Case for Support
Lie group methods and control theory workshop
ICMS Edinburgh: 28th June–1st July 2004

1. Summary

We seek funding for an innovative workshop that brings together two important emerging areas of mathematics and engineering: *Lie-group methods* for the discretization of differential equations and *geometric control theory*. The workshop will be held over four days at the International Centre for Mathematical Sciences (ICMS) in Edinburgh between June 28th and July 1st 2004. The proposed meeting will provide a unique platform for interaction between a collection of international leading researchers that has hitherto never gathered in a similar setting. It will combine the theorists at the forefront of geometric integration with those at the leading edge in the theory of geometric control. And to push these mutually beneficial frontiers further, we will include specialists in the application and use of these techniques in robotics, mechanical control, computer vision and optical motion capture.

2. Scientific organising committee

Uwe Helmke completed his PhD in mathematics at the University of Bremen in 1982/83. From 1984 to 1995 he was first a Lecturer and then Senior Lecturer (”Privatdozent”) in the Department of Mathematics at Regensburg University. In 1995 he was appointed as Full Professor in Pure Mathematics and Chair in Real Analysis at the University of Würzburg. He is a member of the AMS, GAMM, SIAM and a senior member of the IEEE. He has held visiting positions at several other universities, including a postdoctoral fellowship at the Division of Applied Sciences, Harvard University (1983; host: Prof. C.I. Byrnes), as well as frequent research positions at Ben Gurion University of the Negev (host: Prof. P.A. Fuhrmann) and at the Australian National University (hosts: Profs B.D.O. Anderson, J.B. Moore).

Professor Helmke was co-chair of the 1993 International Symposium of the Mathematical Theory of Networks and Systems (MTNS 93) in Regensburg, and since 2003 has also been co-chairman of the tri-annual Oberwolfach meeting in control theory. He has been organiser of several workshops, invited special sessions and minisymposia at international conferences, and is a member of the steering committee of MTNS, the GAMM Special Interest Group in Mathematical Control Theory, and a member of the programme committee for the IFAC Activity Group in Linear Systems. He is on the editorial board of several journals in mathematics, and systems and control theory. Professor Helmke is a well known international expert in mathematical systems and control theory, and optimization. He is also member of two very large European research networks, Nonlinear Control Network (1998–2002) and Control Training Sites (2001–2005). He has been frequently invited or plenary speaker at major international conferences and is an influential figure in the systems engineering and mathematics community. He has been the author or co-author of over 100 publications, including one graduate level textbook with Prof. John Moore.

Webpage: `http://www.mathematik.uni-wuerzburg.de/RM2`, email: `helmke@mail.uni-wuerzberg.de`

Desmond Higham has been a Professor of Mathematics at the University of Strathclyde since October 1999, having formerly been a Reader at Strathclyde and a Lecturer at the University of Dundee. His research interests cover numerical analysis and stochastic computation. He has over 50 refereed publications (see for example: Aves, Griffiths & Higham 2000; Mattingly, Stuart and Higham 2002 and Higham, Mao & Stuart 2002) and is an editor of the *SIAM Journal of Scientific Computing.* He has collaborated with a number of leading researchers in computational mathematics, including Griffiths (Dundee) and Stuart (Warwick, formerly Stanford). His work with Griffiths has been supported by two large EPSRC grants: *Dynamics of Timestepping in the Numerical Analysis of Differential Equations* 1993—1996 [GR/H94634] and *Time-Stepping and Nonlinear Dynamics* 1997–2000 [split into GR/K80228 and GR/M42206]. Higham has been involved in organising a number of workshops of conferences. Most recently, he was responsible for a series of four one-day meetings on “Stochastics and Numerics”, sponsored by the LMS, served on the scientific committee of the ICMS workshop "SDEs and SPDEs : Numerical Methods and Applications", March/April 2003, and organised minisymposia at the Foundations of Computational Mathematics Conference, Minneapolis, August 5–14 2002 and International Conference on Scientific Computing and Differential Equations (SciCADE), Trondheim, June/July 2003.

