F10PC1

Department of Mathematics

Exam paper, comments & Solutions

F10PC1

Pure Mathematics C

Duration: 2 Hours

Semester 1 Exam Diet 2009

Attempt three questions

A University approved calculator may be used
for basic computations, but
appropriate working must be shown to obtain full credit.

Automata Theory 2009

1.

(a) Construct *complete deterministic automata* recognising each of the following languages. You do not have to justify your answers, but each automaton will be marked either right or wrong.

(i) $a^2 + (a^3 + a^6 + a^8)(a^6)^*$.

(ii) $\{x \in (a+b)^* \colon |x|_a \equiv 4 \pmod 5\}$.

(iii) $\{x \in (a+b)^* \colon |x|_a \geq 4\}$.

(iv) $abab(a+b)^*$.

(v) $(a+b)^*abab$.

(b) Construct a complete deterministic finite state automaton that recognises all those binary strings that represent numbers that when divided by 5 leave the remainder 3. The string ε represents 0, and we allow leading 0's. To obtain full marks you must justify your answer.

2.

(a) Construct a complete deterministic automaton with 2 states that recognises the language

$$L = \{x \in (a+b)^* \colon |x| \equiv 0 \pmod 2\}.$$

Construct a complete deterministic automaton with 3 states that recognises the language

$$M = (a+b)^*aa(a+b)^*.$$

Show how to combine your two automata in order to construct a complete deterministic automaton that recognizes the language $L \setminus M$.

(b) Write down a *non-deterministic* automaton with 7 states that recognizes the language

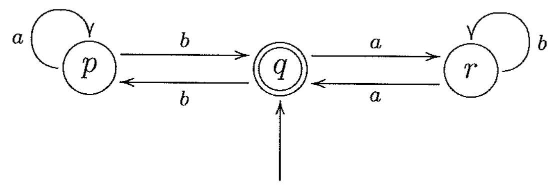
$$a^2(a+b)^*ab(a+b)^*a^2.$$

Use the accessible subset construction to construct a complete deterministic automaton recognizing the same language. To obtain full credit, please show all steps in your application of this algorithm.

Exam questions continue overleaf

3.

(a) Apply the standard algorithm to find a regular expression describing the language recognised by the automaton **A** below. To obtain full credit you must show all steps in the algorithm.



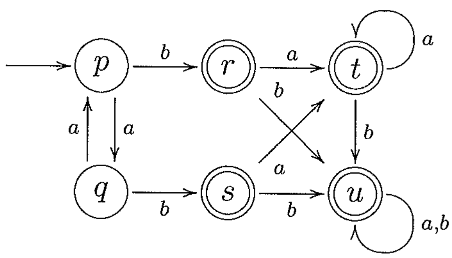
(b) What is a *regular expression*?

State Kleene's Theorem.

Assuming the standard results concerning automata with ε-transitions, prove that the union, product and Kleene star of regular languages are likewise regular.

4.

(a) Apply the minimization algorithm to construct a reduced, accessible automaton that recognizes the same language as the automaton below. To obtain full credit, please show all steps in the algorithm.



(b) Use the *method of quotients* to construct a complete deterministic automaton that recognizes the language $(a+b)^*abaa$. To obtain full credit, please show all steps in your calculations.

Exam paper ends

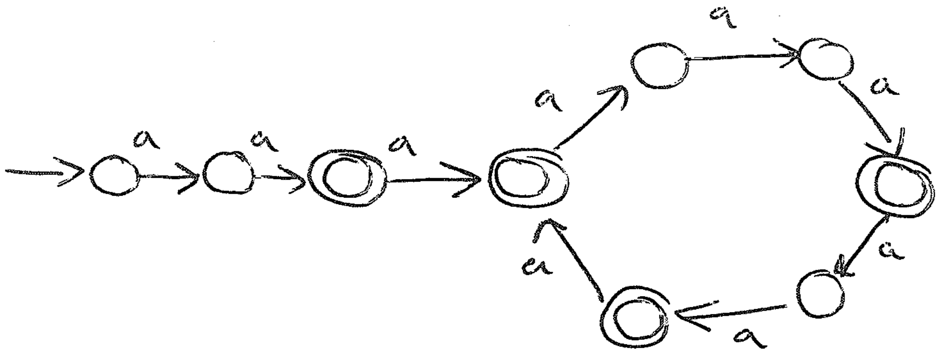
Automata theory

Exam paper 2009

Exam question	Comments
1(a)	All standard types of automata discussed in Chapter 2. Similar to HW1, Q1 and HW3, Q2.
1(b)	I discussed the method for solving this question in class. Similar to HW1, Q2.
2(a)	Standard algorithm: combining automata reflecting boolean operations on languages. Similar to HW2, Q1.
2(b)	Standard algorithm. Similar to HW2, Q2.
3(a)	Standard algorithm. Similar to HW4, Q2 and HW5, Q1, and HW6, Q1.
3(b)	Bookwork.
4(a)	Standard algorithm. Similar to HW5, Q2, and HW6, Q2.
4(b)	Standard algorithm. Similar to HW6, Q3.

