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SEDENTARY GHOST POLES IN HIGHER DERIVATIVE GRAVITY

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Following Antoniadis and Tomboulis [1] we consider the gauge behaviour of the massive spin-2 ghost pole that appears in the propagator of higher derivative gravity theories. In contradistinction to [1] we observe that the pair of complex conjugate poles that appear in the resummed propagator are gauge independent. They are sedentary, that is, under a change in the gauge parameter they do not move. We derive this result using the ubiquitous Nielsen identities [11].

1. Introduction

In this paper we examine the gauge behaviour of the massive spin-two ghost pole that appears in higher derivative theories of gravity, observing that it is gauge parameter independent, contrary to the conclusions of [1]. We point out the analogy between the present case and two examples, that of the Higgs boson mass pole in a spontaneously broken abelian Higgs model and that of the scalar mass pole in a theory with a scalar field coupled to gravity. The latter of these provides a particularly close parallel with the behaviour of the ghost pole (or rather complex poles in the resummed propagator). Finally, we observe that the gauge-parameter independence, which we shall hereafter shorten to gauge independence of the poles means that Antoniadis and Tomboulis' formal unitarity proof will not work.

The plan of the paper is as follows: in sect. 2 we outline the motivation for higher derivative lagrangians and discuss the nature of the massive ghost pole, using a $1/N$ approximation with resummed propagators. We find that the O(1) approximation is gauge independent, (we shall see in sect. 3 why this must be so). The presentation follows that of ref. [1] closely, which the reader is encouraged to consult; in sect. 3 we discuss the Nielsen identities, which govern the gauge parameter dependence for the higher derivative theory and compare these with those for the abelian Higgs model and a gravity/scalar theory, concluding that the massive ghost poles are gauge parameter independent in higher orders too; in sect. 4 we summarize our results and discuss the consequences for the unitarity of the theory.

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2. The massive ghost pole and the $1/N$ expansion

The standard Einstein-Hilbert action for general relativity is given by

$$S = \int d^4x\sqrt{-g}\left(\frac{\gamma}{K^2}R + \frac{\lambda}{K^4}\right) \tag{2.1}$$

where $1/K$ is a mass scale, γ a numerical constant inserted for later convenience and λ the cosmological constant. This is perturbatively non-renormalizable because of the dimensionful coupling constant [2–4] and, when euclideanized, does not provide a bounded path integral. It has therefore been suggested that one should use the most general fourth-order action that is compatible with general covariance

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{\alpha^2}\left(R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}R^2\right) + \beta R^2 + \frac{\gamma}{K^2}R + \frac{\lambda}{K^4}\right], \tag{2.2}$$

where α and β are further numerical constants. Not only is (2.2) renormalizable [5] but it leads to a bounded euclidean path integral when $\beta < 0$ and is asymptotically free [6, 7]. Despite these promising attributes (2.2) apparently suffers from the malaise of non-unitarity, because the bare propagator of the graviton field $h^{\mu\nu}$, which we define by

$$\sqrt{-g}\, g^{\mu\nu} = \eta^{\mu\nu} + \alpha K h^{\mu\nu} \tag{2.3}$$

contains a negative residue spin-2 pole (i.e. a ghost). Unlike the more familiar Faddeev-Popov [8] ghosts which cancel with unphysical components of the original fields, and hence preserve unitarity, the massive spin-2 ghost appears on its own.

For the action in (2.2) the spin-2 part of the graviton propagator is given by [1],

$$D^{(2)}_{\mu\nu\kappa\lambda} = \frac{2}{\alpha^2\gamma}\left[\frac{1}{q^2} - \frac{1}{q^2 - \alpha^2\gamma/K^2}\right]P^{(2)}, \tag{2.4}$$

where the spin-2 projector is (in n dimensions)

$$P^{(2)}_{\mu\nu\kappa\lambda} = \tfrac{1}{2}\left(\theta_{\mu\nu}\theta_{\kappa\lambda} + \theta_{\mu\lambda}\theta_{\nu\kappa}\right) - \frac{1}{n-1}\theta_{\mu\nu}\theta_{\kappa\lambda} \tag{2.5}$$

with

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \tag{2.6}$$

The first term in the brackets in (2.4) is the customary graviton pole at $k^2 = 0$ and the second, with negative residue, at $q^2 = \alpha^2\gamma/K^2$ is the massive ghost. The massive ghost is, however, unstable because the theory contains couplings of the form

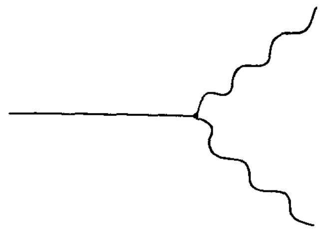

Fig. 1. A massive ghost/graviton vertex. The straight line represents a massive ghost and the wavy line a graviton.

