Prob Sheet 1: First Order Ordinary Differential Equations

Module F13YT2

1. Integration revision. Find $y(x)$, given that
$(i)\ \dfrac{dy}{dx} = \dfrac{e^x}{3+6e^x}$; $\quad (ii)\ \dfrac{dy}{dx} = x^2 \sin x$.

2. Name the type and find the general solution for each of the following first order equations:
$(i)\ \dfrac{dy}{dx} = \dfrac{x^2}{y}$; $\quad (ii)\ \dfrac{dy}{dx} + 3y = x + e^{-2x}$; $\quad (iii)\ x\dfrac{dy}{dx} = x\cos(2x) - y$;
$(iv)\ \dfrac{dy}{dx} = \dfrac{y^2+2xy}{x^2}$; $\quad (v)\ xy^2 - x + (x^2y + y)\dfrac{dy}{dx} = 0$;
$(vi)\ (xe^y + 2y)\dfrac{dy}{dx} + e^y = 0$;
$(vii)\ (\cos x + 2xe^{2y})\dfrac{dy}{dx} + 1 - y\sin x + e^{2y} = 0$
$(viii)\ \dfrac{dy}{dt} + y = 2\,y^3$

3. Show that each of the following equations has two distinct solutions.
$(i)\ \dfrac{dy}{dx} = y^{1/2}$; $\ y(0) = 0$; $\quad (ii)\ y\dfrac{dy}{dx} = x$; $\ y(0) = 0$.
Explain, in each case, why the hypotheses of Picard's Theorem are not satisfied.

4. Find all constant solutions of $\dfrac{dy}{dx} + y^2 = 1$. Hence sketch the solutions satisfying $(i)\ y(0) = 1.1$; $\ (ii)\ y(0) = 0$; $\ (iii)\ y(0) = -1.1$. Check your answer using Maple.

5. Consider the equation $\dfrac{dy}{dx} = y^2 - x$. Determine the points in the xy-plane at which $(i)\dfrac{dy}{dx} = 0$; $(ii)\dfrac{dy}{dx} > 0$; and $(iii)\dfrac{dy}{dx} < 0$. Hence sketch the solution satisfying $y(1) = 1$. Check your answer using Maple.

6. Consider the following first order differential equation

$$\frac{dy}{dx} = y - x. \qquad (*)$$

(a) Determine the points in the xy-plane at which
$(i)\dfrac{dy}{dx} = 0$; $(ii)\dfrac{dy}{dx} > 0$; and $(iii)\dfrac{dy}{dx} < 0$. Hence, sketch the direction field of $(*)$.

(b) Sketch the solution of $(*)$ satisfying the initial condition $y(0) = \frac{1}{2}$ in the direction field you have made for (a) (there is no need to solve the initial value problem explicitly).