Excellent Swimmers

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F. Alouges, ADS, A. Lefebvre, Optimal strokes for low Re swimmers: an example
J. Nonlinear Sci, **18**, 277-302 (2008)

Outline

- 1. Swimming and swimmers
- 2. Hydrodynamics and Reynolds number
- 3. Life at low Reynolds numbers
- 4. Gaits and strokes:
  - A swimming problem
  - A problem of optimal swimming

(theme: empirical vs. algorithmic optimization)

1. Swimming and swimmers

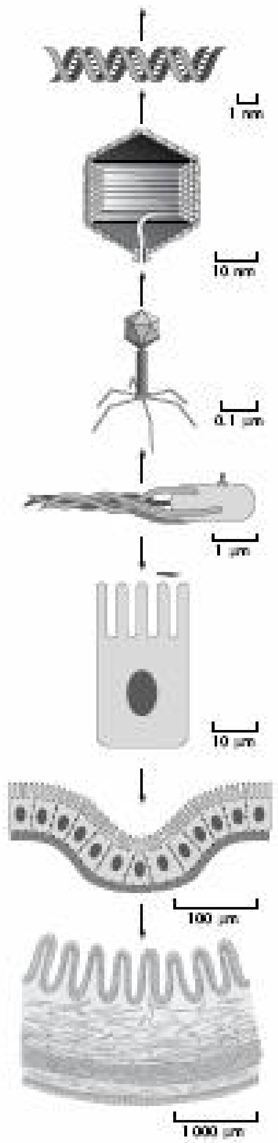
Swimming

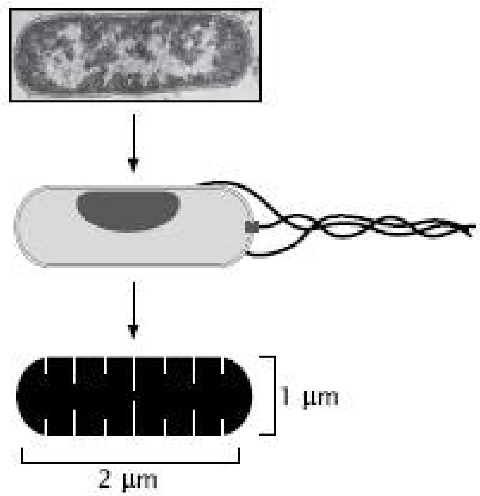
- Swimming: a definition

  the ability to advance in water by performing a cyclic shape change (a *stroke*) in the absence of external propulsive forces

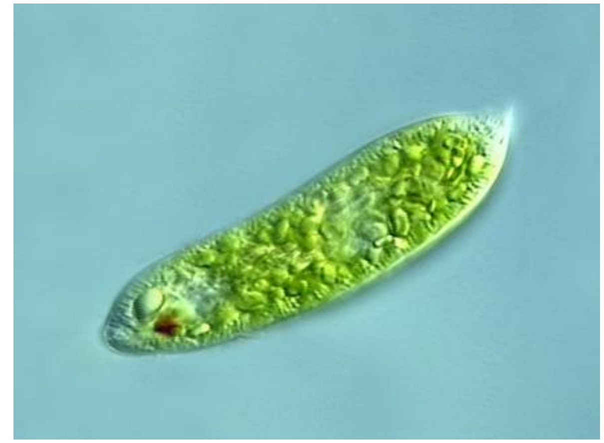
- Nature of shape variables: will focus on time-periodic functions

Microscopic swimmers: bacteria and cells





Escherichia Coli (E. Coli)



Euglenoids

Two movies of metaboly in Eutreptiella sp.

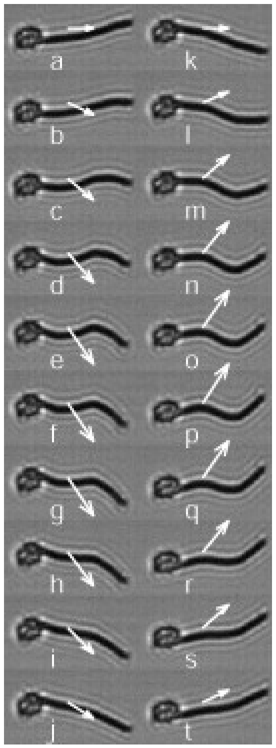
Eutrep2.mov

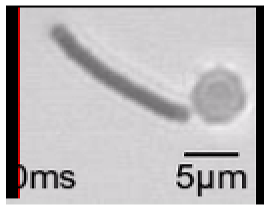
Ccmp389a.mov

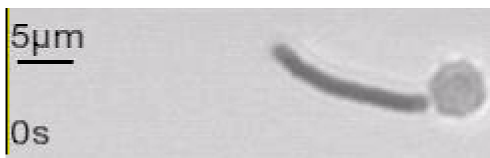
One movie of swimming in Anisonema sp.

