

Week 5: Examples on Series Solutions

Module F13YB1

2004-05

1. By using the method of solution in series find two linearly independent solutions of $y''(x) + y(x) = 0$.

2. Find the solution of

$$y''(x) - 2xy'(x) - 2y(x) = 0; \quad y(0) = 1, \quad y'(0) = 1.$$

3. Determine the nature of the point $x = 0$ of the following equation and hence find two linearly independent solutions.

$$2x^2y''(x) - xy'(x) + (1+x)y(x) = 0$$

4. By using the method of Frobenius find a nonzero solution of the equation

$$x \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0.$$

Is the solution analytic at $x = 0$?

5.* Determine the nature of the point $x = 0$ for the following equation

$$(1+x^2) \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0.$$

Hence find two linearly independent solutions of it.

[10 marks]

6.* By using the method of Frobenius find a nonzero solution of the equation

$$xy'' + (1-2x)y' + (x-1)y = 0.$$

[10 marks]

Hand in solutions to starred questions by 16 November