Triangle-generation in topological D-brane categories

Nils Carqueville

King's College London

Introduction

- closed $\mathcal{N} = (2,2)$ **Landau-Ginzburg models**

Vafa/Warner 1989; Howe/West 1989; Witten 1993; Kontsevich/Kapustin/Li 2002; Orlov 2005; Herbst/Hori/Page 2008

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(Topological) supersymmetric Landau-Ginzburg models

Closed $\mathcal{N} = (2,2)$ LG model:

$$S_\Sigma = \int_\Sigma \mathrm{d}^4\theta \mathrm{d}^2 x K(X, \bar{X}) + \left(\int_\Sigma \mathrm{d}^2\theta \mathrm{d}^2 x W(X) + \text{c. c.} \right)$$

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with conditions

$$D\Pi = g(X)\,, \qquad Q^2(X) = W(X) \cdot \mathbb{1}\,, \qquad Q(X) = \begin{pmatrix} 0 & g(X) \\ f(X) & 0 \end{pmatrix}$$

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Open strings between Q and Q' described by $D_{QQ'}$-cohomology with

$$D_{QQ'} : \phi \longmapsto Q'\phi - \phi Q$$

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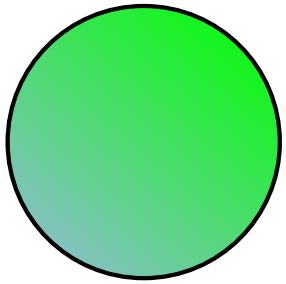
D-branes in LG models



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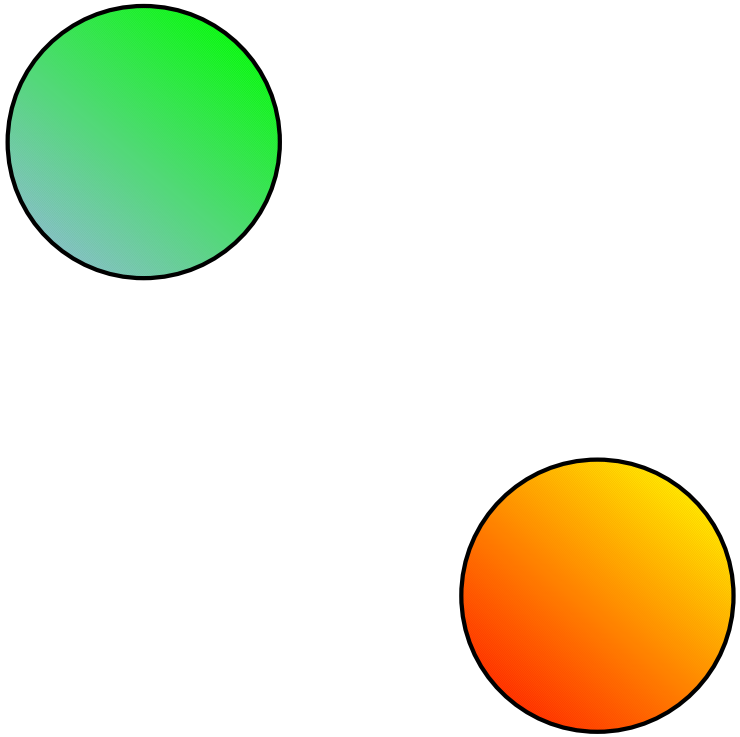