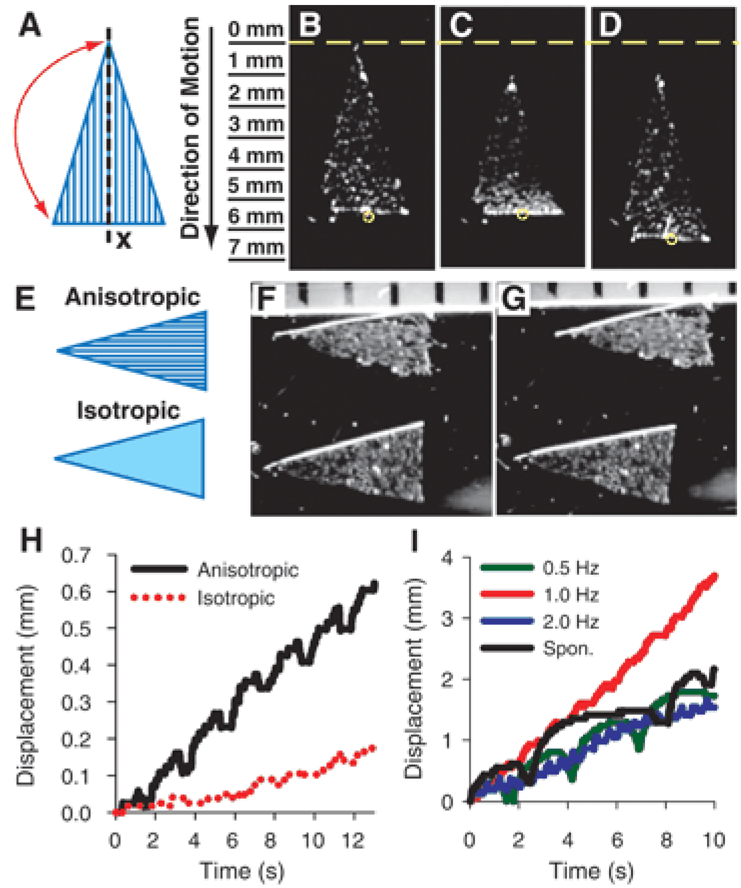
Aniso2.mov

Man made micro swimmers: micro-robots









red blood cell + flexible magnetic filament

polymer film + muscle cells

H. Stone et al., Nature (2005)

G. Whitesides et al., Science (2007)

2. Hydrodynamics:
From Navier(-Stokes) to Stokes via Reynolds

Navier

Reynolds

Stokes

From Navier-Stokes to Stokes

$$\rho\left(\frac{\partial v}{\partial t} + (v \cdot \nabla) v\right) = -\nabla p + \eta \Delta v$$
$$\mathrm{div}\; v = 0$$

Navier-Stokes

Non-dimensionalization

$$x_* = \frac{x}{L}\ ,\ t_* = \frac{t}{T}\ ,\ p_* = \frac{p}{V\eta}\ ,\ u_* = \frac{\mathrm{v}}{V}$$

$$\left[\begin{array}{l} \boxed{\sigma Re \frac{\partial u_*}{\partial t_*} + Re(u_* \cdot \nabla_*) u_*} - \Delta_* u_* + \nabla_* p_* = 0, \\ \mathrm{div}_* u_* = 0 \end{array}\right.$$

$$\boxed{\mathrm{Re} = \frac{VL\rho}{\eta}} \text{ and } \sigma = \frac{\mathrm{L}}{\mathrm{VT}}$$

$$-\eta \Delta v + \nabla p = 0$$
$$\mathrm{div}\; v = 0$$

Stokes

All inertial effects neglected.

Reynolds Number (Re)

$$\mathrm{Re} = \frac{VL\rho}{\eta}$$

Velocity (typical order of magnitude)	V
Diameter (typical size)	L
Mass density of the fluid	ρ
Viscosity of the fluid	η

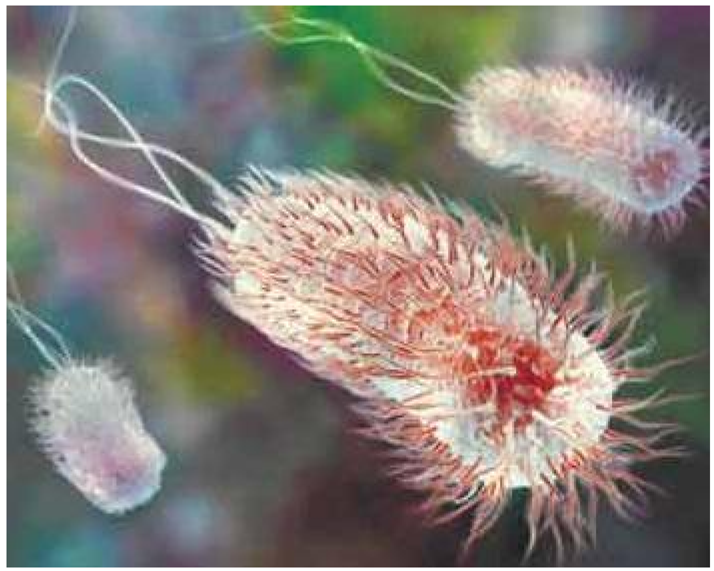
For water at room temperature $\rho/\eta = 10^6\ (m^2s^{-1})^{-1}$.

Re is a dimensionless measure of relative importance of inertia vs. viscosity

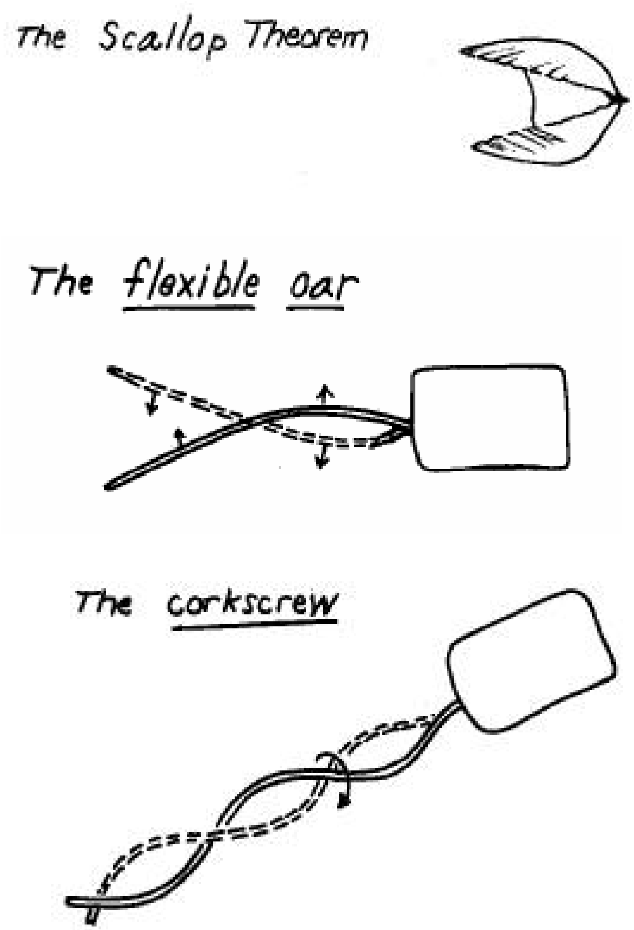
Orders of magnitude for swimmers:

Men, dolphins, sharks:	$L=1m$, $V=1\text{-}10\ ms^{-1}$	$Re=10^6\text{-}10^7$
Bacteria:	$L=1x10^{-6}m$, $V=1\text{-}10x10^{-6}\ ms^{-1}$	$Re=10^{-6}\text{-}10^{-5}$

