FINAL REPORT GR/R91106/01

SDEs and SPDEs: Numerical Methods and Applications 31 March – 4 April 2003 Gabriel Lord¹ Grant Lythe² Tony Shardlow³

A mix of seventy five mathematicians and physicists gathered in Edinburgh for the workshop. The programme consisted of 19 talks of 45 minutes and 19 talks of 25 minutes and allowed plenty of time for interaction and discussion. A timetable and list of participants is appended. The meeting was held at the King's Buildings site of the University of Edinburgh, which offered good facilities for lectures and for discussion. We would like to thank the ICMS for their organisational support of the meeting which was invaluable and appreciated by the participants.

The questionnaire and informal comments from participants during the meeting indicated that they were very satisfied with the lectures, format of the meeting and general organisation. It was noticeable that the mathematicians and physicists appreciated the opportunity to interact and bring the different communities together. A short selection of questionnaire responses is also appended.

Our aim was to bring together leading international researchers and young researchers from two different communities: from those working on the theory of SDEs and SPDEs and those involved in applications. We believe this was a very successful meeting that surpassed our objectives.

Scientific Content

The talks in this workshop focused on stochastic differential equations and their relation to applications. The mathematical talks included Gyongy, who discussed convergence of numerical approximation of SPDEs, Crauel, who gave an introduction to attractors in random dynamical systems, and Debussche/Mattingly, who both discussed invariant measures for the Navier Stokes equations. Talks on practical numerical methods included Moon/Lythe/Menozzi who all spoke on the approximation of exit problems, Buckwar who discussed numerical approximation of delay equations, and Higham who gave an overview of stability theory for SDEs.

Application areas were well represented. Fouque and Sircar discussed applications of SDEs in mathematical finance. Lindenberg and Timofeeva discussed models of chemical reactions and of calcium dynamics in cells. Sánchez/Sancho spoke on phase transitions from the physicist's viewpoint, whilst Bloemker/Maier-Paape dealt with the same issue in the Cahn-Hilliard equation by mathematical techniques. Further application areas include electrical circuits (Winkler), fluid dynamics (Lelievre) and molecular dynamics (Hairer, Moro).

A number of talks were given by Post-Doctoral RAs and Ph.D. students, including Timofeeva, Yan, Bloemker, Hairer, Lelievre.

Outcomes

The workshop resulted in significant interaction between groups and individuals in different research areas and from leading international experts to young UK researchers. This was achieved through the high quality talks and lively informal discussions. An especially successful aspect of the meeting was a significant interplay between the physicists and mathematicians. These observations are bourne out by the comments received: eg Debussche, Doering, Stuart, Lechner. Furthermore a number of new projects and contacts were initiated at the meeting: eg Crauel, Fouque, Sancho, Gyongy, McKane.

¹Department of Mathematics, Heriot-Watt University, Edinburgh EH14 4AS, gabriel@ma.hw.ac.uk.

²Department of Applied Mathematics, University of Leeds, Leeds LS2 9JT, grant@maths.leeds.ac.uk.

³Department of Mathematics, University of Manchester, Manchester M13 9PL. shardlow@maths.man.ac.uk

It is hoped a workshop with similar motivations will be held in the future. Wesley Petersen (Zürich) is currently looking at the possibility of organising such an event in Switzerland.

The meeting appears to have benefitted academics at all levels both within the UK and internationally.

Sample comments from UK academics

Bandrivskyy Excellent. I preliminary agreed a visit to Manchester and we probably will have a few joint seminars with McKane's group in the future.

Gyongy It was a very stimulating successful conference. ... fruitful discussions and have planned some collaborations and visits as a result of the meeting.

Mao Excellent.

McKane Good. I learnt a good deal from the talks ... I started a research project with Charles Doering as a direct consequence of attending the meeting.

Stuart I think the meeting was a great success ... Mixture of physics/numerical/analysis/applications worked very well and was the key to success.

Tretiakov I rate the academic value of the workshop as high, i.e. as well above average ... very nice idea to bring mathematicians and physicists together. I really did enjoy it.

Tribe Excellent. Some inspiring talks.

Veretennikov ...several talks were really excellent, and most of the talks were good and very good. Young researchers also got a place for talks.

SAMPLE COMMENTS FROM SOME UK POST-DOCS AND PHD STUDENTS

Bahar 5/5.

Hairer ... great academic value...

Lechner Excellent! Especially the interchange of physicists and mathematicians was of great importance for me.

Pavliotis ... great conference and I benefitted alot from it. I think there was an excellent mixture of Physics, SPDEs theory and numerics.

Shmatkov Undoubtedly the conference was helpful for me.

Tearne Very worthwhile... Am now planning a visit to Seville ...

Thanawalla Good scientific programme.

Wiberg I thought it was an excellent workshop. The overall quality of the talks was very high.