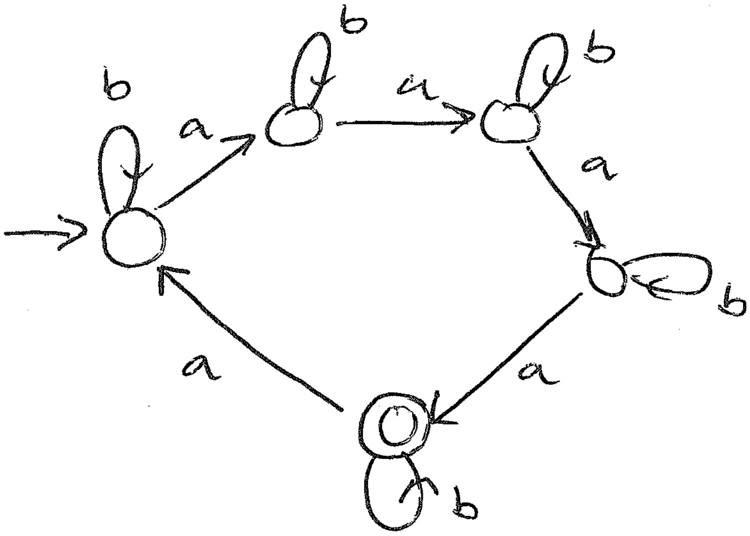
1 (a)

(i)



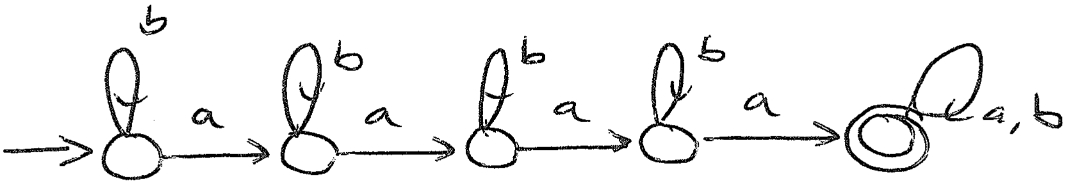
[2]

(ii)

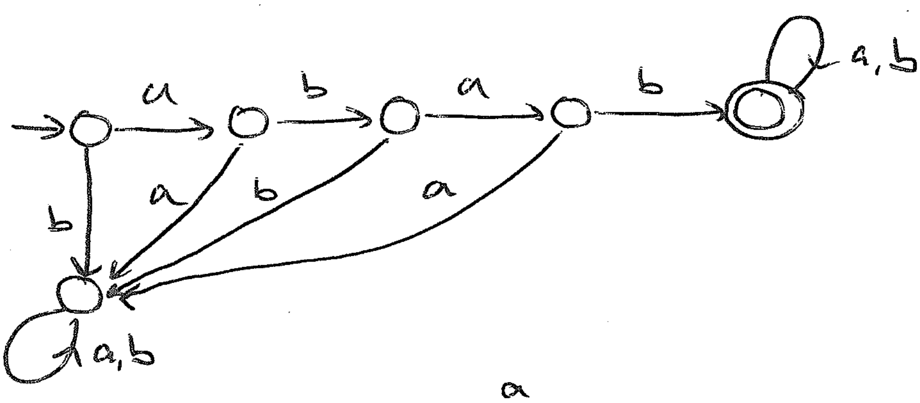


[2]

(iii)

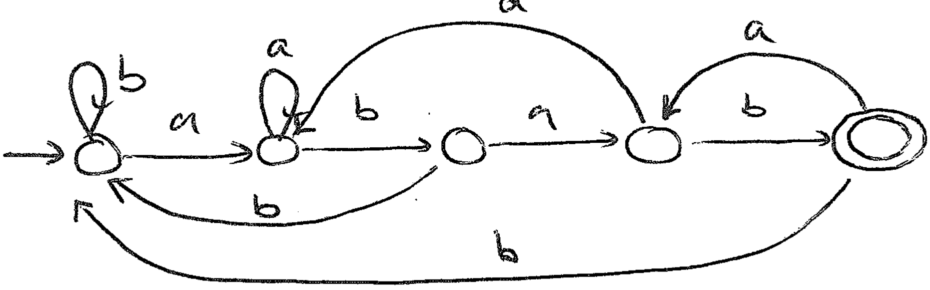


(iv)



[2]

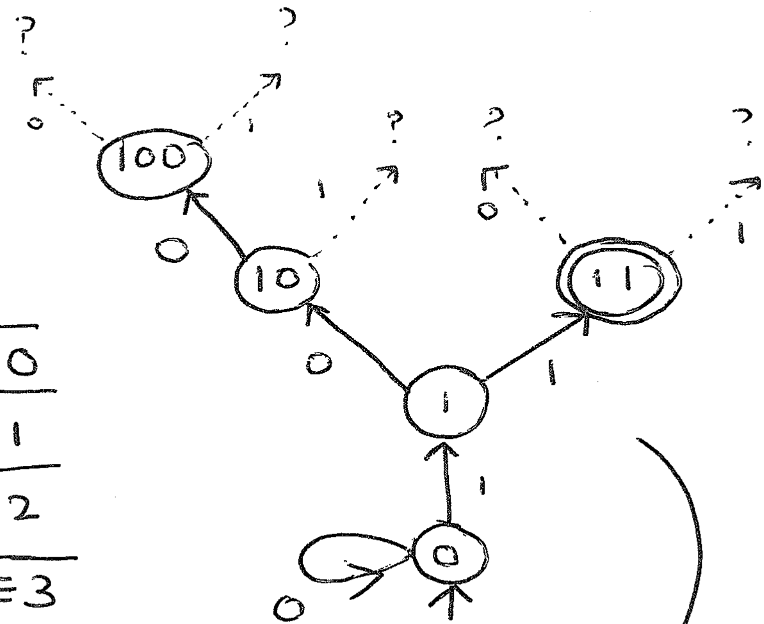
(v)



