Week 3: Second Order ODE's
(linear inhomogenous equations)

Module F13YB1 **2004-05**

1. Three solutions of

$$y''(x) + a_1(x)y'(x) + a_0(x)y(x) = f(x) \qquad (*)$$

are $u_1(x) = x$, $u_2(x) = x + e^x$ and $u_3(x) = 1 + x + e^x$
Determine the solution of (*) satisfying the initial conditions $y(0) = 1$ and $y'(0) = 3$.

2. Find particular solutions of
(a) $y'' + 4y' + 4y = x^2$;
(b) $y'' - 4y' + 3y = 2e^x$;
(c) $y'' + 2y' + 5y = \cos(2x)$;
(d) $y'' + y = \sec(x)$.

3.* Solve the following initial value problem

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = e^{-3x}, \qquad y(0) = 0, \quad \frac{dy}{dx}(0) = 1.$$

[8 marks]

4* Show that the method of variation of the parameters applied to the equation

$$\frac{d^2y}{dx^2} + \omega^2 y = f(x),$$

where $\omega > 0$ is a real constant, leads to the particular solution

$$y(x) = \frac{1}{\omega}\int_0^x f(t)\sin[\omega(x-t)]\,dt.$$

[12 marks]

Hand in by 2 November