(fig. 1) [1] through which it can decay into gravitons. We should therefore not attempt to use the massive part of (2.4) as our ghost propagator but follow the remedy detailed in [1] (and illustrated by an illuminating example taken from Veltman).

The recipe is to use a modified perturbation theory without self-energy parts for the unstable particle, in which the bare propagator for this particle $D_0(q^2)$, is replaced by the complete propagator

$$D(q^2) = \left(D_0(q^2)^{-1} - \pi(q^2)\right)^{-1}, \tag{2.7}$$

where $\pi(q^2)$ is the sum of all the $1PI$ self-energy parts. We must, of course, calculate $D(q^2)$ in some approximation scheme to find the nature of our resummed pole and discuss its gauge properties. The method chosen by Antoniadis and Tomboulis, following the earlier work by Tomboulis [6, 10] on the renormalizability of gravity, was to couple N fermionic fields to the gravitational action (2.2) and carry out a $1/N$ expansion. The fermionic action was given by

$$S_{\mathrm{F}} = \int \mathrm{d}^4x\, \bar{\psi}_a e^{\mu}_m \gamma^m \cdot \tfrac{1}{2} i \overleftrightarrow{\nabla}_{\mu} \psi_a, \tag{2.8}$$

where μ is a world index, m a tangent space index, $a = 1 \dots N$, e^{μ}_m is the vierbein field and

$$\overleftrightarrow{\nabla}_{\mu} = \overrightarrow{\nabla}_{\mu} - \overleftarrow{\nabla}_{\mu}, \tag{2.9a}$$

$$\overrightarrow{\nabla}_{\mu} = \overrightarrow{\partial}_{\mu} + \tfrac{1}{8}[\gamma^m, \gamma^n] e^{\nu}_m \overrightarrow{\partial}_{\mu} e_{n\nu}. \tag{2.9b}$$

We can then expand quantities in our theory in powers of $1/N$ keeping $\alpha^2 N$, β/N and γ/N fixed. The lowest order (in $1/N$) approximation to the complete propagator will be given by (fig. 2). As we can see, at this order only the fermionic loops contribution to the graviton self-energy is required.

If we use dimensional regularization and an n-dimensional counterterm we find that the spin-2 part of the graviton propagator becomes, in this lowest-order $1/N$

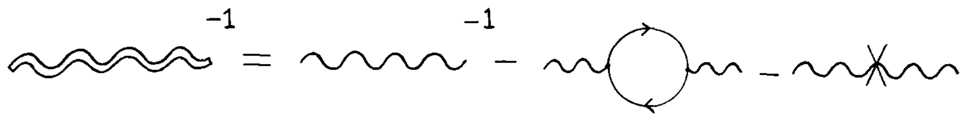

Fig. 2. The leading order in $1/N$ contribution to the complete propagator. The thick wavy line represents the complete propagator, the thin wavy line the bare propagator, the straight arrowed line a fermionic propagator and the cross a counterterm.

approximation

$$D^{(2)}_{\mu\nu\kappa\lambda} = \frac{2iP^{(2)}_{\mu\nu\kappa\lambda}}{q^2\left(\alpha^2\gamma - q^2K^2 - \left(N\alpha^2/20(4\pi)^2\right)q^2K^2\ln(-q^2/\mu^2)\right)}, \tag{2.10}$$

where (2.10) is written in Minkowski space. If we define

$$\Lambda = \mu\exp\left(-\frac{20(4\pi)^2}{\alpha^2 N}\right), \tag{2.11}$$

this may be written as

$$D^{(2)}_{\mu\nu k\lambda} = \frac{2iP^{(2)}_{\mu\nu\kappa\lambda}}{q^2\left(\alpha^2\gamma - \frac{1}{20}\left(N\alpha^2/(4\pi)^2\right)q^2K^2\ln(-q^2/\Lambda^2)\right)}. \tag{2.12}$$

We observe, in addition to the graviton pole at $q^2=0$, a pair of complex conjugate poles at $q^2 = r\mathrm{e}^{\pm i\theta}$, where $r \sim \gamma/NK^2$. The "pole" for the unstable massive ghost has thus split into a pair of complex conjugate poles in the physical riemannian energy sheet. At this order unitarity is satisfied [1] and, as we can see by (2.12), the poles are gauge independent.