3. Life at low Reynolds numbers



The scallop theorem and some consequences

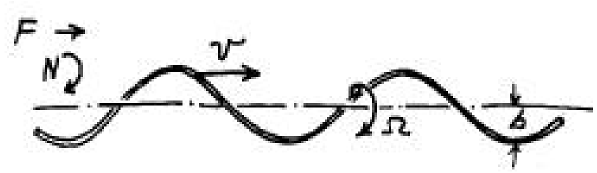


in a flow regime obeying Stokes equations, a scallop cannot advance through the reciprocal motion of its valves

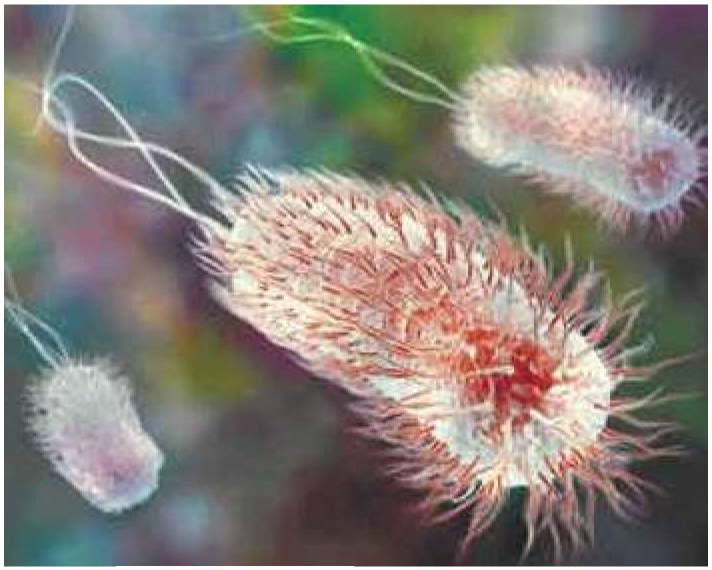
whatever forward motion will be produced by closing the valves, it will be exactly canceled by a backward motion upon reopening them

