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Host lifespan and the evolution of resistance to multiple parasites.

Host are typically challenged by multiple parasites, but to date theory on the evolution of resistance has mainly focused on single infections. We develop a series of models that examine the impact of multiple parasites on the evolution of resistance under the assumption that parasites coexist at the host population scale as a consequence of superinfection. In this way we are able to explicitly examine the impact of ecological dynamics on the evolutionary outcome. We use our models to address a key question of how host lifespan affects investment in resistance to multiple parasites. We show that investment in costly resistance depends on the specificity of the immune response and on whether or not the focal parasite leads to more acute infection than the co-circulating parasite. A key finding is that investment in resistance always increases as the immune response becomes more general independently of whether it is the focal or the co-circulating parasite that exploits the host most aggressively. Long-lived hosts always invest more than short-lived hosts in both general resistance and resistance that is specific to relatively acute focal parasites. However, for specific resistance to parasites that are less acute than co-circulating parasites it is the short-lived hosts that are predicted to invest most. We show that these results apply whatever the mode of defence i.e. whether it is through avoidance or through increased recovery, with or without acquired immunity, or through acquired immunity itself. As a whole, our results emphasise the importance of considering multiple parasites in determining optimal immune investment in eco-evolutionary systems.

Key words: epidemiology, ecology, host resistance, density dependence, superinfection, coexistence, lifespan.

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1. Introduction

In natural settings hosts are subject to attack from a multitude of parasites (Morand and Poulin, 2000; Nunn et al., 2003, 2005). The impact of multiple infection on the evolution of parasite virulence has been well studied (Bremermann and Pickering, 1983; Bonhoeffer and Nowak, 1994; Nowak and May, 1994; van Baalen and Sabelis, 1995; Frank, 1996; Mosquera and Adler, 1998; Gandon et al., 2001) with this theory suggesting that parasite diversity associated with higher parasite virulence (though collective action between co-infecting is arasites can alter this result, see Brown et al. (2002)). Furthermore, of particular interest is that the strength of this effect can decrease with the relatedness of the parasites (Frank, 1996; Gandon and Michalakis, 2002). The role of multiple infections in the evolution of host resistance, on the other hand, is less well studied (Poitrineau et al. (2003); Jokela 10 et al. (2000); Kada and Lion (2015)) with all of the evolution of resistance theory that 11 explicitly takes account of ecological feedbacks restricted to defence against a single parasite 12 (or transient parasite diversity, see Kada and Lion (2015)). Parasites clearly interact directly 13 through competition for susceptible hosts, but when the host evolves resistance to a focal 14 parasite the extent to which the resistance also counters co-circulating parasites constitutes 15 an additional, less obvious interaction between parasites. Therefore, the relationship between 16 parasite diversity and the pattern of evolved resistance is likely to be complex. In particular 17 there is considerable interest in the role that host lifespan plays in determining optimal 18 investment in costly defence (van Boven and Weissing, 2004; Miller et al., 2007; Boots et al., 19 2013; Donnelly et al., 2015) but it is not yet understood how this will depend on the nature 20 of co-circulating parasites. 21

There is a large body of work that examines the evolution of immunity in the context of ecological feedbacks and the presence of a single parasite strain (Bowers et al., 1994; Antonovics and Thrall, 1994; Boots and Haraguchi, 1999; van Baalen, 1998; van Boven and Weissing, 2004; Miller et al., 2007; Boots et al., 2013; Donnelly et al., 2015). In addition there are a few models that have considered parasite (or enemy) diversity in the study of defence (Jokela

et al., 2000; Poitrineau et al., 2003; Kada and Lion, 2015). Poitrineau et al. (2003) explored 27 defence against two natural enemies and examined how cross resistance (synergy in resistance) 28 influences optimal defence investment, while Jokela et al. (2000) focused on how the level of 29 parasite diversity impacts on the optimal level of defence allocation. Both studies consider 30 only evolutionary dynamics and do not incorporate ecological feedbacks so that the role of 31 life-histories that influence evolutionary outcomes through population dynamics is not clear. 32 In Kada and Lion (2015) which included a type of superinfection that did not involve stably coexisting parasites at the host population scale (rather, a rare invading parasite lineage 34 superinfected a stably circulating parasite and vice versa), the co-evolutionary dynamics of 35 recovery resistance and virulence were studied. They found that superinfection can lead to high virulence and high investment in defence but crucially, resistance developed to counter 37 one parasite did not simultaneously feedback to the prevalence of the other in the form of 38 superinfection modelled in Kada and Lion (2015). Here, we make a novel extension to these studies by applying an eco-evolutionary approach to the question of how stable co-circulating 40 parasitic challenges determine natural selection for host resistance achieved through avoidance, 41 recovery and acquired immunity. For the first time in a framework that allows multi-parasite 42 coexistence at the host population scale and encompasses specific as well as non-specific immune response we account for the complex ecological feedbacks between the dynamics of 44 multiple parasites and evolving resistance. In this way, we examine how traits such as host 45 lifespan determine patterns of optimal investment in host defence.

