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NIELSEN IDENTITIES IN THE 't HOOFT GAUGE

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We derive Nielsen identities for the gauge invariance of the effective potential and the physical Higgs mass in the 't Hooft gauge and verify them to one-loop level. In addition to the standard derivation we also show how they may be derived by considering a BRS transformation which acts on the gauge parameter.

1. Introduction

The abelian Higgs model forms a useful laboratory for exploring the gauge dependence of the effective potential. A recent, comprehensive paper by Aitchison and Fraser [1] (hereafter AF) provided explicit one-loop calculations in a restricted class of 't Hooft-like gauges, extending the earlier more formal results of Nielsen [2], who worked in a Fermi gauge. The identities derived covered both the gauge dependence of the effective potential and that of the physical Higgs meson mass. They are of the general form shown below:

$$\xi \frac{\partial V}{\partial \xi} + C(\phi, \xi) \frac{\partial V}{\partial \phi} = 0, \tag{1}$$

$$\xi \frac{\partial m^2}{\partial \xi} + C(\phi, \xi) \frac{\partial m^2}{\partial \phi} = 0, \qquad \text{at} \quad \phi = \phi_0, \tag{2}$$

where V is the effective potential, m the physical Higgs mass, ϕ the c-number classical field, ξ the gauge parameter, ϕ_0 the value of ϕ at the effective potential minimum.

The object $C(\phi, \xi)$ is a field theoretic expression which can be calculated in some expansion scheme. The content of these identities is simply that the implicit (via ϕ) and explicit gauge dependence of V and m^2 cancel out, which means that the minima of the effective potential and the physical Higgs mass are gauge-parameter independent. In their paper AF worked with the gauge-fixing term

$$-\frac{1}{2\xi}\left(\partial_\mu A^\mu + e v_i \Phi_i\right)^2, \tag{3}$$

where Φ is the quantum scalar field and A^μ the quantum gauge field.

They did not use a 't Hooft gauge because they were unable to derive the appropriate Nielsen identity and they conjectured that the derivation was impossible for any gauge-fixing term that contained explicit ξ-dependence inside the brackets. We show here that this is possible for gauges of the form

$$-\frac{1}{2\xi}\left(\partial_\mu A^\mu + e\xi v_i \Phi_i\right)^2. \tag{4}$$

This contains, as a special case, the 't Hooft gauge

$$-\frac{1}{2\xi}\left(\partial_\mu A^\mu + e\xi \varepsilon_{ij}\phi_{j0}\Phi_i\right)^2, \qquad \varepsilon_{ij} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{5}$$

One must exercise a modicum of care in using the 't Hooft gauge because the minimum field expectation value that is being calculated is introduced into the lagrangian in the gauge-fixing term. The field shift that is carried out to calculate the effective potential via Jackiw's method [3] is not to be confused with the ϕ_{j0} appearing in the gauge-fixing term. It is only at the potential minimum that the two are identical.

2. Derivation of the Nielsen identity

The reader is encouraged to consult AF's paper, as we follow their methods closely, but we summarize the relevant details in appendix A. The lagrangian we consider is

$$\begin{aligned}
\mathcal{L}(x) = &-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}(\partial_\mu\Phi_i)(\partial^\mu\Phi_i) \\
&- e\varepsilon_{ij}(\partial_\mu\Phi_i)\Phi_j A^\mu + \tfrac{1}{2}e^2A^2\Phi^2 - \frac{1}{2\xi}\left(\partial_\mu A^\mu + e\xi v_i\Phi_i\right)^2 \\
&+ \tfrac{1}{2}\mu^2\Phi^2 - \frac{\lambda}{4!}\Phi^4 + \partial_\mu\psi^*\partial^\mu\psi - e^2\psi^*\psi\varepsilon_{ij}\xi v_i\Phi_j, \\
&\Phi^2 = \Phi_1^2 + \Phi_2^2, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\end{aligned} \tag{6}$$

The BRS [4] transforms for this lagrangian are

$$\begin{aligned}
&\delta A_\mu = \varepsilon\partial_\mu\psi, \qquad \delta\psi = 0, \\
&\delta\psi^* = -\frac{\varepsilon}{\xi}\left(\partial_\mu A^\mu + e\xi v_i\Phi_i\right), \\
&\delta\Phi_i = \varepsilon e\varepsilon_{ij}\psi\Phi_j.
\end{aligned} \tag{7}$$

