

Section B: Electrical Engineering – Answer ALL questions

EE8) (10 Marks)

The transient current flowing in an RL electrical circuit is a solution of the first order differential equation:

$$\frac{di}{dt} + \frac{R}{L}i = 0.$$

Given that $L=10\text{H}$ and $R=100\Omega$, and that,

$$i(t) = Ke^{-pt},$$

is a solution of a first order differential equation of this type, determine the values of K and p for the case where $i = 5\text{ A}$ at time $t=0$.

EE9) (10 Marks)

The power in a resistive circuit may be calculated using:

$$P = I^2R,$$

where P is the power in the circuit, I is the current flowing in the circuit, and R is the circuit resistance.

The average value of a periodic signal $x(t)$ of period T is:

$$\bar{x} = \frac{1}{T} \int_0^T x(t) dt.$$

For a circuit with a 10Ω resistance, through which a sinusoidal current $I = \sin(\omega t)$ is passing, write down an expression for the average power in the circuit, and hence calculate the average power in watts. [Note that $\sin^2(\omega t) = \frac{1}{2} - \frac{1}{2}\cos(2\omega t)$]

EE10) (10 Marks)

A company sells amplifiers capable of generating 10W, 25W, and 50W of audio power. It has available 100 of the 10W units, of which 15% are defective, 70 of the 25W units of which 10% are defective, and 30 of the 50W units of which 10% are defective.

- What is the probability that an amplifier sold from the 10W units is defective?
- If each wattage amplifier sells in equal likelihood, what is the probability of a randomly selected unit being 50W and defective?
- What is the probability that a unit randomly selected for sale is defective?

END OF ELECTRICAL ENGINEERING SECTION B

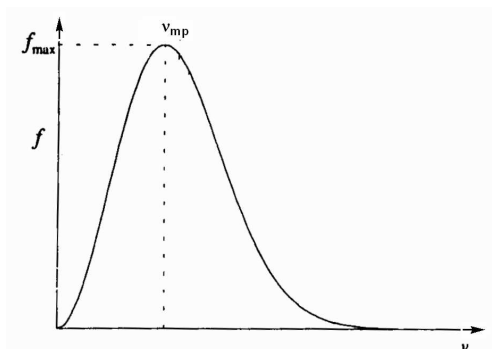
Section B: CHEMISTRY – Answer ALL questions

C8) (10 Marks)

The probability, f , that a gas molecule has a speed, v , is given by $f(v)$ and is called the Maxwell-Boltzmann distribution,

$$f(v) = bv^2e^{-av^2},$$

where a and b depend upon the mass and the temperature of the gas molecules and for a given molecule at a given temperature a and b are positive constants. The distribution looks like



where v_{mp} is the most probable speed at the maximum of the distribution.

- Differentiate f with respect to v and from df/dv find the most probable velocity v_{mp} which corresponds to the maximum of the curve
- Confirm that this is a maximum by differentiating again and finding the sign of d^2f/dv^2 at the value for v_{mp} .

SECTION B CHEMISTRY CONTINUES OVERLEAF

C9) (10 Marks)

When a gas expands at a constant temperature, T , from volume V_1 to V_2 , such that the gas pressure changes slowly from p_1 to p_2 and is always equal to the external applied pressure, then the amount of work done by the gas is given by the integral

$$\text{work} = - \int_{V_1}^{V_2} p \, dV.$$

Assuming the gas is an ideal gas then $pV = nRT$ where V is the volume of n moles of gas and $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ is the gas constant.

(a) Integrate the above equation to find an equation for the work done in terms of the initial and final gas volumes.

(b) Convert this to an expression in terms of the pressures p_1 and p_2 .

(c) If 3 mole of an ideal gas at a temperature of 298 K and at a pressure of 10 atmospheres expands to a final pressure of 1 atmosphere, what is the work done by the gas?

NOTE: You do not need to convert the pressure from atmospheres to any other unit.

C10) (10 Marks)

A chemist measures the bond length of hydrogen chloride gas 8 times (in units of 10^{-10} m)

1.279 1.277 1.277 1.281

1.278 1.283 1.281 1.280

Determine, showing ALL your working:

- The mean of the bond length to 4 significant figures.
- The median of the bond length to 4 significant figures.
- The standard deviation of the bond length.
- The relative standard deviation (defined as the standard deviation / mean value).

END OF CHEMISTRY SECTION B

Section B: PHYSICS – Answer ALL questions

P8) (10 marks)

a) The potential at a distance r from an electric charge q is

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r},$$

where ϵ_0 is a constant. The force on a charge of $+1$ unit is given by

$$F(r) = -\frac{dV}{dr}.$$

Find the force $F(r)$.

b) A light ray meets a plane slab of glass at an angle α to the normal (perpendicular). In the glass it is refracted, at an angle β to the normal. α and β are related by

$$\frac{\sin \alpha}{\sin \beta} = n,$$

where n is a constant (the refractive index of the glass). Find $\frac{d\beta}{d\alpha}$.

P9).

The motion of a particle performing damped, simple harmonic oscillation is given by

$$y = e^{-t} \sin(2t),$$

where y is the displacement from the mean position at time t . Determine the time when the displacement is first maximum and the corresponding value of the maximum displacement.

P10).

The work done by a thermally insulated gas when it expands from volume V_1 to V_2 is given by

$$W = \int_{V_1}^{V_2} p dV,$$

where the pressure p is related to V through the law $pV^{1.4} = \text{constant}$. Evaluate the work done when the initial p and V values are 10^6 Nm^{-2} and 1.0 litre and final volume is 2.5 litres.