The framework of evolutionary invasion analysis (Metz et al., 1996; Geritz et al., 1998) 47 uses explicit ecological dynamics to derive fitness and provides tools for assessing the stability 48 of evolutionary trajectories. Here we use these methods to examine resistance evolution in 40 the presence of multiple parasites. This requires parasite coexistence, here referring to stable 50 persistence of more than one parasite at the host population scale. To achieve this we assume 51 superinfection interaction, where individual hosts infected with a less virulent parasite a 52 are susceptible to infection, with displacement of the original parasite by a more virulent 53 parasite (Nowak and May, 1994). Once co-existing multiple infections are incorporated in 54 host parasite models the question of the specificity of resistance naturally arises, and in 55

this study we examine how the level of cross resistance impacts on the evolution of host resistance to infection. The ecological derivation of host fitness for a range of disease and host characteristics provides clear insight into the effect of co-circulating parasites on host resistance, and demonstrates consistent patterns of investment regardless of the type, or actual mechanism, of resistance.

There has been considerable interest in how host immune investment differs between 61 populations of contrasting lifespans (van Boven and Weissing, 2004; Miller et al., 2007; Boots al., 2013; Donnelly et al., 2015). In particular, a naive view of immune investment is that it et 63 will increase monotonically as lifespan and hence exposure increases. However, recent theory 64 using evolutionary invasion analysis has shown that when ecological feedbacks are included 65 the relationship between life-span and immune investment can be complex (van Boven and 66 Weissing, 2004; Miller et al., 2007; Boots et al., 2013; Donnelly et al., 2015). However, as yet 67 none of this theory has taken account of parasite diversity. Here, by incorporating ecological 68 dynamics we achieve a key aim of our study: an examination of how investment in resistance varies with host lifespan when hosts are challenged by multiple parasites. 70

2. Methods

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(a) Epidemiological Model

We assume a host structure based on susceptible, infected, and recovered/immune sub-73 populations (Kermack and McKendrick, 1927; Macdonald, 1957; Anderson and May, 1979). 74 We extend the classical framework so that susceptible hosts, with density X, can be infected 75 by either hosts with a focal infection, Y_1 or a co-circulating infection, Y_2 . Hosts can recover and 76 gain life-long immunity to the focal infection, Z_1 and related to this the host may be infected 77 by the co-circulating parasite but immune to the focal parasite, Y_2^{Z1} . We allow therefore for 78 a range of resistance mechanisms (to the focal parasite) in the presence of a co-circulating parasite but for simplicity there is no acquired immunity to the co-circulating parasite, see 80 figure 1 for schematic depiction. Nevertheless immunity to the focal parasite and resistance in 81

- ⁸² general can carry over to the co-circulating parasite if it is non-specific. The epidemiological
- ⁸³ dynamics are governed by the following equations:

$$\frac{dX}{dt} = aH - qH^2 - bX - \beta_1 XY_1 - \beta_2 X(Y_2 + Y_2^{Z1}) + (1 - \nu_1)\gamma_1 Y_1 + \gamma_2 Y_2 \tag{1}$$

$$\frac{dY_1}{dt} = \beta_1 X Y_1 - (\alpha_1 + b + \gamma_1) Y_1 - s\beta_2 Y_1 (Y_2 + Y_2^{Z1})$$
(2)

$$\frac{dY_2}{dt} = \beta_2 X (Y_2 + Y_2^{Z1}) - (\alpha_2 + b + \gamma_2) Y_2 + s\beta_2 Y_1 (Y_2 + Y_2^{Z1})$$
(3)

$$\frac{dZ_1}{dt} = \nu_1 \gamma_1 Y_1 - bZ_1 - \sigma \beta_2 Z_1 (Y_2 + Y_2^{Z1}) + \gamma_2 Y_2^{Z1}$$
(4)

$$\frac{dY_2^{Z1}}{dt} = \sigma\beta_2 Z_1 (Y_2 + Y_2^{Z1}) - (\alpha_2 + b + \gamma_2) Y_2^{Z1}$$
(5)