SAMPLE COMMENTS FROM INTERNATIONAL PARTICIPANTS

Buckwar The academic value was very high, a good mixture across topics ...and experience and age of the participants.

Crauel The academic value of the workshop was high... Altogether this made a very fruitful mixture. Completed work ... planned visits.

Debussche Excellent. I particularly like the fact that mathematicians from various fields and also non mathematicians were invited.

Doering Excellent, it involved a unique meeting of several scientific communities with many common interests but far too little contact and regular communication.... I made new contact with some Spanish scientists (Garcia and Moro).

Fouque Excellent. I met Peter Kloeden for the first time and I will most certainly visit him ... **Hausenblas** It was a good mixture between numerics and other topics.

Kloeden First rate. ... Having written papers on fractional Brownian motion several years ago, ... it was pleasing to hear of activity in this area in finance.

Kuske ... nice combination of mathematicians, physicists, chemits, computational types, etc.

Lindenberg I did learn a lot, especially from mathematicians that made an effort to make their material understandable to less mathematically expert listeners.... It was good to bring together mathematicians and physicists, ...

Mitsui Excellent. I enjoyed it very much. ... I could establish a collaboration plan ...

Moon It was very high quality meeting.

Sancho I found it very profitable for my research. I started discussions with Stuart and Pavliotis:. Schurz The academic value of the workshop is very high and I can see good impacts on our research ... I could initiate 2 new collaborations ...

Schwab Started discussing with Prof. Kloeden ...

Sircar Very stimulating mix of topics and applications.

Timetable

	Monday 31 March	
9:15-10:00	Doering	Duality and wavefront propagation in a stochastic
		Fisher-KPP equation
10:00-10:45	Kuske	Multi-scale analysis for stochastic bifurcations
10.45-11.15	Coffee	
11:15-12:00	Lindenberg	Sub-diffusion-limited reactions
12:00-12:25	Kloeden	Order barrier for numerical methods for monotone SDEs
12.25-2.00	Lunch	
2:00-2:45	Sancho	Multiplicative noise-induce phase transitions: Ito versus Stratonovich.
2:45-3:10	Maier-Paape	Spinodal Decomposition for the Cahn-Hilliard-Cook equation
3:10-3:35	Moro	On the mesoscopic description of microscopic particle models using stochastic differential equations.
3.35-4.15	Coffee	
4:15-4:40	Buckwar	Euler-Maruyama and Milstein approximations for
		stochastic functional differential equations with dis-
		tributed memory term
4:40-5.05	Moon	Adaptive Monte Carlo Algorithm for Killed Diffusion
5.05-5:30	Menozzi	New results on the approximation of killed processes
	Tuesday 1 April	
9:15-10:00	Van den Broeck	Brownian ratchets versus Langevin theory
10:00-10:45	Tretyakov	Quasi-symplectic methods for Langevin equations
10.45-11.15	Coffee	
11:15-12:00	McKane	Using optimal paths to calculate stochastic outcomes
12:00-12:25	Timofeeva	Sparks and waves in a stochastic fire-diffuse-fire model of Ca ²⁺ release
12.25-2.00	Lunch	
2:00-2:45	Schwab	Numerical Solution of elliptic PDEs with stochastic data
2:45-3:10	Yan	Finite element method for stochastic parabolic partial differential equations
3:10-3:35	Zouraris	Galerkin finite element approximations of Elliptic SPDEs
3.35-4.15	Coffee	
4:15-4:40	Lelievre	Micro-macro simulations of polymeric fluid
4:40-5:25	Stuart	Fitting SDEs to partially observed dynamical systems

	Wednesday 2 April	
9:15-10:00	Crauel	Random dynamical systems and SPDE
10.00-10.25	Langa	Asymptotic behaviour of nonautonomous Lotka-
		Volterra equations
10:25-10:50	Zhao	Stable/Unstable manifold theorem for SPDEs
10.50-11.15	Coffee	
11:15-12:00	Robinson	The structure of random attractors
12:00-12:25	Bloemker	Pattern formation below the threshold of stabil-
		ity
12.25-13.00	Hairer	Non-equilibrium stationary states
	Thursday 3 April	-
9:15-10:00	Fouque	Multi-scale stochastic volatility asymptotics
10:00-10:45	Sánchez	Reconciling analytics and numerics on the one-
		dimensional, stochastic sine-Gordon equation
10.45-11.15	Coffee	
11:15-12:00	Debussche	Ergodicity for the 3D stochastic Navier–Stokes
		equations
12:00-12:25	Hausenblas	Numerical approximation of the stochastic
		Navier Stokes equation
12.25-2.00	Lunch	-
2:00-2:45	Higham	Stability Issues in Long Term Simulations of
		Stochastic Differential Equations
2:45-3:10	Mao	Stability of numerical solutions to hybrid SDEs
3:10-3:35	Schurz	Stability of Some Numerical Methods Along
		Lyapunov Functionals for SDEs
3.35-4.15	Coffee	
4:15-4:40	Shevchenko	Rate of convergence of discrete-time approxima-
		tions for solutions of SDEs in Hilbert spaces
4:40-5:25	Mattingly	Ergodicity and scales in 2D Navier Stokes equa-
		tion
	Friday 4 April	
10:00-10:45	Sircar	Fractional Brownian Motion Approximation of
		Financial Markets with Inert Investors
10.45-11.15	Coffee	
11:15-12:00	Gyongy	On the numerical approximation of stochastic
		Partial Differential Equations
12:00-12:25	Winkler	Numerical Integration of Stochastic DAEs in
		Circuit Analysis
12.25-2.00	Lunch	
2:00-2:45	Lythe	Exponential time stepping for SDEs
2:45-3:10	Mannella	Exponentially fast MC simulations
3.55-4.15	Coffee	

