

Week 1: First Order Ordinary Differential Equations

Module F13YB1

2004-05

0. Integration revision. Find $y(x)$, given that

(i) $\frac{dy}{dx} = \frac{e^x}{3 + 6e^x}$; (ii) $\frac{dy}{dx} = x^2 \sin x$.

1. Name the type and find the general solution for each of the following first order equations:

(i) $\frac{dy}{dx} = \frac{x^2}{y}$; (ii) $\frac{dy}{dx} + 3y = x + e^{-2x}$; (iii) $x \frac{dy}{dx} = x \cos(2x) - y$;

(iv) $\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}$; (v) $xy^2 - x + (x^2y + y) \frac{dy}{dx} = 0$;

(vi) $(xe^y + 2y) \frac{dy}{dx} + e^y = 0$.

2. Show that each of the following equations has two distinct solutions.

(i) $\frac{dy}{dx} = y^{1/2}$; $y(0) = 0$; (ii) $y \frac{dy}{dx} = x$; $y(0) = 0$.

Explain, in each case, why the hypotheses of Picard's Theorem are not satisfied.

3. Find all constant solutions of $\frac{dy}{dx} + y^2 = 1$. Hence sketch the solutions satisfying

(i) $y(0) = 1.1$; (ii) $y(0) = 0$; (iii) $y(0) = -1.1$. Check your answer using Maple.

4. Consider the equation $\frac{dy}{dx} = y^2 - x$. Determine the points in the xy -plane at which

(i) $\frac{dy}{dx} = 0$; (ii) $\frac{dy}{dx} > 0$; and (iii) $\frac{dy}{dx} < 0$. Hence sketch the solution satisfying $y(1) = 1$.

Check your answer using Maple.

5*. Consider the following first order differential equation

$$\frac{dy}{dx} = y - y^3. \quad (*)$$

(a) Find all constant solutions and hence sketch the direction field of (*).

(b) Sketch the solution of (*) satisfying the initial condition $y(0) = \frac{1}{2}$ in the direction field you made for (a) (there is no need to solve the initial value problem explicitly).

[5 marks]

6*. Find the general solutions of the following first order differential equations.

(a) $(\sin x + x^2 e^y - 1) \frac{dy}{dx} + y \cos x + 2x e^y = 0$.

(b) $\frac{dy}{dt} + y = y^4$,

[15 marks]

hand in by 19 October