

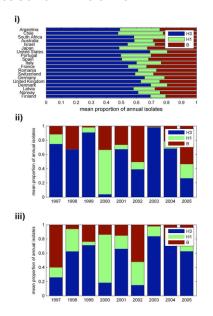
Drift and shift

- Globally, influenza causes three to five million cases of severe illness per year, resulting in up to half a million deaths.
- Antigenic drift: mutations cause small changes in surface antigens generating a variety of strains. The population has partial immunity due to previous exposure. The dominant strains cause an epidemic.
- Antigenic shift: re-assortment, e.g. haemagglutinin from an avian strain introduced to genome of human influenza, produces a virus with novel surface antigens. Everybody is susceptible and a pandemic may happen.

Pandemic influenza

- 1889: Russian flu (H2N2?) kills around 1 million.
- 1918: Spanish flu (H1N1) kills up to 40 million.
- 1957: Asian flu (H2N2) kills up to 4 million.
- 1968: Hong Kong flu (H3N2) kills around 700 thousand.
- 1989: UK flu epidemic kills 29 thousand.
- * HA: Haemagglutinin. NA: Neuraminidase.
- 2003: SARS kills 774.

Seasonal influenza



Annual influenza epidemics

- H3: Influenza A (H3N2).
 Appeared in 1968 and displaced H2N2.
- H1: Influenza A (H1N1).
 Caused 1918 pandemic.
 Reappeared in a 1977 but did not displace H3N2.
- B: Influenza B. Only infects humans. Two distinct 'lines' in circulation, Victoria (mainly in Asia) and Yamagata (worldwide). Figures from Finkelman et al. 2007.

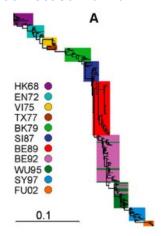
Why the seasonal pattern?



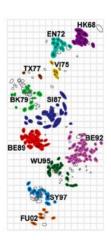
Picture: Lance Jennings

- Influenza spreads from a tropical reservoir in (birds?) and humans (not B?). (Finkelman, others)
- Increased crowding in winter and increased transmission due to temperature, humidity, less sunlight, etc. (Cannell et al. 2008, others)
- Transmission dynamics and/or virus evolution generate periodic behaviour. (Dushoff et al. 2004, Recker et al. 2007, others)

Between-season drift



 Phylogenetic tree of the HA1 nucleotide sequences.
 Figures from Smith et al. 2004.



 Antigenic map of influenza A (H3N2) virus from 1968 to 2003.

New Zealand sentinel GP surveillance

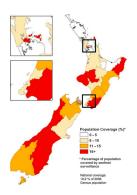
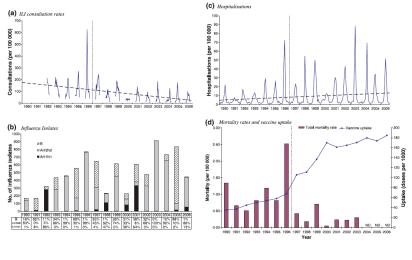


Figure from Huang et al. 2008.
 Influenza and Other Respiratory
 Viruses 2:139-45.

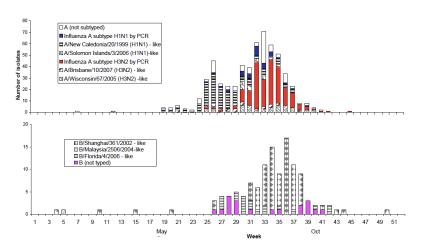
- Operated by ESR* as part of the WHO global programme for influenza surveillance.
- Sentinel practices record the daily number of consultations for ILI (influenza like illness) from May to September, plus some data.
- Respiratory samples from patients with ILI are sent to a laboratory for identification (3 per week).
- Results to WHO Flunet.
 - *Institute of Environmental Science and Research

New Zealand data 1990 - 2006



Figures from Huang et al. 2008.

