

## Week 7/8: Laplace Transform and Boundary Value Problems

Module F13YB1

2004-05

1. Solve the following boundary value problem by using an appropriate Green's function:

$$y''(x) + 4y(x) = f(x); \quad y(0) = 0, \quad y'(1) = 0.$$

2. Consider the boundary value problem

$$y''(x) = f(x); \quad y(-1) = 0, \quad y(1) = 0.$$

- (a) Find the Green's function.  
(b) Show that the Green's function you found in (a) satisfies

$$\frac{\partial^2 G}{\partial s^2}(x, s) = \delta(s - x).$$

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- 3.\* A mass of 1 kg is attached to a spring with spring constant 3 N/m and is free to oscillate in a viscous fluid with damping constant 4 Nsec/m. At time  $t = 0$  the mass is stretched 0.25m downwards from its equilibrium position and released from rest. At time  $t = 3$  the mass is subjected to a downwards impulse of 2 Ns. Write down and solve a differential equation describing the displacement of the particle. [ 8 marks]

- 4.\* Solve the following boundary value problem by using an appropriate Green's function:

$$x^2 y''(x) + x y'(x) - y(x) = x^2; \quad y(1) + y'(1) = 0 = y(2) - 2y'(2), \quad 1 \leq x \leq 2$$

[12 marks]

Hand in solutions to starred questions by 30 November