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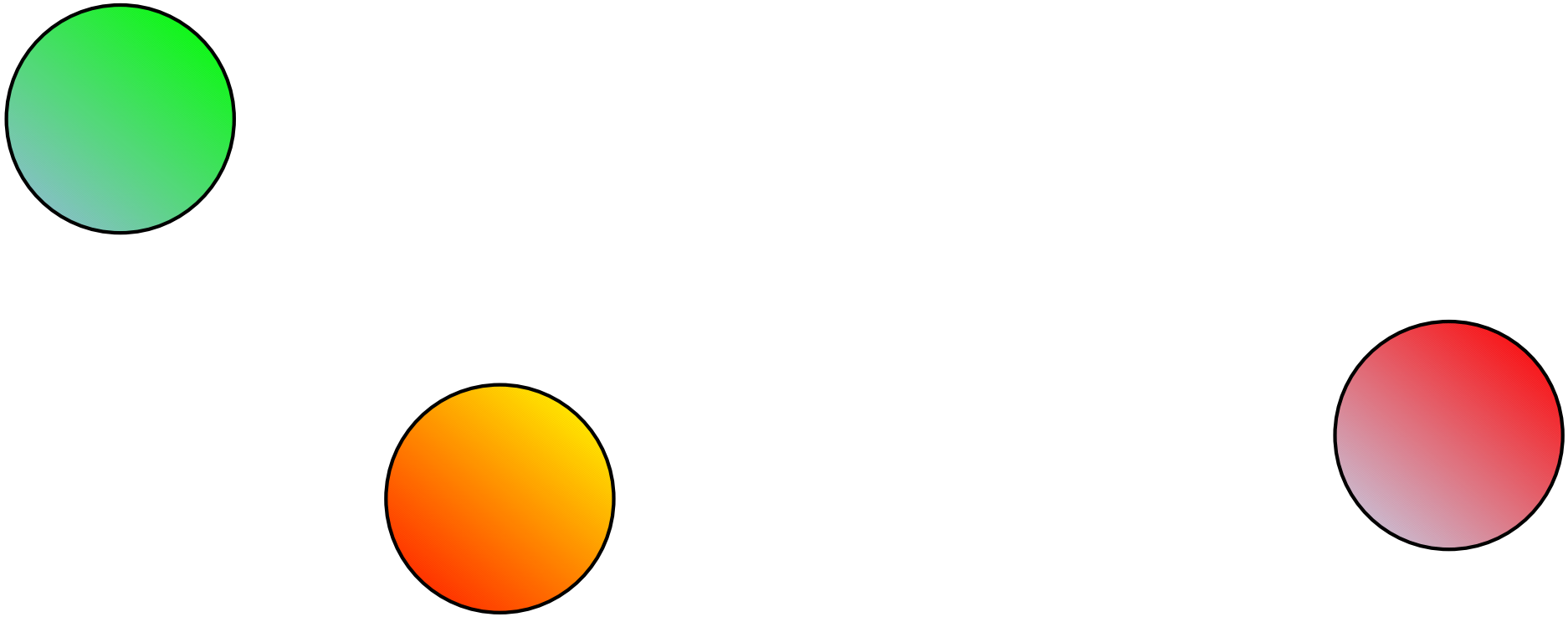