New Zealand data - 2007



• 646 samples: 31% H1N1, 44% H3N2, 25% B.

Figure from Lopez & Huang 2007.

A New Zealand model

- A three-strain epidemic model to guide vaccination policy.
- The proportion of the population infectious:

$$\frac{\mathrm{d}y_S}{\mathrm{d}t} = \mathcal{R}_0 x_S y_S - y_S$$

where $S \in \Omega = \{H1, H3, B\}$.

• The function $x_S(t) = 1 - u_S + (1 - \mathcal{P})(u_S - z_S)$, for some $0 \leq \mathcal{P} \leq 1$, is the *relative susceptibility* of the population. A proportion z_S is specifically protected, and a proportion u_S is non-specifically protected.

Assumptions

- Neglect virus evolution within season.
- No cross-subtype protection between seasons.
- No cross-type protection within seasons.

Within-season immunity - H1N1

• The proportion of the population *specifically protected*:

$$\frac{\mathrm{d}z_{H1}}{\mathrm{d}t} = \mathcal{R}_0 y_{H1} \left(1 - z_{H1} \right)$$

• The proportion of the population protected:

$$\frac{du_{H1}}{dt} = \mathcal{R}_0 (y_{H1} + y_{H3}) (1 - u_{H1}) \qquad u_{H1}(0) = z_{H1}(0)$$

$$u_{H1}(t) = 1 - \frac{(1 - z_{H1}(t)) (1 - z_{H3}(t))}{1 - z_{H3}(0)}$$

The relative susceptibility of the population is

$$x_{H1}(t) = (1 - z_{H1}(t)) \left(1 - \mathcal{P} + \mathcal{P} \frac{1 - z_{H3}(t)}{1 - z_{H3}(0)}\right)$$

What is \mathcal{P} ?

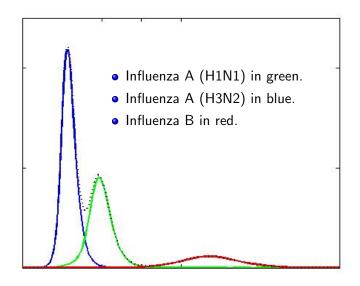
- It appears that there is no cross-immunity among subtypes although the evidence is somewhat conflicting. (Andreasen 2003).
- Transient nonspecific immunity is paramount in the generation of such dynamics, and in limiting diversity within subtypes - although other mechanisms that induce nonspecific inter-strain competition (...) may also contribute. ... A number of studies support the existence of short-lived nonspecific within-subtype and between-subtype immune responses (Ferguson et al. 2003).
- Some degree of generalized immunity or ecological interference may be needed to account for these interannual subtype patterns (Koelle et al. 2006).

What is \mathcal{R}_0 ?

- While the estimated \mathcal{R}_0 remains obscure, epidemiologists have directly measured its father, secondary attack rates, for more than 5 decades. For a highly infectious virus, secondary attack rates for influenza are surprisingly low. (Cannell *et al.* 2008).
- Andreasen (2003) $\mathcal{R}_0 \simeq 3 10$.
- Boni *et al.* (2004) $1.2 < \mathcal{R}_0 < 6$.
- Dushoff *et al.* (2004) $9.6 < \mathcal{R}_0 < 10.4$ and sinusoidal.
- Ferguson *et al.* (2003) $\mathcal{R}_0 = 5$.

- Gog & Grenfell (2002) $\mathcal{R}_0 \leqslant 5$.
- Gog et al. (2003) $\mathcal{R}_0 = 20$.
- Koelle *et al.* (2006) $\mathcal{R}_0 = 5$.
- Minayev & Ferguson (2008) $\mathcal{R}_0 = 2$.
- Recker et al. (2007) $\mathcal{R}_0 \simeq 4$.

