TASEP in presence of a localised dynamical constraint

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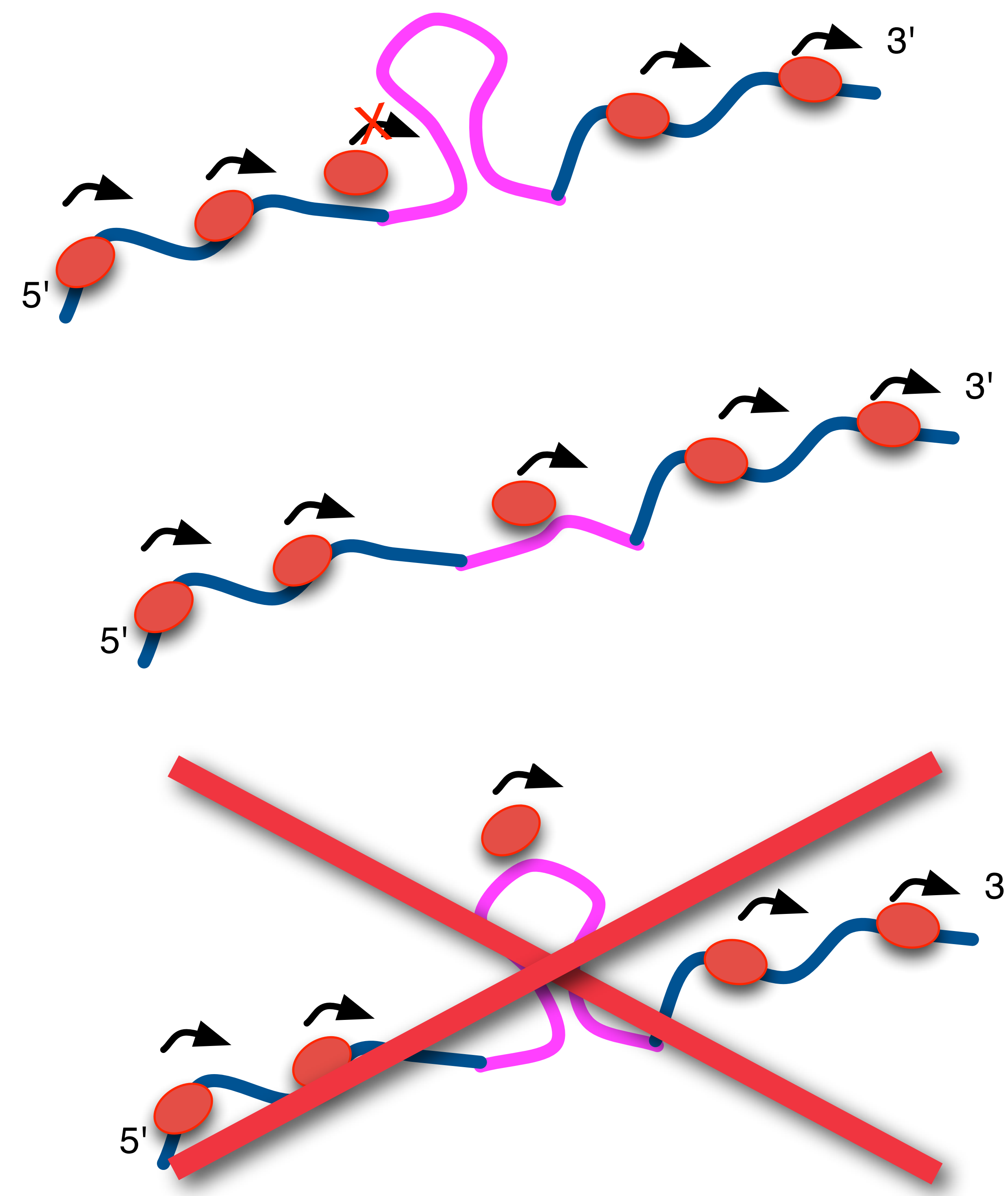
Outline

- Biological motivation
- The model
- Mean Field analysis and results

TASEP and Tranlsation

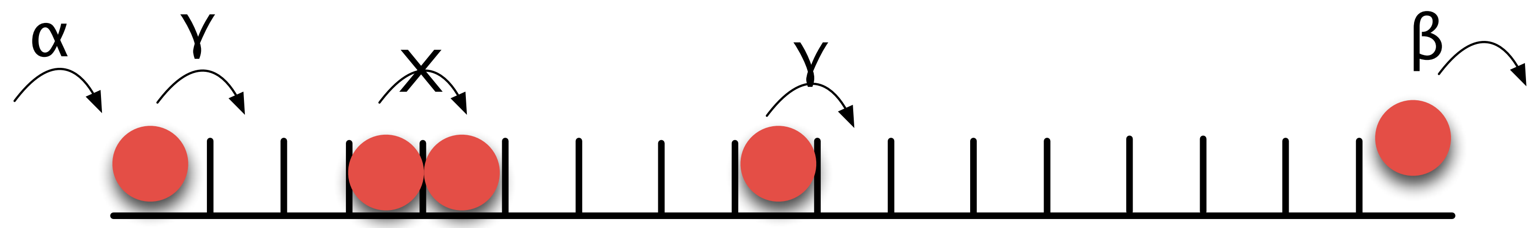
- **TASEP** has been originally proposed for mRNA translation.[1]
- **mRNA is complex : secondary structures exists** regulating the translation.
- Such regions can be active or not, depending on their **folding status**.
    - **folded** -> **inhibited** translation.
    - **unfolded** -> translation is **allowed**.
    - if a **ribosome** is **initiating** the translation, the **secondary strucrure** stays **unfolded**.

