Felix Klein

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Felix Christian Klein (25 April 1849 – 22 June 1925) was a German mathematician, known for his work in group theory, function theory, non-Euclidean geometry, and on the connections between geometry and group theory. His 1872 Erlangen Program, classifying geometries by their underlying symmetry groups, was a hugely influential synthesis of much of the mathematics of the day.

Felix Klein

Born	25 April 1849 Düsseldorf, Rhine, Prussia
Died	22 June 1925 (aged 76) Göttingen, Hanover, Germany
Nationality	German
Fields	Mathematics
Institutions	Universität Erlangen Technische Hochschule München Universität Leipzig Georg-August-Universität Göttingen
Alma mater	Rheinische Friedrich-Wilhelms-Universität Bonn
Doctoral advisor	Julius Plücker and Rudolf Lipschitz
Doctoral students	Ludwig Bieberbach Maxime Bôcher Oskar Bolza Frank Nelson Cole Henry B. Fine Erwin Freundlich Robert Fricke Philipp Furtwängler Axel Harnack Adolf Hurwitz Edward Kasner Ferdinand von Lindemann Alexander Ostrowski Hermann Rothe William Edward Story Edward Van Vleck

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Life

Klein was born in Düsseldorf, to Prussian parents; his father was a Prussian government official's secretary stationed in the Rhine Province. He attended the Gymnasium in Düsseldorf, then studied mathematics and physics at the University of Bonn, 1865–1866, intending to become a physicist. At that time, Julius Plücker held Bonn's chair of mathematics and experimental physics, but by the time Klein became his assistant, in 1866, Plücker's interest was geometry. Klein received his doctorate, supervised by Plücker, from the University of Bonn in 1868.

Plücker died in 1868, leaving his book on the foundations of line geometry incomplete. Klein was the obvious person to complete the second part of Plücker's *Neue Geometrie des*

Raumes, and thus became acquainted with Alfred Clebsch, who had moved to Göttingen in 1868. Klein visited Clebsch the following year, along with visits to Berlin and Paris. In July 1870, at the outbreak of the Franco-Prussian War, he was in Paris and had to leave the country. For a short time, he served as a medical orderly in the Prussian army before being appointed lecturer at Göttingen in early 1871.

	Henry Seely White Alexander Witting Grace Chisholm Young Walther von Dyck
Known for	Function theory Klein bottle
Notable awards	De Morgan Medal (1893) Copley medal (1912)

Erlangen appointed Klein professor in 1872, when he was only 23. In this, he was strongly supported by Clebsch, who regarded him as likely to become the leading mathematician of his day. Klein did not build a school at Erlangen where there were few students, and so he was pleased to be offered a chair at Munich's Technische Hochschule in 1875. There he and Alexander von Brill taught advanced courses to many excellent students, e.g., Adolf Hurwitz, Walther von Dyck, Karl Rohn, Carl Runge, Max Planck, Luigi Bianchi, and Gregorio Ricci-Curbastro.

In 1875 Klein married Anne Hegel, the granddaughter of the philosopher Georg Wilhelm Friedrich Hegel.

After five years at the Technische Hochschule, Klein was appointed to a chair of geometry at Leipzig. There his colleagues included Walther von Dyck, Rohn, Eduard Study and Friedrich Engel. Klein's years at Leipzig, 1880 to 1886, fundamentally changed his life. In 1882, his health collapsed; in 1883–1884, he was plagued by depression.

His career as a research mathematician essentially over, Klein accepted a chair at the University of Göttingen in 1886. From then until his 1913 retirement, he sought to re-establish Göttingen as the world's leading mathematics research center. Yet he never managed to transfer from Leipzig to Göttingen his own role as the leader of a school of geometry. At Göttingen, he taught a variety of courses, mainly on the interface between mathematics and physics, such as mechanics and potential theory.

The research center Klein established at Göttingen served as a model for the best such centers throughout the world. He introduced weekly discussion meetings, and created a mathematical reading room and library. In 1895, Klein hired David Hilbert away from Königsberg; this appointment proved fateful, because Hilbert continued Göttingen's glory until his own retirement in 1932.