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All parameters are non-negative and the total host density is given by $H = X + Y_1 + Y_2 + Y_2 + Y_1 + Y_2 + Y_2$ 85 $Y_2^{Z1} + Z_1$. All hosts produce susceptible offspring at rate a which is limited by intra-specific 86 crowding, q. Hosts die at natural death rate b. In addition, infected hosts suffer additional 87 disease induced mortality (virulence) at rate α_1 for the focal parasite and α_2 for the cocirculating parasite. The dynamics of transmission and recovery are shown in schematic form 89 in Figure 1. In detail we assume that transmission of infection is a mass action process 90 between susceptible and infected types, with transmission coefficient β_1 for the focal infection 91 and β_2 for the co-circulating infection. Virulence is assumed to be correlated with the rate 92 at which parasites exploit individual hosts. As a consequence, individuals infected with the 93 less virulent parasite are susceptible to infection by the more virulent parasite (since the 94 competitive advantage of high host exploitation leads to competitive replacement within the 95 host i.e. superinfection, see e.g. Nowak and May (1994)). If $\alpha_2 > \alpha_1$ the more aggressive co-96 circulating parasite superinfects the focal parasite and this is the situation represented by 97 equations 1-5 and depicted as model 1 in figure 1. If $\alpha_1 < \alpha_2$ the focal parasite is more 98 virulent and superinfects the co-circulating one and this is depicted as model 2 in figure 1 (for 99 brevity the equations for this model are not shown but it is simply the above model with the 100

direction of superinfection reversed and a transmission coefficient for the superinfection term 101 of β_1 rather than β_2). The superinfection coefficient s controls the strength of the interaction 102 and for our purposes $0 \le s \le 1$. Infected hosts recover at rate γ_1 from the focal infection and 103 γ_2 from the co-circulating infection, with a proportion of recoveries from the focal infection, 10 $\nu_1 \in (0, 1)$, becoming immune to the focal parasite and the remaining individuals returning to 105 a susceptible state. Immunity to the focal parasite can carry over to the co-circulating parasite 106 if it is non-specific. This occurs if $\sigma < 1$ and implies that immunity to the focal parasite reduces the likelihood of infection to the co-circulating parasite, see figure 1 which shows that infection 10 by the co-circulating parasite of class Z_1 occurs at σ the rate of that of X. 109

This general model form can be used to capture a wide range of classical infection 110 scenarios. For example, if $\nu_1 = 0$ the model represents a Susceptible–Infected–Susceptible 111 (SIS) framework, where there is no immune memory and recovered individuals are 112 completely susceptible to both infections. On the other hand if $\nu_1 = 1$ we have the 113 Susceptible–Infected–Recovered (SIR) model with specific ($\sigma = 1$) or non-specific ($\sigma < 1$) life-11 long immunity (though, for simplicity, the structure due to the co-circulating parasite remains 115 SIS). In this SIR example specificity (of acquired immunity) is denoted by σ i.e. if σ is high 116 then specificity is high. In all the other forms of resistance, specificity is denoted by c, and is 11 defined as a parameter in the host trait that resists the co-circulating infection (i.e. $\beta_2 = \beta_2(c)$ 118 for avoidance and $\gamma_2 = \gamma_2(c)$ in the case of recovery) and here high values of c correspond to low 119 specificity (see later for more details). The fundamental forms of host defence can be defined 120 as follows (Boots et al., 2013): (i) avoidance reduces the probability of becoming infected and 121 resistant hosts therefore have a lower transmission rate (β_1) , (ii) recovery increases the rate of 122 clearance of infection (γ_1) and (iii) acquired immunity increases the probability of recovering 123 to a life-long immune state (ν_1). We first consider routes of innate resistance, i.e. avoidance 12 and recovery (i and ii above) in an SIS setting, then in an SIR setting with specific life-long 12 immunity and later evolution of acquired immunity itself. 126