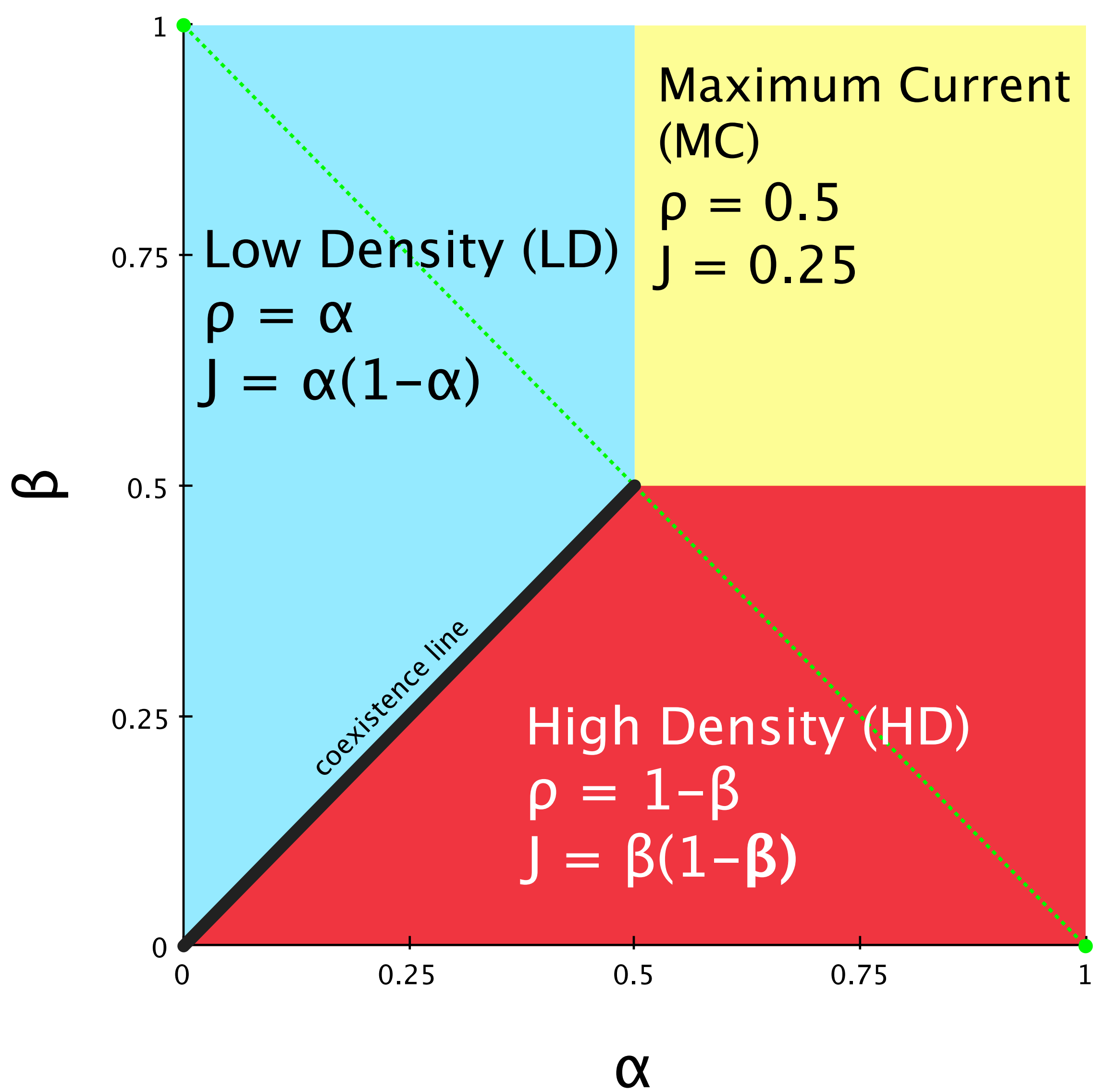


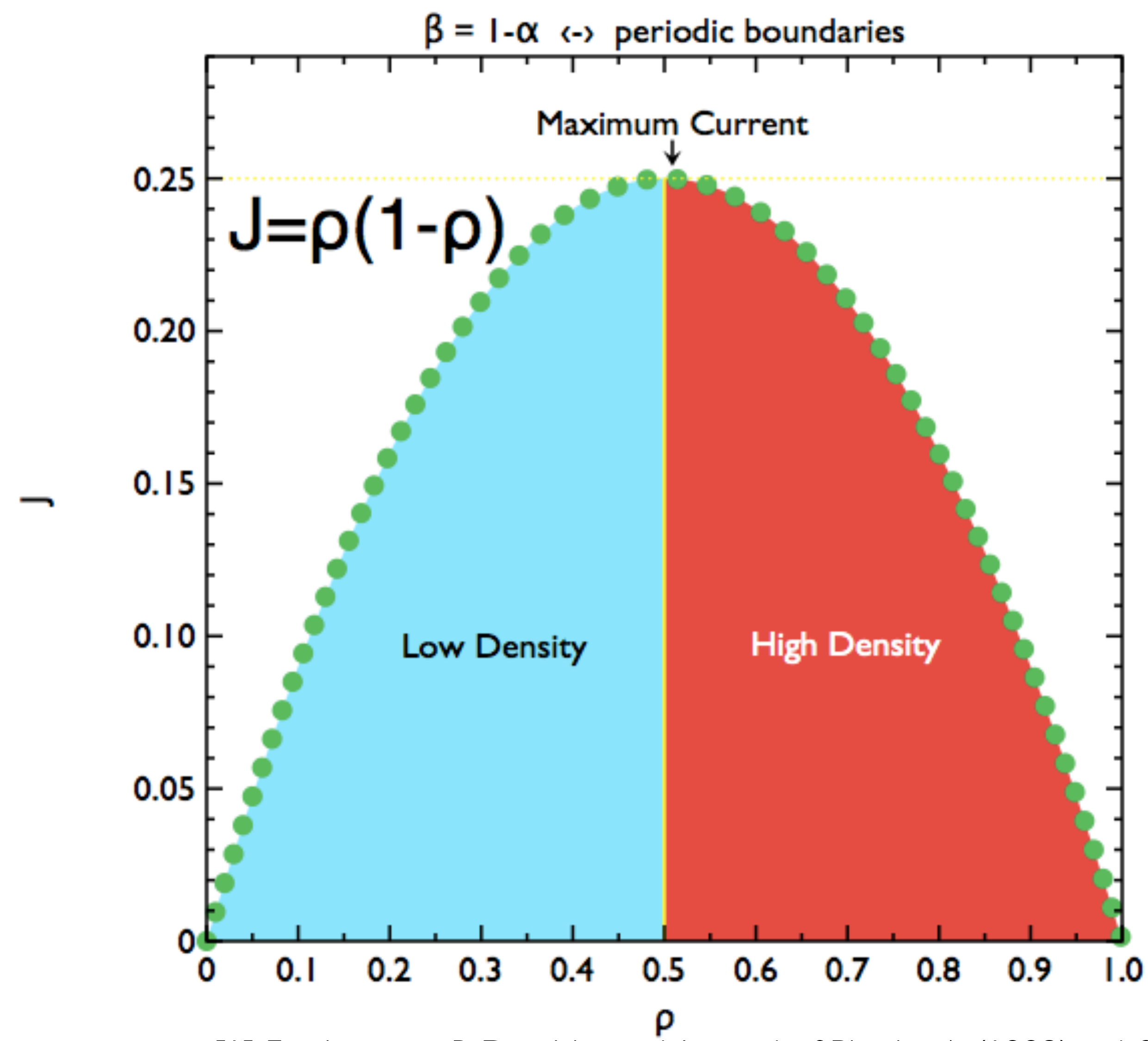
[1] MacDonald CT, Gibbs JH, Pipkin AC. Biopolymers. 1968;6:1–25.

Totally Asymmetric Simple Exclusion Process

- If the folding/unfolding process is not present, we can model the process as a TASEP:





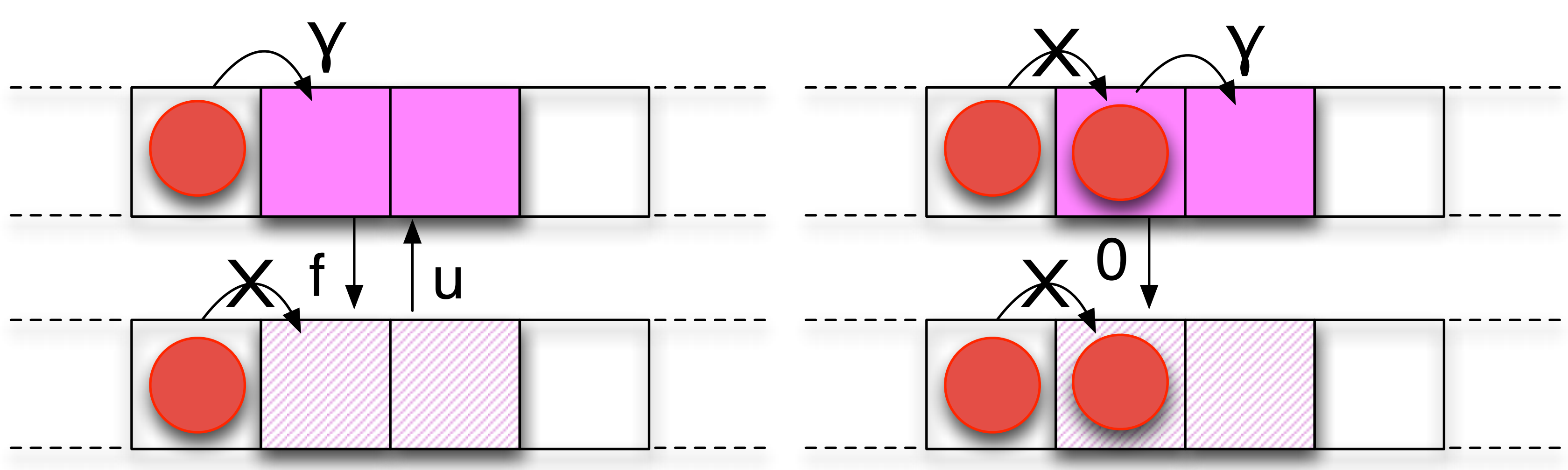


[1] For instance B Derrida et al. Journal of Physics A (1993) vol. 26 pp. 1493
[2] Chou et al. Reports on Progress in Physics (2011) vol. 74 pp. 116601

- exact results [1]
- paradigm for non equilibrium physics and biological processes [2]

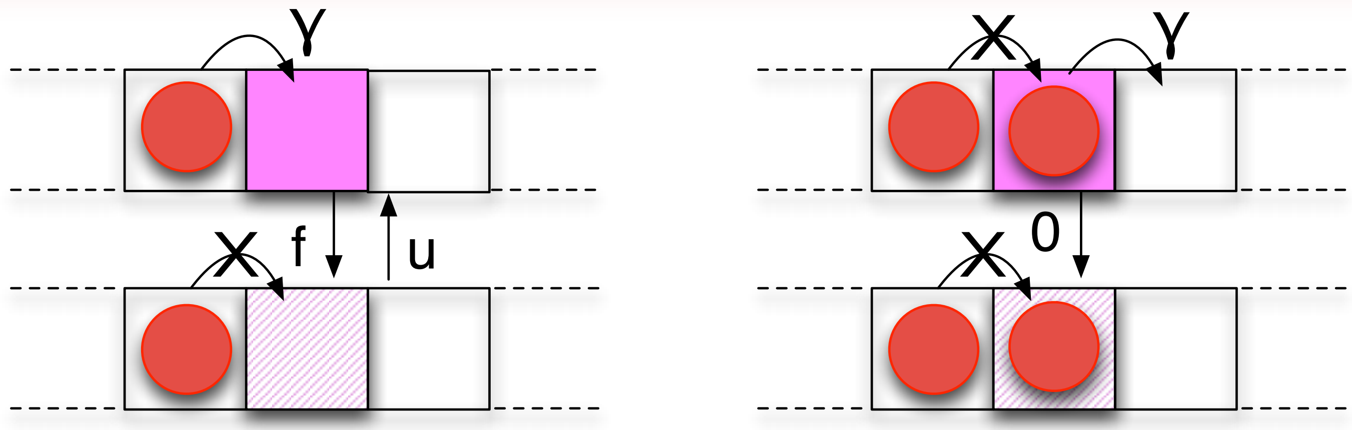
Introducing a *folding region*

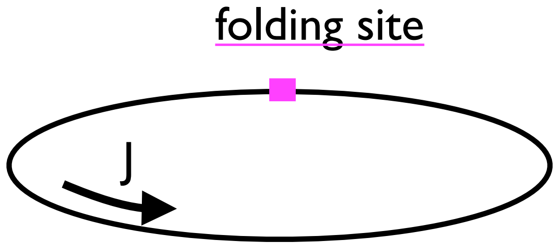
- Localisation of an extended defect
  - internal dynamics
  - coupling with particles presence.
- Effects on the dynamics
  - periodic boundaries
  - open boundaries