The Nielsen identities are derived by performing a BRS transform on the augmented generating functional (appendix A)

$$\tilde{Z}_k = \int [D\Phi_\alpha] \exp\left(i \int \mathrm{d}^4x \left(\mathcal{L}(x) + K_\alpha Q_\alpha + J_\alpha \Phi_\alpha + hO \right) \right)$$

$$= \int [D\Phi_\alpha] \exp(i\tilde{S}_k), \tag{8}$$

where $\Phi_\alpha(x)$ is the generic field, $J_\alpha(x)$ the generic source, $Q_\alpha(x)$ the BRS charge, $\tilde{}$ denotes the presence of O, k denotes the presence of $K_\alpha Q_\alpha$, $h(x)$ the source for O. We choose the operator O as

$$O = -\tfrac{1}{2}\psi^*\left(\partial_\mu A^\mu - e\xi v_i \Phi_i\right). \tag{9}$$

Consider the BRS transform of O: $\delta O = \varepsilon \overline{O}$,

$$\delta O = \varepsilon \left[\frac{1}{2\xi}\left((\partial_\mu A^\mu)^2 - e^2\xi^2 (v_i\Phi_i)^2 \right) + \tfrac{1}{2}\psi^* \Box \psi - \tfrac{1}{2} e^2 \xi \psi^* \psi \varepsilon_{ij} v_i \Phi_j \right]. \tag{10}$$

Using the equations of motion for the ghost field we obtain

$$\delta O = \varepsilon \left[\frac{1}{2\xi}\left((\partial_\mu A^\mu)^2 - e^2\xi^2 (v_i\Phi_i)^2 \right) + \tfrac{1}{2}\psi^* \eta - e^2 \xi \psi^* \psi \varepsilon_{ij} v_i \Phi_j \right]. \tag{11}$$

One now compares this with

$$\xi \frac{\partial \mathcal{L}}{\partial \xi} = \left[\frac{1}{2\xi}\left((\partial_\mu A^\mu)^2 - e^2\xi^2 (v_i\Phi_i)^2 \right) - e^2 \xi \psi^* \psi \varepsilon_{ij} v_i \Phi_j \right]. \tag{12}$$

We see that, to within a term which vanishes when we consider the effective potential, $\xi \partial\mathcal{L}/\partial\xi$ has been expressed in terms of a BRS-transformed operator. This enables us to follow AF's proof through to arrive at the embryonic first Nielsen identity

$$\xi \frac{\partial V(\phi_i, \xi)}{\partial \xi} - \int \mathrm{d}^4x \, \frac{\partial \Gamma(O(x))}{\delta K_j(0)} \frac{\partial V(\phi_i, \xi)}{\partial \phi_j} = -\frac{e\xi v_i \phi_i}{\Omega} \int \mathrm{d}^4x \int \mathrm{d}^4z \, \frac{\delta \Gamma(O(x))}{\delta \psi_c^*(z)}, \tag{13}$$

where Ω is the spacetime volume, ϕ_i the classical field, $\Gamma(O)$ the 1PI generating functional with one insertion of O.

With a gauge of the form above (4) there are, however, two stationary points [5] of the effective potential, only one of which satisfies $v_i \phi_i = 0$, which would cast (13) into the required form. At tree level we have

$$\phi_{i0} = \varepsilon_{ij} \frac{v_j}{|v|} \left(\frac{6\mu^2}{\lambda} \right)^{1/2},$$

$$\phi_{i0} = \frac{v_i}{|v|} \left[\frac{6}{\lambda} \left(\mu^2 - \xi e^2 v^2 \right) \right]^{1/2}. \tag{14}$$

However, as has been elucidated by Fukuda and Kugo [6], the second solution is spurious, in that it does not correspond to a vanishing expectation value of the gauge field. This can be gauged away only at the expense of an x-dependent vacuum expectation value for ϕ_i, which takes us out of the context of the effective potential.