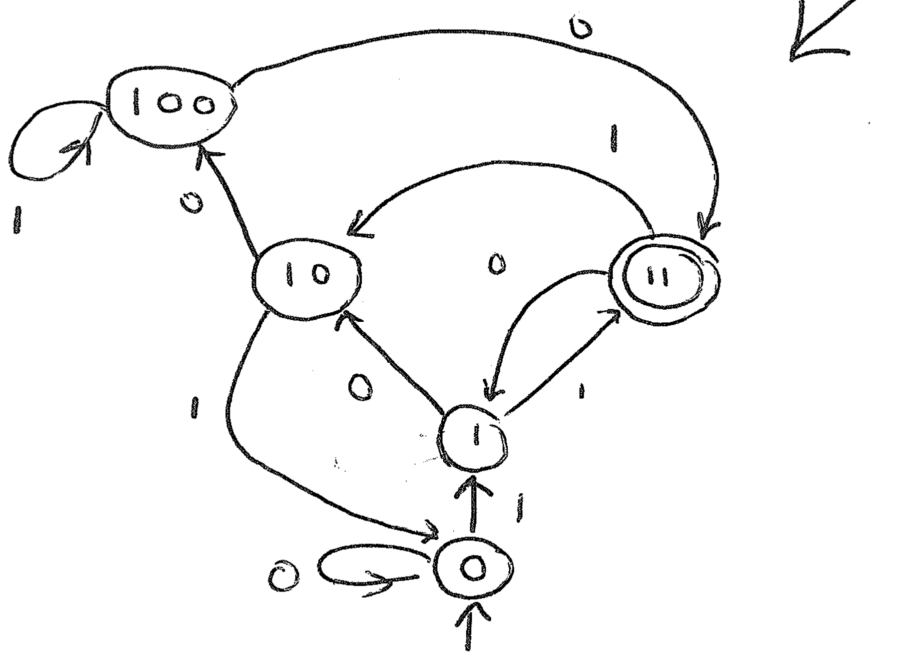
[2]

1(b)

0	0
1	1
2	10
3	11
4	100
5	101 ≡ 0
6	110 ≡ 1
7	111 ≡ 2
8	1000 ≡ 3
9	1001 ≡ 4



Skeleton

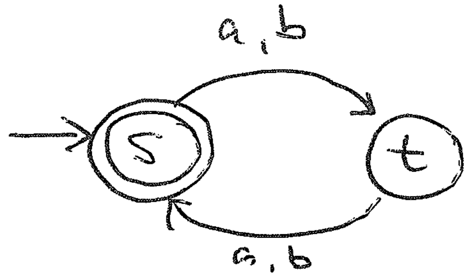


fill in missing transitions using table in top left

[10]

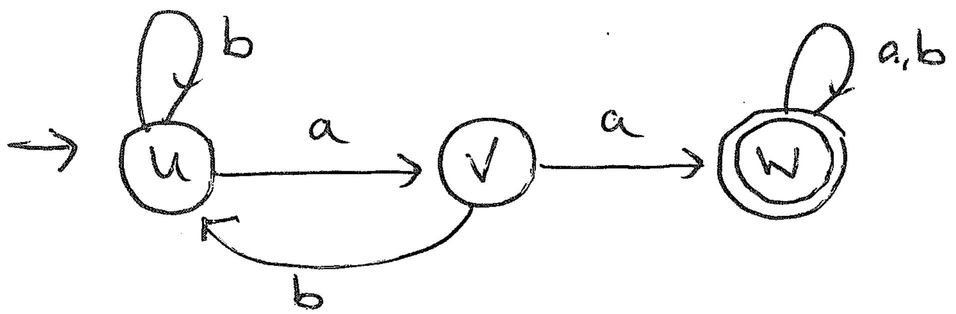
2(a)

$L = \{x \in (a+b)^* : |x| \equiv 0 \pmod 2\}$. $\underline{A}$ s.t. $L(\underline{A}) = L$ is:



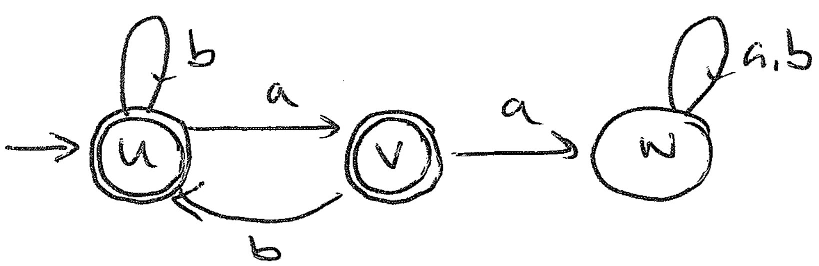
[2]