Periodic Boundaries

1 site long folding region

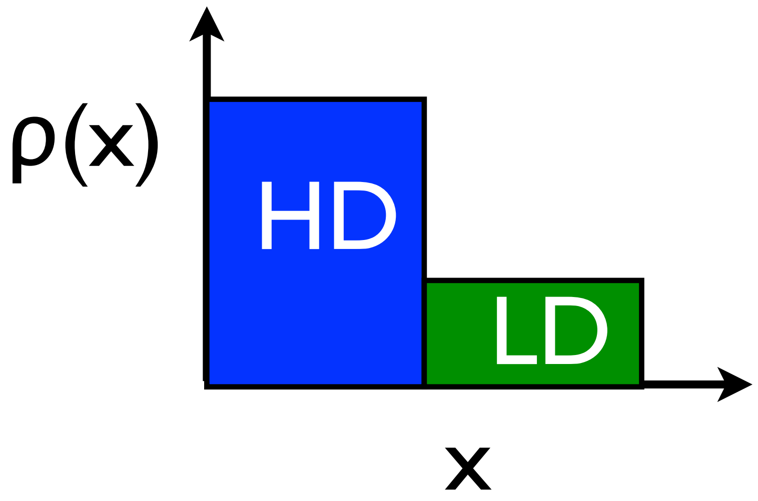




- Ring of **L sites**;
- **N particles** fixing the density **ρ=N/L**
- **1 site** with internal dynamics
- Simulations performed using the **Gillespie Algorithm.**

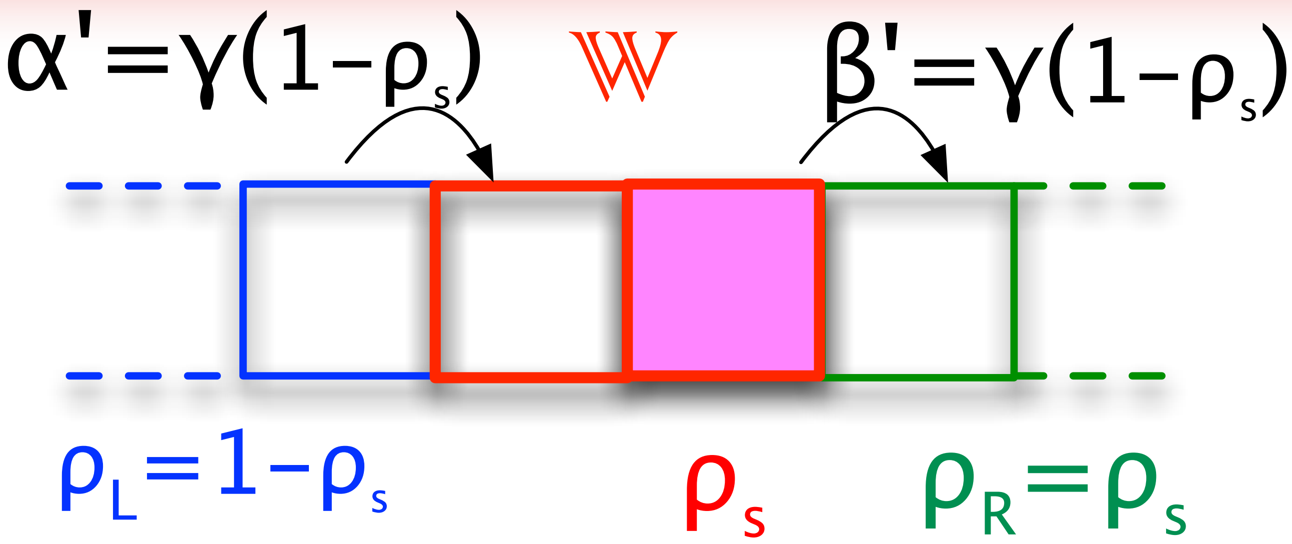
Finite-Segment Mean Field approach

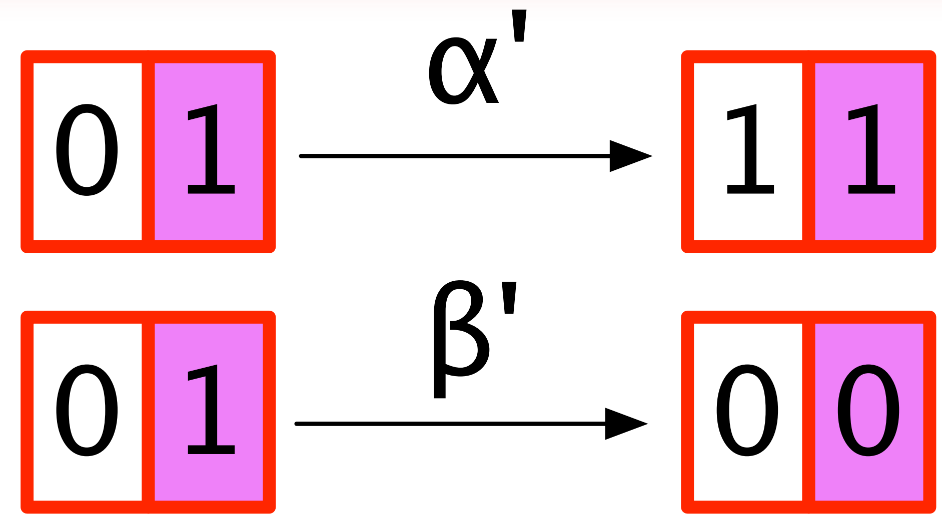
- The folding region is a defect:
  - slows down the dynamics
  - the system is split in two phases
    - High density ρ_{HD} before the defect
    - Low density ρ_{LD} after
  - by spatial continuity of the current through the boundary $\rho_{HD}(1-\rho_{HD}) = \rho_{LD}(1-\rho_{LD})$
    - if we have a splitting -> $\rho_{H} = 1-\rho_{LD}$



similar to Janowsky et Lebowitz Physical Review A (1992) vol. 45 (2) pp. 618

Finite-Segment Mean Field approach





- The folding site **s** has density ρ_s and makes incoming particles wait before it.
- the system is analysed in three parts , conserving the current spatial continuity:
  - high density **$1\text{-}\rho_s$** (LEFT)
  - the **folding region + one site before** it (MIDDLE) : decoupling the folding process and the injection of particles.
  - low density ρ_s (RIGHT)
- The MIDDLE region is governed by an exact transition matrix $\mathbb{W}$ for the Probability **$P(\{n_{s-1},n_s\})$**. $\mathbb{W}$ depends ρ_s, f, u , γ.

*J J Dong et al 2009 J. Phys. A: Math. Theor. 42 015002

Finite-Segment Mean Field approach

- 2 sites -> 6 possible states : {empty, full}x{empty & open, full & open, closed}