Under Klein's editorship, *Mathematische Annalen* became one of the very best mathematics journals in the world. Founded by Clebsch, only under Klein's management did it first rival then surpass *Crelle's Journal* based out of the University of Berlin. Klein set up a small team of editors who met regularly, making democratic decisions. The journal specialized in complex analysis, algebraic geometry, and invariant theory (at least until Hilbert killed the subject). It also provided an important outlet for real analysis and the new group theory.

Thanks in part to Klein's efforts, Göttingen began admitting women in 1893. He supervised the first Ph.D. thesis in mathematics written at Göttingen by a woman; she was Grace Chisholm Young, an English student of Arthur Cayley's, whom Klein admired.

Around 1900, Klein began to take an interest in mathematical instruction in schools. In 1905, he played a decisive role in formulating a plan recommending that the rudiments of differential and integral calculus and the function concept be taught in secondary schools. This recommendation was gradually implemented in many countries around the world. In 1908, Klein was elected chairman of the

MATHEMATISCHE ANNALI

IN VERBINDUNG MIT C. NEUMANN

BEGRÜNDET DURCH

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Prof. K. VonderMühll zu Leipzig

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Weierstraß

1815-1897

Development of Mathematics in The 19th Century

FELIX KLEIN

TRANSLATED BY M. ACKERMAN

APPENDIX "KLEINIAN MATHEMATICS FROM AN ADVANCED STANDPOINT," BY R. HERMANN

Weierstrass, on the other hand, came from Catholic circles. He was born on October 31, 1815, in Ostenfeld in Muensterland, where his father was a paymaster. The father is said to have converted to Catholicism. Whether or not this is so, the environment in which Weierstrass grew up was decidedly Catholic, and this circumstance greatly influenced his development. Because of it he spent many years in places that are till then unknown in the history of mathematics. This is shown by a collocation of some dates of his life:

1829-34: He attended the *gymnasium* at Paderborn.

1839-40: He studied at the Academy in Muenster under Gudermann (also a Catholic). In Muenster he also passed his probationary year as form-master (*Oberlehrer*).

1842-48: He was a *gymansium* teacher in Deutsch-Crone (West Prussia) at the Catholic *Progymnasium* (a secondary school with a curtailed curriculum).

1848-54/55: He held the same position at the Collegium Hoseanum, a seminary in Braunsberg (East Prussia).

From these dates we learn that Weierstrass spent the years of greatest creative power--between the ages of 30 and 40--far from all scientific life, almost completely out of reach of any mathematical stimulus, in small, indeed the smallest places, whose names are otherwise hardly ever heard.

Of a quite different tone, for us still wholly unclarified, is his life during his student years 1834-38 in Bonn. At the University of Bonn, more mixed in religions, Weierstrass studied at first not mathematics but jurisprudence. At the same time he was active in the Corps Saxonia. It is said of him that every evening he was one of the merriest fellows at the tavern and was never absent from the fencing room. How this is compatible with the rest of his development is wholly unintelligible to me. Lampe praised the later Weierstrass for the "free sense with which Weierstrass treated life to a certain extent as a sovereign"; this manner may have been taken over from his student years in Bonn.

There was virtually no mathematical stimulation for Weierstrass in Bonn. Until 1836 Muenchow taught there; as a representative of the old school he united astronomy, physics and mathematics. His successor Pluecker, who still joined mathematics and physics, certainly could not devote much time to the former. And Weierstrass attended hardly any of his lectures. Driven by an invincible inclination, Weierstrass began private mathematical studies--he had already been advised on Steiner's works in Crelle's *Journal* while in Paderborn. Jacobi's "*Fundamenta nova*" had appeared in 1829, but was still quite new and attracting much attention. Weierstrass worked through it with enormous effort--he had no preliminary knowledge--and resolved to deepen these studies. He heard that in Muenster Gudermann was comprehensively studying

A full list is found at the end of Volume 3 of the *Werke*. Here I would like to name only the general cycle that Weierstrass followed: analytic functions, elliptic functions, applications of elliptic functions, hyperelliptic or abelian functions.