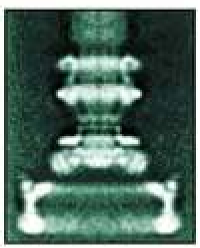
G.I. Taylor, 1951

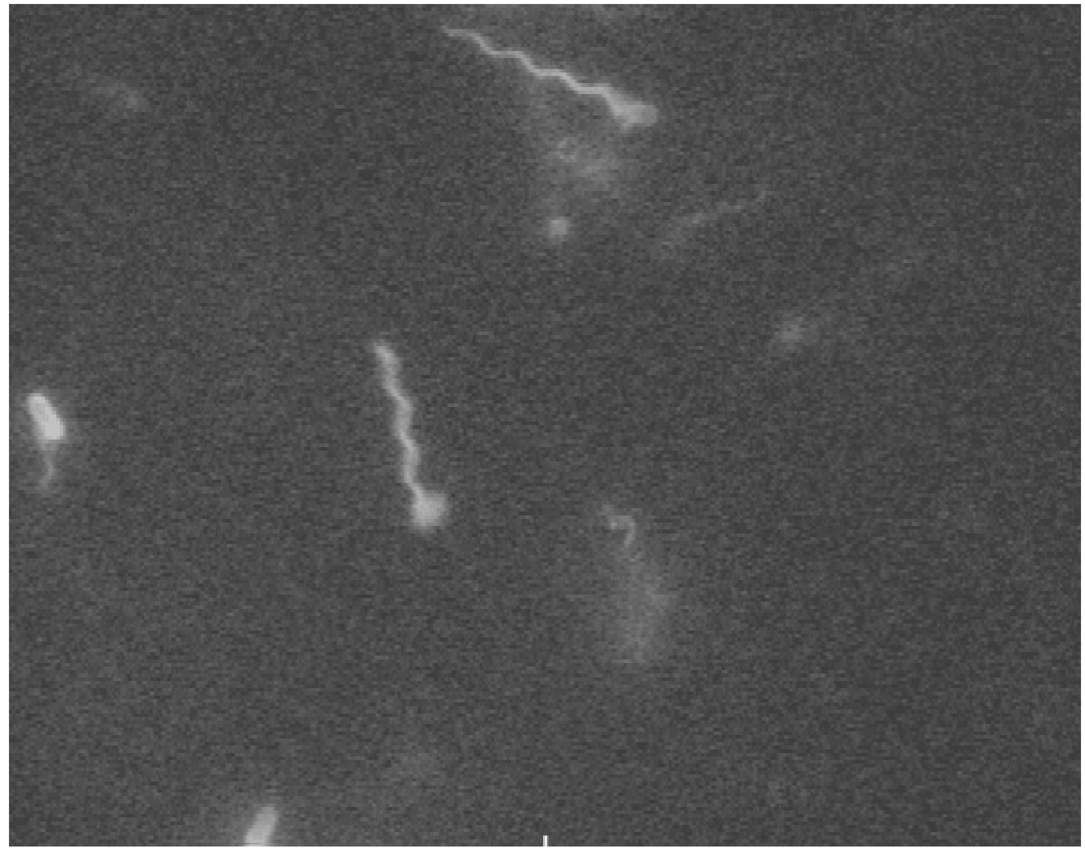
H. Berg, 1973



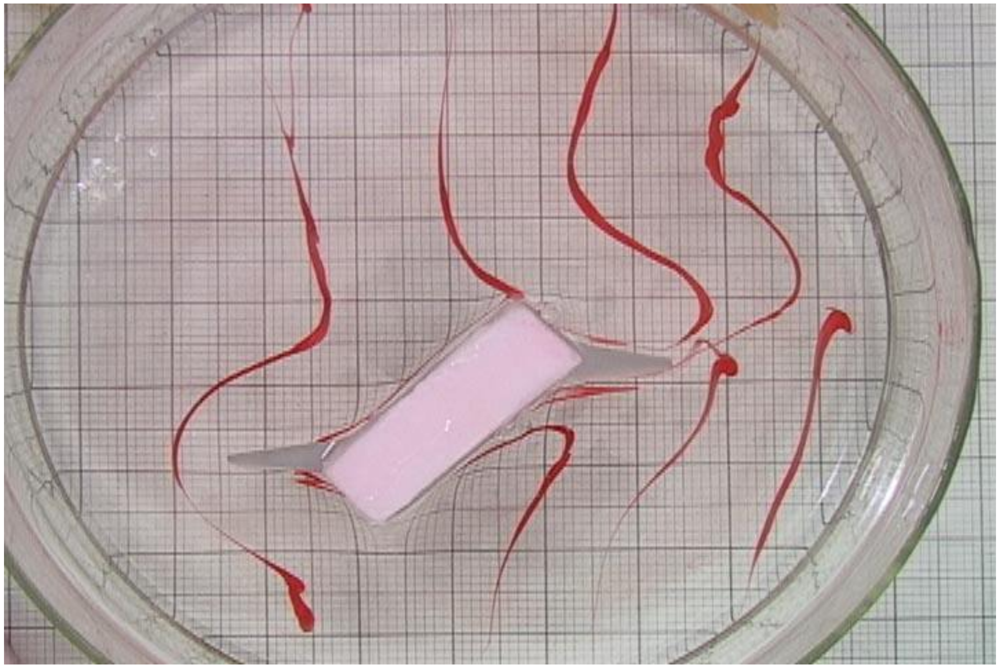
And indeed, here is the rotary motor

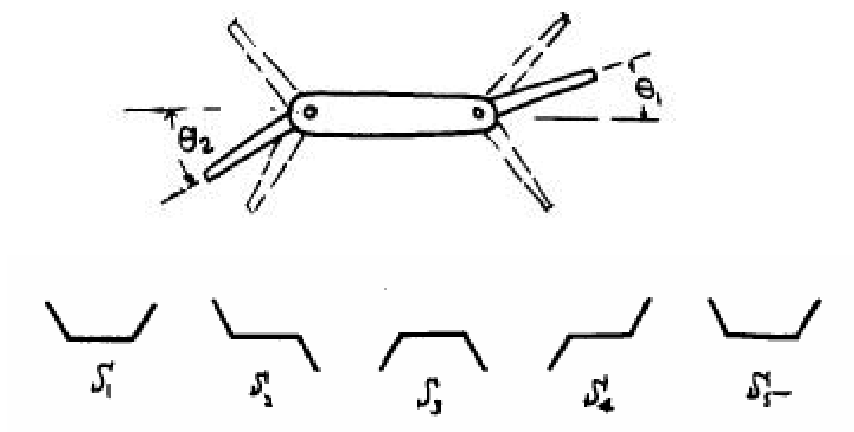


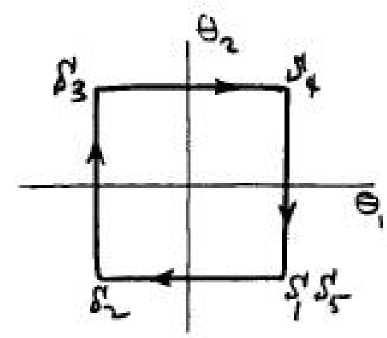




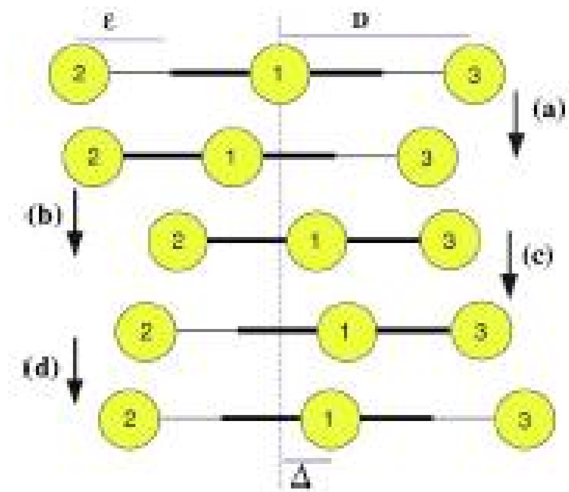
How to beat the scallop theorem:
1. The three-link swimmer of Purcell







How to beat the scallop theorem:
2. The three-sphere swimmer of Golestanian (2004)



.... and the story continues (pushmepullyou, ...)

4. Optimal strokes

WWW.OLYMPIC.ORG - Official website of the Olympic Movement

ATHLETES | PROFILES > RICHARD DOUGLAS FOSBURY

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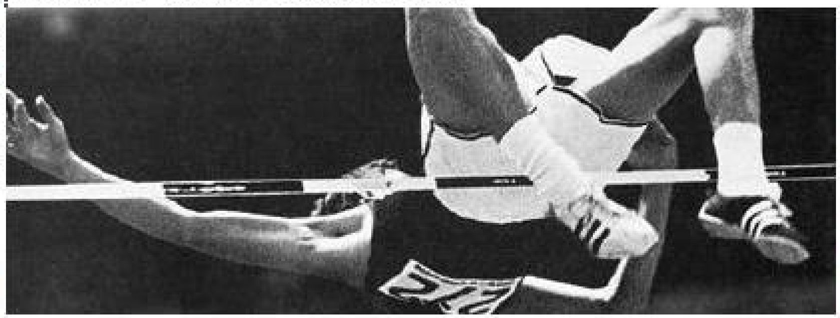

Richard Douglas FOSBURY

Dick Fosbury revolutionised the high jump

The 1968 Mexico City Olympics marked the international debut of Dick Fosbury and his celebrated "Fosbury flop," which would soon revolutionize high-jumping. At the time, jumpers took off from their inside foot and swung their outside foot up and over the bar. Fosbury's technique began by racing up to the bar at great speed and taking off from his right (or outside) foot. The he twisted his body so that he went over the bar head first with his back to the bar. While the coaches of the world shook their heads in disbelief, the Mexico City audience was absolutely captivated by Fosbury and shouted, "Olé" as he cleared the bar. Fosbury cleared every height through 2.22 metres without a miss and then achieved a personal record of 2.24 metres to win the gold medal. By 1980, 13 of the 16 Olympic finalists were using the Fosbury flop. It has since been shown that, unbeknownst to Fosbury, the first person to use the flop technique was actually a jumper from Montana named Bruce Quande, who was photographed flopping over a bar in 1963.

