Linear Functional Analysis, First Edition

Misprints and futher comments:

- Page 5, Defn 1.4: display should read $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$.
- Page 13, line -5: change $x \in A$ to $x \in M$.
- Page 13–14: Definition 1.23 and Theorem 1.25 (e) have been changed to read as follows:

  Definition 1.23
  Let (M, d) be a metric space. For any $x \in M$ and any number $r > 0$, the set

  $$B_x(r) = \{y \in M : d(x, y) < r\}$$

  will be called the *open ball* with centre x and radius r. If $r = 1$ the ball $B_x(1)$ is said to be an open *unit* ball. The set $\{y \in M : d(x, y) \le r\}$ will be called the *closed ball* with centre x and radius r. If $r = 1$ this set will be called a closed *unit* ball.

  Theorem 1.25 (e)
  For any $x \in M$ and $r > 0$, the "open" and "closed" balls in Definition 1.23 are open and closed in the sense of Definition 1.24. Furthermore,

  $$\overline{B_x(r)} \subset \{y \in M : d(x, y) \le r\},$$

  but these sets need not be equal in general (however, for most of the spaces considered in this book these sets are equal, see Exercise 2.12).
- Page 14, line -11: change $\overline{B_x(r)}$ to $\overline{B_x(1)}$.
- Page 17: change the third sentence of Theorem 1.35 to:
  In particular, if $\mathbb{F} = \mathbb{R}$ then the numbers $\sup\{f(x) : x \in M\}$ and $\inf\{f(x) : x \in M\}$, exist and are finite.
- Page 17: in Definition 1.36 change 'the function $f - g$ is continuous' to 'the function $|f - g|$ is continuous'.
- Page 40, lines -4, -5: change $n \ge \mathbb{N}$ to $n \ge N$.
- Page 55, line 4: change $\{a_n b_n\}$ to $\{a_n \overline{b}_n\}$.
- Page 57, line 14: change Lemma 3.12b to Lemma 3.12c.
- Page 59, Solution to Ex. 3.16: the displayed formulae should read

  $$\|f + g\|^2 + \|f - g\|^2 = 4 + 1 = 5,$$
  $$2(\|f\|^2 + \|g\|^2) = 2(1 + 1) = 4.$$
- Page 78, line -3: change Exercise 3.51 to Example 3.51.
- Page 80, line -6: change $\sum_{n=1}^{k}$ to $\sum_{n=1}^{\infty}$.

- Page 80, line -4: change $\sum_{n=1}^{k}$ to $\sum_{n=1}^{N}$.
- Page 82, line 4 of proof: change g_2 to g_1.
- Page 83, line -8: change g_1 to f_δ.
- Page 83, line -7: change g_1 to f.
- Page 85, Ex. 3.28 (c): delete the term $(2^n n!)^2$ on the right hand side of the formula (the calculation in the solution is correct, but needs to be divided by $(2^n n!)^2$ to yield the formula in the question).
- Page 89, line -10: change $f \in C_{\mathbb{C}}[0,1]$ to $f \in C_{\mathbb{F}}[0,1]$.
- Page 106, line -6: change 0 to $\{0\}$.
- Page 108, line 8: change 'part (a)' to 'part (b)'.
- Page 115, line -9: change $S \in B(X,Y)$ to $S \in B(Y,X)$.
- Page 116, line 3: change X to Y.
- Page 118, line -8: change $y \in B(X)$ to $y \in X$
- Page 119, line -7: change $\{x_n\} \in X$ to $\{x_n\}$ in X
- Page 133, lines -8, -9: change $-\lambda\overline{\lambda}I$ to $+\lambda\overline{\lambda}I$.
- Page 118: a mixture of notations is used in the final paragraph. Initially, operators C, D, E, F are mentioned in the text, but A, B, C, D are used in the first display, and then C, D, E, F are used in the second display. This occurred due to a decision to change notation, and then not implementing this change completely. This can be corrected by consistently using either the letters A, B, C, D or C, D, E, F throughout this paragraph. A similar mixture of notations, with a similar correction, also arises in Exercises 6.18, 6.19, and in their solutions.
  In addition, in the solution of Exercise 6.18, I_U, I_V should be changed to I_X, I_Y.
- Page 194, line -3: change Exercise 6.7 to Exercise 6.8.
- Page 204, line 6: change g to f.
- Page 204: Unless $\alpha_0 = \alpha_1 = 0$, the space Y_i is not a linear subspace, and T_i is not a linear operator (the first draft of the book dealt only with the case $\alpha_0 = \alpha_1 = 0$, but was then changed since the Green's function material needs $\alpha_0 \neq 0$).

  The simplest fix for this is to observe that we don't actually use linearity of T_i. The proof of Lemma 7.16 actually shows that $T_i : Y_i \to X$ is a bijection from the set Y_i to the linear space X. Lemma 7.17 then shows that T_i transforms the initial value problem (7.14), (7.15) into the inhomogeneous, linear Volterra equation (7.18) (this proof also does not use linearity). We then use linear methods to solve the Volterra equation, and the bijectivity of T_i shows that the solution of the Volterra equation corresponds to a solution of the initial value problem.