(b) Population Dynamics

A key measure of the ability of a parasite to spread in a host population is R_0 , the basic reproduction number, given here by

$$R_0^i = \beta_i X^0 / (\alpha_i + b + \gamma_i) \tag{6}$$

for parasite *i* in the absence of the alternative parasite. In equation 6 X^0 represents the equilibrated density of susceptible hosts in the absence of any infection (i.e. the host carrying capacity, $X^0 = (a - b)/q$). A second key measure is endemic disease prevalence, the frequency of infected individuals in the equilibrium host population. In single infection models of this type, whether the population structure is *SI*, *SIS*, *SIR* (i.e. our model with s = 0) or *SIRS*, prevalence at the endemic equilibrium satisfies,

$$\alpha \frac{Y}{H} = a - qH - b \tag{7}$$

i.e. prevalence scaled by virulence equals per capita host population turnover (i.e. density dependent net reproduction). However, when there are two infections in the population, as per the model represented by equations 1-5, per capita turnover at equilibrium equals the sum of the prevalences of the two infections weighted by their respective rates of virulence,

$$\alpha_1 \frac{Y_1}{H} + \alpha_2 \frac{Y_2}{H} = a - qH - b \tag{8}$$

Therefore, as per single infection models, equilibrium infection in the host population is determined by the supply of susceptible individuals (i.e. turnover) but with the key difference that host turnover is shared amongst the multiple infections. One consequence of equation 8 is that coexistence of parasites means that equilibrium prevalence of any one parasite is always less than it would be if it were circulating in the host population alone. A condition for the

stable coexistence of parasites at the host population scale can be found in Nowak and May(1994) for a similar model.

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(c) Trade-off

There is strong empirical evidence for the association of resistance with physiological costs 148 through the diversion of resources to the development and maintenance of the resistance. For 149 example, in Fuxa and Richter (1989) the percentage of eggs that hatch as well as the number 150 produced per female were all lower in fall armyworm lines selected for resistance to NPV. 151 Longer development time, reduction in egg viability as well as an increase in pupal weight 152 were a consequence of selection for resistance to a granulosis virus in *Plodia interpunctella* 153 (Boots and Begon, 1993). There is also evidence of reduced larval competitive ability in 15 immune-selected Drosophila melanogaster (Kraaijeveld and Godfray, 1997). Taken together 155 these studies represent a sound basis for assuming that costs to resistance can be manifested 156 in reduced host reproduction or reduced competitive ability. In this study we assume an 15 association between level of resistance and reproduction rate such that recovery, avoidance 158 and acquired immunity are all positive decreasing functions of host reproduction rate. This is 159 consistent with the majority of previous studies that examine the evolution of resistance to parasites (see Boots et al. (2009)). 161

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(d) Specificity of Immune Response

We begin by considering an SIS framework where the focal parasite is less virulent 163 than the co-circulating parasite (i.e. $\alpha_1 < \alpha_2$). Hosts invest in costly resistance, $0 \le \theta(a) \le 1$, 164 through avoidance of the focal infection (i.e. $\hat{\beta}_1 = \beta_1(1 - \theta(a))$) and resistance may carry 16 over to the co-circulating infection depending on the specificity of resistance $(0 \le c \le 1, \text{ when }$ 166 c=0 the resistance is specific to the focal infection), i.e. $\hat{\beta}_2 = \beta_2(1-c\theta(a)))$. As c increases 167 the resistance becomes more general. Alternatively resistance can be through recovery (i.e. 168 $\hat{\gamma}_1 = \gamma_1(1+\theta(a))$ and $\hat{\gamma}_2 = \gamma_2(1+c\theta(a)))$. Similarly the focal infection can be more virulent than the co-circulating parasite for each of the above cases (i.e. $\alpha_1 > \alpha_2$). When it comes to 170 an SIR framework we consider all of the above cases but, for brevity, only present results for 171

cases where the focal parasite is less virulent than the co-circulating parasite. Finally in an 172 SIR framework resistance may be through acquired immunity, corresponding to $\hat{\nu}_1 = \theta(a)\nu$. 173 For convenience we view specificity of acquired immunity not in terms of the probability 174 of clearance of the co-circulating infection to an immune state, but rather as the decrease 17 in transmission of the co-circulating infection to individuals who are immune to the focal 176 infection. For this reason, specificity in acquired immunity is a fixed coefficient, σ , in equations 177 4-5 with $\sigma = 1$ when resistance is specific or $\sigma < 1$ when it is not specific. For simplicity, we do not allow the less intuitive case where σ exceeds 1 (i.e. resistance developed to counter a 179 focal parasite is more effective against a co-circulating parasite). See table 1 for a summary 180 of the cases studied. 181