In higher orders one will have to consider graviton loops containing the complex ghost poles so there is the possibility of gauge dependence for the pole values and a loss of unitarity. We shall discuss the former at some length in the following section and outline its relation with the latter. As explicit calculations would be very onerous we resort to the Ward identities.

3. Gauge-dependence and Nielsen identities

In a gauge theory a pole mass m^2_{pole} will be gauge-independent if

$$\left.\frac{\mathrm{d}\,m^2_{\text{pole}}}{\mathrm{d}\,\xi}\right| = 0, \tag{3.1}$$

where ξ is any gauge parameter and we have assumed that we are calculating m^2 from Γ, the effective action, so the vertical bar represents setting all the semiclassical fields, denoted generically here by $\bar{\phi}$

$$\bar{\phi} = \frac{\delta W[J]}{\delta J} \tag{3.2}$$

to their vacuum expectation values. Now, $\bar{\phi}$ is calculated using the gauge fixed action

$$\exp\left(\frac{i}{\hbar} W[J]\right) = \int [\mathrm{D}\,\Phi] \exp\left[\frac{i}{\hbar}\left(S + S_{\text{gauge-fixing}} + \int J\phi\right)\right] \tag{3.3}$$

and m^2 will in general depend on the vacuum expectation values of ϕ, so m^2 may contain an implicit ξ dependence through $\bar{\phi}$. We can thus write (3.1) as

$$\left.\frac{\partial m^2_{\text{pole}}}{\partial \xi}\right| + \left.\frac{\partial m^2_{\text{pole}}}{\partial \bar{\phi}}\right|_{\xi} \frac{\mathrm{d}\,\bar{\phi}}{\mathrm{d}\,\xi} = 0, \tag{3.4}$$

where ∂ is an explicit derivative. The criterion for gauge independence when the second term in (4.4) is non-zero is thus not simply

$$\left.\frac{\partial m^2_{\text{pole}}}{\partial \xi}\right| = 0. \tag{3.5}$$

Nielsen [11], using the BRS [12] invariance of the theory was able to derive identities of the form (3.4) for physical quantities (such as pole masses) in a spontaneously broken abelian Higgs model. In his notation he found

$$\xi \left.\frac{\partial m^2_{\text{pole}}}{\partial \xi}\right| + C(\bar{\phi}, \xi) \left.\frac{\partial m^2_{\text{pole}}}{\partial \bar{\phi}}\right|_{\xi} = 0, \tag{3.6}$$

where $C(\bar{\phi}, \xi)$ is now explicitly calculable as a field theoretic expression.

The simplest method of deriving the Nielsen identities is to extend the BRS invariance of the theory to include a transformation acting on the gauge parameter we are interested in [13, 14], i.e.

$$\delta_{\text{BRS}}\xi = \varepsilon\chi, \tag{3.7}$$

where ε is the anticommuting BRS parameter and χ is also anticommuting. We must also include appropriate compensating terms in the action to maintain the BRS invariance.

$$S_\chi = \int \chi P, \tag{3.8}$$

where P is some dimension 3, ghost number-1 expression local in the fields. The Ward identities for the theory are now [13]

$$S(\Gamma)+\chi\frac{\partial\Gamma}{\partial\xi}=0, \tag{3.9}$$

where

$$S(\Gamma)=\int\left(\frac{\delta\Gamma}{\delta\bar{\phi}}\frac{\delta\Gamma}{\delta\rho}+\frac{\delta\Gamma}{\delta\rho_{c}}\frac{\delta\Gamma}{\delta\bar{C}}+\bar{B}\frac{\delta\Gamma}{\delta\bar{C}^{*}}\right). \tag{3.10}$$

$\bar{\phi}$ is a physical field, ρ a source which couples to $\delta_{\mathrm{BRS}}\phi$, $\bar{C}$ a ghost, ρ_c a source which couples to $\delta_{\mathrm{BRS}}C$, $\bar{C}^*$ an antighost and $\bar{B}$ an auxiliary field used to fix the gauge. Schematically

$$S_{\text{gauge-fixing}}=\int\left(\tfrac{1}{2}\xi B^{2}+BF+\text{ghosts}\right), \tag{3.11}$$

where F is the gauge-fixing function. The Nielsen identity is obtained by differentiating (3.9) with respect to χ and setting $\chi=0$

$$S\left(-\frac{\partial\Gamma}{\partial\chi}\right)+\frac{\partial\Gamma}{\partial\xi}=0. \tag{3.12}$$

To obtain the equivalent for m^2_{pole} we differentiate (3.12) twice with respect to $\bar{\phi}$ and set all the fields equal to their vacuum expectation values.

