Fractons

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CompPhys18

Plan of talk

3D Plaquette Ising (Gonihedric) Model

Toric Code (and anyons)

Toric Code from gauging a global symmetry

X-Cube model (and fractons)

X-Cube model from gauging a subsystem symmetry

Plaquette Ising/Gonihedric model

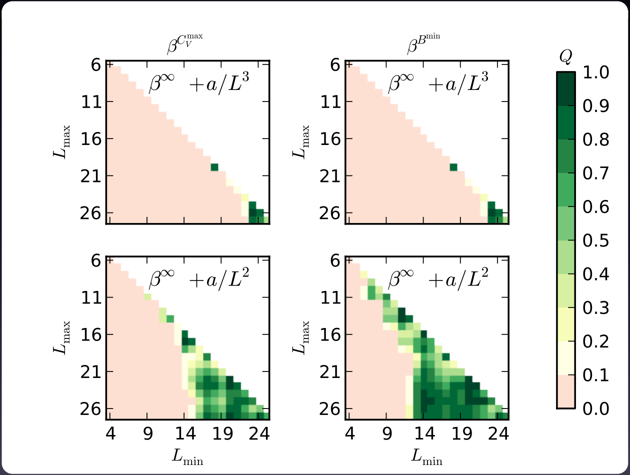
Hamiltonian

$$\mathcal{H} = -\sum_{\Box} \sigma_i \sigma_j \sigma_k \sigma_l$$

Spins at vertices of 3D cubic lattice

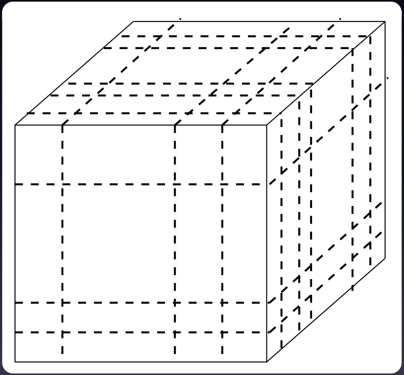
Strong first order phase transition

Scaling oddity (Janke/Mueller)



Standard $1/L^3$ gives much poorer quality

Typical Ground state



1st Order FSS with Exponential Degeneracy

Normally q is constant

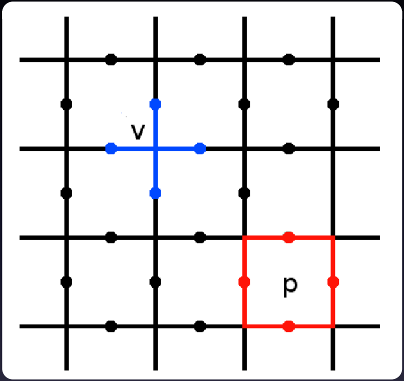
If instead $q \propto e^L$ ($q = e^{(3 \ln 2) L}$), as in Gonihedric model

$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{\ln q}{L^d \Delta e} + \dots$$

becomes

$$\beta^{C_V^{\max}}(L) = \beta^{\infty} - \frac{3 \ln 2}{L^{d-1} \Delta e} + \dots$$

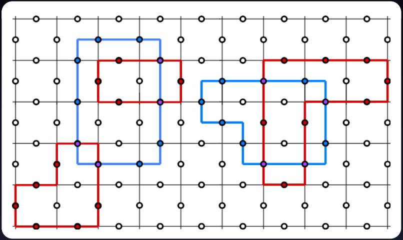
Toric Code



$$A_v = \prod_{i \in v} \tau_i^x, \quad B_p = \prod_{i \in p} \tau_i^z$$

$$H = -J_v \sum_v A_v - J_p \sum_p B_p$$

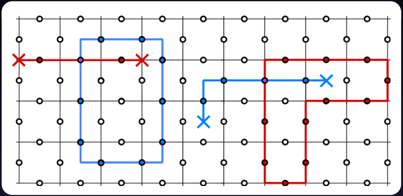
Toric Code: Ground State



$$|\xi_0\rangle = \prod_v \frac{1}{\sqrt{2}} (\mathbb{1}_v + A_v) \underbrace{|0\rangle \otimes \ldots \otimes |0\rangle}_{N_e \text{times}}$$

A "Loop Soup"

Toric Code: Excitations



Defects (i.e. quasiparticles) appear on the end of strings

$$W_e = \prod \tau_z. \quad W_m = \prod \tau_x$$

Braiding excitations reveals anyonic behavior

Toric Code: Anyons

e, m bosonic w.r.t. themselves

Take e for a walk around m, gives -1 phase $\implies$ anyons

Other interesting properties, topological degeneracy of ground state etc

Gauging: From here to there:

