Random mappings with a given number of cyclical points

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Abstract

In this paper we consider a random mapping, $\hat{T}_n$, of the finite set $\{1, 2, ..., n\}$ into itself for which the digraph representation $\hat{G}_n$ is constructed by: (1) selecting a random number, $\hat{L}_n$, of cyclic vertices, (2) constructing a uniform random forest of size n with the selected cyclic vertices as roots, and (3) forming 'cycles' of trees by applying a random permutation to the selected cyclic vertices. We investigate $\hat{k}_n$, the size of a 'typical' component of $\hat{G}_n$, and, under the assumption that the random permutation on the cyclical vertices is uniform, we obtain the asymptotic distribution of $\hat{k}_n$ conditioned on $\hat{L}_n = m(n)$. As an application of our results, we show in Section 3 that provided $\hat{L}_n$ is of order much larger than $\sqrt{n}$, then the joint distribution of the normalized order statistics of the component sizes of $\hat{G}_n$ converges to the Poisson-Dirichlet(1) distribution as $n \to \infty$. Other applications and generalizations are also discussed in Section 3.

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