To make these considerations a little more explicit, consider the abelian Higgs model with an R_ξ type gauge-fixing, for which we take the lagrangian to be

$$\begin{aligned}\mathscr{L}=&-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{1}{2}(\partial_{\mu}\phi_{i})(\partial^{\mu}\phi^{i})\\&-e\epsilon_{ij}(\partial_{\mu}\phi_{i})\phi_{j}A^{\mu}+\tfrac{1}{2}e^{2}A^{2}\phi^{2}+\tfrac{1}{2}m^{2}\phi^{2}\\&-(\lambda/4!)\phi^{4}+\tfrac{1}{2}\xi B^{2}+B\left(\partial_{\mu}A^{\mu}+e\epsilon_{ij}\xi\langle\phi_{j}\rangle\phi_{i}\right)\\&+\partial_{\mu}C^{*}\partial^{\mu}C-e^{2}\xi C^{*}C\epsilon_{ij}\epsilon_{il}\langle\phi_{l}\rangle\Phi_{j},\end{aligned} \tag{3.13}$$

where ϕ now represents only the scalar fields of the theory, $i=1,2$, $\epsilon_{12}=-\epsilon_{21}=1$, $\epsilon_{11}=\epsilon_{22}=0$ and B is the auxiliary field which may be eliminated via its equations of motion to obtain the customary gauge fixing term

$$-\frac{1}{2\xi}\left(\partial_{\mu}A^{\mu}+e\epsilon_{ij}\xi\langle\phi_{j}\rangle\phi_{i}\right)^{2}. \tag{3.14}$$

If we introduce the extra BRS transform $\delta\xi = \epsilon\chi$, we need to add

$$\mathscr{L}_\chi = \chi\left(\tfrac{1}{2}C^*B + C^*\epsilon_{ij}\langle\phi_j\rangle\phi_i\right) = \chi P, \tag{3.15}$$

to the lagrangian. The equivalent of (3.12) is then

$$\int\left(\frac{\delta\Gamma}{\delta\bar{A}_\mu}\frac{\delta\Gamma(P)}{\delta\rho_\mu} + \frac{\delta\Gamma}{\delta\bar{\phi}_i}\frac{\delta\Gamma(P)}{\delta L_i} + \bar{B}\frac{\delta\Gamma(P)}{\delta\bar{C}^*} + \frac{\partial\bar{B}}{\partial\chi}\frac{\delta\Gamma}{\delta\bar{C}^*}\right) - \frac{\partial\Gamma}{\partial\xi} = 0, \tag{3.16}$$

where L_i is a source that couples to $\delta_{\mathrm{BRS}}\phi_i$ and (after eliminating B),

$$P = -\frac{C^*}{2\xi}\left(\partial_\mu A^\mu - e\xi\epsilon_{ij}\langle\phi_j\rangle\phi_i\right). \tag{3.17}$$

If we take the 1 direction as the physical Higgs scalar, differentiating (3.16) twice with respect to $\bar{\phi}_1$ gives the Nielsen identity for the inverse propagator, which at the pole reads

$$\left(\frac{\partial}{\partial\xi} - \int\frac{\delta\Gamma(P)}{\delta L_1}\frac{\delta}{\delta\bar{\phi}_1}\right)\frac{\delta^2\Gamma}{\delta^2\bar{\phi}_1} = 0, \tag{3.18}$$

which implies

$$\left(\frac{\partial}{\partial\xi} - \int\frac{\delta\Gamma(P)}{\delta L_1}\frac{\delta}{\delta\bar{\phi}_1}\right)m^2_{\mathrm{pole}} = 0. \tag{3.19}$$