Within-season profile



The final size equation

• The proportion of the population infected with variant S in the season is $\mathcal{Z}_S = z_S(\infty) - z_S(0)$, where

$$\mathcal{R}_0 m_S \mathcal{Z}_S + \log \left(1 - \frac{\mathcal{Z}_S}{1 - z_S(0)} \right) = 0$$

and

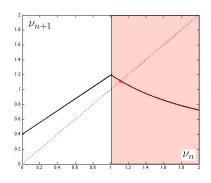
$$m_S = rac{1}{\mathcal{Z}_S} \int_{z_S(0)}^{z_S(\infty)} rac{x_S(t)}{1 - z_S(t)} \, \mathrm{d}z_S(t)$$

• For influenza A (H1N1): (and similarly for H3N2)

$$m_{H1} = 1 - \mathcal{P} + rac{\mathcal{P}}{\mathcal{Z}_{H1}} \int_{z_{H1}(0)}^{z_{H1}(\infty)} rac{1 - z_{H3}(t)}{1 - z_{H3}(0)} \, \mathrm{d}z_{H1}(t)$$

• For influenza B, $m_B = 1$ (assuming no interaction with A).

Between seasons - influenza B



 Fixed point has 11% of population infected with influenza B.

- Immunity to influenza B lasts on average 4.8 years. Define $\mu_B = (1 b) d_B$.
- Effective \mathcal{R}_0 . Let

$$\nu_n = \mathcal{R}_0 \left(1 - z_B(0) \right)$$

at the start of year n. Then

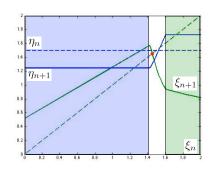
$$\nu_{n+1} = (1 - \mu_B) \mathcal{R}_0 + \mu_B \Phi(\nu_n)$$

with
$$\Phi(\nu) = -\mathcal{W}(-\nu e^{-\nu})$$

and $y = \mathcal{W}(x)$ if $ye^y = x$.

(Lambert W-function. Ma & Earn 2006)

Between seasons - influenza A



- Green is ξ (H1N1).
- Blue is η (H3N2).
- The fixed point is <u>unstable</u> with $\mathcal{R}_0 = 2$.

- Immunity to H1N1 lasts 3.5 years, and to H3N2 1.7 years.
- Effective \mathcal{R}_0 . Let

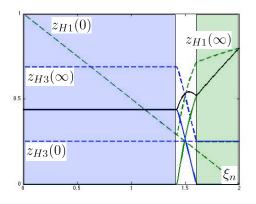
$$\xi_n = \mathcal{R}_0 (1 - z_{H1}(0))$$

 $\eta_n = \mathcal{R}_0 (1 - z_{H3}(0))$

at the start of year n. Then

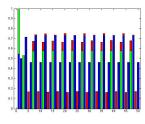
$$\xi_{n+1} = (1 - \mu_{H1}) \mathcal{R}_0 + \frac{\mu_{H1}}{m_{H1}} \Phi (m_{H1} \xi_n)$$
$$\eta_{n+1} = (1 - \mu_{H3}) \mathcal{R}_0 + \frac{\mu_{H3}}{m_{H3}} \Phi (m_{H3} \eta_n)$$

Within seasons - influenza A

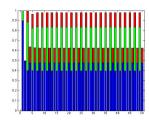


• Epidemic size as a function of effective \mathcal{R}_0 . $\xi_n = \mathcal{R}_0 (1 - z_{H1}(0))$

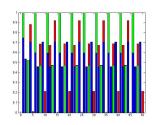
Repeated epidemics



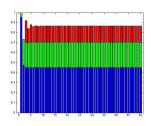
 $\mathcal{R}_0 = 1.5 \quad \text{period 4}$