Search

ATHLETES
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IDENTITY CARD
Born :
6 March 1947
Birthplace:
Portland (United States)
Nationality:
United States
Sport :
Athletics

VIDEO GALLERY

Mexico
1968
D. Fosburv

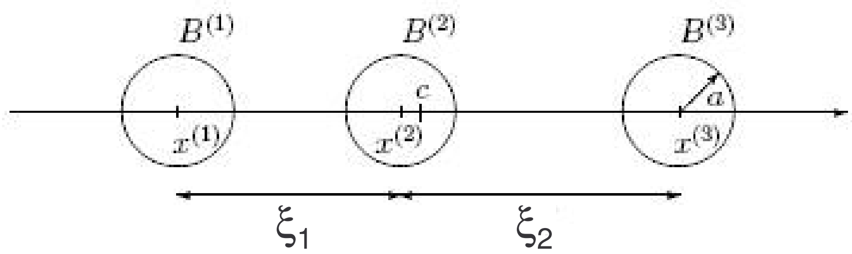
ATTENDANCE AT THE OLYMPIC GAMES
Mexico 1968

AWARDS
Olympic medals:
Gold: 1
Details

Race for life

- Optimal strokes for sports is fun, but Low Re regime not relevant for this. So why are optimal strokes interesting in low Re regime?
- Understand biologically relevant swimming mechanism and their optimality (how is it realized?; what is being optimized?)
- Race for life (selection of optimal patterns out of evolutionary pressure)
- Optimal control of micro- and nano-robots for minimally invasive diagnostic tools, drug delivery, surgery, DNA repair, …….

Mathematical swimming in a nutshell



$$\xi_1 = x^{(2)} - x^{(1)}$$
$$\xi_2 = x^{(3)} - x^{(2)}$$

shape

$$c = \frac{1}{3}\sum_{i=1}^{3} x^{(i)}$$

position

Swimming is .....

$\xi_1(t)$, $\xi_2(t)$ periodic of period T

executing cyclic shape changes

producing

$$\Delta c = \int_0^T \dot{c}\,dt \neq 0$$

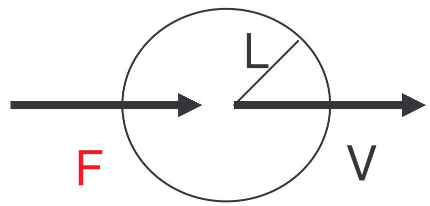
position change after one period

under

$f^{(1)} + f^{(2)} + f^{(3)} = 0$

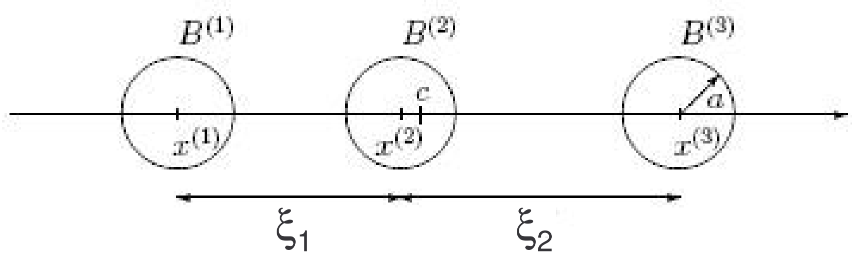
no external force (self-propulsion)

Positional change from shape change



Stokes formula

$$F = 6\pi\eta LV$$

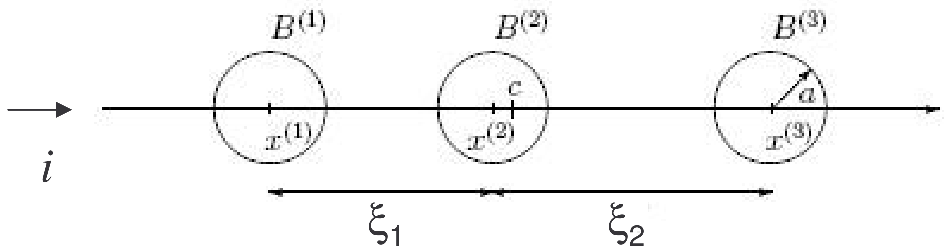


$$v^{(i)} = \dot{x}^{(i)} \leftrightarrow \dot{\xi}_1, \dot{\xi}_2, \dot{c}$$

$(f^{(1)}, f^{(2)}, f^{(3)})^T = R(\xi_1,\xi_2) \; (v^{(1)}, v^{(2)}, v^{(3)})^T$ R a symm. pos. def. 3x3 matrix

$f^{(1)} + f^{(2)} + f^{(3)} = 0$ (self-propulsion)

General axisymmetric case



v_{Dir} Dirichlet data on $\partial\Omega$

$DN_\xi[v_{Dir}]=\sigma\, n$ surface force on $\partial\Omega$

$$0 = i \cdot \int_{\partial\Omega} DN_\xi[v_{Dir}]\ dA$$

$$= \varphi_1(\xi, \not{c})\,\dot{\xi}_1 + \varphi_2(\xi, \not{c})\,\dot{\xi}_2 + \underbrace{\varphi_3(\xi, \not{c})}_{>0}\,\dot{c}$$

by transl. inv.