This is just Nielsen's result with

$$C(\bar{\phi},\xi) = -\xi\int\frac{\delta\Gamma(P)}{\delta L_1}. \tag{3.20}$$

One can verify (3.19) order by order in a loop expansion, for example [14,15]. $C(\bar{\phi},\xi)$ receives its first contribution at one-loop order because it has two bilinear field insertions

$$\begin{aligned} C(\bar{\phi},\xi) = -i\hbar\int&\langle 0|T(i/\hbar)^2 \\ &\times\left[-\tfrac{1}{2}C^*(x)\left(\partial_\mu A^\mu(x) - e\xi\epsilon_{ij}\langle\phi_j\rangle\phi_i(x)\right)eC(0)\phi_2(0)\right] \\ &\times\exp\left(\frac{i}{\hbar}S\right)|0\rangle \end{aligned} \tag{3.21}$$

and one can see that both the first and second terms in

$$\xi \frac{\partial m^2_{\text{pole}}}{\partial \xi}\bigg| + C(\bar{\phi}, \xi) \frac{\partial m^2_{\text{pole}}}{\partial \bar{\phi}}\bigg|_{\xi} = 0 \tag{3.6}$$

contribute. (3.6) means that under a change $\xi \to \xi + \delta\xi$, $\langle\phi\rangle$ changes to $\langle\phi\rangle + (C(\langle\phi\rangle, \xi)/\xi)\delta\xi$ to maintain the gauge independence. Although the results are for bare quantities they translate directly to the renormalized case for a gauge invariant regularization scheme, such as dimensional regularization.

As we can write

$$\langle\phi\rangle + \frac{C(\langle\phi\rangle, \xi)}{\xi}\delta\xi = \Omega\langle\phi\rangle, \tag{3.22}$$

we see that the change in $\langle\phi\rangle$ induced by the change in ξ is a global rescaling and corresponds to a change in our energy scale ($[\langle\phi\rangle] = \text{mass}$) which does not affect observable physics*. If we had considered an unphysical pole, such as the longitudinal part of the gauge-boson propagator we would have found

$$\xi \frac{\partial m^2_{\text{pole}}}{\partial \xi}\bigg| + C(\bar{\phi}, \xi) \frac{\partial m^2_{\text{pole}}}{\partial \bar{\phi}_1}\bigg|_{\xi} = \int d^4x\, d^4z\, \Delta(x, z), \tag{3.23}$$

where $\Delta(x, z)$ is some non-zero field theoretic expression. (3.23) means

$$\frac{d\, m^2_{\text{pole}}}{d\,\xi} \neq 0 \tag{3.24}$$

and, as we know that we can define a gauge invariant S-matrix, such gauge-dependent poles disappear from the spectrum.

Another example of a gauge-independent mass pole is given by a scalar field coupled to Einstein gravity [16]

$$S = \int d^4x\sqrt{-g}\Bigg[\left(\frac{\gamma}{K^2}R + \frac{\lambda}{K^4} + \tfrac{1}{2}(\nabla_\mu\phi)(\nabla_\nu\phi)g^{\mu\nu} - \tfrac{1}{2}m^2\phi^2\right)$$
$$+ \tfrac{1}{2}\xi B_\mu B_\nu \eta^{\mu\nu} + B_\mu F^\mu + C_\mu^* M^\mu_\nu C^\nu\Bigg], \tag{3.25}$$

* In this context it is important to note that the Nielsen identity reads

$$\left(\frac{\partial}{\partial\xi} + C(\bar{\phi}, \xi)\frac{\partial}{\partial\bar{\phi}}\right)A = 0,$$

where A is any physical quantity and C is the same for all of them. One does not, therefore, find inconsistently different scalings for different quantities.

where C^{ν} is the ghost, C_{μ}^{*} the antighost, B_{μ} the auxiliary field, F^{μ} the gauge-fixing function and M the Faddeev-Popov matrix. By following the same steps that led to (3.19) we find

$$\left(\frac{\partial}{\partial \xi} - \int \frac{\delta \Gamma(P)}{\delta L} \frac{\delta}{\delta \bar{\phi}} - \int \frac{\delta \Gamma(P)}{\delta \rho_{\mu\nu}} \frac{\delta}{\delta \bar{h}^{\mu\nu}} \right) m_{\text{pole}}^{2} = 0 , \tag{3.26}$$

where m_{pole}^{2} is the pole mass squared of the scalar field, L couples to the BRS variation of ϕ

$$\delta_{\text{BRS}} \phi = -\varepsilon C^{\alpha} \partial_{\alpha} \phi \tag{3.27}$$

