

## Weeks 6: Examples on Laplace Transforms

Module F13YB1

2004-05

1. Find the Laplace transforms of the following functions of  $t$ :  
 $(t + 1)^2$ ;  $\sin^2(t)$ ;  $t \sin(t)$ ;  $e^t \sin(t)$ ;  $te^{-t}$ .

2. Find the inverse Laplace transforms of

$$\frac{s+1}{s^2+2s+2}; \quad \frac{1}{s^2-3s+2}; \quad \frac{s}{(s-1)^2(s^2+4)}; \quad \frac{s}{s^2+4s+8}.$$

3. By using Laplace transforms solve the following initial value problem:

$$y'' - 2y' + 2y = \cos(t); \quad y(0) = 1, \quad y'(0) = 0.$$

4. Find the Laplace transforms of the following functions:

$$f(t) = \begin{cases} 0 & \text{if } 0 \leq t < \frac{\pi}{2} \\ \sin t & \text{if } t \geq \frac{\pi}{2} \end{cases} \quad g(t) = \begin{cases} 1 & \text{if } 0 \leq t < 2 \\ 2 & \text{if } t \geq 2. \end{cases}$$

5. Solve the initial value problem

$$y'' + y = f(t); \quad y(0) = 0, \quad y'(0) = 0$$

where

$$f(t) = \begin{cases} 1 & \text{if } 0 \leq t < 1 \\ 0 & \text{if } t \geq 1. \end{cases}$$

6. Solve the following equation with the help of the Convolution Theorem:

$$y(t) = 1 + \int_0^t y(x) \sin(t-x) dx,$$

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7.\* By using Laplace transforms solve the following initial value problems:

(i)  $y'' + y' - 6y = 2e^{3t}$ ;  $y(0) = 0$ ,  $y'(0) = 0$ .

(ii)  $y'' + 4y' + 4y = u_3(t)$ ;  $y(0) = 2$ ,  $y'(0) = 0$ .

(iii)  $e^{-t} = y(t) + 2 \int_0^t y(t-\tau) \cos(\tau) d\tau$

[20 marks]

Hand in solutions to starred questions by 23 November