Alternatively, we can make the transformation

$$v = u - \alpha_0 - \alpha_1(s - a).$$

Substituting this into the initial value problem (7.14), (7.15) converts it to the following initial value problem

$$\begin{aligned} v'' + q_1 v' + q_0 v &= h, \\ v(a) = v'(a) &= 0, \end{aligned}$$

where $h(s) = f(s) - (\alpha_0 + \alpha_1(s-a))q_0(s) - \alpha_1 q_1(s)$ (as on p. 205). Applying the arguments given in the book to this transformed problem does give a linear subspace Y_i and a linear transformation T_i. Notice also that this transformation has actually crept into the formulation of the Volterra equation (7.18) on p. 205.

- Page 222, line 8: change $\frac{1}{n}$ to $\frac{1}{n+1}$.
- Page 222, Soln 2.4: $\alpha = -\dfrac{r}{\|x\|}$ is also a solution.
- Page 223, lines 10, 11 and 13: change $\frac{1}{n}$ to $\frac{1}{n+1}$.
- Page 225, line −5: change $\sum_{n=1}^{n}$ to $\sum_{n=1}^{k}$.
- Page 227, lines −8, −9: change dot product . to inner product $(\cdot, \cdot)$.
- Page 232, Soln. 4.1: change X to $[0, 1]$.
- Page 233, Soln. 4.2 (a): change g to h throughout this part of the solution; also, change $\int_X$ to $\int_0^1$.
- Page 237, line 10: change $\lim_{x\to 0}$ to $\lim_{n\to\infty}$.
- Page 242, Solution 5.19: The given solution is incomplete. At one point the solution says 'Hence if $|\lambda| < 2$ then $\lambda \in \sigma(T) \ldots$', but the argument given to prove this in fact only shows that at least one of $\lambda, -\lambda$ must belong to $\sigma(T)$. It will complete the proof if we can show that $\lambda \in \sigma(T) \iff -\lambda \in \sigma(T)$.

  To do this we define $S : \ell^2 \to \ell^2$ by

  $$S(x_1, x_2, x_3, x_4, \ldots) = (x_1, -x_2, x_3, -x_4, \ldots).$$

  Then $S \in B(\ell^2)$ and $S^2 = I$ (so S is invertible), and we have

  $$\begin{aligned} STS(x_1, x_2, x_3, x_4, \ldots) &= ST(x_1, -x_2, x_3, -x_4, \ldots) = S(0, 4x_1, -x_2, 4x_3, -x_4, \ldots) \\ &= (0, -4x_1, -x_2, -4x_3, -x_4, \ldots) \\ &= -T(x_1, x_2, x_3, x_4, \ldots). \end{aligned}$$

  Hence,

  $$\begin{aligned} \lambda \notin \sigma(T) &\iff S(T - \lambda I)S \ \text{ is invertible} \\ &\iff STS - \lambda I \ \text{ is invertible} \\ &\iff -T - \lambda I \ \text{ is invertible} \\ &\iff T + \lambda I \ \text{ is invertible} \\ &\iff -\lambda \notin \sigma(T), \end{aligned}$$

  which completes the proof.

- Page 243, line −2: change $\frac{1}{2}\sigma(S) + 1$ to $\frac{1}{2}(\sigma(S) + 1)$.
- Page 246, line 2: change the given B^* to $B^* = \dfrac{1}{\sqrt{2}} \begin{bmatrix} 2+i & 2-i \\ -2i-1 & -1+2i \end{bmatrix}$.
- Page 250, line 9: change α_n to α_m.
- Page 253, line 10: change λ^n to λ^{n-1}.

The following misprints occurred in the first printing but have been corrected in the second printing (you can tell if your copy is the second printing if it says '2nd printing' below 'Printed in Great Britain' on the page preceding the Preface; the first printing says nothing here).

- Page 11: part (d) of Definition 1.17 should read $d(x, z) \leq d(x, y) + d(y, z)$.
- Page 48, line −13: change Corollary 2.17 to Theorem 2.16.
- Page 54, line 1 of Example 3.7: change F^k to $L^2(X)$.
- Page 60, line 5: change Corollary 1.58 to Corollary 1.57.
- Page 66, part (d) of proof: Suppose that $0 \neq x \in A^\perp$ and let $y = \|x\|^{-1}x$. Changing x to y in the rest of the proof shows that $y = 0$, which is a contradiction and so proves that $x = 0$.
- Page 67, lines 1–2: interchange A and B.
- Page 70, line 19: change part (f) to part (b).
- Page 75, line −6: change (x, e_n) in right summand to α_n.
- Page 77, line −12: change Lemma 3.13 to Lemma 3.17.
- Page 82, line 4 of proof: change g_2 to g_1.
- Page 92, line 10: change $(a - b)$ to $(b - a)$.
- Page 142, line −6: change x to $\{x_n\}$ (x is not defined).
- Page 229, Solution 3.19: change U to Y.
- Page 231, Solution 3.26: change U to Y.