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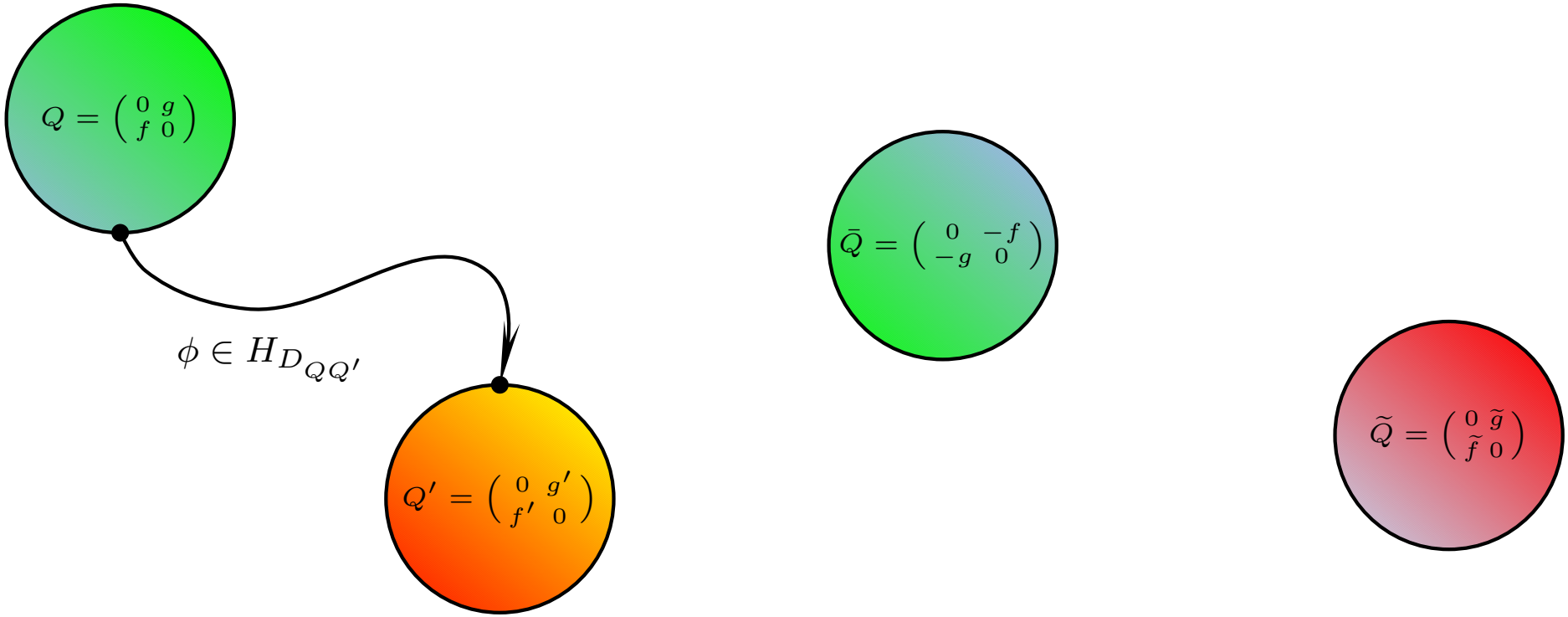

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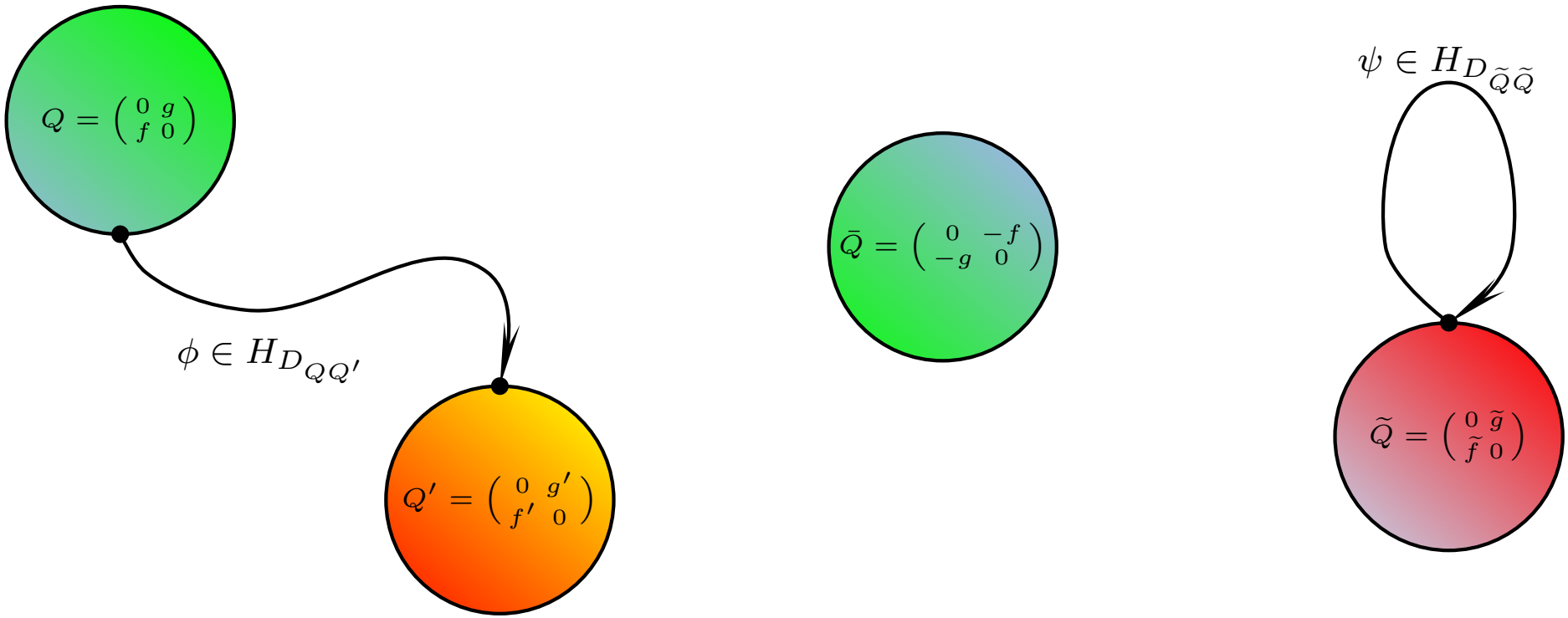