$$\hat{\rho}_s = 1 - \rho_s$$

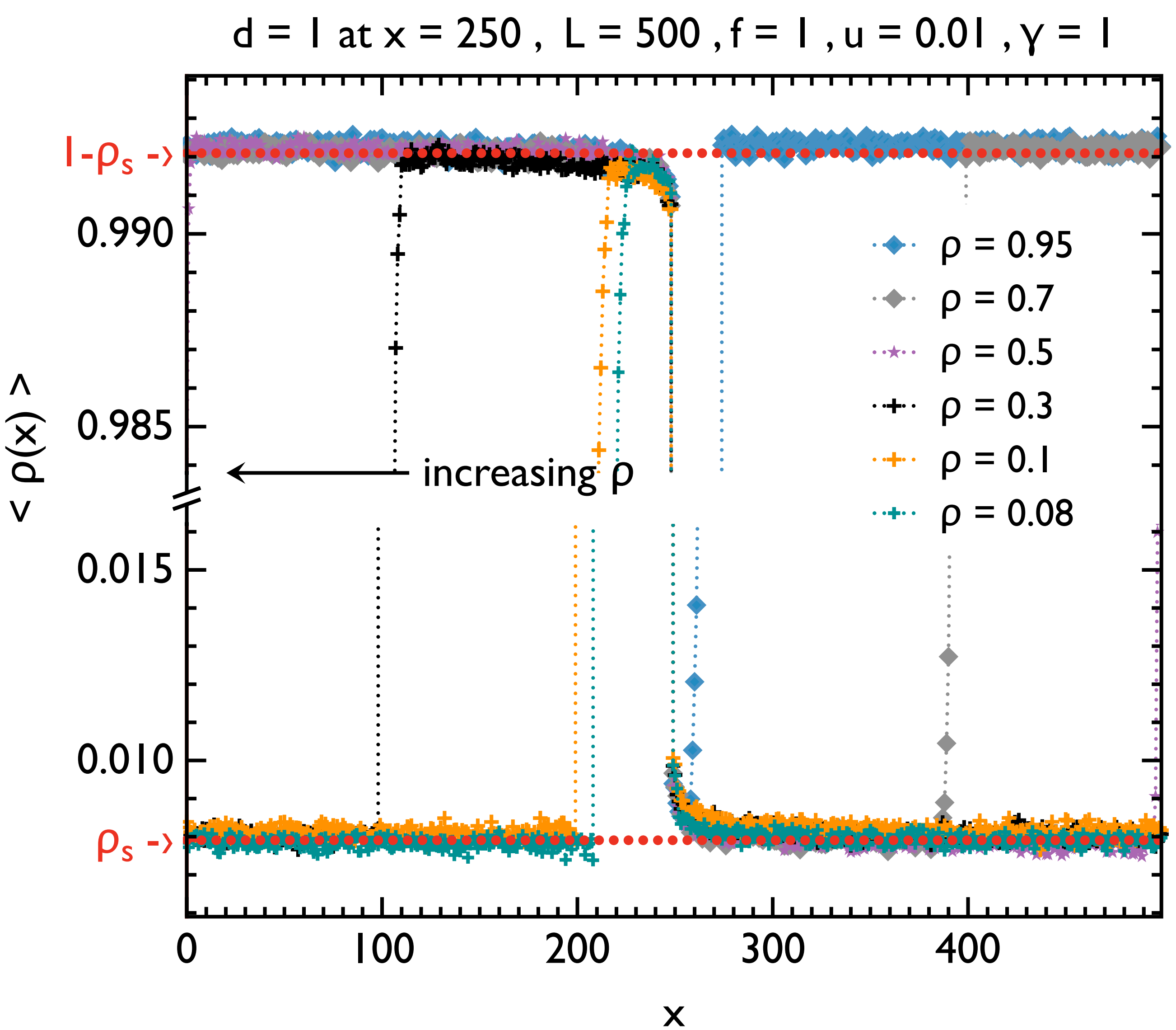
$$\mathbb{W} = \begin{pmatrix} -f-\gamma\hat{\rho}_s & u & 0 & 0 & \gamma\hat{\rho}_s & 0 \\ f & -u-\gamma\hat{\rho}_s & 0 & 0 & 0 & 0 \\ \gamma\hat{\rho}_s & 0 & -f-\gamma & u & 0 & \gamma\hat{\rho}_s \\ 0 & \gamma\hat{\rho}_s & f & -u & 0 & 0 \\ 0 & 0 & \gamma & 0 & -2\gamma\hat{\rho}_s & 0 \\ 0 & 0 & 0 & 0 & \gamma\hat{\rho}_s & -\gamma\hat{\rho}_s \end{pmatrix}$$

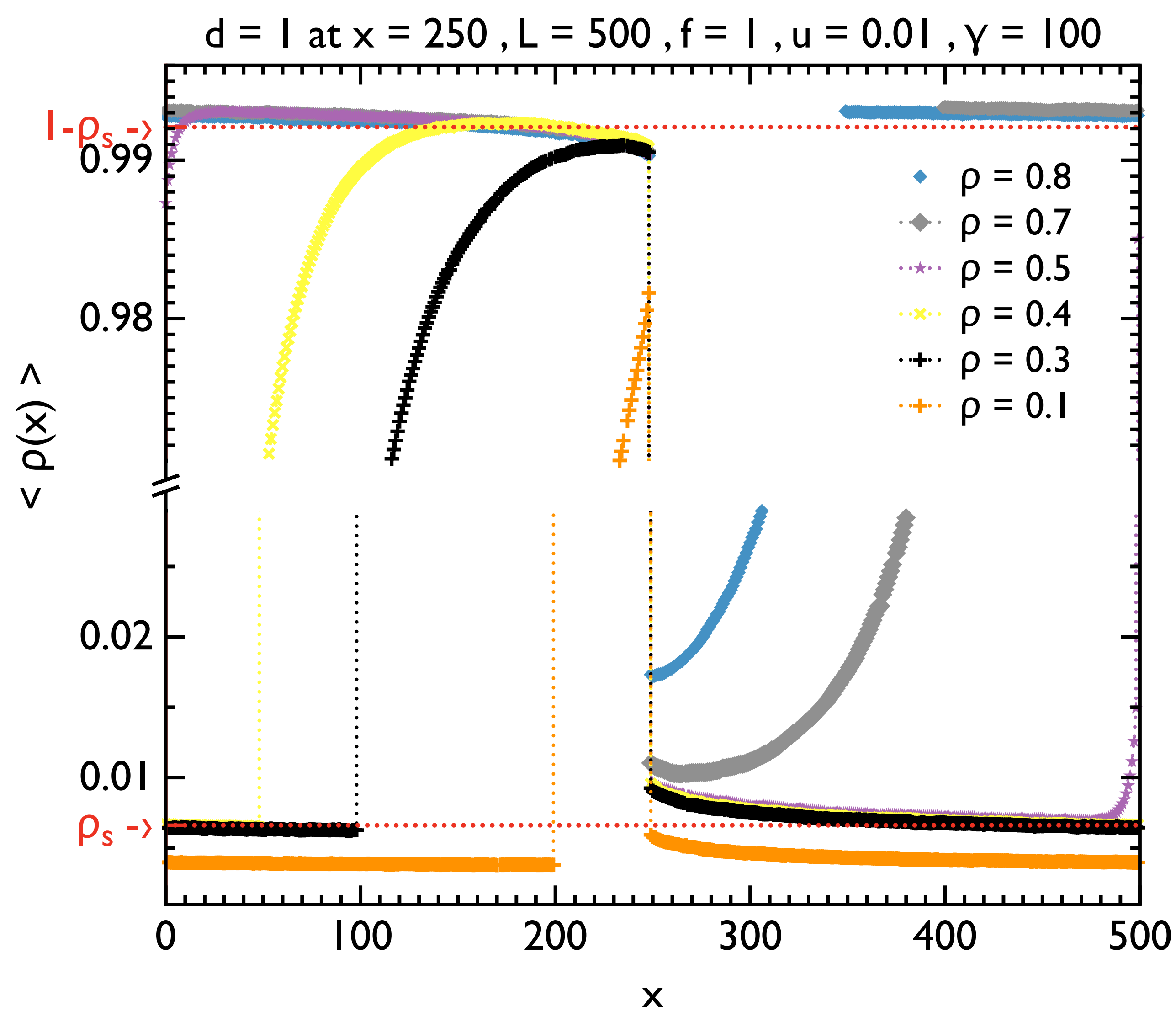
- Stationary state : $\mathbb{W}P = 0$
  - one obtains the probability $P(n_s=1)$ as a function of ρ_s,γ,u,f.
  - solve $P(n_s=1) = \rho_s$
  - get $\rho_s(\gamma,u,f)$.
  - The corresponding value for the current is $J_{max} = \rho_s(1-\rho_s)$