Participants

Name	Institution
Ares, Saul	Universidad Carlos III de Madrid
Atkinson, Richard	Birmingham University
Bahar, Arifah	University of Strathclyde
Bandrivsky, Andriy	Lancaster University
Bloemker, Dirk	Institut für Matematik
Brzesniak, Zdzislaw	University of Hull
Buchmann, Fabian	ETH Zürich
Buckwar, Evelyn	Humboldt-Universität zu Berlin
Castro, Mario	Universidad Pontificia Comillas de Madrid
Chen, Xiaoli	University of Edinburgh
Crauel, Hans	Technische Universität Ilmenau
Crisan, Dan	Imperial College London
Debussche, Arnaud	ENS de Cachan
Doering, Charles	University of Michigan
Famelis, Ioannis	National Technical University of Athens
Fouque, Jean-Pierre	North Carolina State University
Gilsing, Hagen	Humboldt University Berlin
Gyongy, Istvan	Edinburgh University
Hairer, Martin	University of Warwick
Hausenblas, Erika	University of Salzburg
Higham, Desmond J	University of Strathclyde
Homer, Martin	University of Bristol
Horváth-Bokor, Rózsa	University of Veszprm
Kloeden, Peter	Johnann Wolfgang Goethe University
Kuske, Rachel	University of British Columbia
Langa, Jose A	University of Seville
Larsson, Stig	Chalmers University of Technology
Lechner, Patrick	University of Bath
Lekagul, Tanapan	Imperial College
Lelievre, Tony	Ecole Nationale des Ponts et Chaussés
Lindenberg, Katja	University of California San Diego
Lord, Gabriel J.	Heriot-Watt University
Lythe, Grant	University of Leeds
Maier-Paape, Stanislaus	RWTH-Aachen
Malham, Simon	Heriot-Watt University
Mandica, Simone	University of Bath
Mannella, Riccardo	University of Pisa
Mao, Xuerong	University of Strathclyde
Mattingly, Jonathan	IAS/Duke University
McKane, Alan J	University of Manchester
Menozzi, Stephane	Université Paris VI-CMAP
Mitsui, Taketomo	Nagoya University
Moon, Kyoung-Sook	Royal Institute of Technology Stockholm

Moro, Esteban Universidad Carlos III de Madrid

Onono, Etuka Strathclyde University
Pavliotis, Grigorios University of Warwick

Petersen, Wesley ETH Zürich

Quer, Lluís Universitat de Barcelona Raghib, Michael University of Glasgow

Retkute, Renata Galsgow Caledonian University

Robinson, James University of Warwick Sabanis, Sotirios University of Edinburgh

Sánchez, Angel Universidad Carlos III de Madrid

Sancho, Jose M Universitat de Barcelona Schurz, Henri Southern Illinois University

Schwab, Christoph
Seynaeve, Bert

ETH Zürich
K U Leuven

Shardlow, Tony University of Manchester

Shevchenko, Georgiy Kyiv Taras Shevchenko National University

Shmatkov, Anton

Sircar, Ronnie

Stuart, Andrew

Tearne, Oliver

Thanawalla, Rutang

Timofeeva, Yulia

Tretyakov, Michael

University of Edinburgh

Princeton University

University of Warwick

Heriot-Watt University

University of Loughborough

University of Leicester

Tribe, Roger Warwick Mathematics Institute

Van den Broeck, Christian
Veretennikov, Alexander

University of Limburg
University of Leeds

von Schwerin, Erik Royal Institute of Technology Stockholm

Wiberg, Petter University of Warwick Wiese, Anke Heriot-Watt University

Winkler, Renate
Yan, Yubin

Humboldt-Universitä zu Berlin
Chalmers University of Technology

Yuan, Chenggui Strathclyde University
Zhao, Huaizhong Loughborough University
Zouraris, Georgios University of the Aegean