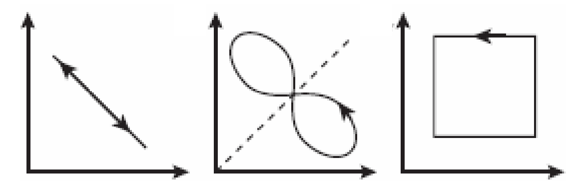
Then $\dot{c}$ is uniquely determined and depends linearly on $\dot{\xi}_1$ and $\dot{\xi}_2$

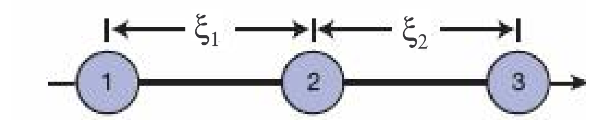
$$\frac{dc}{dt} = V_1(\xi)\frac{d\xi_1}{dt} + V_2(\xi)\frac{d\xi_2}{dt}$$

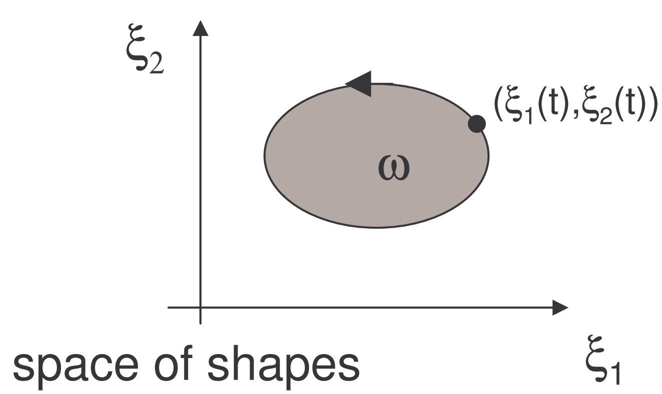
$$V_i(\xi) = -\varphi_i(\xi) / \varphi_{n+1}(\xi)$$

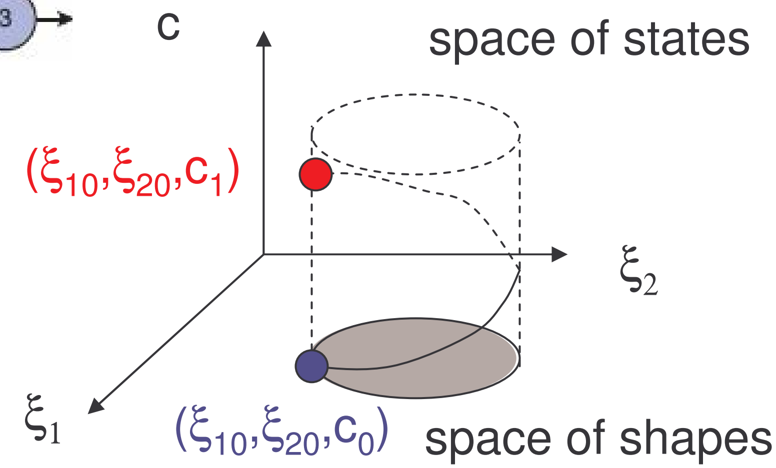
Swimming with one formula

$$\Delta c = \int_0^T \dot{c}\, dt = \int_0^T \left(V_1 \frac{d\xi_1}{dt} + V_2 \frac{d\xi_2}{dt}\right) dt = \int_\omega \mathrm{curl}\, V \, d\xi_1 d\xi_2$$





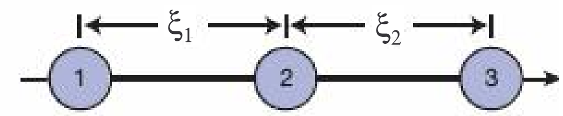




Swimming rests on the differential form $V_1 d\xi_1 + V_2 d\xi_2$ not being exact

Controllability

Nonholonomic motion planning vs. steering a control system

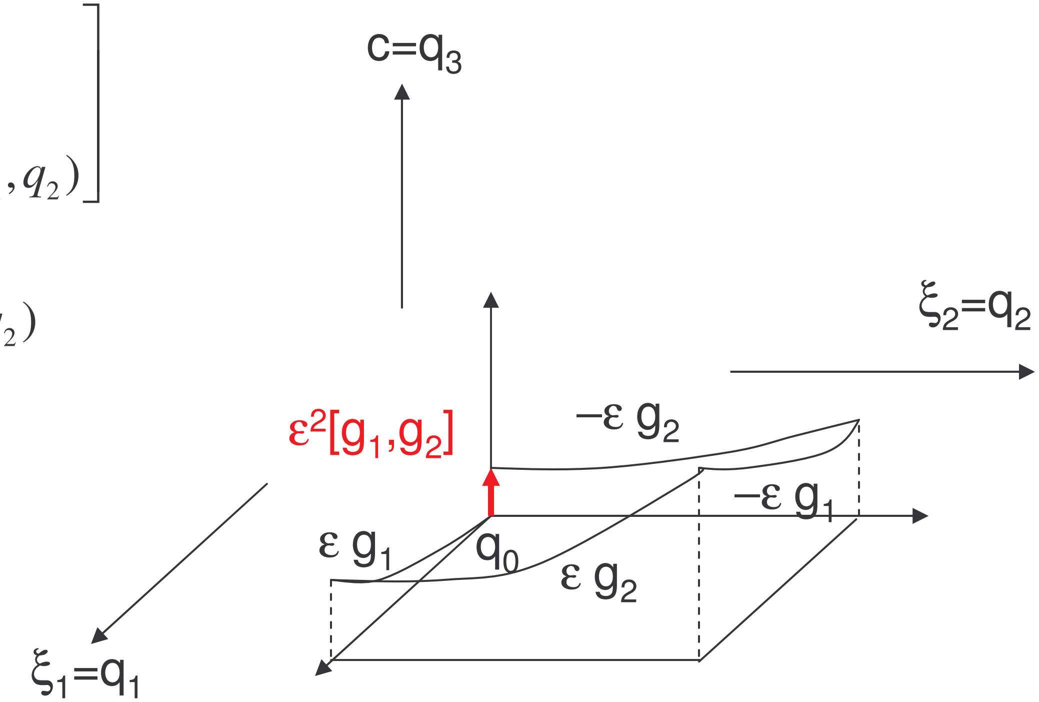


$$\dot{\xi}_1 = u_1$$
$$\dot{\xi}_2 = u_2$$
$$\dot{c} = V_1(\xi)\,u_1 + V_2(\xi)\,u_2$$

$$\dot{q} = \frac{d}{dt}\begin{bmatrix} q_1 := \xi_1 \\ q_2 := \xi_2 \\ q_3 := c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ V_1(q_1,q_2) \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 1 \\ V_2(q_1,q_2) \end{bmatrix} u_2 = \sum_{i=1}^{2} g_i(q) u_i$$

$$[g_1, g_2] = (\frac{\partial}{\partial q} g_2) g_1 - (\frac{\partial}{\partial q} g_1) g_2 = \begin{bmatrix} 0 \\ 0 \\ \mathrm{curl}\, V(q_1,q_2) \end{bmatrix}$$

$$\det[g_1 \mid g_2 \mid [g_1,g_2]](q_1,q_2,q_3) = \mathrm{curl}\, V(q_1,q_2)$$



The three-spere swimmer
is globally controllable (Lie-bracket
generating or totally nonholonomic
control system)