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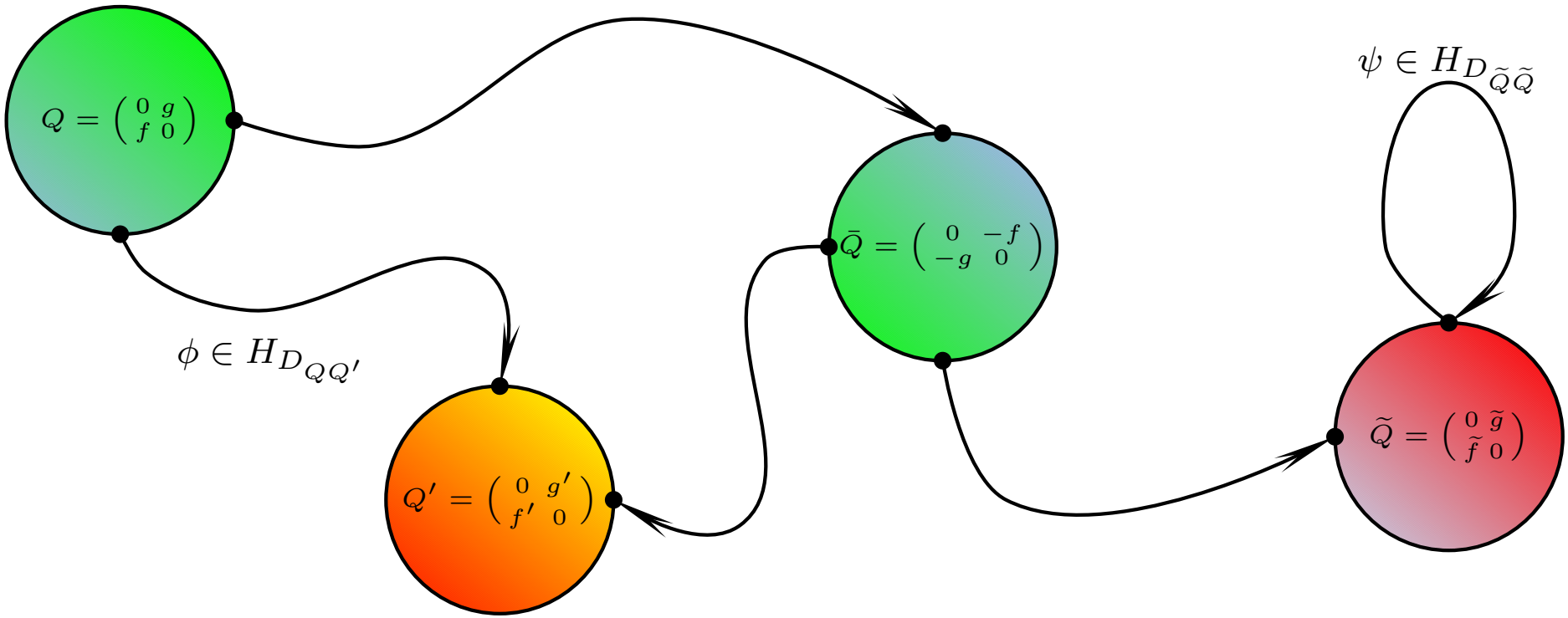

