Week 4: The oscillating spring

Module F13YB1 **2004-05**

1. An object of mass 1 kg is attached to a spring with spring constant 1 N/m and is immersed in a viscous fluid with damping constant 2 Nsec/m. At time $t = 0$ the mass is lowered $\frac{1}{4}$ m and given an initial velocity of 1 m/sec in the upward direction. Show that the mass will overshoot its equilibrium position once and then return to equilibrium. Sketch the position of the mass as a function of time.

2. A spring is stretched 2 m by a mass of 2 kg. The mass is immersed in a viscous fluid with damping constant 4 Nsec/m and is acted on by an external force of $4\cos(2t)$ N in the downwards direction. Determine the steady state response of the system. (You may assume $g = 10$ m s$^{-}2$).

3. An object of mass 4 kg is attached to a spring with spring constant 64 N/m and is acted on by an external force $f(t) = A\cos^3(pt)$ in the downwards direction. Ignoring air resistance, find all values of p at which resonance occurs.

3.* An object of mass 1 kg is attached to a spring with spring constant 4 Nm^{-1}. The spring is immersed in a viscous liquid with damping constant 1 N sec m^{-1} and an external force $F(t) = 2\cos\omega t$ N is applied to the mass, where $\omega \geq 0$.

(a) Write down the differential equation governing the motion of the object and find its general solution.

(b) Consider now the steady state solution. Give its amplitude R as a function of ω. Find the frequency ω_{max} at which the amplitude $R(\omega)$ attains its maximum value. Sketch the function $R(\omega)$ and mark the value of ω_{max} in your sketch.

(c) Define the term resonance and find the value of the amplitude R at the resonance frequency. Mark the resoncance frequency in the sketch you made for (b).

[14 marks]

4.* A particle of mass 1 kg is attached to a spring with spring constant 9 N/m and is acted on by an external force $\cos(3t)$ N in the downward direction at time t. Ignoring air resistance, write down the differential equation describing the motion of the particle and find its general solution. If the particle is initially in the equilibrium position and at rest show that its displacement at time $t = \dfrac{3\pi}{2}$ sec is $\dfrac{\pi}{4}$ m.

[6 marks]

Hand in solutions to starred questions by 9 November