Webpage: `http://www.maths.strath.ac.uk/ aas96106+`, email: `djh@maths.strath.ac.uk`

Arieh Iserles (Chair) is the Professor of Numerical Analysis of Differential Equations at University of Cambridge and his previous appointments and visiting positions have included King's College, Cambridge, University of Arizona, California Institute of Technology, Universidad de Valladolid and Norwegian University of Science and Technology, where he was the Lars Onsager Visiting Professor. He is the Managing Editor of *Foundations of Computational Mathematics* and of *Acta Numerica,* a Honorary Editor of *LMS Journal of Computation and Mathematics*, member of the Board of Directors of Society for Foundations of Computational Mathematics and of IMACS, a member of EPSRC College and a recipient of the Lars Onsager Medal. He has been the chair of SFoCM (1997–99) and his recent organisational experience includes FoCM conferences in 1997, 1999 (where he was the chair), 2002 and 2005, SciCADE conferences in 1997 and 2001, a special semester on foundations of computational mathematics at MSRI, Berkeley, in 1998, a workshop on "Group theory and numerical analysis" at CRM, Montreal, in 2003 and an LMS/EPSRC instructional course in 2002.

Professor Iserles's research interests comprise numerical analysis of ordinary and partial differential equations, functional equations, approximation theory, theory of orthogonal polynomials and special functions and, more recently, numerical geometric integration. He has more than 150 publications, including Iserles (1996), Iserles & Nørsett (1999), Iserles, Munthe-Kaas, Nørsett & Zanna (2000) and Iserles (2002). Professor Iserles is a frequent plenary speaker at international conferences and his recent invitations have included the AR Mitchell Lecture at the Biennial Conference in Numerical Analysis, Dundee, 2001, and the opening lecture of the SciCADE'03 conference in Trondheim, 2003.

Webpage: `http://www.damtp.cam.ac.uk/user/ai/`, email: `A.Iserles@damtp.cam.ac.uk`

Luciano Lopez received his degree in Mathematics in 1980 at University of Bari where he is employed as Full Professor in Numerical Analysis since November 1995. He is member of the Editorial Board of the new international *Mediterranean Journal of Mathematics,* published by Birkhäuser. He co-organised the workshop on Structural Dynamical Systems in Linear Algebra and Control, SDS'01, 1–4 July 2001, Bari, Italy (40 participants), and is co-organising the following minisymposia and workshops: ICCS2003, Saint Petersburg, 2–4 June 2003, and Technical Session on Numerical Methods for Structured Systems; Structural Dynamical Systems in Linear Algebra and Control, SDS'03, 22–25 June 2003, Bari, Italy (40 participants). He has been leader of several national projects in Numerical Analysis supported by CNR, INDAM and the University of Bari. Professor Lopez is an author of more than 50 papers published in international journals, e.g. Del Buono & Lopez (2001), Dieci & Lopez (2003) and Chu, Del Buono, Lopez & Politi (2003). His recent research interests are in geometric numerical methods for control problems and numerical methods for Lyapunov exponents.

Webpage: `http://www.dm.uniba.it/ lopez`, email: `lopezl@dm.uniba.it`

Simon Malham (local organiser) has been a lecturer in the Mathematics Department at Heriot-Watt University since September 2000. After completing his PhD at Imperial College in 1993, he spent three years as a Visiting Assistant Professor at University of Arizona where he continued his thesis work on regularity assumptions for the Navier–Stokes equations. In 1995 he co-organised Arizona Days conference on *Nonlinear Dynamics* at the Center for Nonlinear Studies, Los Alamos National Laboratory, with 30 participants. Since 1996 he also held lectureships at Nottingham University and Imperial College and his research interests developed to include reaction-diffusion phenomena (Malham & Oliver), geometric geophysical fluid mechanics (Ford, Malham & Oliver 2002), resolution of spectra and Lie-group integration methods (Aparicio, Malham & Oliver 2003). He has been the colloquium organiser at both Nottingham and Heriot-Watt for a total of 4 years.