Periodic Boundaries

Results

Mean field and simulation comparison



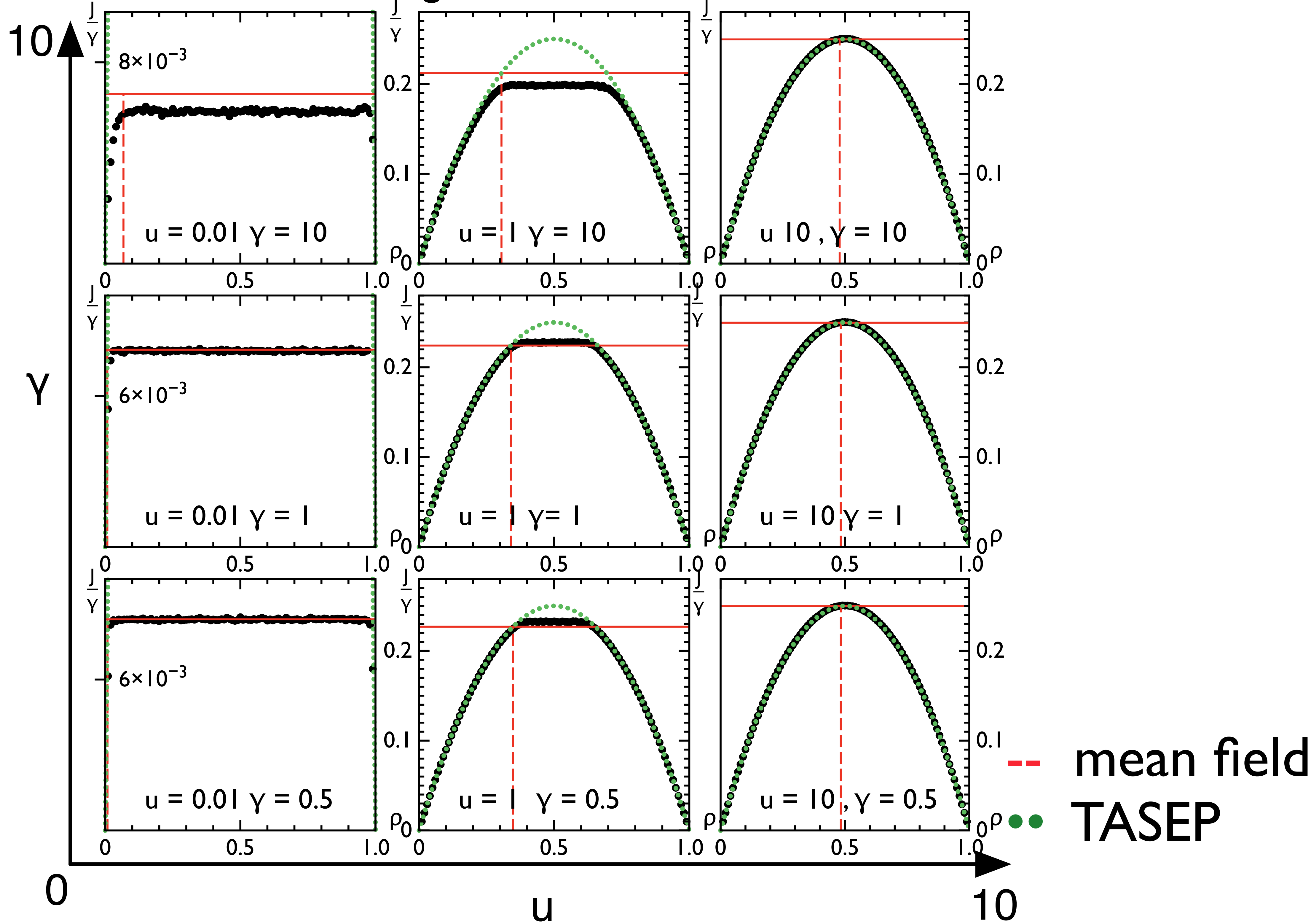


the system is split in two phases
(= MF hypothesis)

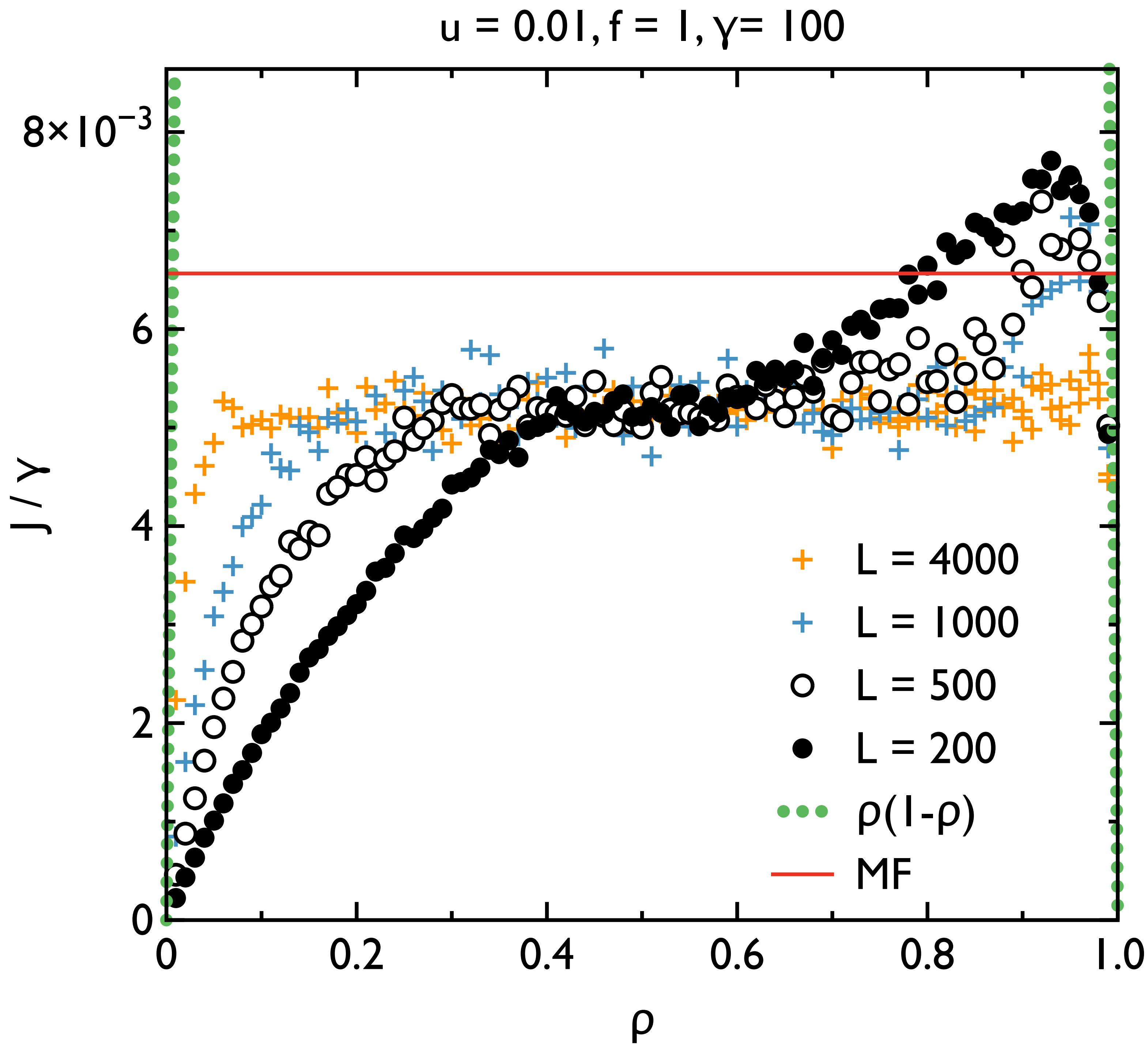
non flat profiles, non diffusive shocks dynamics
(≠ MF hypothesis)