$M = (a+b)^* a^2 (a+b)^*$. $\underline{B}$ s.t. $L(\underline{B}) = M$ is:



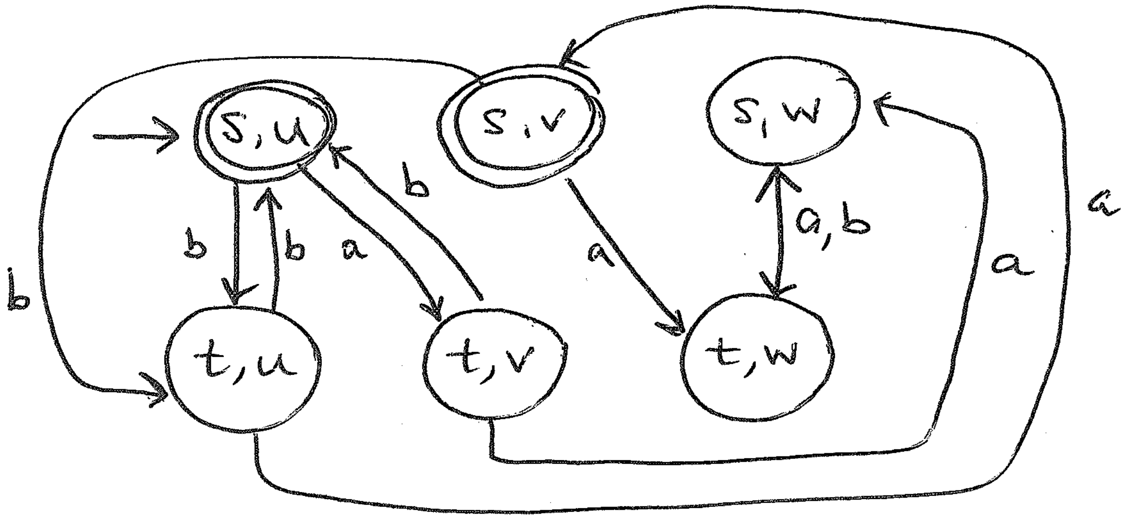
[2]

machine for the complement of M



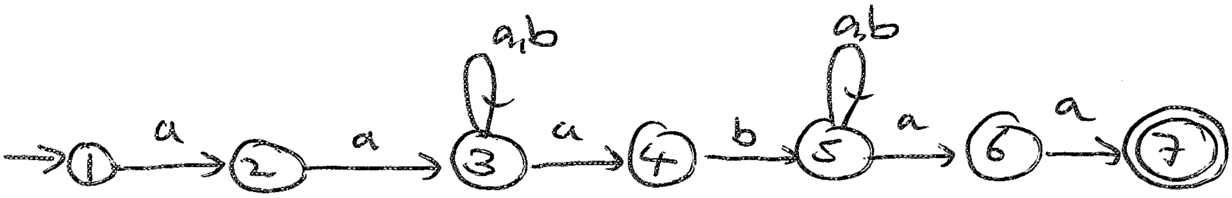
[1]