Fukuda and Kugo have further shown that the direction of spontaneous symmetry breaking is unchanged by higher-order corrections. Thus by choice we can take

$$v_i = v e_i, \qquad e_i = (0,1),$$

$$\phi_{i0} = \phi_0 \eta_i, \qquad \eta_i = (1,0), \tag{15}$$

discarding the spurious solution and considering the effective potential as a function of $\phi = \phi_1$ only. Writing $C(\phi, \xi) = -\int \mathrm{d}^4x\, \delta\Gamma(O(x))/\delta K(0)$,

$$\xi \frac{\partial V(\phi, \xi)}{\partial \xi} + C(\phi, \xi) \frac{\partial V(\phi, \xi)}{\partial \phi} = 0. \tag{1}$$

3. One-loop verification of effective potential identity

Expanding (1) order by order in $\hbar$ we obtain

$$\xi \frac{\partial V^{(1)}}{\partial \xi} + C^{(1)}(\phi, \xi) \frac{\partial V^{(0)}}{\partial \phi} = 0. \tag{16}$$

The superscripts denote the order in $\hbar$. We can calculate both $V^{(1)}$ and $C^{(1)}$ by Jackiw's functional method. Starting with the latter,

$$C(\phi, \xi) = -i\hbar \int \mathrm{d}^4x \, \langle 0 | \mathrm{T} \left(\frac{i}{\hbar} \right)^2 \left[-\tfrac{1}{2} \psi^*(x) \left(\partial \cdot A(x) - e\xi v \Phi_2(x) \right) \right.$$

$$\left. \times e \psi(0) \Phi_2(0) \exp\left(\frac{i}{\hbar} S_{\text{eff}}[\phi, \Phi] \right) \right] | 0 \rangle, \tag{17}$$

$$S_{\text{eff}}[\phi, \Phi] = S_{\mathcal{L}}[\phi + \Phi] - S_{\mathcal{L}}[\phi] - \int \mathrm{d}^4x \, \Phi(x) \left. \frac{\delta S_{\mathcal{L}}}{\delta \Phi} \right|_{\Phi = \phi}. \tag{18}$$

The one-loop term is

$$C^{(1)}(\phi,\xi)=i\hbar\int \mathrm{d}^4x\,\langle 0|\mathrm{T}\left(\frac{i}{\hbar}\right)^2\tfrac{1}{2}\psi^*(x)\left(\partial\cdot A(x)-e\xi v\Phi_2(x)\right)e\psi(0)\Phi_2(0)|0\rangle. \tag{19}$$

We use the propagators in appendix B to evaluate this:

$$C^{(1)}=\tfrac{1}{2}ie\int \text{đ}^4k\left[\frac{i}{k^2+e^2\xi v\phi}\,\frac{-ik^2(\xi\phi+\xi v)}{D_N}+\frac{-ie^2\xi v}{\left(k^2+e^2\xi v\phi\right)}\,\frac{i\left(k^2-\xi e^2\phi^2\right)}{D_N}\right], \tag{20}$$

$$D_N=k^4-k^2\left(m_2^2-2e^2\xi v\phi\right)+e^2\phi^2\left(e^2\xi^2v^2+\xi m_2^2\right),\qquad m_2^2=\tfrac{1}{6}\lambda\phi^2-\mu^2.$$

We simplify this to

$$C^{(1)}=\tfrac{1}{2}ie^2\xi\int \text{đ}^4k\,\frac{(2v+\phi)k^2-e^2\xi v\phi^2}{\left(k^2+e^2\xi v\phi\right)D_N}. \tag{21}$$

We now consider the one-loop effective potential

$$V^{(1)}(\phi,\xi)=i\int \text{đ}^4k\left[\ln\left(k^2+e^2\xi v\phi\right)-\tfrac{3}{2}\ln\left(-k^2+e^2\phi^2\right)\right.$$
$$\left.-\tfrac{1}{2}\ln\left(k^2-m_1^2\right)-\tfrac{1}{2}\ln D_N\right],\qquad m_1^2=\tfrac{1}{2}\lambda\phi^2-\mu^2. \tag{22}$$

So,

$$\xi\frac{\partial V^{(1)}}{\partial\xi}=i\int \text{đ}^4k\left[\frac{e^2\xi v\phi}{k^2+e^2\xi v\phi}-\frac{1}{2}\,\frac{\left[2k^2e^2\xi v\phi+2e^4\phi^2\xi^2v^2+e^2\phi^2m_2^2\xi\right]}{D_N}\right], \tag{23}$$

which equals

$$\xi\frac{\partial V^{(1)}}{\partial\xi}=-\tfrac{1}{2}ie\xi\phi m_2^2\int \text{đ}^4k\,\frac{(2v+\phi)k^2-v\xi e^2\phi^2}{\left(k^2+e^2\xi v\phi\right)D_N}. \tag{24}$$