He lectured on other subjects, like synthetic geometry or the calculus of variations, the latter being more often repeated in later years.

According to my recollections--I had come to Berlin in 1869 and was there in 1869/70--Weierstrass's position was one of absolute authority; his audience accepted his teachings as an incontestable norm, often without having rightly understood it in its deeper sense. No doubt was allowed to arise; and a control was possible only with difficulty since Weierstrass cited very little. In his lectures he had set himself the goal of presenting a coherent system of well-ordered thoughts. Thus he would begin with a methodical building up from below, arranging the progression so that in the sequel he would have to refer back only to himself.

Along with Lie, I myself--and I now regret it--attended none of Weierstrass's courses, out of a spirit of opposition; and in the seminar I pursued my own ideas. But I did write down one of Weierstrass's lectures on elliptic functions and years later often used it for my own works on this subject.

Weierstrass came gradually to be regarded throughout the scientific world as an incomparable authority (see Mittag-Leffler's address to the Paris Congress in 1900, where (p. 131) he quotes Hermite as saying: "Weierstrass est notre maître à tous" [Weierstrass is the master of us all]).

Yet in the end, Weierstrass was not spared the disappointment of seeing his doctrines assailed (see his letter to Kowalewska of March 24, 1885, communicated by Mittag-Leffler in *Acta Math.* Vol. 39, p. 194 ff). A new direction in mathematics arose with Kronecker, who, on the basis of philosophical considerations, acknowledged the existence of only the integral, or at most the rational, numbers and wanted totally to ban irrational numbers--a direction that regarded the foundations of Weierstrass's function theory as unsatisfactory. This is, within the framework of science, the same change as we can see carried out in literature and art, often in rapid succession. It is much to be regretted that Weierstrass--probably because of Kronecker's personal polemics--suffered so acutely from the reaction that become noticeable in the last years of his life--a suffering that is perceptible in the letter just cited. Now we would almost like to say that he shouldn't have taken it so hard; it is only an instance of

Weierstrass σ

$$\sigma(u) = \sqrt{\frac{\pi}{2\omega}} \frac{1}{\sqrt[4]{(e_1-e_2)(e_1-e_3)(e_2-e_3)}}$$

$$\times \vartheta_1\left(\frac{u}{2\omega}\middle|\tau\right) e^{\frac{u^2\eta}{2\omega}}$$

$$\omega \to \dot{\omega} = \alpha\omega + \beta\omega'$$
$$\omega' \to \dot{\omega}' = \gamma\omega + \delta\omega'$$
$$\eta \to \dot{\eta} = \alpha\eta + \beta\eta'$$
$$\eta' \to \dot{\eta}' = \gamma\eta + \delta\eta'$$
$$\tau \to \dot{\tau} = \frac{\gamma + \delta\tau}{\alpha + \beta\tau}$$
$$\alpha\delta - \beta\gamma = 1$$

