Equilibrium shapes for anistropic surface energies

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Abstract

The equilibrium shape of a crystal under the action of an external field is determined by the minimization under a volume constraint of a total free energy, consisting in a surface tension term plus a bulk potential term.

In the absence of the potential term equilibrium shapes can be explicitly characterized in terms of the surface tension energy and are obtained by translation and scaling of a unique convex set, known as the Wulff shape of the crystal. In the particular case of an isotropic surface tension the Wulff shape is the Euclidean ball, and the minimization problem under consideration amounts the classical isoperimetric problem.

Very little is known about geometric properties of equilibrium shapes of crystals in presence of a potential term. We provide some satisfactory answers in the case of small potential terms, establishing the convexity and the proximity of equilibrium shapes to Wulff shapes. We then discuss some results about two-dimensional crystals that raise some interesting open problems.