Results

1 folding site , L = 500 , f = 1



L-dependent J(ρ) at very large γ/u

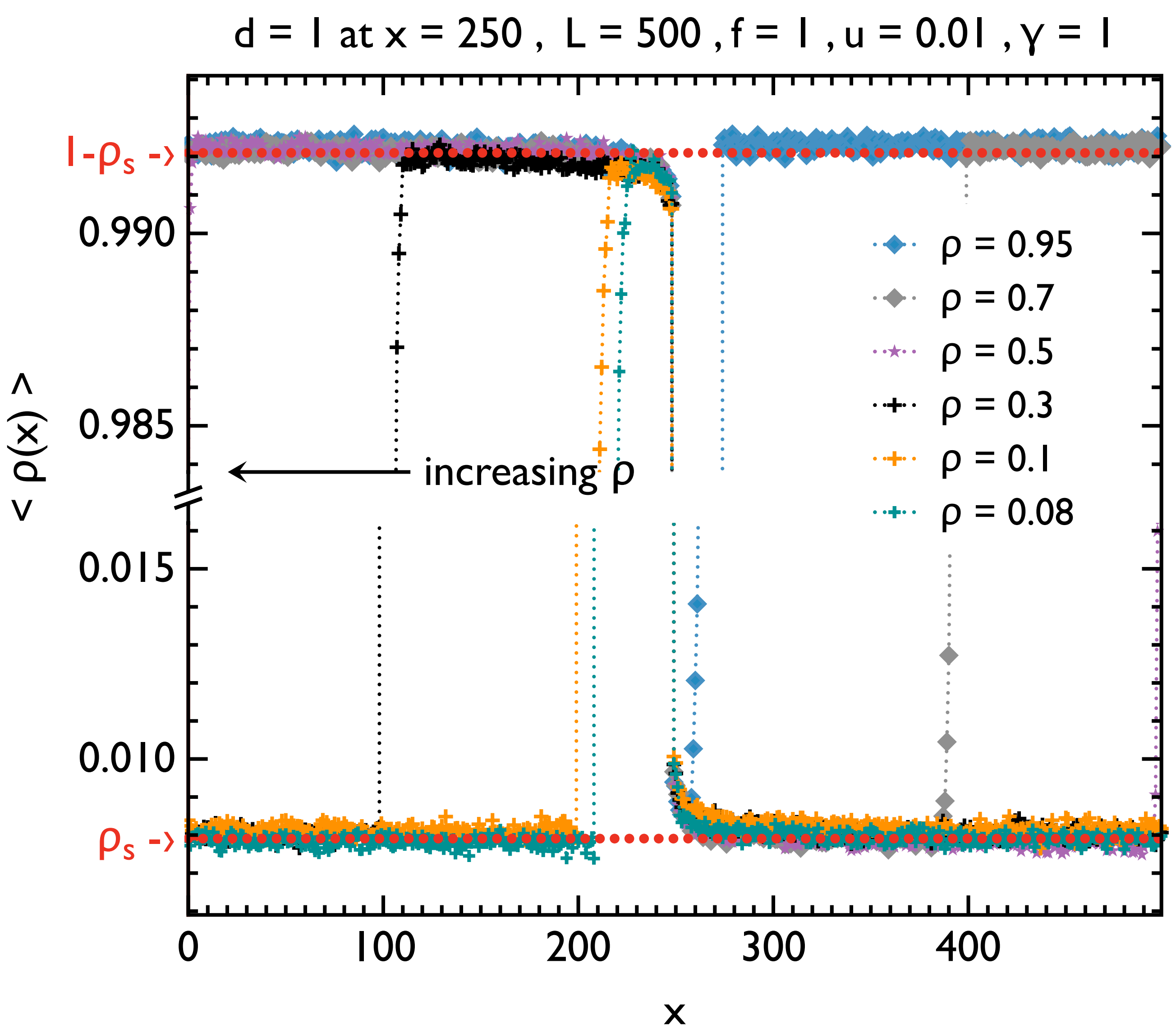


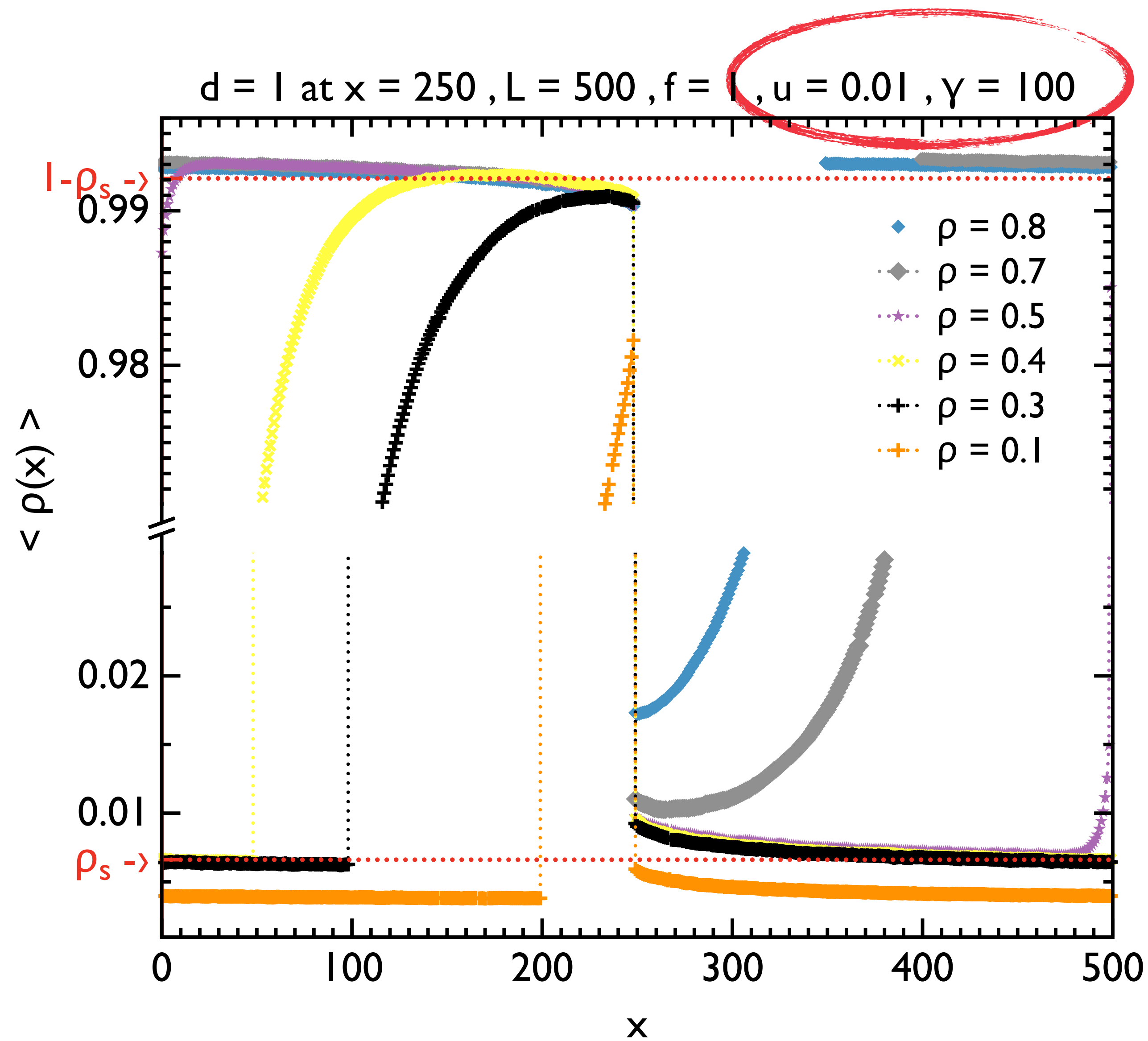
- strong correlations
  - **enhanced** particles flow at high density (>0.5)
  - **reduced** particle flow at low density (<0.5)
  - **slow** convergence to L -> ∞ plateaux