1866-1956

H. F. Baker. Abel theorem and the allied theory of theta functions, Cambridge 1897, reprinted in 1995

H. F. Baker. Multiply Periodic Functions Cambridge, 1907

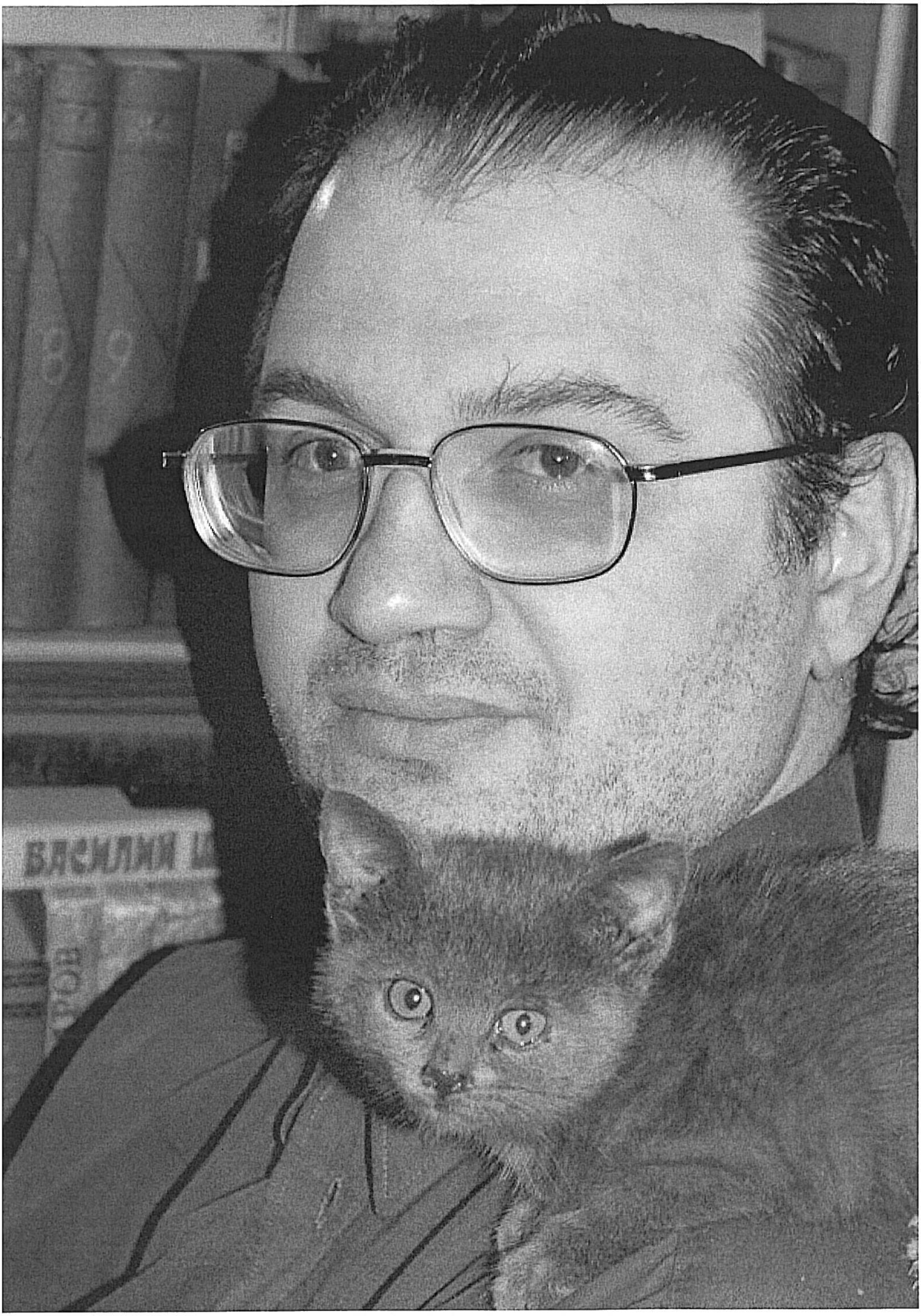
H. Baker: $g=2$

$$\frac{\sigma(u+v)\,\sigma(u-v)}{\sigma^2(u)\,\sigma^2(v)}$$

$$= \wp_{22}(u)\wp_{12}(v) - \wp_{12}(u)\wp_{22}(v)$$

$$+ \wp_{11}(v) - \wp_{11}(u)$$

$$\wp_{ij} = -\frac{\partial^2}{\partial u_i \partial u_j} \ln \sigma(u)$$



D. Leykin: $g \geq 2$

$$\frac{\sigma(u+v)\,\sigma(u-v)}{\sigma^2(u)\,\sigma^2(v)} = \mathrm{Pf}\big(M(u,v)\big)$$