Problem Sheet 6 for Oscillations and Waves

Module F12MS3 **2007-08**

1 A string of length 10 m, with a total mass 1 kg is fixed at its endpoints. The tension in the string is $\tau = 40$ N and the string is allowed to oscillate freely in one transverse direction.

(a) Write down the equation governing the transverse displacement of the string. Take care to define all variables you use.

(b) Find all normal modes of the string and sketch the lowest three modes at time $t = 0$.

(c) For each of the normal modes give the wavelength, angular frequency, period and frequency.

2 Suppose that $f : [-L, L] \to \mathbb{R}$ is a continuous and odd function i.e. $f(-x) = -f(x)$. Show that $\int_{-L}^{L} f(x)dx = 0$.

3 Find the Fourier series of each the following functions. In each case sketch the graph of the function to which the Fourier series converges over an x-range of three periods of the Fourier series.

(a) $g(x) = |x|, \quad -\pi < x < \pi,$

(b) $h(x) = 1 - x^2, \quad -1 < x < 1.$

4 Consider the wave equation

$$\frac{1}{v^2}\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}$$

for a real-valued function $z(t, x)$.

(a) Show that if $f : \mathbb{R} \to \mathbb{R}$ is an arbitrary twice differentiable function of one variable then

$$z(t, x) = f(x - vt) + f(x + vt) \tag{1}$$

satisfies the wave equation.

(b) Consider the "box" function

$$f(w) = \begin{cases} 0 & \text{if} \quad |w| > 1 \\ 1 & \text{if} \quad |w| \leq 1. \end{cases} \tag{2}$$

Sketch the corresponding solution (1) of the wave equation for the parameter value $v = 1$ at times $t = 0, 1, 3$.