-- mean field

•• TASEP

Mean field failure for highly correlated density profiles





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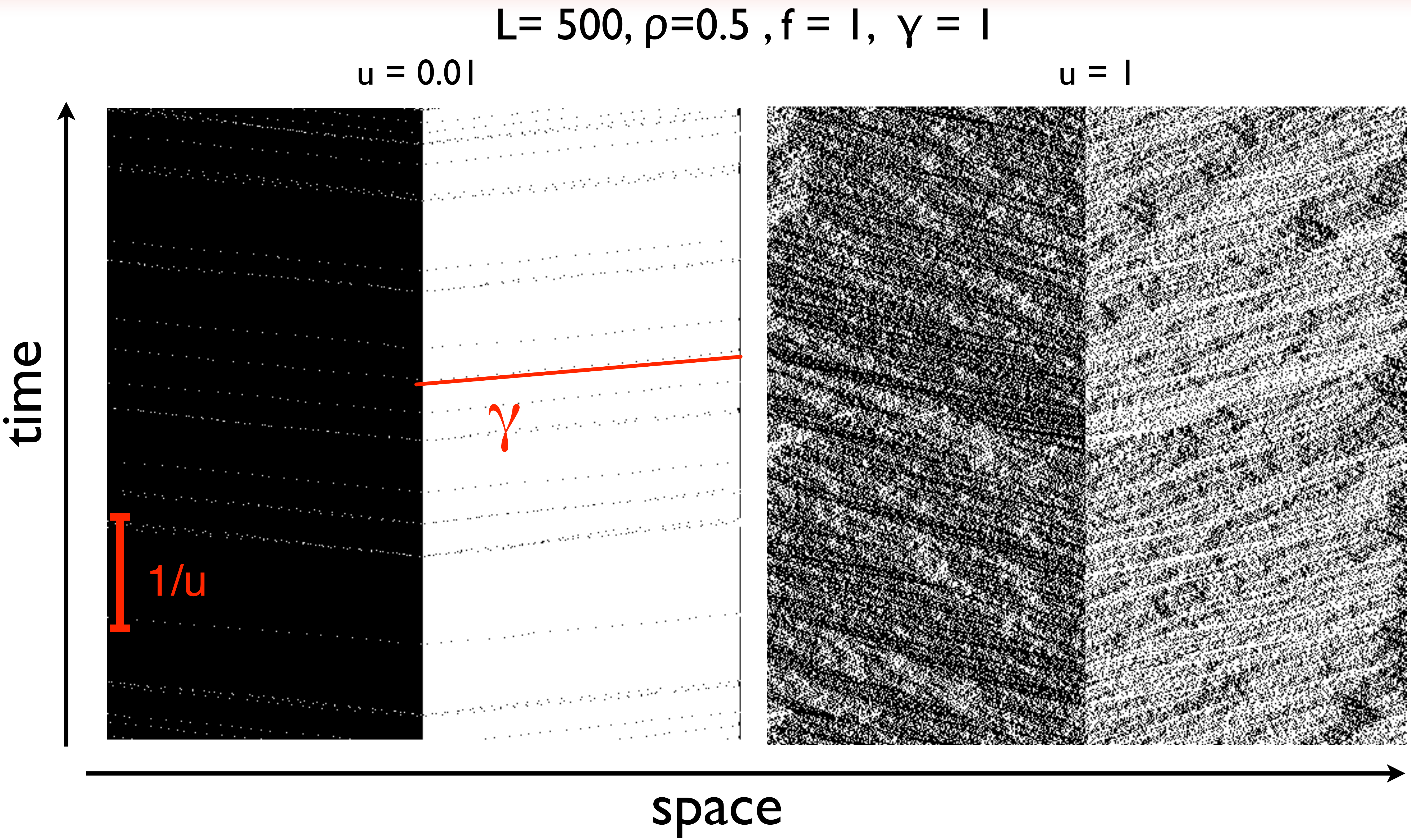
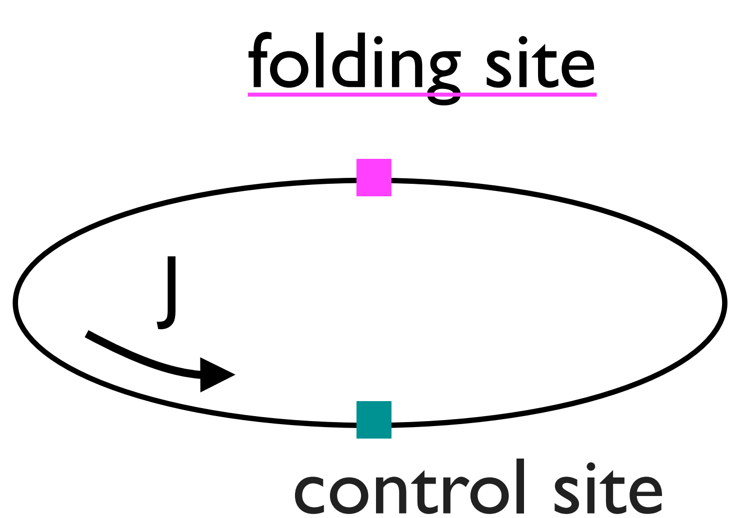
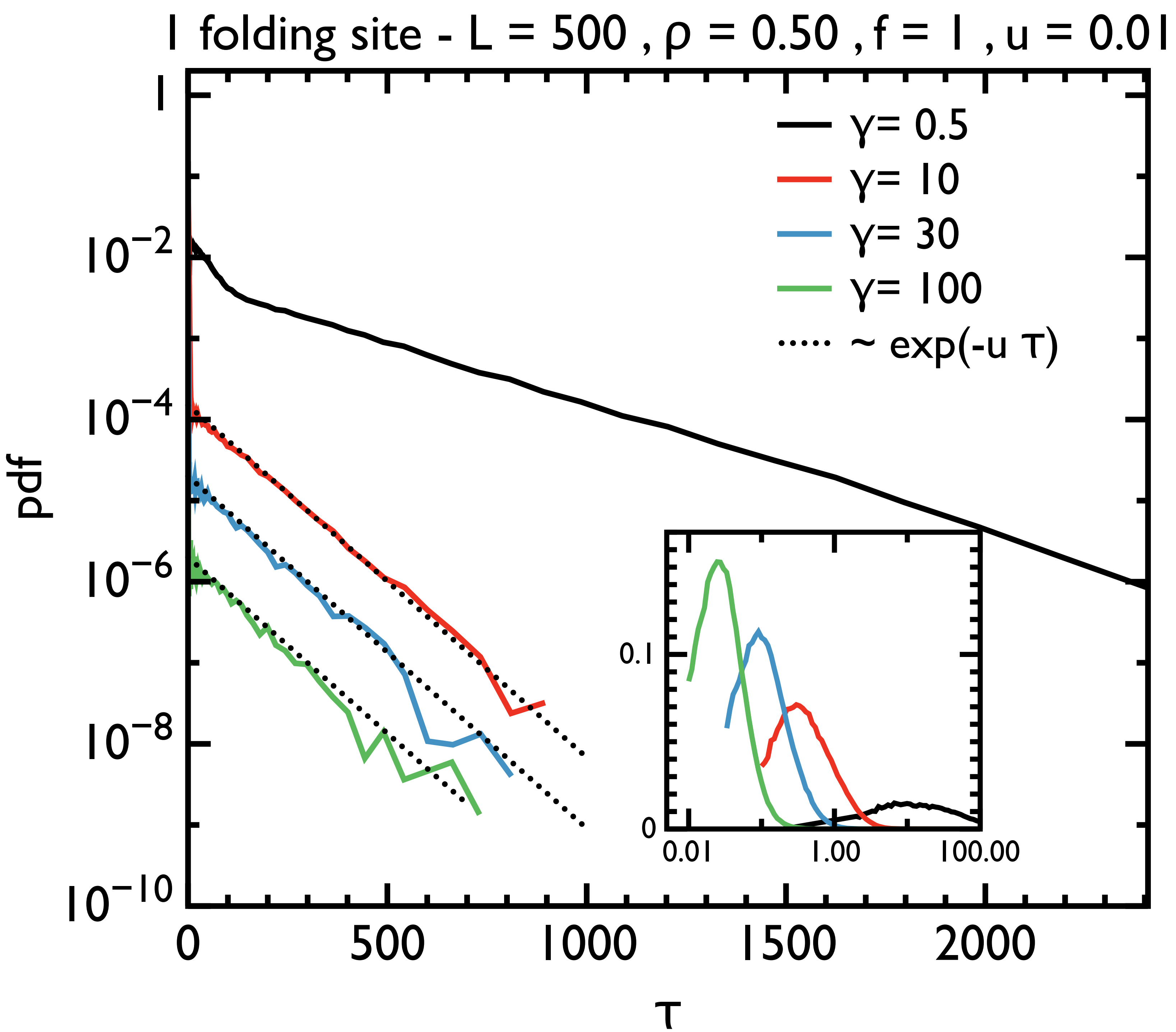
Small unfolding rates lead to intermittency



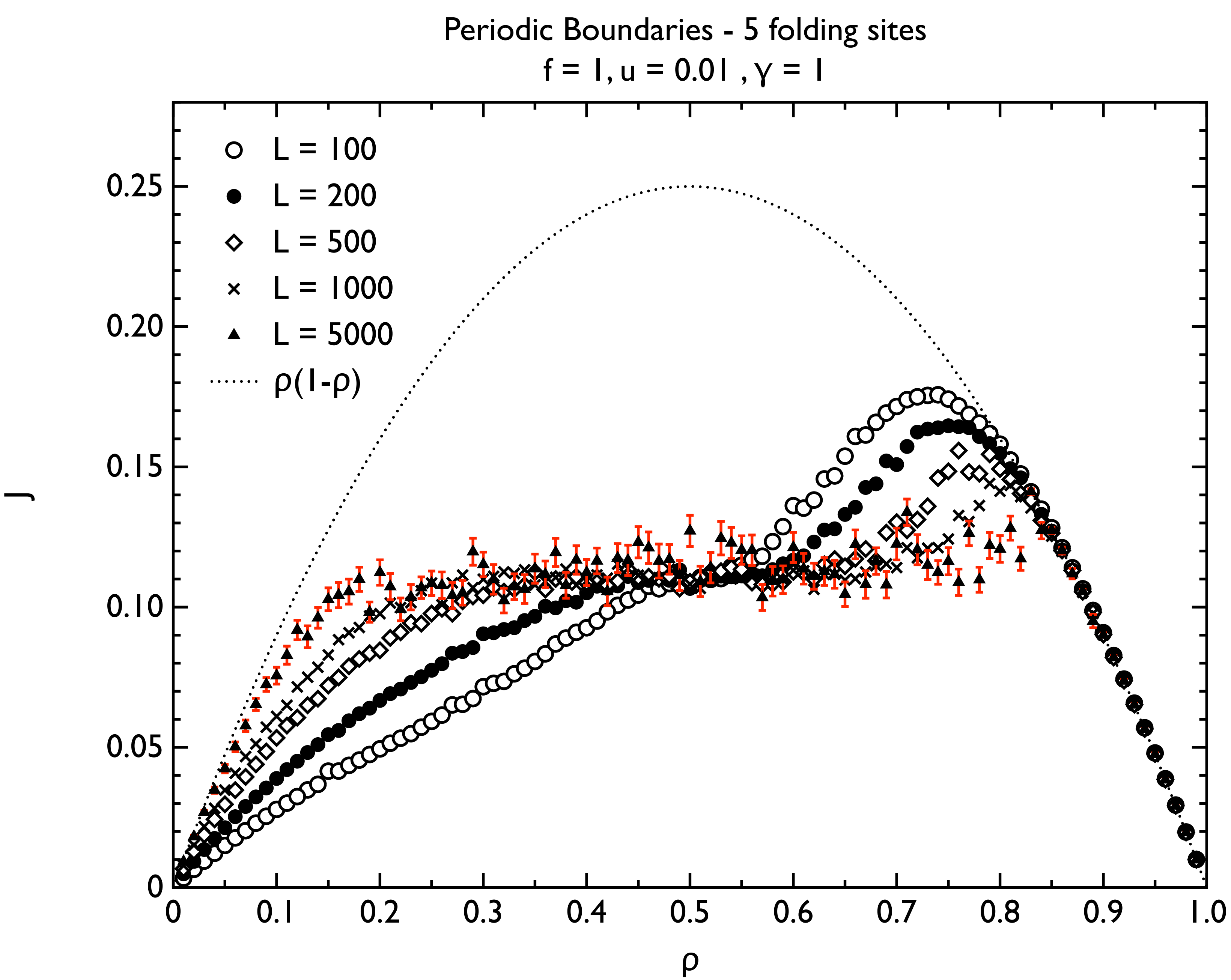
Passage times distributions

- Distr. of the time lag between two successive particles passing through a control site
  - two typical times
  - high γ/u increase the time separation





