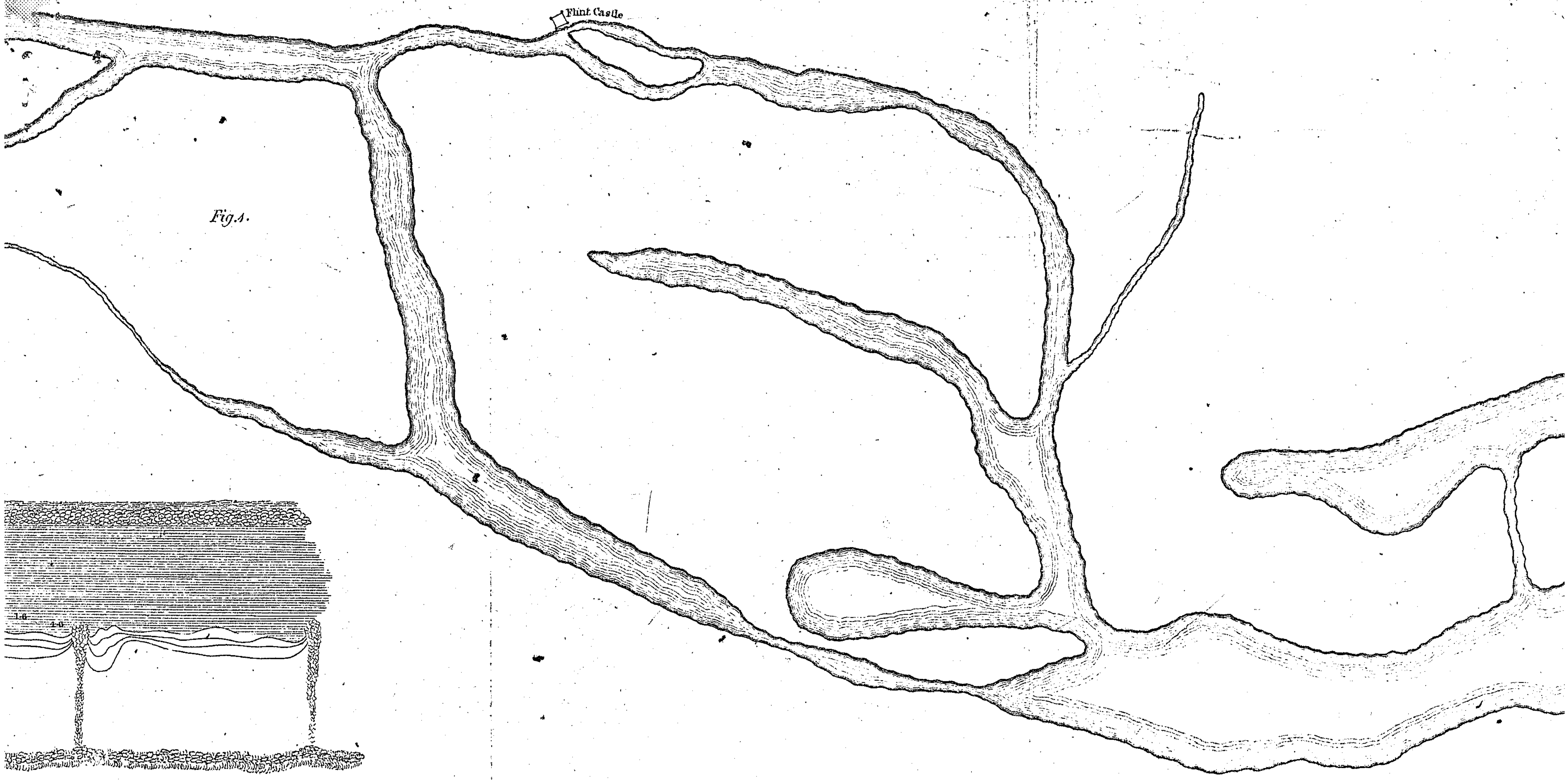

Fig. 1.

Fig. 3.