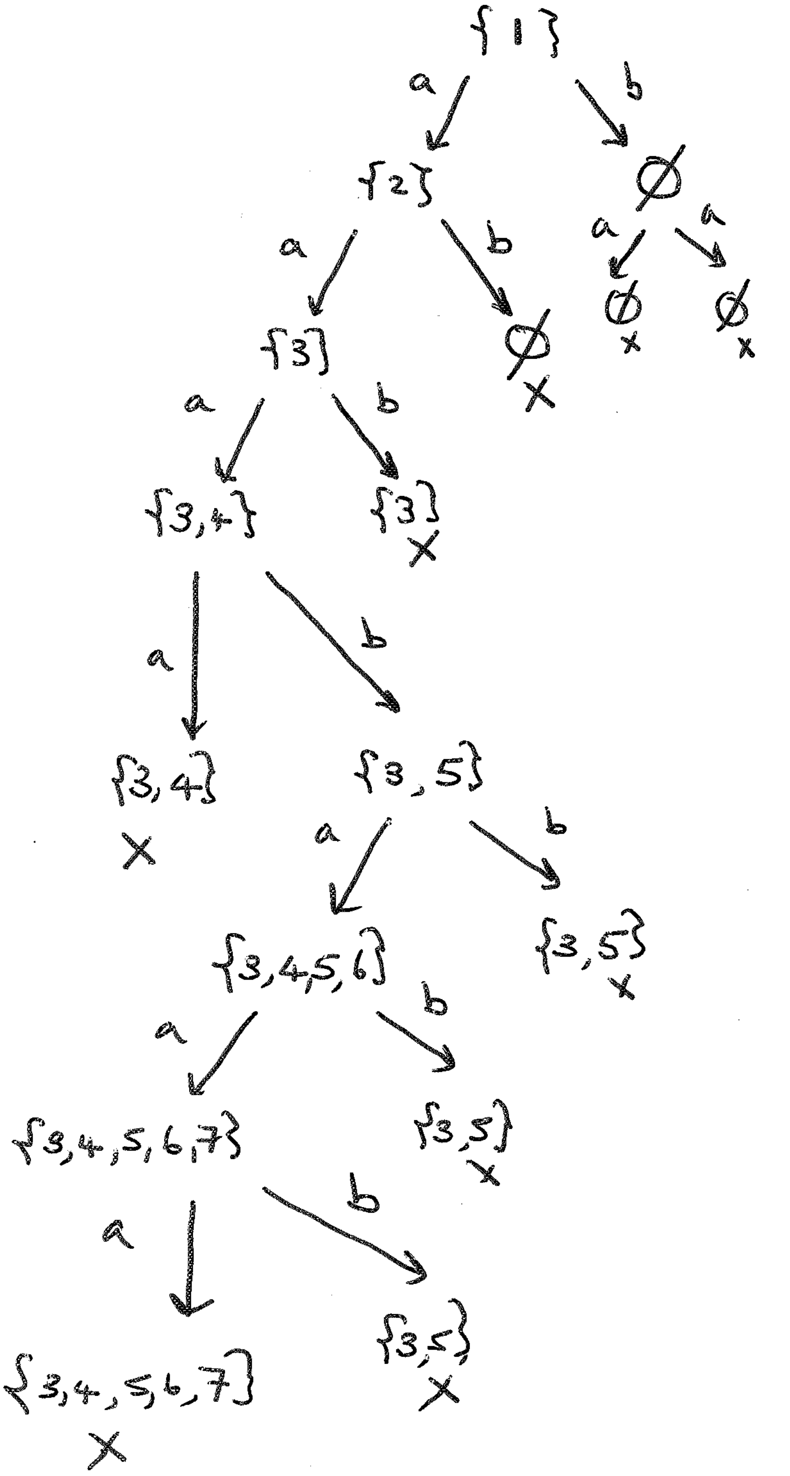
Machine for $L \setminus M$

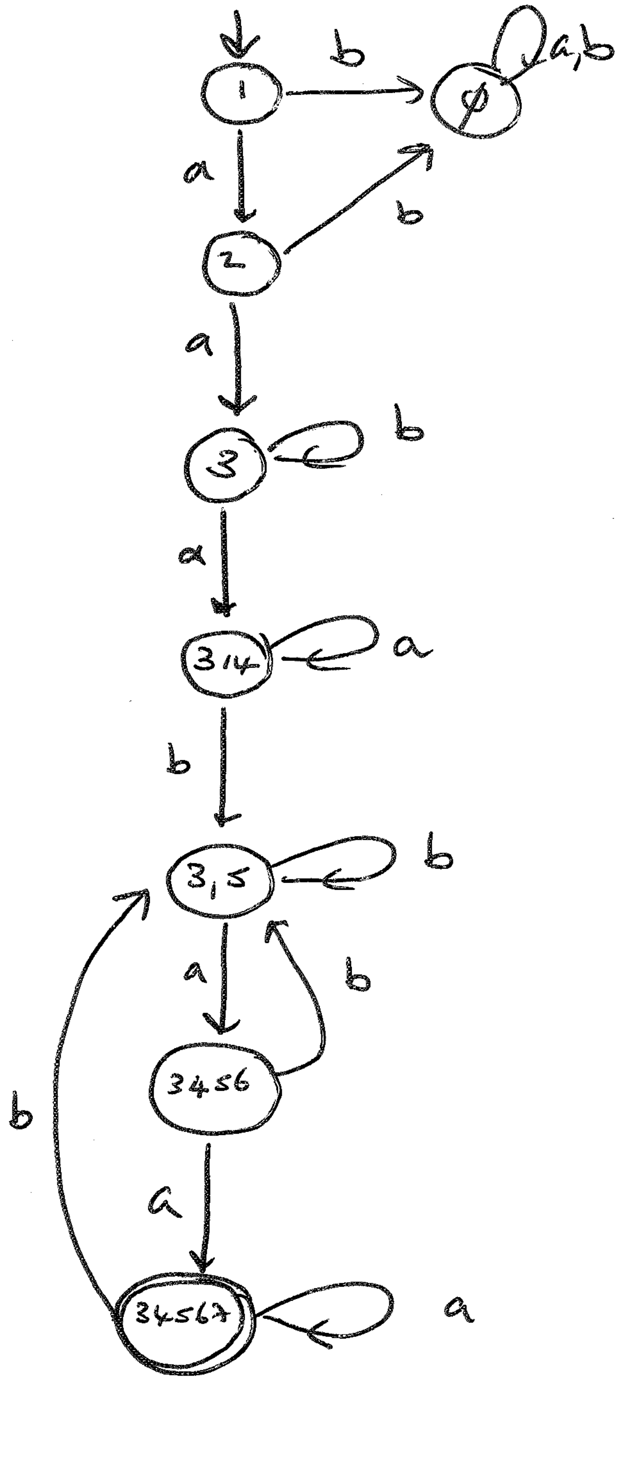


[5]

2 (b).