 $\mathcal{R}_0 = 3.0 \;\;$ period 2



 $\mathcal{R}_0 = 2.0 \quad \text{period } 10$

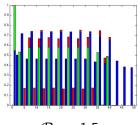


 $\mathcal{R}_0 = 4.0$ period 1

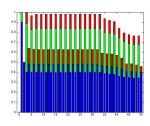
Vaccination

- The WHO make twice-yearly recommendations on vaccine formulation:
 - in February for the next Northern Hemisphere winter;
 - in September for the next Southern Hemisphere winter.
- The Australian Influenza Vaccine Committee meets in October, with representatives from New Zealand and South Africa.
- New Zealand adopts the AIVC recommendation. In 2007 this was:
 - an A/Solomon Island/3/2006 (H1N1) like strain;
 - an A/Brisbane/10/2007 (H3N2) like strain;
 - a B/Florida/4/2006 like strain.
- Free influenza vaccination has been available to those 65+ since 1997, and other risk groups since 1999.

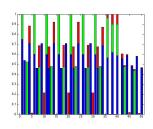
Repeated epidemics with vaccination



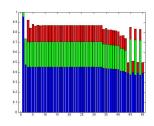




 $R_0 = 3.0$

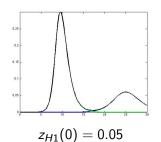


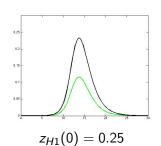
$$\mathcal{R}_0=2.0\,$$

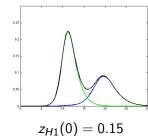


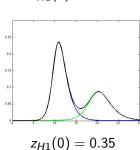
$$R_0 = 4.0$$

Within season epidemics: effect of prior exposure



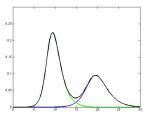




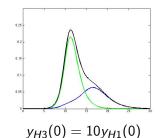


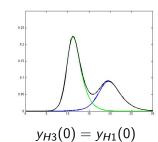
- $z_{H3}(0) = 0.25$
- $y_{H1}(0)$ = 10^{-6}
- $y_{H3}(0) = 10^{-6}$

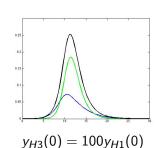
Within season epidemics: effect of initial infection



$$y_{H1}(0) = 10y_{H3}(0)$$







- $z_{H1}(0) = 0.15$
- $z_{H3}(0) = 0.25$
- min $y_S(0) = 10^{-6}$

The two-strain toy model

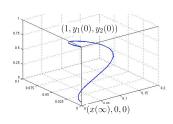
 The proportion not infected this year

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\mathcal{R}_0 x \left(y_1 + y_2 \right)$$

- Assume complete cross-immunity, $\mathcal{P}=1$.
- Proportion immune to strain 1 is $x_1(t) = x_1(0)x(t)$.

$$\frac{dy_1}{dt} = x_1(0)\mathcal{R}_0 x y_1 - y_1 \frac{dy_2}{dt} = x_2(0)\mathcal{R}_0 x y_2 - y_2$$

New joint work with Viggo Andreasen.

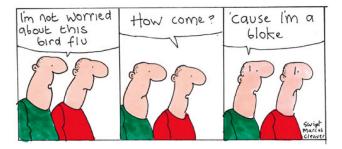


- Given \mathcal{R}_0 , $x_1(0)$ and $x_2(0)$, we have a map $(y_1(0), y_2(0)) \rightarrow x(\infty)$.
- Epidemic peaks are at

$$\phi = \frac{y_2}{y_1} = -\frac{x_1(0)\mathcal{R}_0 x - 1}{x_2(0)\mathcal{R}_0 x - 1}$$

Future work

- Use information from the toy model to refine the final size equation.
- Tune the model parameters (using data!).
- Extend to a three age-group model.
- Make sensible pronouncements on the vaccination policy.



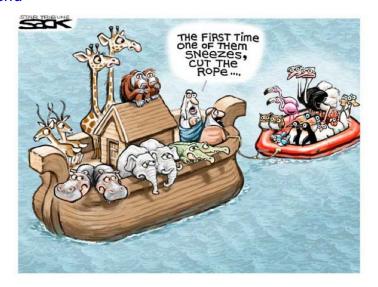
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The end



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