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3. Results

Using the next generation method (Diekmann et al., 1990; van den Driessche and Watmough, 183 2002; Hurford et al., 2010), see supporting information S1, we derive a proxy for invasion 184 fitness, denoted $s_r(m)$, for the set of models outlined in the *methods* section for each of the cases 18 detailed in table 1. Under the assumptions of adaptive dynamics (Metz et al., 1996; Geritz 18 et al., 1998) a population will evolve through small, rare mutations in the direction of the 187 gradient of the invasion fitness until an evolutionary singularity, where the mutant derivative 188 of invasion fitness is zero, is reached (alternatively the evolving population may reach the 189 limit of the phenotypic range). Evolutionary singularities can be classified according to their 190 evolutionary and convergence stability properties (Metz et al., 1996). If a singularity is both 191 evolutionary and convergence stable it is an uninvadable evolutionary attractor and an end 192 point of evolution (Eshel, 1983). We wish to examine how the position of such singularities, 193 which is determined by selection pressures, change when model parameters, in particular 194 host lifespan, are varied. The results presented throughout are obtained using mathematical 195 software for symbolic computation (Maple). They are additionally supported with simulations of the adaptive dynamics process whereby population dynamics of interacting resident and 197 mutant host sub-populations are numerically solved with mutants, of similar effect to residents, 198

randomly introduced on a time scale slower than that of the ecological dynamics (for further detail on adaptive dynamics simulations see Donnelly et al. (2015)).

For SIS innate resistance we use the invasion fitness proxy, $s_r(m)$, for hosts bearing a 20 mutant investment phenotype, $\theta^m(a^m)$, to locate evolutionary attractors and to show how the 202 evolved level of the resistance phenotype varies with host lifespan, see figure 2a - d. This can 203 be shown when resistance is specific (black curves, figure 2a - d) and also when resistance 20 is non-specific (grey curves, figure 2a - d). It can also be shown when resistance is through avoidance (figure 2a & b, i.e. cases 1-4 table 1) and when resistance is through recovery 20 (figure 2c & d, i.e. cases 5-8 table 1), when the resistance is developed primarily to counter 207 a relatively avirulent focal infection (figure 2a & c) or to counter a relatively virulent focal infection (figure 2b & d). The resulting graphs indicate that regardless of the route of innate 20 resistance, investment increases with host lifespan except when it is specific to an avirulent 210 infection. 21

Focusing on the case where resistance evolves to counter an avirulent focal infection we 212 show that these results extend to an SIR framework, arising through the presence of acquired 213 immunity specific to the avirulent focal infection (i.e. $\sigma = 1$), see figure 3a & b for avoidance, 214 i.e. cases 9-10 table 1, and see figure 3c & d for recovery, i.e. cases 11-12 in table 1. As the 21 proportion of immune individuals in the population increases (due to changing the value of ν , 216 i.e. the probability of inducing acquired immunity upon recovery, from $\nu = 0$ represented by a 217 black curve to $\nu = 1$ represented by a light grey curve) there is no qualitative change, though 21 the overall magnitude of investment tends to decrease (because recovery to immunity decreases 219 prevalence, reducing the need for resistance). Finally, we analyse optimal acquired immunity 220 developed to counter the less virulent parasite. Here, the mutant investment phenotype is 221 $\nu_1^m(a^m)$ and immunity extends to the virulent infection if $\sigma < 1$. When immunity is non-222 specific, investment increases with increasing lifespan, when immunity is specific investment 223 decreases with increasing lifespan, see figure 4a, i.e. cases 13 - 14 in table 1. 224

As a whole, the results show that resistance to a relatively avirulent focal infection in the presence of a co-circulating virulent infection varies with host lifespan in a manner that is dependent on the specificity, but significantly, is not dependent on the route of resistance.

In general, investment increases as the level of specificity in resistance decreases. We provide 228 a further illustration of this in figure 4b - d where curves are given for different lifespans 229 for evolving avoidance, 4b, recovery, 4c and acquired immunity, 4d, respectively. Investment 230 is greater at low lifespans when resistance is specific (c is low) but investment is greater at 231 long lifespans when resistance is relatively general (c is high). Therefore, there is a level of 232 specificity for each form of resistance below which investment decreases with increasing host 233 lifespan and above which investment increases with increasing host lifespan. This transition 234 occurs for relatively small values of specificity for the innate forms of resistance (i.e. avoidance 235 and recovery) compared to the relatively high value of specificity at which it occurs for acquired 236 immunity. 237