and $\rho_{\mu\nu}$ couples to the BRS variation of $h^{\mu\nu}$

$$\delta_{\text{BRS}} h^{\mu\nu} = \varepsilon \left[\partial^{\mu} C^{\nu} + \partial^{\nu} C^{\mu} - \eta^{\mu\nu} (\partial_{\alpha} C^{\alpha}) - \partial_{\alpha} (C^{\alpha} h^{\mu\nu}) + h^{\alpha\nu} \partial_{\alpha} C^{\mu} + h^{\alpha\mu} \partial_{\alpha} C^{\nu} \right] , \tag{3.28}$$

where, for this theory, we have defined $\sqrt{-g}\, g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}$. As $\langle \phi \rangle = 0$ here the second term in (3.26) will vanish [16] but the third need not because we can have

$$\int \frac{\delta \Gamma(P)}{\delta \rho_{\mu\nu}} = -\tau (\bar{\phi}, \bar{h}, \xi) \eta^{\mu\nu} \tag{3.29}$$

and explicit calculations show that $\tau \neq 0$ [17]. We can write (3.26) as

$$\left. \frac{\partial m_{\text{pole}}^{2}}{\partial \xi} \right| + \tau \eta^{\mu\nu} \left. \frac{\partial m_{\text{pole}}^{2}}{\partial \bar{h}^{\mu\nu}} \right|_{\xi} = 0 . \tag{3.30}$$

This lends itself to the same sort of interpretation as (3.6), under a change $\xi \to \xi + \delta\xi$ we preserve gauge independence with the compensating change

$$\langle h^{\mu\nu} \rangle \to \langle h^{\mu\nu} \rangle + \tau \eta^{\mu\nu} \delta \xi . \tag{3.31}$$

However, we have defined

$$\langle \sqrt{-g}\, g^{\mu\nu} \rangle = \eta^{\mu\nu} ,$$

so $\langle h^{\mu\nu} \rangle = 0$ and we re-interpret (3.31) as a (global) rescaling of $\eta^{\mu\nu}$ (cf. (3.22)).

$$\eta^{\mu\nu} \to (1 + \tau \delta \xi) \eta^{\mu\nu} . \tag{3.32}$$

In this case we have changed our length scale, which again has no effect on observable physics.

After consideration of these two examples we now arrive at the pole we were originally interested in, the massive ghost pole in higher derivative gravity. Before writing down the Nielsen identity we observe that the complex nature of the pole(s) is not a problem because all we require is a zero of the inverse propagator, whether this is at real or complex momenta is irrelevant. We also observe that our use of a $1/N$ expansion scheme presents no problems (one can work order by order as with the ordinary loop expansion which is an expansion in $\hbar$). The BRS transformations for (2.2) in the conventions of [1] are given by

$$\begin{aligned}\delta_{\mathrm{BRS}} h^{\mu\nu} = {} & \varepsilon K\left[\partial^\mu C^\nu + \partial^\nu C^\mu - \eta^{\mu\nu}(\partial_\alpha C^\alpha)\right] \\ & + \varepsilon\alpha K^2\left[(\partial_\alpha C^\mu) h^{\alpha\nu} + (\partial_\alpha C^\nu) h^{\alpha\mu} - (\partial_\alpha h^{\mu\nu}) C_\alpha - h^{\mu\nu}(\partial_\alpha C^\alpha)\right], \end{aligned} \tag{3.33a}$$

$$\delta_{\mathrm{BRS}} C^\alpha = -\varepsilon K^2\, \partial_\beta C^\alpha C^\beta, \tag{3.33b}$$

$$\delta_{\mathrm{BRS}} \bar{C}_\alpha = \varepsilon K B_\alpha, \tag{3.33c}$$

where the difference between (3.33a) and (3.28) comes from the different definitions of $h^{\mu\nu}$ and where we have chosen some gauge-fixing similar to that in (3.25). The N fermionic fields also have the appropriate BRS transformation properties.