Webpage: `http://www.ma.hw.ac.uk/ simonm`, email: `simonm@ma.hw.ac.uk`

The International Centre for Mathematical Sciences (ICMS) is a joint institute of Edinburgh and Heriot-Watt universities and was established in 1990. It has organised over 80 meetings, almost always held in Edinburgh and often at its headquarters at 14 India Street. It has established a reputation internationally for regular high-quality events, including a variety of meetings, workshops, symposia and conferences across all areas of mathematics and its applications.

Webpage: `http://www.ma.hw.ac.uk/icms`, email: `icms@maths.ed.ac.uk`

3. Objectives

We propose to organise a first-of-a-kind workshop that combines two important emerging areas of mathematics and engineering: *Lie-group methods* for solving differential equations and *geometric control theory.*

The goal of Lie-group methods is to discretise differential equations that evolve on homogeneous manifolds whilst retaining this critical geometric invariant. Problems of this kind abound in geometric control theory and control engineering and, indeed, the control-engineering community was the first to address itself to this set of ideas.

Given the pace of developments and the nature of underlying problems and challenges, we believe that the two communities, namely computational mathematicians with an interest in geometric integration techniques and control theorists focusing on issues of optimal geometric control, are now 'in phase'. This is the ideal time to bring them together for the first time and to generate a common agenda and synergy.

- The "Lie group methods" community is eager to learn the class of control problems that exist and the typical difficulties that are likely to arise. Insofar as we can penetrate the barriers of different terminology and mathematical traditions, we understand that many important control systems can be described as evolving in nonlinear configuration spaces, rather than in an Euclidean space, and that this restriction is essential in describing underlying geometric constraints. Moreover, many of such configuration spaces share homogeneous-manifold and Hamiltonian structure: precisely the setting of much of geometric numerical integration.
- The "control theory" community would like to know what the very latest Lie-group methods and Lie–Poisson solvers can do: how accurate and computationally efficient are they? What are potential sources of error or divergence from any inherent geometric constraints? Can we use them for the implementation of real-time feedback control? Will they allow us to streamline and optimise real-time calculations to optimise and fine-tune feedback stabilisation for complex and high-dimensional systems?

The primary intention during the four days of the workshop is, at the first instance, to learn each others' language and terminology and, once we can address issues together, to understand what each community is doing. This is a tall order and we expect the learning curve to be fairly steep in the first two days, when we each try to master the basic agenda of the "other side". Yet, given the enthusiasm in both communities, we have every reason to believe that everybody will raise to this challenge. With this in mind, we have invited the leading international experts in mathematical geometric control theory and geometric numerical integration as well as specialists in specific applications.

Insofar as the structure of the workshop is concerned, we envisage five–six substantive talks each day, with plenty of time for informal discussion and interaction. Given the nature of the workshop, there will be neither short talks nor poster sessions and we do not expect all participants to present talks.

4. Background

4.1. Lie-group methods. Lie group methods are discretization algorithms for solving differential equations whose solution is known to evolve on a Lie group or, with greater generality, which can be described in terms of group actions. The purpose of such methods is to try to preserve numerically the inherent Lie group (geometric) structure, preferably to machine accuracy. The last ten years have seen an explosion in the interest and study of these "geometric integration" techniques—see Crouch & Grossman (1993), Munthe-Kaas (1998), Iserles, Munthe-Kaas, Nørsett & Zanna (2000). Crouch and Grossman pioneered numerical schemes based on Runge–Kutta methods, acting on a rigid frame, for solving differential equations on general manifolds. They were motivated by problems from robotics and control theory. Munthe-Kaas applied Runge–Kutta schemes to the corresponding system of differential equations that evolve in the underlying Lie algebra and in consequence developed what are now called Runge–Kutta–Munthe-Kaas schemes. They preserve Lie group structure in systems of nonlinear differential equations, evolving them in the underlying Lie algebra. At the same time over the last ten years, S. Blanes, F. Casas, A. Iserles, H.Z. Munthe-Kaas, S.P. Nørsett, A. Zanna and others have developed in a systematic manner Lie-group integrators based on the Magnus, Fer and Cayley expansions and addressed a wide spectrum of practical questions pertaining to the implementation of such techniques, in particular issues of effective multivariate quadrature.