Now note that

$$\frac{\partial V^{(0)}}{\partial\phi}=m_2^2\phi. \tag{25}$$

Adding the terms up we obtain

$$\xi \frac{\partial V^{(1)}}{\partial \xi} + C^{(1)}(\phi, \xi) \frac{\partial V^{(0)}}{\partial \phi} = 0. \tag{26}$$

This verifies the first Nielsen identity in a gauge of the form

$$-\frac{1}{2\xi}\left(\partial_\mu A^\mu + e\xi v_i \Phi_i\right)^2. \tag{4}$$

4. One-loop verification of mass identity

The mass^2 to one loop is given by

$$m^{2(1)} = m_1^2 + \sum^{(1)}\left(m_1^2\right). \tag{27}$$

$-i\Sigma$ is the part of the one-loop Higgs self-energy tensor which is the coefficient of $n_i n_j$ (the projector onto the physical Higgs space). To simplify calculations, following AF we expand in powers of e^2 and λ, choosing $\lambda \sim O(e^4)$. This allows us to write to order $e^2\lambda$

$$m^{2(1)} = m_1^2 + \sum^{(1)}(0) + m_1^2 \left.\frac{\partial \sum^{(1)}(p^2)}{\partial p^2}\right|_{p^2=0}, \tag{28}$$

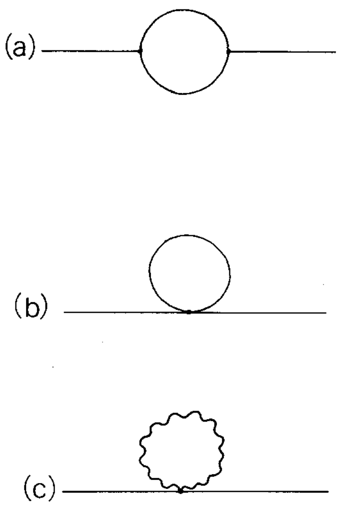


Fig. 1. Graphs removed by expansion scheme. (a) Too high order in λ. (b) p^2 independent. (c) p^2 independent.

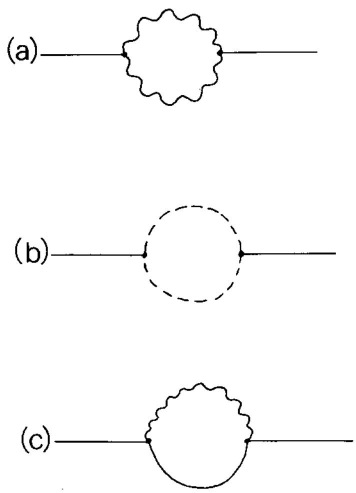


Fig. 2. Graphs to be calculated.

i.e.

$$m^{2(1)} = \frac{\partial^2 V^{(1)}}{\partial \phi^2} + m_1^2 \left. \frac{\partial \sum^{(1)}(p^2)}{\partial p^2} \right|_{p^2=0} . \tag{29}$$

We now choose to work in the 't Hooft gauge proper:

$$\underline{v} = (0, -\phi_0), \qquad \underline{\phi}_0 = (\phi_0, 0). \tag{30}$$

The expansion scheme and the choice of gauge eliminate the graphs of fig. 1 and the Higgs-photon mixing graphs respectively. This leaves the three graphs of fig. 2 to calculate, which we have done here using the $\overline{\text{MS}}$ subtraction scheme with M as the arbitrary renormalisation mass:

$$\left. \sum_{(2a)}^{(1)\prime}(0) \right|_{\phi=\phi_0} = \frac{e^2 \xi}{16\pi^2} \left[\frac{3}{\xi - 1} \ln \xi - \tfrac{11}{6} \right],$$

$$\left. \sum_{(2b)}^{(1)\prime}(0) \right|_{\phi=\phi_0} = \frac{e^2 \xi}{16\pi^2} \left[\tfrac{1}{6} \right],$$

$$\left. \sum_{(2c)}^{(1)\prime}(0) \right|_{\phi=\phi_0} = \frac{e^2 \xi}{16\pi^2} \left[\tfrac{2}{3} + \ln \frac{e^2 \xi \phi_0^2}{M^2} - \frac{3}{\xi - 1} \ln \xi \right]. \tag{31}$$

Adding the terms up we obtain

$$\sum_{\text{total}}{}^{(1)\prime}(0)\bigg|_{\phi=\phi_0} = \frac{e^2\xi}{16\pi^2}\left[\ln\frac{e^2\xi\phi_0^2}{M^2} - 1\right]. \tag{32}$$