END OF PHYSICS SECTION B

Section B: MECHANICAL ENGINEERING – Answer ALL questions

ME8) (10 Marks)

The dynamics of a decelerating car when a parachute is launched to slow the car down is governed by the following differential equation,

$$\frac{dv}{dx} = -\frac{C_D A_{chute} \rho_{air}}{2m_{car}} v,$$

where $C_D=1.2$ is the drag coefficient, $\rho_{air}=1.22 \text{ kg/m}^3$ is the density of air, $A_{chute} = 4\text{m}^2$ is the area of the parachute, $m_{car}=4000 \text{ kg}$ is the mass of the car, x is the distance coordinate and v is the speed of the car.

Defining A as :

$$A = \frac{C_D A_{chute} \rho_{air}}{2m_{car}}$$

(a) Calculate the value of A .

(b) Show that

$$v = e^{-Ax}$$

satisfies the differential equation given above.

SECTION B MECHANICAL ENGINEERING CONTINUES OVERLEAF

ME9) (10 Marks)

The ideal specific enthalpy, h , is a thermodynamic property used in many areas of thermodynamics, such as the design of jet engines and is given by the relationship,

$$h(T) = \int_{T_{ref}}^T c_p(x) dx,$$

where c_p is the specific heat capacity at constant pressure, T is the temperature and T_{ref} is a reference temperature.

If c_p has units of J/(kg C) what are the units of specific enthalpy?

Suppose the specific heat capacity for Nitrogen is well approximated by a cubic polynomial,

$$c_p(T) = c_0 + c_1T + c_2T^2 + c_3T^3,$$

where T is in degrees centigrade and the coefficients in the cubic polynomial are given in the following table,

Coefficient	Value
c_0	1.0302
c_1	0.0001
c_2	1×10^{-7}
c_3	-5×10^{-11}

If we take the reference temperature to be 0°C , evaluate the above integral between the temperatures 0°C and 800°C to give the ideal specific enthalpy for a temperature of 800°C .

SECTION B MECHANICAL ENGINEERING CONTINUES OVERLEAF

ME10) (10 Marks)

The table below presents some reliability data for time to failure of 100 electricity generators. f_i is the number of generators that failed during quarter i (quarter=3 months). F_i is the total number that had failed by the end of quarter i and R_i is the total number that were still functioning at the end of quarter i .

- a) Sketch a histogram of times to failure.
- b) Sketch the cumulative frequency of time to failure.
- c) What is the probability that a generator will fail in the first two quarters?
- d) What is the mean time to failure?

Generator Reliability			
Quarters (3 Month intervals)	f_i	F_i	R_i
1	3	3	97
2	15	18	82
3	54	72	28
4	23	95	5
5	4	99	1
6	1	100	0

END OF MECHANICAL ENGINEERING SECTION B

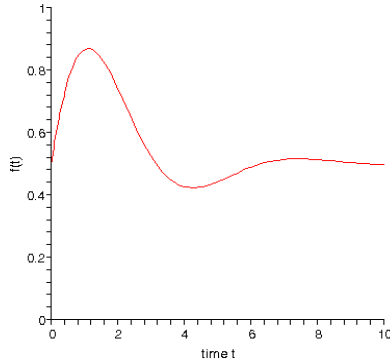
Section B: CHEMICAL ENGINEERING – Answer ALL questions

CE8) (10 Marks)

The response from a control system is described by the following equation:

$$f(t) = a + be^{mt} + ce^{nt} \sin(\omega t),$$

where a, b, c, n, m and ω are constants. An example of this is:



- Differentiate $f(t)$ with respect to t .
- If $b=0$, find the an expression satisfied by the values of t which are the stationary points of $f(t)$.

CE9) (10 Marks)

- The specific heat capacity of SO_2 gas as a function of temperature is:

$$C_p = a + bT + cT^{-2}$$

where the constant are:

a	b	c
46.222	7.8712×10^{-3}	-7.7037×10^{-5}

Calculate the average specific heat capacity for this gas over the temperature range $T_1 = 290\text{K}$ to $T_2 = 490\text{K}$.

Note that C_p is in kJ/kmol-K

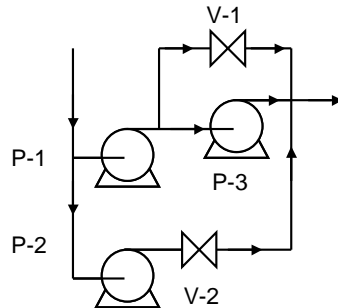
- Use the trapezium rule to find the value of $t = -k \int_{X_1}^{X_2} \frac{dX}{N}$ over the range

$X_1 = 2.5$, $X_2 = 0.5$ where $k=4.0$. Data for X and N is given as:

X	3	2.5	2.0	1.5	1.0	0.5
N	5	4.9	4.54	4.0	3.3	3.0

SECTION B CHEMICAL ENGINEERING CONTINUES OVERLEAF
CE10) (10 Marks)

In a process, liquid is pumped by a system consisting of three pumps and two valves, configured as shown below:



The probability that each item will function normally is given below:

Pumps		Valves	
P-1	0.95	V-1	0.8
P-2	0.90	V-2	0.75
P-3	0.85		

For the process to be working, liquid must be pumped through either P-1 and V-2, or through P-1 and P-2 or P-1 and V-1. From the probability data above, calculate the probability of the following sequences NOT working:

- P-1 and V-2
- P-1 and P-3
- P-1 and V-1

Hence determine the probability that liquid will be pumped through this system.

END OF CHEMICAL ENGINEERING SECTION B

NOTE FROM ANDY WHITE

This question is clearly not correct but I did not want to wait any longer to distribute this sample exam. I suggest you consult the Chemical Engineering staff for help. My guess is the question should say

For the process to be working, liquid must be pumped through either a) P-2 and V-2, or b) P-1 and P-3 or c) P-1 and V-1. Hence determine the probability that liquid will be pumped through this system.