Much of this work is highly interdisciplinary, blending techniques from differential geometry, numerical analysis and graph theory. It also demonstrates how synergy of numerical analysts and control engineers leads to substantive progress: Rigid-frame methods were developed by Peter Crouch and Robert Grossman, who are theoretical control engineers, but their original construction was cumbersome and restricted to low orders. This has been perfected by Arne Martinsen and Brynjulf Owren, both numerical analysts, who have extended an approach, based on graph theory, that was originally introduced by John Butcher in the context of Runge–Kutta methods. Interestingly, this technique was also recently used by Christian Brouder, Alain Connes and Dirk Kreimer in mathematical physics, in elucidating the Hopf-algebraic structure of the renormalisation group (Connes & Kreimer, 1998).

Most Lie-group methods and, indeed, many other geometric integrators, evolve the underlying system in a Lie algebra and this typically requires the evaluation of a matrix exponential so that the answer lies in the Lie group. Most standard methods fail the latter requirement and this has motivated the development of new computational algorithms—see Iserles & Zanna (2002). This underlies another area of joint interface with geometric control theory, namely a range of problems in the linear numerical algebra of structured matrices.

4.2. Nonlinear Control theory. Nonlinear control has its foundations in Riemannian and sub-Riemannian geometry; viz. the integrability conditions for distributions on manifolds developed by Chow, Frobenius and Nagano. The utility of such differential geometric methods in control was first recognized in the early 70s by, for example, R. Hermann, R.W. Brockett, A. Krener, H. Sussmann and V. Jurdjevic. They derived Lie algebraic conditions for local controllability/observability of nonlinear systems. Moreover, a widely applicable and natural generalisation of the Frobenius theorem was found—the orbit theorem of Stefan and Sussmann. In the 1980s, a systematic attempt by Isidori, Krener, Nijmeijer and others helped to create a nonlinear control theory that is similar in scope to the well established and rich theory of linear systems. These early contributions quickly led to an explosion of research on nonlinear systems that continues to this day. While most of the current theory has been local, there is now also a rapidly growing part of the theory devoted to global aspects; e.g. controllability/observability of systems on Lie groups and homogeneous manifolds. The motivation for such research stems from application areas, including mechanics, vision and numerical analysis.

The investigation of controllability/observability of right-invariant systems on Lie groups and homogeneous manifolds can be traced back to the early work by Brockett, Elliot, Boothby, Jurdjevic & Sussmann and Kupka in the 1970s. The theory applies well to systems without drift terms—in the presence of drift terms the theory becomes considerably more complicated and is only currently partially understand. A difficult example of a control system arises in nuclear magnetic resonance, and quantum control, with interesting time-optimal control problems on the unitary group to be solved for the associated Schrödinger equation of a finite spin system.

Similarly, control problems on Lie groups arise for example in mechanics (control of the rigid body dynamics; nonholonomic systems), computer vision (estimation of pose and camera parameters in stereo matching) and robotics (inverse kinematic and grasping problems). In a different direction, completely integrable mechanical systems are often best analysed via associated isospectral flows. An important example here is the Toda lattice. The analysis of such isospectral flows on Lie algebras, also in connection with holonomic and nonholonomic systems, has attracted several researchers in control theory lately; we mention in particular Bloch, Brockett, Krishnaprasad and Marsden. It has also lead to interesting connections to numerical analysis (eigenvalue algorithms; e.g. QR or Jacobi algorithms) and control theory (isodynamical flows, computation of balanced realisations, sensitivity optimisation); see for example Helmke & Moore (1994).

For many control tracking problems arising in applications the dynamics has to be integrated on the constraint manifold, to achieve efficient numerical schemes that can be combined online with the control algorithms. Thus the successful combination of numerical integration methods and control on a manifold requires a careful combination of the two fields and presents a considerable challenge for the future. It appears thus to be a timely task to have for the first time a conference that attempts to bring these different communities together.