a,b
a,b
a
a
a
b
a
a
1
2
3
4
5
6
7


{1}
{2}
{3}
{3,4}
{3}
{3,4}
{3,5}
{3,4,5,6}
{3,5}
{3,4,5,6,7}
{3,5}
{3,4,5,6,7}
{3,5}

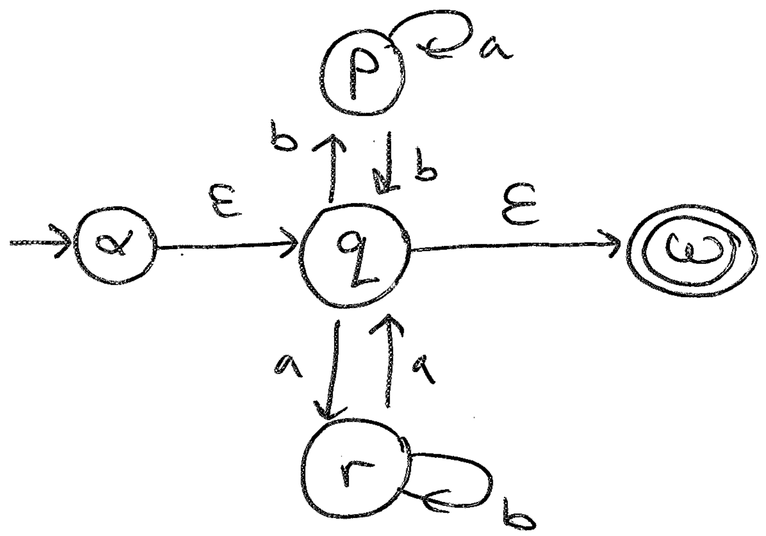

1
2
3
3,4
3,5
3456
34567

[6]

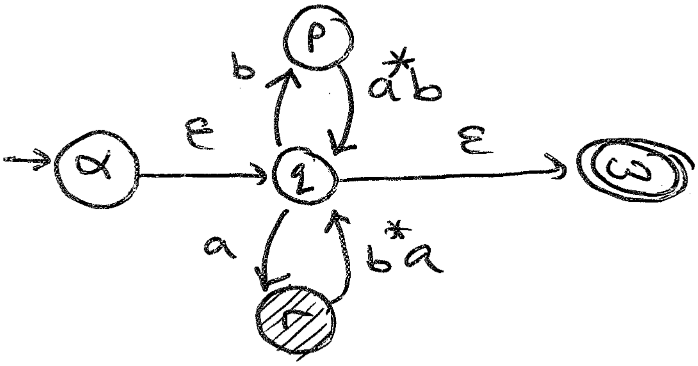
[4]

3 (a)