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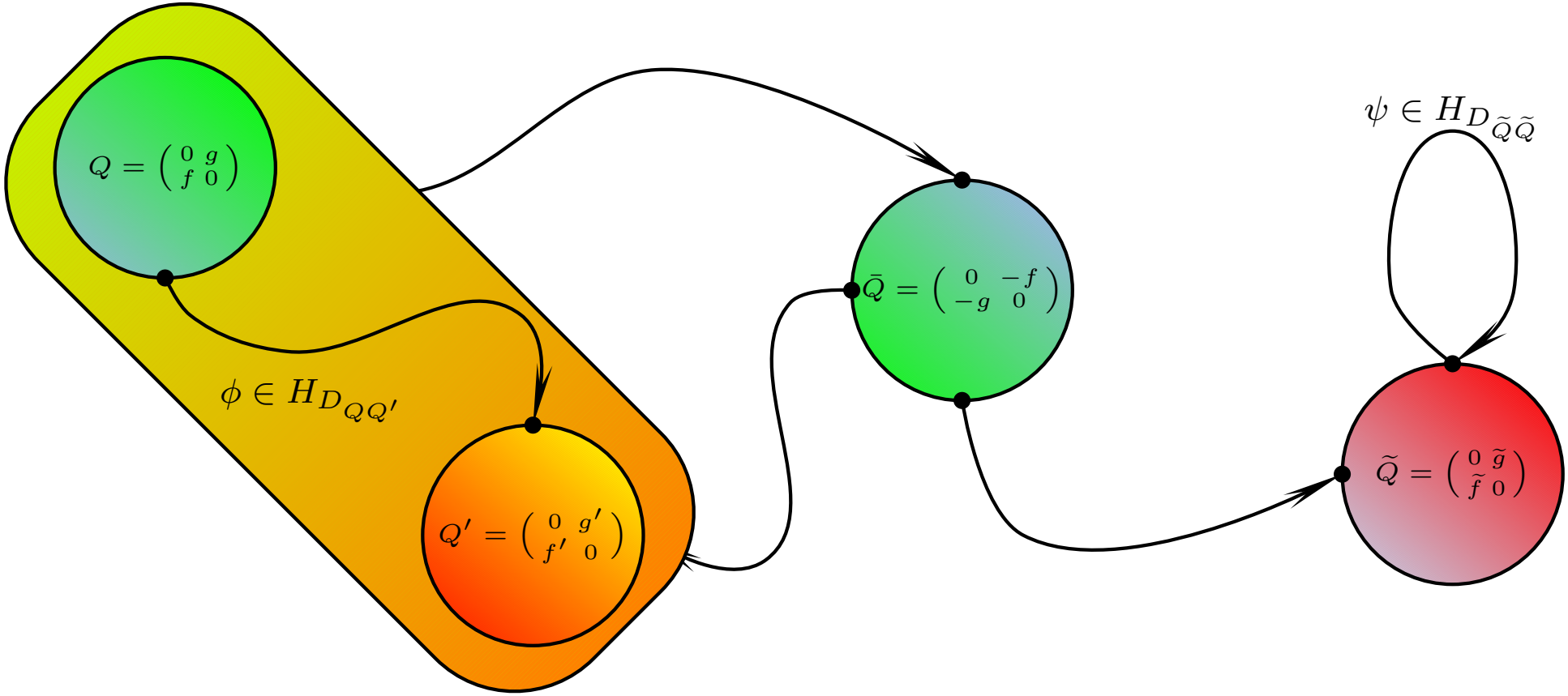