5. The workshop

The meeting will take place from June 28th to July 1st 2004 at the International Centre for Mathematical Sciences (ICMS) in Edinburgh. The ICMS have a great deal of experience organising high quality scientific meetings. All the scientific organisers have experience in organising meetings. We expect 35$\pm$5 participants to come for the whole meeting.

We plan to use the ICMS headquarters at 14 India Street to hold the meeting and the venue for lectures. Participants can be accommodated in the nearby Pollock Halls of Residence (of the University of Edinburgh) which have all the usual facilities including a range of styles of accommodation and spaces for informal discussions to take place. The city of Edinburgh is very well connected for UK and international travel.

Relevance to beneficiaries. This meeting brings together in a novel fashion, two communities which mutually recognise the benefit of their interaction. There is a broad array of important applications covered by the scope of this workshop—robotics, mechanical control, computer vision and optical motion capture. The combination of scientists and engineers envisaged at the workshop, who will work together to solve mutual problems of interest with such applications, will benefit science, technology and industry in the UK and elsewhere.

Timeliness. The pace of recent scientific progress has slightly stretched the two communities, and at this time it is easy to identify a case for and a desire within both scientific/academic communities to rediscover their common interests and heritage.

Dissemination. This meeting provides the opportunity for researchers and practitioners in both fields to share the latest ideas, methods and techniques. It is hoped that previously intractable problems for one community might become accessible with the knowledge and experience of the other. Just as importantly, new and interesting problems will certainly be raised, discussed and taken away for the participants to work on and solve in the future.

As is usual, we envisage the inclusion of standard *workshop webpage* with lists of participants, talks, abstracts and pointers to existing reviews and resources—as well as sponsorship logos such as for EOARD (US Air Force; see below).

We intend to make postscript or pdf versions of the presented talks available online. In fact we shall endeavour to try to convert electronic talks and scan overheads into downloadable formats to make them also available online.

6. Participants and budget

Key participants. We have listed below the leading international researchers whom we believe are key participants for the workshop. The organisers have approached all the key international participants and they have *all* agreed to come and give talks. They bring the top world–wide expertise to the UK in both mathematical geometric control theory and geometric numerical integration. (Some other experts we invited such as Roger Brockett, Peter Crouch and Chris Doran said they regretfully could not attend for personal/timing reasons).

- **Sergio Blanes** (Valencia): Lie group methods, symplectic and splitting methods, mathematical physics;
- **Tony Bloch** (Michigan): geometry, dynamical systems, control theory;
- **Moody Chu** (North Carolina): numerical methods, inverse problems and control;
- **Nicoletta Del Buono** (Bari): numerical methods, geometric integration, control theory;
- **Paul Fuhrmann** (Ben-Gurion University): control theory, algebraic system theory, behaviours, observer theory, functional models;
- **Knut Hueper** (National ICT Australia Ltd): control theory, optimal dextrous hand grasping, computer vision;
- **P.S. Krishnaprasad** (Maryland): geometric control theory, geometric mechanics, Lie groups and distributed parameter systems;
- **Clyde Martin** (Texas Tech): control theory, public health, environmental issues, economics;
- **Hans Munthe-Kaas** (Bergen): geometric integration, coordinate-free numerical methods, parallel computing;
- **Marcel Oliver** (Intl. Univ. Bremen): shallow-water theory, geometric and geophysical fluid mechanics, numerical analysis and Lie-group methods.
- **Brynjulf Owren** (Trondheim): numerical solution of differential equations, geometric integration;
- **Tudor Ratiu** (Lausanne): geometry, dynamical systems, fluid dynamics;
- **Antonella Zanna** (Bergen): isospectral flows, Lie-group methods, symplectic systems, Cayley, Magnus and Fer expansions.

Key UK participants.