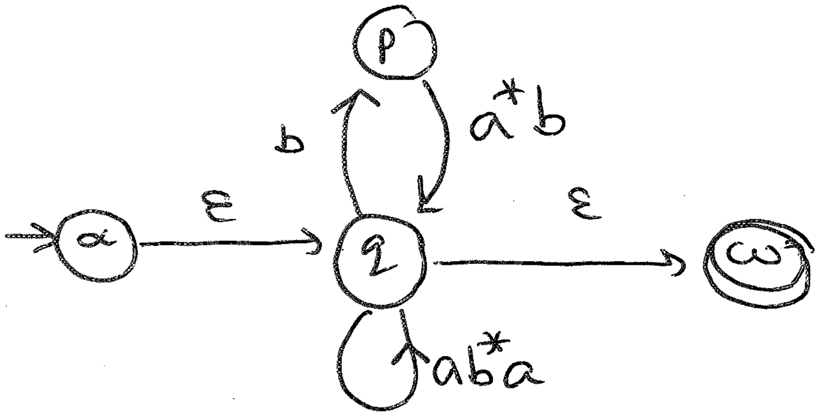
First we must normalize the machine



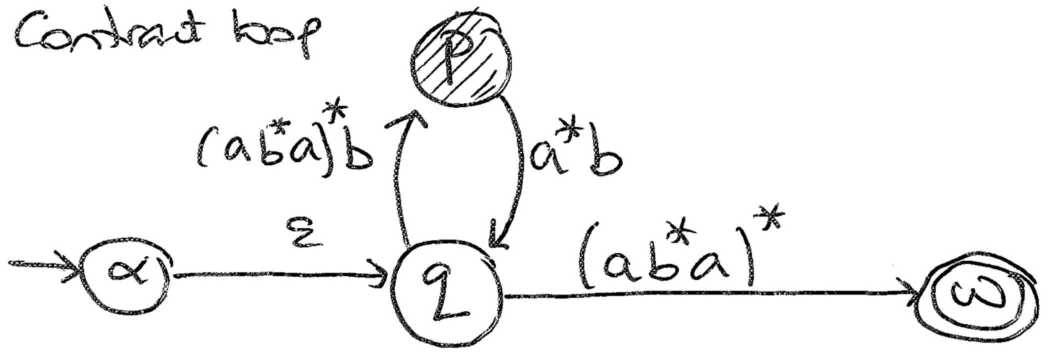
Next we contract loops



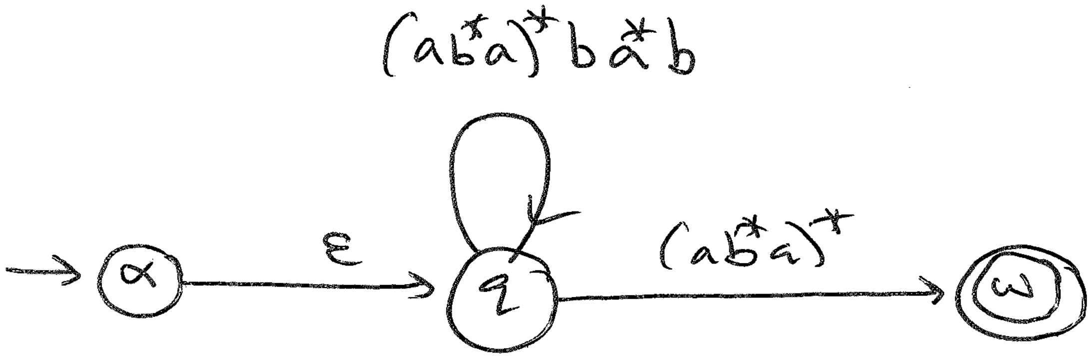
Next eliminate state r



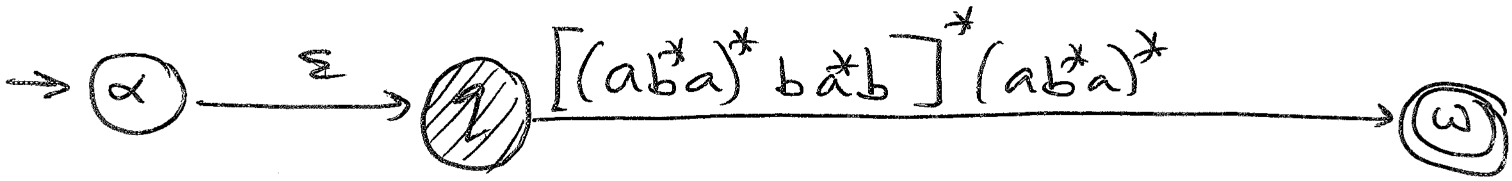
Contract loop



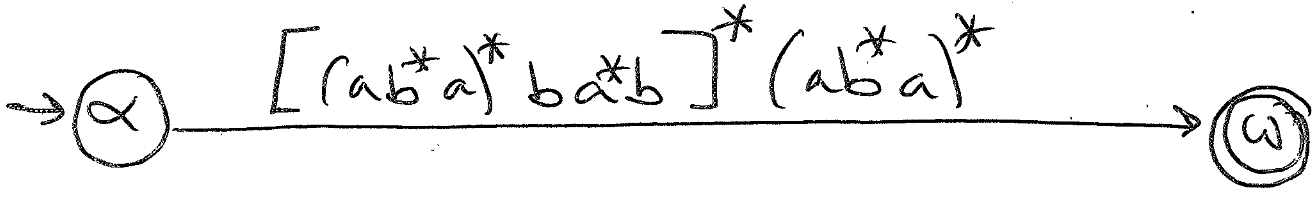
Now eliminate state p



Contract loop



Finally eliminate state q



This is the required regular expression.

3(b) Let $A = \{a_1, .., a_n\}$ be a finite alphabet.
A regular expression over A is constructed according to the following recipe:

(RE1) $\emptyset$ and ε are regular exp's.

(RE2) a_i for $1 \leq i \leq n$ is a reg. exp.

(RE3) if r and s are reg. exp's so are
$(r \cdot s)$, $(r + s)$ and (r^*).

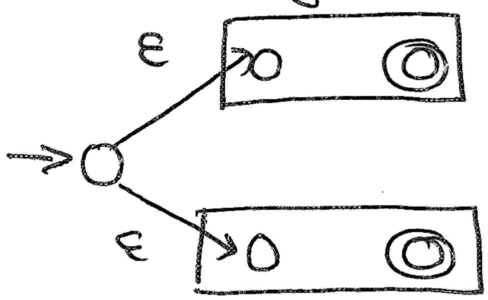
(RE4) These are the only reg. exp's.

[3]

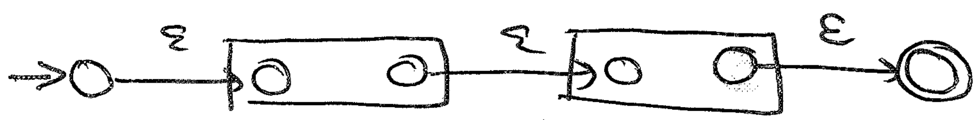
Kleene's Theorem: A language is recognisable if, and only if, it can be described by means of a reg. exp.