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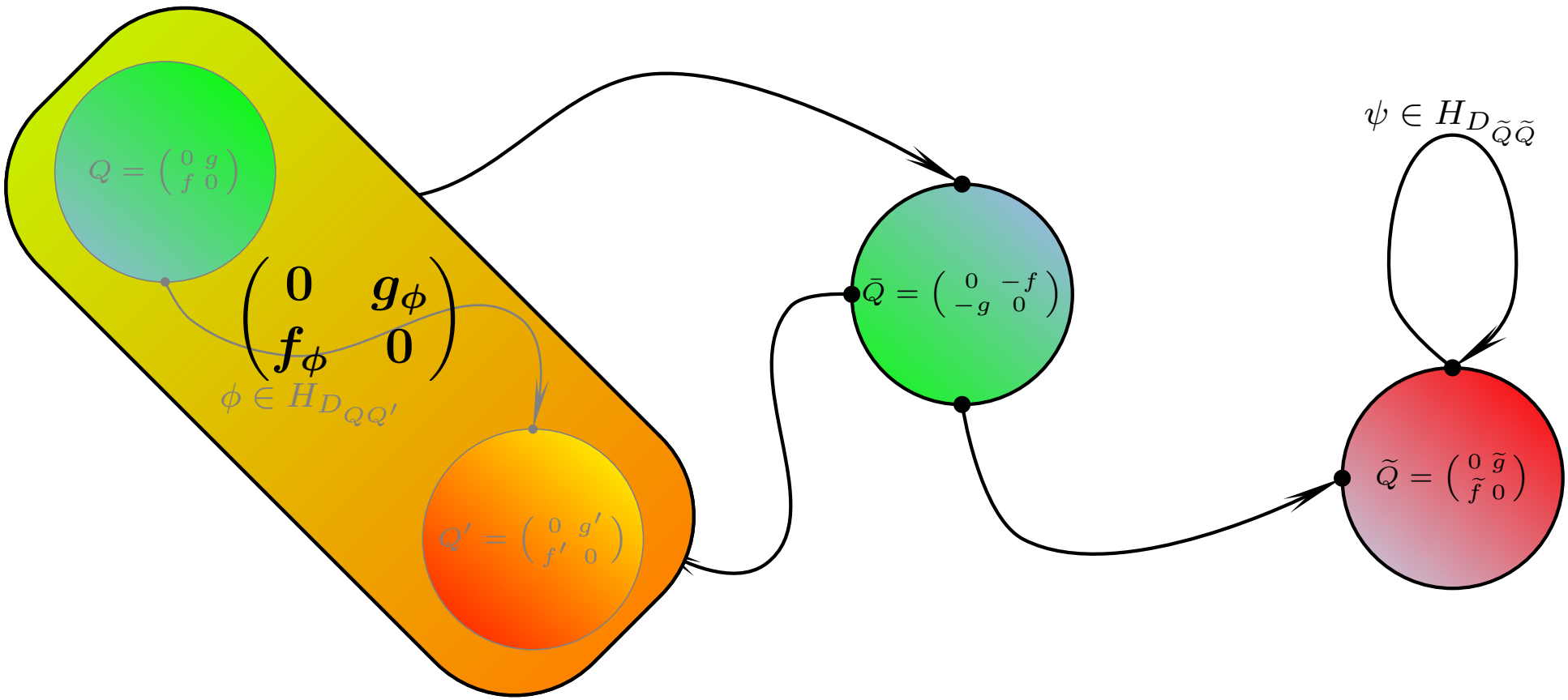