Working to $O(\hbar)$, ϕ_0 is the classical minimum of the potential. Now note

$$m_1^2|_{\phi=\phi_0} = \tfrac{1}{3}\lambda\phi_0^2, \qquad \frac{\partial m_1^2}{\partial\phi}\bigg|_{\phi=\phi_0} = \lambda\phi_0 = \frac{\partial m^{2(0)}}{\partial\phi}\bigg|_{\phi=\phi_0},$$

$$\frac{\partial^2 V^{(1)}}{\partial\phi^2}\bigg|_{\substack{v=-\phi_0\\ \phi=\phi_0}} = \frac{e^2\xi\lambda\phi_0^2}{32\pi^2}\left[\tfrac{1}{3}\ln\frac{e^2\xi\phi_0^2}{M^2} - \tfrac{1}{3}\right]. \tag{33}$$

So

$$\xi\,\frac{\partial m^{2(1)}}{\partial\xi}\bigg|_{\phi=\phi_0} = \frac{e^2\xi\lambda\phi_0^2}{32\pi^2}\left[\ln\frac{e^2\xi\phi_0^2}{M^2}\right]. \tag{34}$$

In our expansion scheme and gauge

$$C^{(1)}(\phi,\xi)|_{\substack{v=-\phi_0\\ \phi=\phi_0}} = -\frac{e^2\xi\phi_0}{32\pi^2}\ln\frac{e^2\xi\phi_0^2}{M^2}. \tag{35}$$

Once again the terms (34) and (35) sum to zero, verifying the mass identity to one loop:

$$\xi\frac{\partial m^{2(1)}}{\partial\xi} + C^{(1)}(\phi,\xi)\,\frac{\partial m^{2(0)}}{\partial\phi}\bigg|_{\phi=\phi_0} = 0. \tag{36}$$

5. Alternative derivation of identities

Some recent work by Piguet and Sibold [7] makes it possible to derive the Nielsen identities within the framework of a set of BRS transformations which also operate on ξ, the gauge parameter. This removes the rather ad hoc introduction of the operator O and places the Nielsen identities in the wider context of a set of identities derived in ref. [7]. They showed that, in a pure Yang-Mills theory,

$$S(\Gamma) + \chi\frac{\partial\Gamma}{\partial\xi} = 0, \tag{37}$$

where

$$\delta\xi = \varepsilon\chi \qquad (\chi \text{ Grassmann variable}),$$

$$S(\Gamma) = \mathrm{Tr}\int \mathrm{d}^4x\left(\frac{\delta\Gamma}{\delta\rho^\mu}\frac{\delta\Gamma}{\delta A_{\mu c}} + \frac{\delta\Gamma}{\delta\sigma}\frac{\delta\Gamma}{\delta\psi_c} + B\frac{\delta\Gamma}{\delta\psi_c^*}\right),$$

ρ^μ is the source for δA_μ, σ the source for $\delta\psi$.

The auxiliary field B is introduced in order that the gauge fixing, in a Fermi gauge, may be written in the form

$$\mathcal{L}_{\mathrm{GF}} = \tfrac{1}{2}\xi B^2 + B(\partial \cdot A) + \tfrac{1}{2}\chi\psi^* B + \partial_\mu \psi^* D_\mu \psi. \tag{38}$$

To obtain the effective action precursor of the Nielsen identity we differentiate (37) w.r.t. χ and set χ to 0:

$$S\left(-\frac{\partial \Gamma}{\partial \chi}\right) + \frac{\partial \Gamma}{\partial \xi} = 0. \tag{39}$$

We now repeat the process in more detail for the abelian Higgs model in the 't Hooft gauge, which we have been considering, translating the gauge fixing and BRS transforms into the language of Piguet and Sibold:

$$\begin{aligned}\mathcal{L}_{\mathrm{GF}} = {} & \tfrac{1}{2}\xi B^2 + B\left(\partial_\mu A^\mu + e\xi v_i \Phi_i\right) + \partial_\mu \psi^* \partial^\mu \psi \\ & - e^2 \xi \psi^* \psi \varepsilon_{ij} v_i \Phi_j + \tfrac{1}{2}\chi\psi^* B + e\chi\psi^* v_i \Phi_i. \end{aligned} \tag{40}$$