[2]

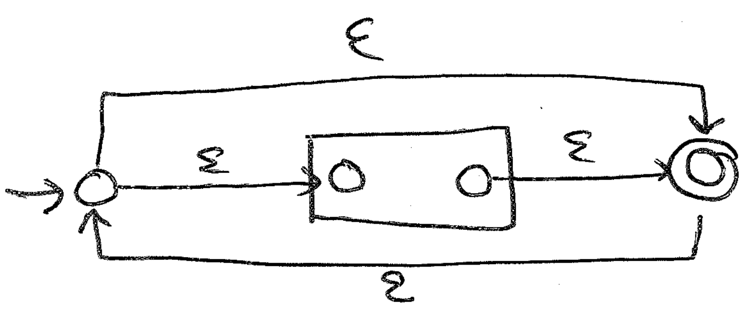
The following pictures are all that is needed: [5]



union



product



Kleene star

4(a)

	p	q	r	s	t	u
p	✓	① ✓	x'	x'	x'	x'
q		✓	x'	x'	x'	x'
r			✓	② ✓	③ ✓	④ ✓
s				✓	⑤ ✓	⑥ ✓
t					✓	⑦ ✓
u						✓

x' initialization

①

	p	q
a	q	p
b	r	s

②

	r	s
a	t	t
b	u	u

③

	r	t
a	t	t
b	u	u

④

	r	u
a	t	u
b	u	u

⑤

	s	t
a	t	t
b	u	u

⑥

	s	u
a	t	u
b	u	u

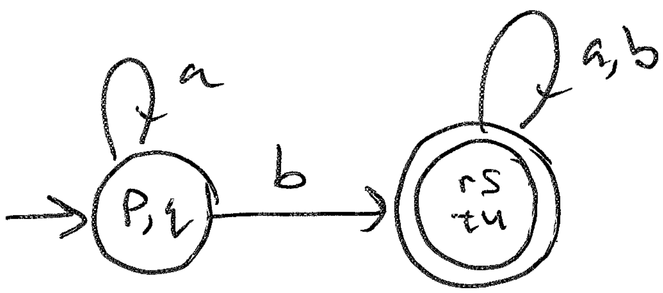
⑦

	t	u
a	t	u
b	u	u

The equivalence classes are therefore

{p, q}, {r, s, t, u}. [8]

$\underline{A}^r$:



[2]

4(b) Let $L = (a+b)^* abaa.$

(1) $\bar{\varepsilon}^1 L = L = L_0.$

(2) $\bar{a}^1 L = \bar{a}^1 (a+b)^* \cdot abaa + \delta((a+b)^*)(\bar{a}^1 \cdot abaa)$

$= (a+b)^* abaa + baa$

$= L + baa = L_1$

(3) $\bar{b}^1 L = \bar{b}^1 (a+b)^* \cdot abaa + \delta((a+b)^*)(\bar{b}^1 \cdot abaa)$

$= L = L_0.$

(4) $\bar{a}^1 L_1 = \bar{a}^1 L + \bar{a}^1 \cdot baa$

$= L_1 + \emptyset = L_1.$

(5) $\bar{b}^1 L_1 = \bar{b}^1 L + \bar{b}^1 \cdot baa$

$= L_0 + aa = L_2.$

(6) $\bar{a}^1 L_2 = \bar{a}^1 L_0 + \bar{a}^1 \cdot aa$

$= L_1 + a = L_3.$

(7) $\bar{b}^1 L_2 = \bar{b}^1 L_0 + \bar{b}^1 \cdot aa = L.$

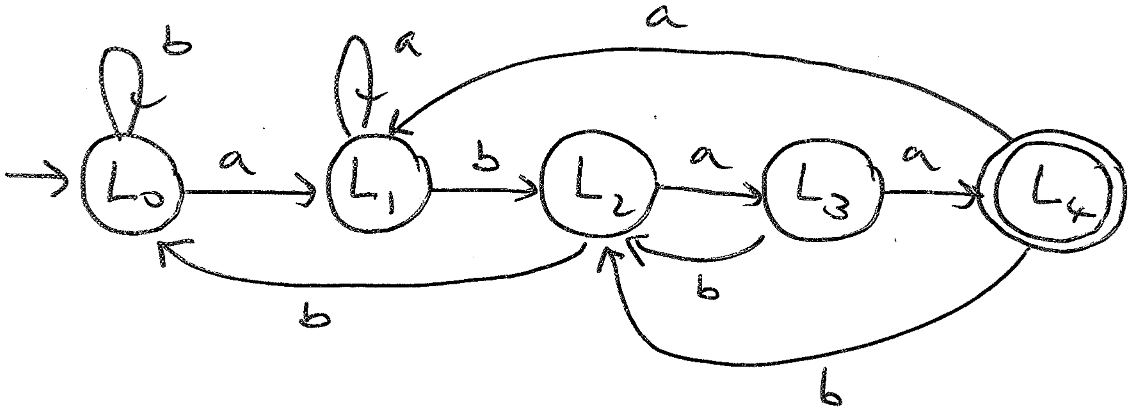
(8) $\bar{a}^1 L_3 = \bar{a}^1 L_1 + \bar{a}^1 \cdot a = L_1 + \varepsilon = L_4.$

(9) $\bar{b}^1 L_3 = \bar{b}^1 L_1 + \bar{b}^1 \cdot a = L_2.$

(10) $\bar{a}^1 L_4 = \bar{a}^1 L_1 + \bar{a}^1 \varepsilon$

$= \bar{a}^1 L_1 = L_1.$

(11) $\bar{b}^1 L_4 = \bar{b}^1 L_1 + \bar{b}^1 \varepsilon = \bar{b}^1 L_1 = L_2.$ [8]



[2]