Swimming at max efficiency (geodesic strokes)

Rescale to unit time interval:

$$\mathrm{Eff}^{-1} = \frac{\int_0^1 \sum_{i=1}^{3} f^{(i)} \cdot \dot{x}^{(i)}(t)\, dt}{6\pi L\eta\ \Delta c^2} = \frac{\int_0^1 \int_{\partial\Omega} DN_{\xi}[\mathrm{v}_{\mathrm{Dir}}]\cdot \mathrm{v}_{\mathrm{Dir}}\, dA\, dt}{6\pi L\eta\ \Delta c^2} = \frac{\int_0^1 G(\xi)\dot{\xi}\cdot\dot{\xi}\, dt}{6\pi L\eta\ \Delta c^2}$$

Minimize expended power at given $\Delta c = \int_0^1 V(\xi)\cdot\dot{\xi}$

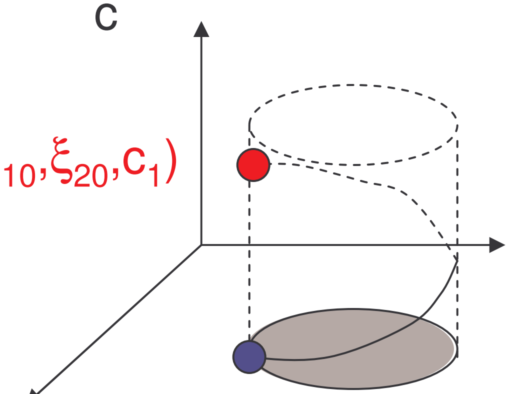
Introduce Lagrange multiplier λ

$$\int_0^1 G(\xi)\dot{\xi}\cdot\dot{\xi} - \lambda \int_0^1 V(\xi)\cdot\dot{\xi}$$

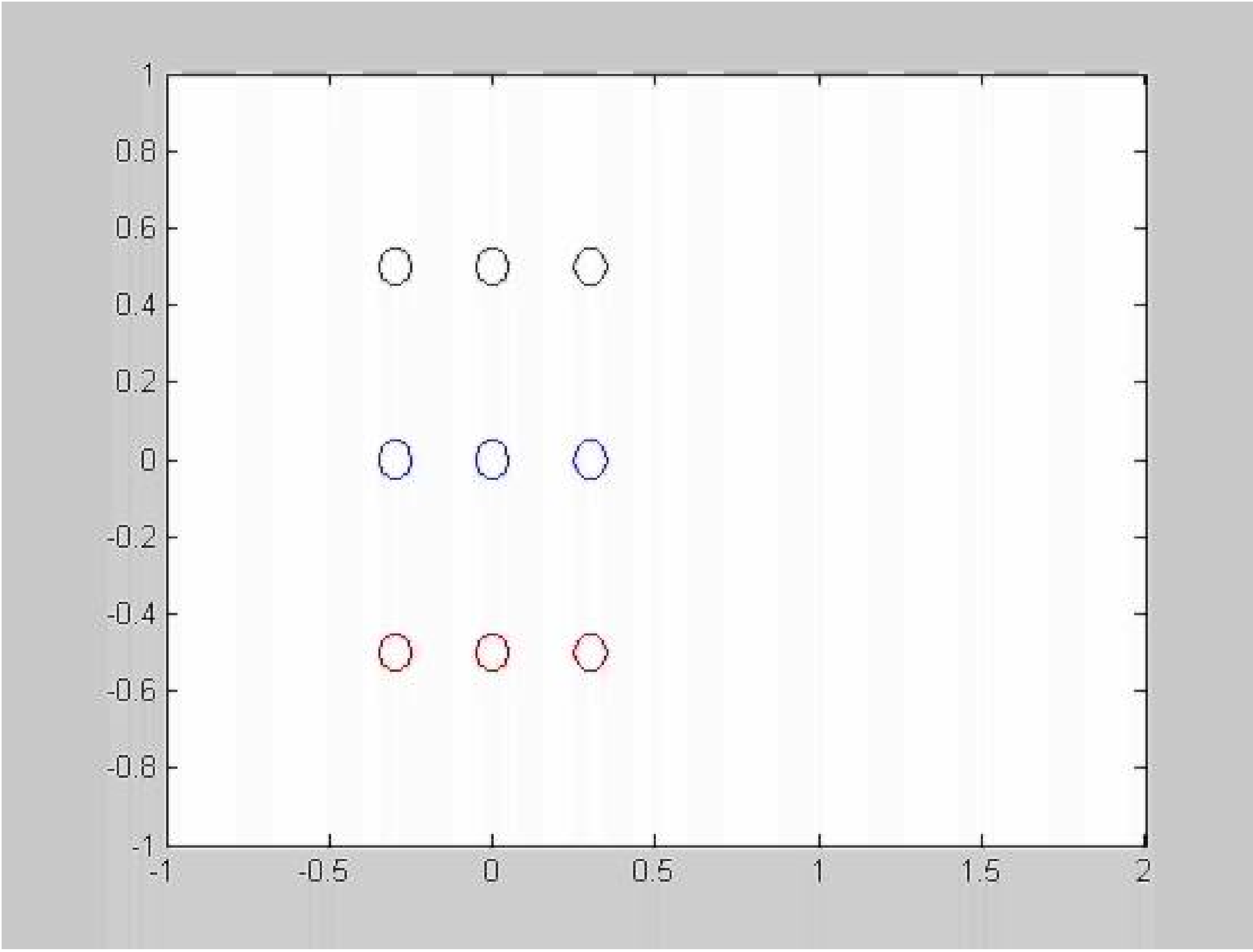
Euler-Lagrange eqn.

$$-\frac{d}{dt}(G\dot{\xi}) + \frac{1}{2}\begin{pmatrix} \partial_1 G\dot{\xi}\cdot\dot{\xi} \\ \partial_2 G\dot{\xi}\cdot\dot{\xi} \end{pmatrix} + \lambda\, \mathrm{curl} V(\xi)\, \dot{\xi}^{\perp} = 0$$