If we denote, as before, the insertion of an operator P in Γ as $\Gamma(P)$:

$$\frac{\partial}{\partial \chi}\Gamma = \Gamma\left(\tfrac{1}{2}\psi^* B + e\psi^* v_i \Phi_i\right). \tag{41}$$

Using $\delta\mathcal{L}/\delta B = 0$ to eliminate the auxiliary field B we obtain

$$\begin{aligned}\frac{\delta \mathcal{L}}{\delta B} &= \xi B + (\partial \cdot A + e\xi v_i \Phi_i) + \tfrac{1}{2}\chi\psi^*, \\ B &= -\frac{1}{\xi}(\partial \cdot A + e\xi v_i \Phi_i) - \frac{1}{2\xi}\chi\psi^*. \end{aligned} \tag{42}$$

This allows us to rewrite $\partial\Gamma/\partial\chi$:

$$\frac{\partial}{\partial \chi}\Gamma = \Gamma\left(-\frac{1}{2\xi}(\partial \cdot A - e\xi v_i \Phi_i)\right) = \Gamma(O'). \tag{43}$$

The operator insertion is precisely $1/\xi$ times that of the operator O which we had to construct in the previous derivation. The BRS transforms for our theory are

$$\begin{aligned} &\delta\psi^* = \varepsilon B, \qquad \delta\psi = 0, \qquad \delta B = 0, \\ &\delta A_\mu = \varepsilon \partial_\mu \psi, \qquad \delta\Phi_i = \varepsilon e \varepsilon_{ij} \psi \Phi_j, \\ &\delta\xi = \varepsilon\chi. \end{aligned} \tag{44}$$

Identity (37) becomes

$$\int \mathrm{d}^4x \left(\frac{\delta\Gamma}{\delta A_{\mu c}} \partial_\mu \psi_c + \frac{\delta\Gamma}{\delta K_i} \frac{\delta\Gamma}{\delta\phi_{ic}} + B \frac{\delta\Gamma}{\delta\psi_c^*} \right) + \chi \frac{\partial\Gamma}{\partial\xi} = 0. \tag{45}$$

Substitute for B:

$$\int \mathrm{d}^4x \left(\frac{\delta\Gamma}{\delta A_{\mu c}} \partial_\mu \psi_c + \frac{\delta\Gamma}{\delta K_i} \frac{\delta\Gamma}{\delta\phi_{ic}} - \frac{1}{\xi}(\partial\cdot A_c + e\xi v_i \phi_{ic}) \frac{\delta\Gamma}{\delta\psi_c^*} - \frac{1}{2\xi}\chi\psi^* \frac{\delta\Gamma}{\delta\psi_c^*} \right) + \chi \frac{\partial\Gamma}{\partial\xi} = 0. \tag{46}$$

Differentiate w.r.t. χ and set $\chi = 0$:

$$\int \mathrm{d}^4z \int \mathrm{d}^4x \left(\frac{\delta\Gamma(O')}{\delta A_{\mu c}} \partial_\mu \psi_c + \frac{\delta\Gamma(O')}{\delta K_i} \frac{\delta\Gamma}{\delta\phi_{ic}} + \frac{\delta\Gamma}{\delta K_i} \frac{\delta\Gamma(O')}{\delta\phi_{ic}} \right.$$
$$\left. - \frac{1}{\xi}(\partial\cdot A_c + e\xi v_i \phi_{ic}) \frac{\delta\Gamma(O')}{\delta\psi_c^*} - \frac{1}{2\xi} \frac{\delta\Gamma}{\delta\psi_c^*} \psi_c^* \right) - \frac{\partial\Gamma}{\partial\xi} = 0. \tag{47}$$

Multiplying through by ξ, specializing to x-independent ϕ_c and setting the other fields to 0 gives

$$\xi \frac{\partial V}{\partial\xi} - \int \mathrm{d}^4x \frac{\delta\Gamma(O(x))}{\delta K_j(0)} \frac{\partial V}{\partial\phi_{jc}} = -\frac{e\xi v_i \phi_{ic}}{\Omega} \int \mathrm{d}^4x\, \mathrm{d}^4z \frac{\delta\Gamma(O(x))}{\delta\psi_c^*(z)}. \tag{13}$$

This rederives the first Nielsen identity.