The procedure that leads to (3.19) and (3.26) then gives us

$$\left(\frac{\partial}{\partial\xi} - \int \frac{\delta\Gamma(P)}{\delta\rho_{\alpha\beta}} \frac{\delta}{\delta\bar{h}^{\alpha\beta}}\right) \frac{\delta^2\Gamma}{\delta\bar{h}^{\mu\nu}\delta\bar{h}^{\kappa\lambda}} = 0, \tag{3.34}$$

where this time we have differentiated with respect to $\bar{h}$ rather than $\bar{\phi}$. We have dropped the contribution of the N matter fields to (3.34) as it vanishes for similar reasons to the second term in (3.26). We can extract the massive spin two ghost poles from (3.34) by multiplying by $P^{(2)\mu\nu\kappa\lambda}$ and setting $q^2 = M^2$ or M^{2*}, where M^2 and M^{2*} are the complex conjugate pole masses. This gives

$$\left(\frac{\partial}{\partial\xi} - \int \frac{\delta\Gamma(P)}{\delta\rho_{\alpha\beta}} \frac{\delta}{\delta\bar{h}^{\alpha\beta}}\right) M^2 = 0 \tag{3.35}$$

and the equivalent for M^{2*}. Antoniadis and Tomboulis calculated the lowest order contribution to

$$-\int \frac{\delta\Gamma(P)}{\delta\rho_{\alpha\beta}} = \tau'\eta^{\alpha\beta} \tag{3.36}$$

in their $1/N$ expansion, using the gauge fixing function

$$F^\mu = e(\partial^2)\left(\partial_\nu h^{\mu\nu} - \frac{\zeta}{e(\partial^2)}\partial^\mu h^\lambda_\lambda\right), \tag{3.37}$$

where

$$e(\partial^2) = K^2\left(\partial^2 - \frac{1}{K^2}\right), \tag{3.38}$$

which was chosen to provide both good IR and UV properties. They found

$$\tau' = \frac{i\alpha}{32\pi^2}\frac{1}{K}\frac{1}{1-2\zeta/e(q^2)}. \tag{3.39}$$

They also considered the gauge with $\zeta/e(q^2)$ replaced by ζ and found

$$\tau' = \frac{i\alpha}{32\pi^2}\frac{1}{K}\frac{1}{(2\zeta+1)^2}. \tag{3.40}$$

We can see from (3.39) and (3.40) why the O(1) pole masses we found in sect. 2 were gauge-independent. Remembering that

$$\sqrt{-g}\,g^{\mu\nu} = \eta^{\mu\nu} + \alpha K h^{\mu\nu} \tag{2.3}$$

we can absorb αK into $\delta/\delta\bar{h}^{\alpha\beta}$ to see that τ', with a factor of α^2 in front, is of $O(1/N)$. The O(1) version of (3.35) is thus

$$\frac{\partial M^{2\,(O(1))}}{\partial\xi} = 0 \tag{3.41}$$

and similarly for M^{2*}. Both terms in (3.35) will contribute in higher orders and we find behaviour exactly analogous to our second example, that of the gravity/scalar system. A change $\xi \to \xi + \delta\xi$ induces a global scaling

$$\eta^{\mu\nu} \to (1 + \tau'\delta\xi)\eta^{\mu\nu} \tag{3.42}$$

which is a change in our length scale. Once again our pole mass(es), in this case $M^{2(*)}$ are left unchanged and are therefore gauge-independent.

The independence of the pole masses from the second gauge parameter ζ that we introduced could just as easily be demonstrated by using the BRS transform

$$\delta\zeta = \varepsilon\beta \tag{3.43}$$

and the appropriate compensating terms. We would then find an equation of the form

$$\frac{\partial M^2}{\partial\zeta} + \tau''\eta^{\alpha\beta}\frac{\partial M^2}{\partial\bar{h}^{\alpha\beta}} = 0, \tag{3.44}$$

where

$$\tau''\eta^{\alpha\beta} = -\int \frac{\partial}{\partial\beta}\frac{\delta\Gamma}{\delta\rho_{\alpha\beta}}. \tag{3.45}$$

4. Discussion

The case for unitarity of higher derivative gravity theories that was put forward in ref. [1] rested on two foundations. The first was the instability of the massive ghost and the consequent need to employ a complete propagator perturbation expansion, which leads to a pair of complex poles appearing in the first sheet of the complex energy plane. The second was that these poles were gauge-dependent and, by arguments analogous to those used to demonstrate the disappearance of gauge dependent real poles, they did not contribute to physical amplitudes. The crucial factor in any such demonstration is the ability to move the unphysical poles around by varying the gauge parameters. We have seen, however, that the massive ghost poles are gauge-independent in a manner reminiscent of physical poles in a spontaneously broken abelian Higgs model and a gravity/scalar theory, so the formal proof of unitarity contained in ref. [1] is incorrect.

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