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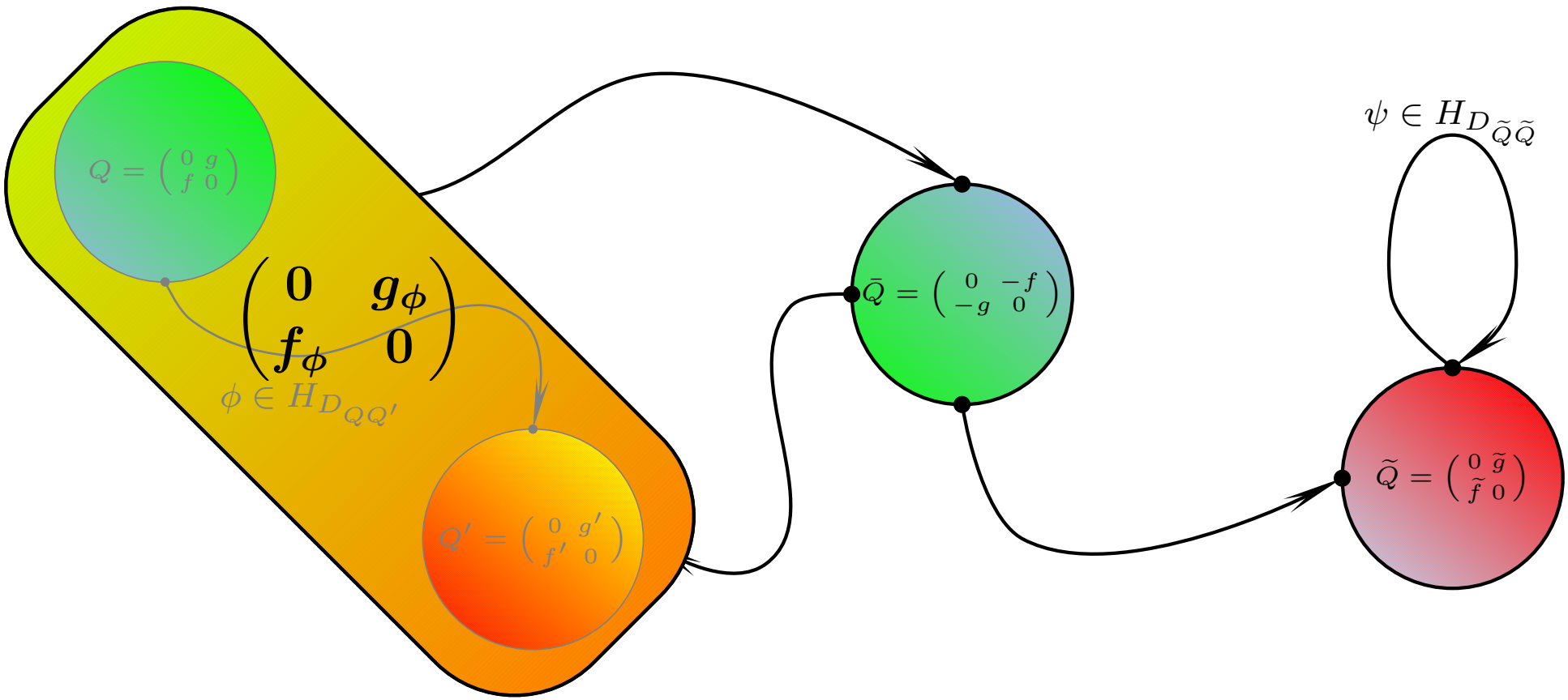

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Fact. The D-brane category $\mathrm{MF}(W)$ of matrix factorisations together with shift functor T and distinguished triangles

$$Q \xrightarrow{\phi} Q' \longrightarrow \mathrm{C}(\phi) \longrightarrow \bar{Q}$$

is **triangulated**.

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- ☐ compute all $\psi \in H(\mathrm{C}(\phi), \mathcal{Q}_{\text{other}})$
- ☐ if $\mathrm{End}(\mathrm{C}(\psi)) = 0$, then $\bar{Q}$ and Q' condense to $\mathcal{Q}_{\text{other}}$

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A_n:	x^{n+1}	$\mathbb{Z}_{n+1}$	
D_n:	$x^2y + y^{n-1} + z^2$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	for n even
		$\mathbb{Z}_4$	for n odd
E_6:	$x^3 + y^4 + z^2$	$\mathbb{Z}_3$	
E_7:	$x^3 + xy^3 + z^2$	$\mathbb{Z}_2$	
E_8:	$x^3 + y^5 + z^2$	$\{0\}$	

Schreyer 1987; Kajiura/Saito/Takahashi 2005; Kapustin/Li 2003; Brunner/Gaberdiel 2005; Keller/Rossi 2006

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Result: All branes can be generated by Q_1 and Q_n:

$$\mathrm{MF}(x^{n+1}) = \mathrm{tria}(Q_1) = \mathrm{tria}(Q_n)$$

Schreyer 1987; Kajiura/Saito/Takahashi 2005; Kapustin/Li 2003; Brunner/Gaberdiel 2005; Keller/Rossi 2006