Appendix A

The first term in the Nielsen identity (1) is $\xi\partial V/\partial\xi$, so we consider $\xi\partial\Gamma/\partial\xi$, noting that by virtue of it being an explicit differentiation,

$$\frac{\partial\Gamma}{\partial\xi} = \frac{\partial W}{\partial\xi}. \tag{A.1}$$

$\xi\partial\Gamma/\partial\xi$ generates 1PI Green functions with the insertion

$$\int \mathrm{d}^4x \left(\frac{1}{2\xi}\left((\partial\cdot A)^2 - e^2\xi^2 (v_i \Phi_i)^2 \right) - e^2\xi\psi^*\psi\varepsilon_{ij} v_i \Phi_j \right). \tag{A.2}$$

We cannot quite generate this insertion from a BRS transformed operator but the extra $\frac{1}{2}\psi^*\eta$ term vanishes when we consider the effective potential. We now denote the insertion by $\overline{O}(x)$ and write

$$\int [D\Phi_\alpha]\overline{O}(x)\exp\left(i\int d^4x\,(\mathcal{L}+J_\alpha\Phi_\alpha)\right)$$
$$=\frac{\delta}{\delta h(x)}\int d^4z\int [D\Phi_\alpha]h(z)\overline{O}(z)\exp\left(i\int d^4x\,(\mathcal{L}+J_\alpha\Phi_\alpha)\right). \tag{A.3}$$

We write the $\tilde{Z}_k$ of (8) more explicitly:

$$\tilde{Z}_k=\int [DA_\mu][D\psi][D\psi^*][D\Phi_i]\exp(i\hat{S}_k),$$
$$\hat{S}_k=\int d^4x\,\left(\mathcal{L}+K_ie\psi\varepsilon_{ij}\Phi_j+J_\mu A^\mu+j_i\Phi_i+\eta^*\psi+\psi^*\eta+hO\right). \tag{A.4}$$

Carry out a BRS transform on $\tilde{Z}_k$:

$$\int d^4z\,[DA_\mu]\dots[D\Phi_i]$$
$$\times\left(J_\mu\partial^\mu\psi-\eta\frac{1}{\xi}(\partial\cdot A+e\xi v_i\Phi_i)+j_ie\psi\varepsilon_{ij}\Phi_j+h\overline{O}\right)\exp(i\tilde{S}_k)=0, \tag{A.5}$$

i.e.

$$\int d^4z\left(J_\mu\partial^\mu\frac{\delta}{\delta\eta^*}-\eta\frac{1}{\xi}\left(\partial_\mu\frac{\delta}{\delta J_\mu}+e\xi v_i\frac{\delta}{\delta j_i}\right)+j_i\frac{\delta}{\delta K_i}\right)\tilde{Z}_k$$
$$=-i\int d^4z\,[DA_\mu]\dots[D\Phi_i]h(z)\overline{O}(z)\exp(i\tilde{S}_k). \tag{A.6}$$

Legendre transforming the LHS we obtain

$$\tilde{\Gamma}_k=\tilde{W}_k-\int d^4x\,\left(J_\mu A^\mu_c+\eta^*\psi_c+\psi^*_c\eta+j_i\phi_{ic}\right),$$

$$\tilde{W}_k=-i\hbar\ln\tilde{Z}_k,$$

where A_c, etc. are the classical fields.

$$\int d^4z\left(-\frac{\delta\tilde{\Gamma}_k}{\delta A_{\mu c}}\partial_\mu\psi_c+\frac{\delta\tilde{\Gamma}_k}{\delta\psi^*_c}\frac{1}{\xi}(\partial\cdot A_c+e\xi v_i\phi_{ic})-\frac{\delta\tilde{\Gamma}_k}{\delta\Phi_{ic}}\frac{\delta\tilde{\Gamma}_k}{\delta K_i}\right)$$
$$=-\frac{1}{\tilde{Z}_k}\int d^4z\,[DA_\mu]\dots[D\Phi_i]h(z)\overline{O}(z)\exp(i\tilde{S}_k). \tag{A.7}$$

Functionally differentiate w.r.t. $h(x)$ and then set $h=0$, which removes the tildes.