Large folding regions have stronger effects

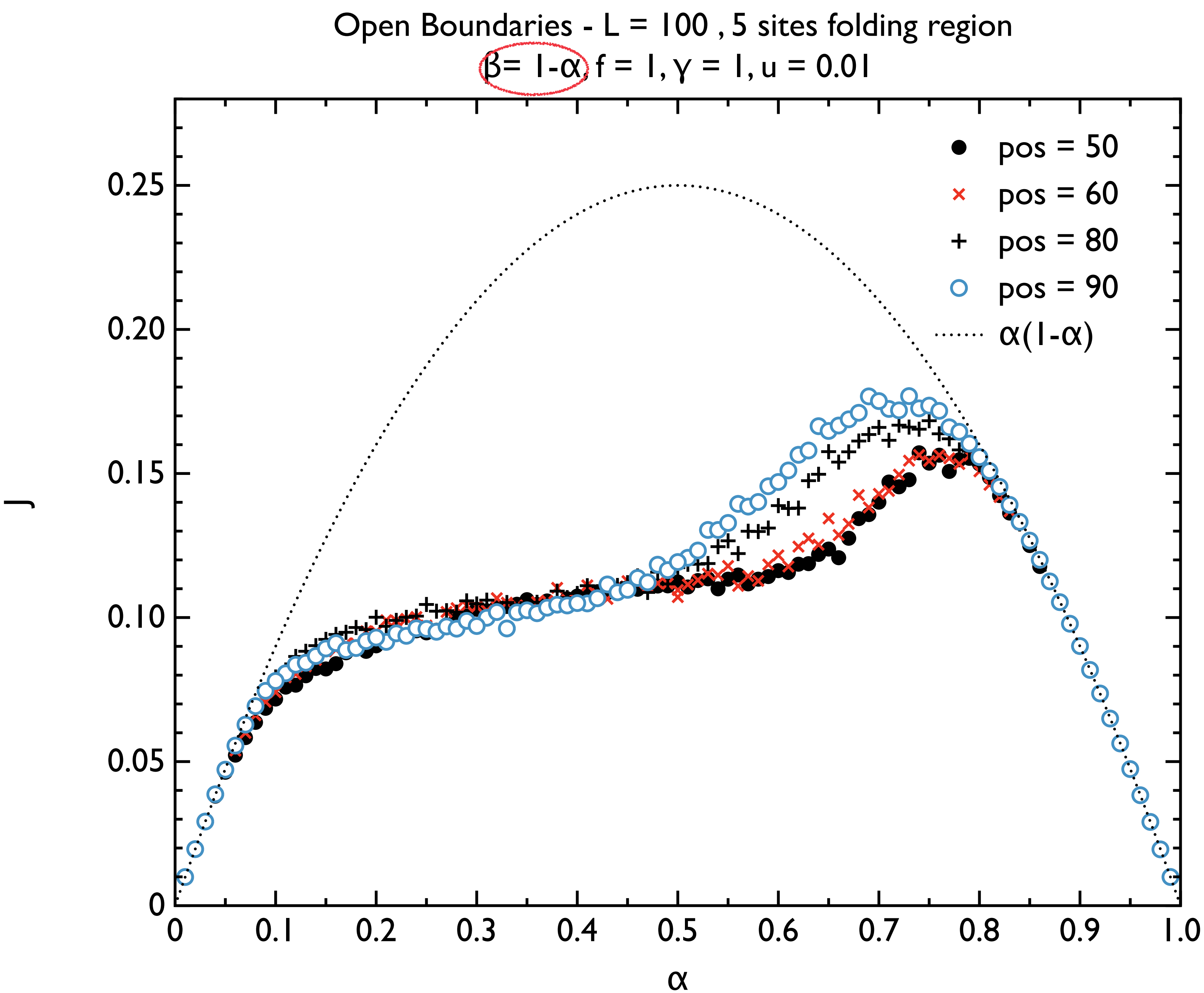


- $d>1$, L -> ∞
  - more particles configurations leading to an open folding region
    - higher currents for a given set {**u**,**f**,**γ**}
- $d>1$, finite L
  - strong correlations
    - **enhanced** particles flow at high density (>0.5)
    - **reduced** particle flow at low density (<0.5)
    - **slow** convergence to L -> ∞ plateaux
    - complex domain wall diffusion within the lattice

Some Open Boundaries

Results

Finite size effects depend on the size of the LD part



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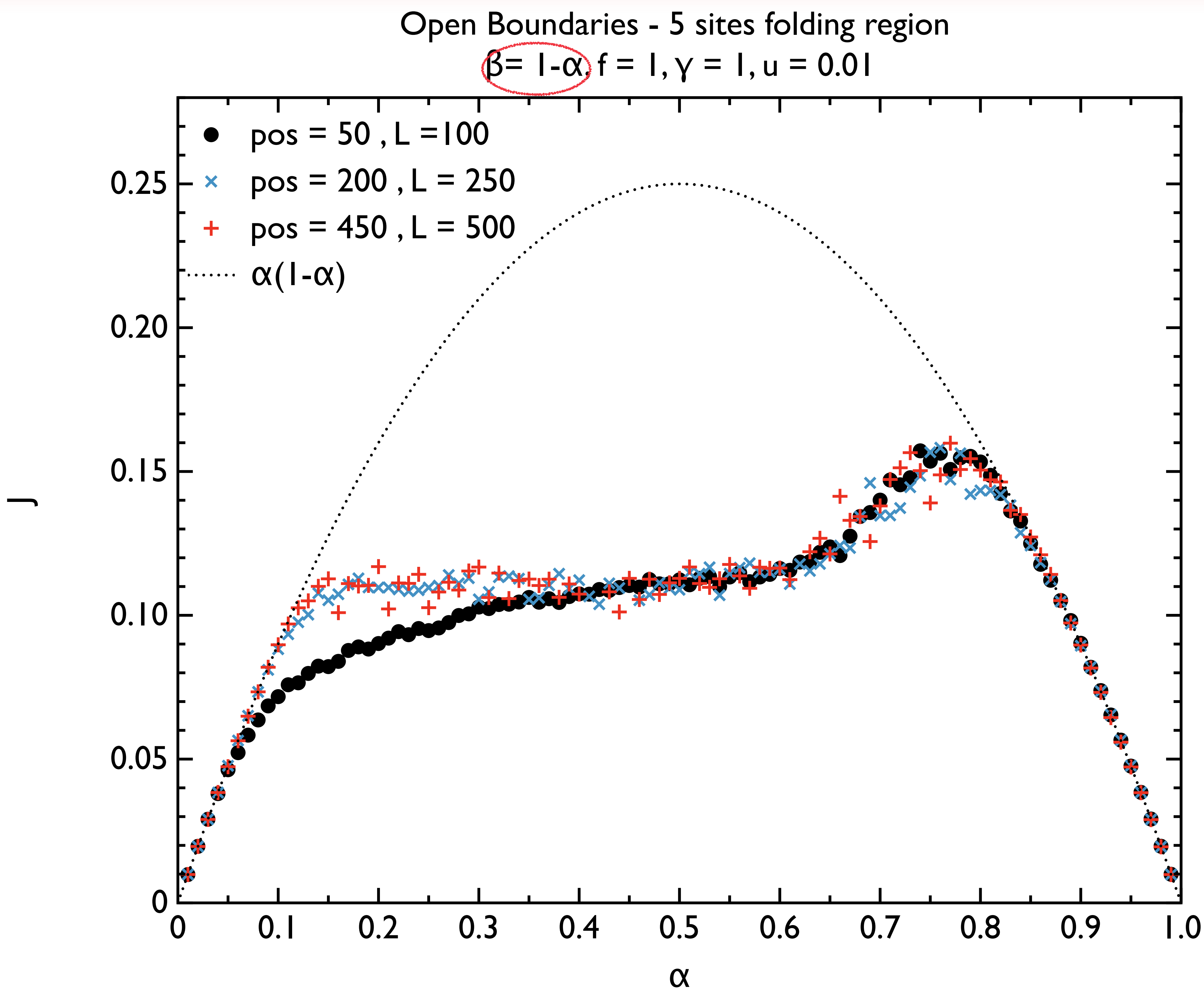

Conclusions

- MF description of the homogeneous flow regime.
- Intermittent dynamics at high **γ/u :**
  - 2 typical waiting times -> 2 typical microscopic currents
- Current enhancement/reduction depending on the extension of the LD phase.
  - strong size effects;
  - relevant for biology : mRNA length 10^2 - 10^3 bases ~ 10^2 of codons.
  - long range density profiles tails