- **Chris Budd** (Bath): nonlinear PDEs and Lie groups, dynamical systems, electrostatics, optics, industrial problems;
- **Joan Lasenby** (Cambridge): applications of geometric algebra in engineering, computer vision and optical motion capture, modelling and tracking of articulated human motion, optimization in signal processing, image processing;
- **Harmut Logemann** (Bath): systems and control theory;

- **Liz Mansfield** (Kent): variational structure and symmetry preserving numerical schemes, computer algebra techniques for solution of differential and difference equations;
- **E.P. Ryan** (Bath): systems and control theory, nonsmooth analysis; differential equations/inclusions, adaptive control;
- **Stuart Townley** (Exeter): control theory, integral control, tracking, stabilization, neural networks;
- **Claudia Wulff** (Surrey): dynamical systems with symmetry, Hamiltonian systems, nonlinear PDEs and pattern formation, numerics of dynamical systems.

Further UK participants. We are enthusiastic to invite any interested individuals whom we have not identified. These may be control engineers working in academic engineering departments or in industry who have a particular interest in geometric control and the specific topic of the meeting—for example from such groups in Southampton and the Control Theory and Applications Centre in Coventry (CTAC). Indeed the ICMS has a fund from the London Mathematical Society, some of which will be released to support up to five further UK participants. We intend to advertise the workshop via the internet and word-of-email.

Young and developing UK participants. Another important aspect of this workshop is to try to encourage as many young and/or developing UK researchers to attend the workshop as possible. We would like to reserve places for up to 10 further PhD students or postdoctoral researchers (or researchers starting to develop in this area) who show a relevant interest in attending. Places at the workshop could be allocated on a competitive basis; though this will depend on the number of expressions of interest.

Costs. Our calculations are based on four days subsistence for UK participants and six days for others (we hope that international travellers will arrive on the preceding Saturday June 26th to take advantage of cheap air fares). The ICMS estimate an average subsistence of around £60 per day based on en-suite accommodation in student accommodation or two-star B&B in Edinburgh, all meals provided. The average costs per participant and the total costs of the meeting are summarized in the following tables.

Date	**B&B**	**Day**	**Evening**	**Subsistence**
Sat 26th	38	0	15	
Sun 27th	38	7	15	
Mon 28th	38	7	15	
Tues 29th	38	7	15	
Weds 30th	38	7	25	
Thurs 1st	38	7	15	
Total cost	228	35	100	**363**
Without Sat night				**300**

TABLE 1. Average subsistence costs per participant.

Country	**Travel**	**Subsistence**	**Sub-total**	**Number**	**Total**
UK	110	300	410	8	3280
Europe	260	360	620	10	6200
E. USA	400	360	760	2	1520
C. USA	600	360	960	2	1920
Australia	750	360	1110	1	1110
					14,030
Local Org.	0	100	100	1	100
Other UK	110	300	410	10	4100
					4200
Total direct costs					**18,230**
ICIAM '99					5000
LMS fund					1000
EOARD (US Air Force): $5000					2800
Total requested from EPSRC					**9,430**

TABLE 2. Total costs for the meeting.

The total costs for the key participants is thus **£14,030**. Including up to ten young UK participants, the grand total direct costs are **£18,230**. The total requested from the EPSRC is **£5,000**.

Secured funding. We have *already secured* some funds for the meeting:

- ICMS has a fund from the London Mathematical Society which helps UK-based mathematicians to attend ICMS meetings. It is their intention to release up to £1,000 from the fund to support the local costs of approximately five further UK mathematicians who wish to attend this meeting.
- ICIAM '99 Fund committee have agreed to release £5,000 towards the expenses of the meeting.
- European Office of Aerospace Research and Development (EOARD)—which is a branch of the Air Force Office of Scientific Research (AFSOR) of the US Air Force—have already paid ICMS \$5,000 $\approx$ £2,800 in support of the meeting from their Conference Support Program (CSP).

Hence the total requested from the EPSRC is £**5,000**. We expect that some of the key participants do have some of their own funds for travel from grants of their own and we hope that the *shortfall* of £**4,430** will be absorbed in this way. However the majority of the key participants have indicated that they cannot attend without travel and subsistence funding.

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