$(\Gamma_k(O(x)) = \Gamma_k \quad \text{with } O \text{ insertion})$,

$$\int d^4z \left(-\frac{\delta\Gamma_k(O(x))}{\delta A_{\mu c}} \partial_\mu \psi_c + \frac{\delta\Gamma_k(O(x))}{\delta\psi_c^*} \frac{1}{\xi}(\partial\cdot A_c + e\xi v_i\phi_{ic}) - \frac{\delta\Gamma_k(O(x))}{\delta\phi_{ic}} \frac{\delta\Gamma_k}{\delta K_i} - \frac{\delta\Gamma_k}{\delta\phi_{ic}} \frac{\delta\Gamma_k(O(x))}{\delta K_i} \right)$$

$$= -\frac{1}{Z_k} \int [DA_\mu]\dots[D\Phi_i]\overline{O}(x)\exp(iS_k). \tag{A.8}$$

But

$$\frac{1}{Z_k}\int [DA_\mu]\dots[D\Phi_i]\overline{O}(x)\exp(iS_k) = \xi\frac{\partial W_k}{\partial\xi} + \tfrac{1}{2}\int d^4x\, \eta(x)\frac{\delta W_k}{\delta\eta(x)}$$

$$= \xi\frac{\partial\Gamma_k}{\partial\xi} + \tfrac{1}{2}\int d^4x\, \frac{\delta\Gamma_k}{\delta\psi_c^*(x)}\psi_c^*(x). \tag{A.9}$$

At this stage specialize to the effective potential, which reduces (A.8) to:

$$\xi\frac{\partial V}{\partial\xi} - \int d^4x\, \frac{\delta\Gamma(O(x))}{\delta K_j(0)}\bigg|_{\substack{\phi_{jc}(z)=\phi_j \\ \text{other fields } 0}} \frac{\partial V}{\partial\phi_j}$$

$$= -\frac{ev_i\phi_{ic}\xi}{\Omega}\int d^4x \int d^4z\, \frac{\delta\Gamma(O(x))}{\delta\psi_c^*(z)}. \tag{A.10)/(8}$$

The mass identity follows by considering $\xi(\partial/\partial\xi)(\delta^2\Gamma/\delta\phi_{1c}\delta\phi_{1c})$ (and noticing a convolution). This gives, eventually,

$$\left(\xi\frac{\partial}{\partial\xi} + C(\phi,\xi)\frac{\partial}{\partial\phi}\right)\Delta^{-1}_{\text{phys}}(p^2)\bigg|_{\phi=\phi_0} = 2\Delta^{-1}_{\text{phys}}\int d^4r\, e^{ip\cdot r}\int d^4z\, F(z,r)\bigg|_{\phi=\phi_0},$$

$$F(z, x-y) = \frac{\delta^2\Gamma(O(x))}{\delta K_1(z)\,\delta\phi_{1c}(y)}\bigg|_{\phi_{1c}=\phi}. \tag{A.11}$$

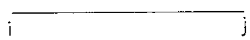
At the potential minimum $\Delta^{-1}_{\text{phys}}(p^2)$, the inverse propagator for the physical Higgs is zero, so we arrive at

$$\left.\left(\xi\frac{\partial}{\partial\xi}+C(\phi,\xi)\frac{\partial}{\partial\phi}\right)m^2\right|_{\phi=\phi_0}=0. \tag{A.12}$$

Appendix B

The propagators (for calculating $C^{(1)}$, $V^{(1)}$ and Σ) are

$$\frac{i}{k^2-e^2\varepsilon_{ij}\xi v_i\phi_j},$$

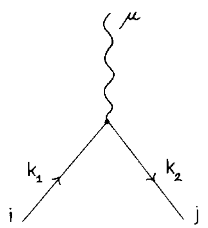


$$\frac{i\left(k^2-\xi e^2\phi^2\right)}{D_N}\left(\delta_{ij}-\eta_i\eta_j\right)+\frac{i\eta_i\eta_j}{k^2-m_1^2},$$

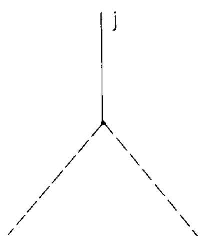
$$-iC\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)-iD\frac{k_\mu k_\nu}{k^2},$$

$$C=\frac{1}{k^2-e^2\phi^2},\qquad D=\frac{\xi\left(k^2-m_2^2-e^2\xi v^2\right)}{D_N}.$$

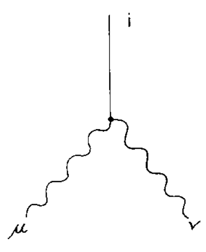
The vertices (those used in calculations only) are



$$-e\varepsilon_{ij}(k_1+k_2)^\mu,$$



$$-ie^2\xi v_i\varepsilon_{ij},$$



$$2ie^2\phi_i g_{\mu\nu}.$$

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