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Appendix

Triangulated categories

Definition. Let $\mathcal{T}$ be an additive category with an additive automorphism $T : \mathcal{T} \longrightarrow \mathcal{T}$ and a class of *distinguished triangles* of the form

$$X \xrightarrow{u} Y \xrightarrow{v} Z \xrightarrow{w} TX$$

Then $\mathcal{T}$ is **triangulated** iff the following axioms hold:

(TR1a) $X \xrightarrow{\mathbb{1}_X} X \longrightarrow 0 \longrightarrow TX$ is a distinguished triangle.

(TR1b) Any triangle isomorphic to a distinguished triangle is distinguished.

(TR1c) Any $X \xrightarrow{u} Y$ can be completed to a distinguished triangle $X \xrightarrow{u} Y \xrightarrow{v} Z \xrightarrow{w} TX$.

(TR2) $X \xrightarrow{u} Y \xrightarrow{v} Z \xrightarrow{w} TX$ is distinguished iff $Y \xrightarrow{v} Z \xrightarrow{w} TX \xrightarrow{-Tu} TY$ is distinguished.

Triangulated categories

(TR3) For any f, g in

$$\begin{tikzcd}
X \arrow[r] \arrow[d, "f"] & Y \arrow[r] \arrow[d, "g"] & Z \arrow[r] \arrow[d, dashed, "h"] & TX \arrow[d, "Tf"] \\
X' \arrow[r] & Y' \arrow[r] & Z' \arrow[r] & TX'
\end{tikzcd}$$

there is $h : Z \longrightarrow Z'$ to complete the *morphism of triangles*.

(TR4) Any diagramme of type "upper cap" can be completed by a diagramme of type "lower cap" to an *octahedron diagramme*:

$$\begin{tikzcd}
X' \arrow[dr, "{[1]}"] \arrow[dd, "{[1]}"'] & & Z \\
& Y \arrow[ur] \arrow[dl] & \\
Z' \arrow[rr, "{[1]}"] & & X \arrow[uu] \arrow[ul]
\end{tikzcd}
\qquad
\begin{tikzcd}
X' \arrow[dd, "{[1]}"'] & & Z \arrow[ll] \arrow[dl] \\
& Y' \arrow[ul] \arrow[dr, "{[1]}"] & \\
Z' \arrow[ur] \arrow[rr, "{[1]}"] & & X \arrow[uu]
\end{tikzcd}$$

Octahedron diagramme

The resulting octahedron can also be displayed in the following way:

$$
\begin{tikzcd}
 & Y \arrow[dr] \arrow[ddl] & \\
X' \arrow[ur, "{[1]}"] \arrow[rr, dotted, no head] \arrow[d, "{[1]}"'] & & Z \arrow[dl, dotted, no head] \\
Z' \arrow[rr, "{[1]}"] \arrow[dr] & & X \arrow[u] \arrow[ul] \\
 & Y' \arrow[ur, "{[1]}"'] \arrow[uul, dotted, no head] &
\end{tikzcd}
$$

Physical interpretation

A distinguished triangle

$$X \longrightarrow Y \longrightarrow Z \longrightarrow TX$$

can be interpreted as *"the branes X and Z may bind via the potentially tachyonic string $Z \longrightarrow TX$ to form Y"*, denoted as $X, Z \leftrightsquigarrow Y$.

- X binds with no brane to produce X, i. e. $X, 0 \leftrightsquigarrow X$.
- If there is a string $X \longrightarrow Y$, then there is a brane Z such that $X, Z \leftrightsquigarrow Y$.
- If $X, Z \leftrightsquigarrow Y$, then $TX, Y \leftrightsquigarrow Z$, so TX is the anti-brane of X.
- (TR2) and (TR4) together describe consistent decompositions of brane systems:

$$Z' \leftrightsquigarrow TX, Y \leftrightsquigarrow TX, Z, T^{-1}X' \leftrightsquigarrow Y', T^{-1}X' .$$