Here - Quantum Transverse Ising:

$$\mathcal{H} = -\beta \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Gauge the global $\mathbb{Z}_2$ symmetry

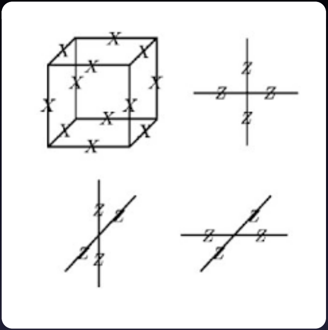
$$\mathcal{H} = -\beta \sum_{\langle i,j \rangle} \sigma_i^z \tau_{ij}^z \sigma_j^z - h \sum_i \sigma_i^x - J_p \sum_{\square} \tau_i^z \tau_j^z \tau_k^z \tau_l^z$$

$\beta \to 0$, gauge invariance: $\sigma_i^x \prod_{i \in v} \tau_i^x = 1$

There - Toric Code:

$$\mathcal{H} = -h \sum_v A_v - J_p \sum_p B_p$$

The X-cube Model

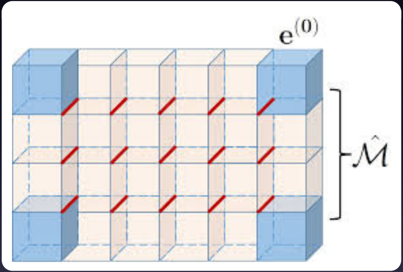


$$A_c = \prod_{i \in \square} \tau_i^x, \quad B_i^{xy,yz,xz} = \prod_{j \in +,i} \tau_j^z$$

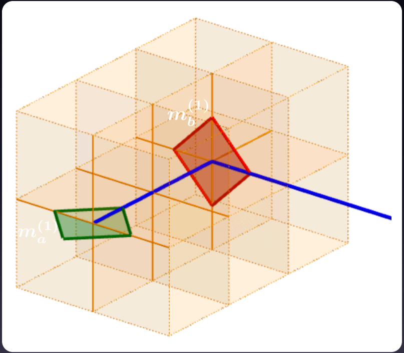
$$H = -J_{\square} \sum A_{\square} - J_{xy} \sum_i B_i^{xy} - J_{yz} \sum_i B_i^{yz} - J_{xz} \sum_i B_i^{xz}$$

Toric Code: Fractons

Electric excitations τ_z



Magnetic excitations τ_x



Pics c/o Vijay et.al.

From here to there: Subsystem Symmetry Gauging

Here - Quantum Transverse Plaquette Ising:

$$\mathcal{H} = -\beta \sum_{\square} \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z - h \sum_i \sigma_i^x$$

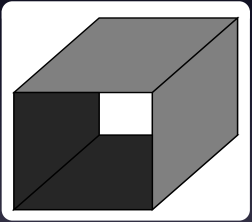
Gauge the $\mathbb{Z}_2$ subsystem symmetry

$$\mathcal{H} \;=\; -\beta \sum_{\square} \tau_{\square}^z \sigma_i^z \sigma_j^z \sigma_k^z \sigma_l^z - h \sum_i \sigma_i^x + \ldots$$

A picture, or two, is worth a thousand words

From here to there: Subsystem Symmetry Gauging II

Equivalent of plaquette flux term in 2D is matchbox (not cube)



Gives $B_i^{xy,yz,xz} = \prod_{j\in +,i} \sigma_j^z$ flux terms

From here to there: Subsystem Symmetry Gauging

There (almost)

$$
\begin{aligned}
\mathcal{H} \;\; = \;\; & -\beta \sum_{\square} \tau^z_{\square} \sigma^z_i \sigma^z_j \sigma^z_k \sigma^z_l - h \sum_i \sigma^x_i \\
& -J_{xy} \sum_i B^{xy}_i - J_{yz} \sum_i B^{yz}_i - J_{xz} \sum_i B^{xz}_i
\end{aligned}
$$

$\beta \to 0$, gauge invariance: $\sigma^x_i \prod_i \tau^x_i = 1$

There

$$
H = -h \sum A_{\cube} - J_{xy} \sum_i B^{xy}_i - J_{yz} \sum_i B^{yz}_i - J_{xz} \sum_i B^{xz}_i
$$

2 second, 2 line summary

Gauge global $\mathbb{Z}_2$, get Toric Code, anyons etc

Gauge subsystem symmetry, get fractons
(reduced mobility anyons)

References

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