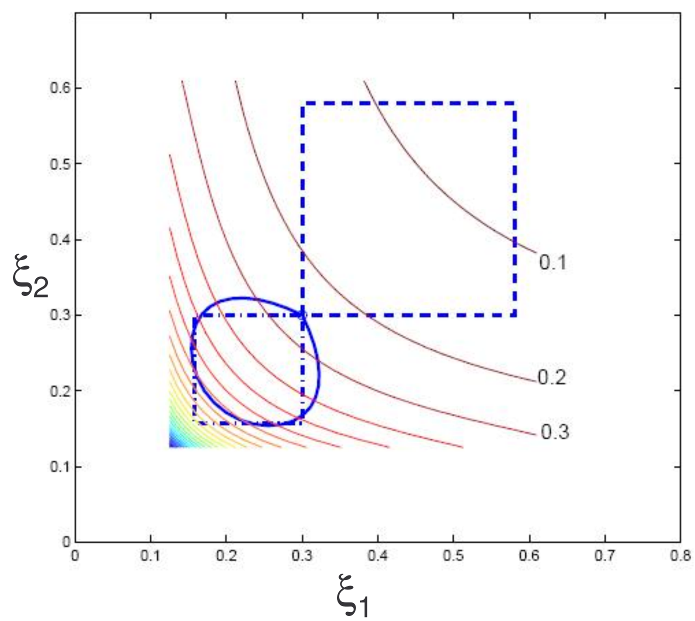
Optimal paths are sub-riemannian geodesics.
Optimal strokes exist.
They can be computed numerically.



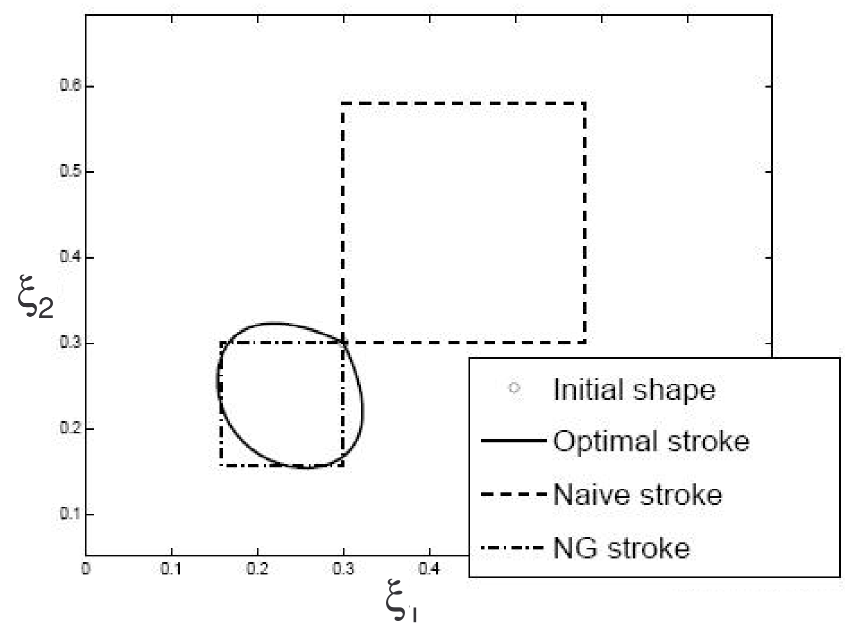
Three sphere swimmers: a race



Optimal stroke

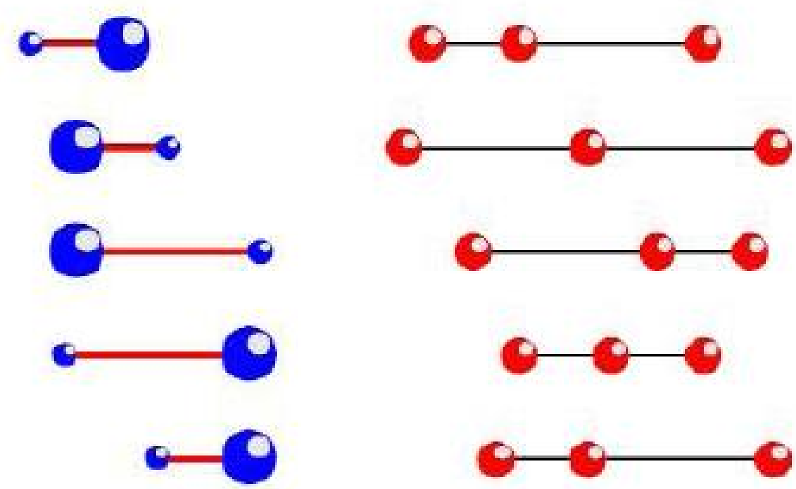


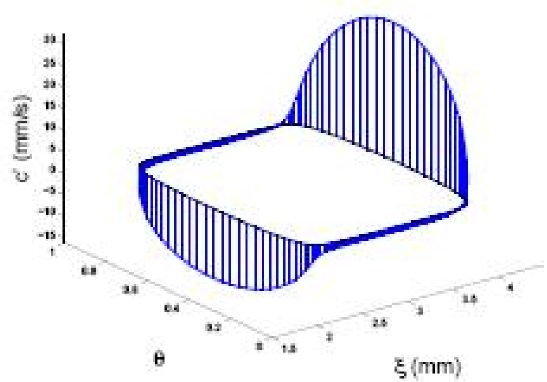
Level curves of curl V(ξ_1,ξ_2)



Optimal stroke maximizes flux of curl V at given energy input, or minimizes power consumption at given flux of curl V

Pushmepullyou





Positional change during optimal stroke for PMPY

Summary and perspectives

Eutrep2.mov

Eutreptiella sp. (Euglenoids)

- Self propulsion yields the key equation(s) giving positional change from shape change
- Swimming problem as a problem of controllability
- Optimal strokes of low Re swimmers computable; sub-riemannian geodesics

- Is this micro-organism swimming ?
- Is there something being optimized by the stroke of this micro-organism ?

Happy Birthday John!

John Hall Thorpe (1874-1947)
A Wild Flower Bunch, c1925.