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4. Discussion

Hosts are typically challenged by multiple parasites, but to date theory on the evolution of 239 resistance has mostly focused on single infections. We have developed a series of models that 240 have examined the impact of multiple parasites on the evolution of resistance with explicit feedbacks between the ecological and evolutionary dynamics. Our key assumption is that parasites coexist as a consequence of superinfection which assumes that a more virulent 243 parasite can replace a less virulent parasite within an individual host. Our results show 244 that co-circulating parasitism dramatically impacts on the evolution of resistance to a focal 24 parasite. In particular, the specificity of the resistance with respect to co-circulating parasites 246 is critical to the outcome. A key, intuitive, result is that investment in resistance increases 247 as the immune response becomes more general. This finding is related to those of previous studies that considered the impact of multiple enemies on resistance evolution in the absence 249 of ecological dynamics. Jokela et al. (2000) considered the evolution of resistance for different 250 levels of parasite diversity. They showed that specific host resistance is less effective when 251 faced with a diverse range of parasites and therefore that host resistance increases as parasite 252 diversity decreases. Poitrineau et al. (2003) examined the evolution of defence to two separate 253 enemies and considered scenarios of synergy or interference in defence response, showing that 25 investment increases as the level of synergy increases. Our finding that resistance increases 25 as immune investment becomes more general is related to these results, and extends them to 256 systems including explicit feedbacks between the ecological and evolutionary dynamics. 257

Risk of infection by pathogens and parasites has led hosts to evolve a wide range of defence mechanisms from behavioural strategies (Joop et al., 2014) to the bio-chemical cascades of the complement system and the memory B and T cells of acquired vertebrate immunity (Schmid-Hempel, 2002; Frank, 2002). Intuition suggests that the longer a host lives the more it is likely to benefit from immunity. This observation has been used to explain macro-evolutionary patterns of investment such as the lack of acquired immunity in invertebrates (Ricklefs and Wikelski, 2002; Tieleman et al., 2005) and is supported by a number of empirical studies. For example, a positive correlation between immunity and lifespan in avian hosts has been

demonstrated for humoral, cell mediated, and constitutive immune responses (Ardia, 2005; 266 Tella et al., 2002; Versteegh et al., 2012; Lee et al., 2008). Theoretical models that have 267 examined the evolution of resistance in the face of a single parasite make the assumptions 26 underlying this intuition explicit. They have provided some support for this pattern but also deviate from it in important ways (van Boven and Weissing, 2004; Miller et al., 2007; Boots 270 et al., 2013; Donnelly et al., 2015). For example, in contradiction to the intuition, optimal 271 resistance in hosts capable of permanent acquired immunity can be maximal at intermediate lifespan (Boots et al., 2013; Donnelly et al., 2015) and in the case of innate resistance this can 273 be true even in the absence of acquired immunity (Miller et al., 2007; Donnelly et al., 2015). 274 However, a key aspect of these studies is that host populations are burdened by only one 27 infection. Here we address the key question of how optimal investment changes with lifespan 27 in the face of co-circulating parasites. 277

When a host population is challenged by multiple parasites the investment in immunity 27 is critically dependent on the specificity of the defence. When the resistance is relatively 279 general, then investment increases with host lifespan. In contrast, when immunity is specific 280 the pattern of investment relative to host lifespan depends on the nature of the co-circulating 281 parasite. If the co-circulating parasite is less aggressive in exploiting the host than the focal infection, then investment increases with lifespan. However, if the co-circulating parasite is 28 more aggressive, then specific immune investment decreases as host lifespan increases because 284 the ratio of infected individuals with the co-circulating parasite to individuals with the focal parasite increases (since there is a higher incidence of superinfection at high host lifespans). 286 These patterns are true in our model when the evolving resistance is innate in a host incapable 287 of immune memory, is innate in a host responding additionally with immune memory or when 288 the evolving resistance is itself acquired. This is an important insight since it shows that the life-history patterns will depend on the nature of the co-circulating parasite, and the 290 specificity of the response, but not the mode of resistance itself, which is in stark contrast to 201 single infection models where patterns fundamentally depend on the type of resistance (i.e. 29 innate vs acquired) but not the exact mode (for example avoidance vs recovery within the 293 innate type) (van Boven and Weissing, 2004; Miller et al., 2007; Boots et al., 2013; Donnelly 294

et al., 2015). Therefore, a key implication of our work is that, in contrast to single infection models, the classic idea that more investment should occur in longer-lived hosts is generally supported when there are multiple parasites.

What are the underlying processes that lead to these different findings (i.e. in the effect of host lifespan on optimal immune defence) when host are faced by multiple rather than single 299 parasites? Single infection models deviate from the intuition that investment increases with 300 lifespan because of two important effects that are undermined by the presence of co-circulating infections (see detailed analysis in Donnelly et al. (2015)). In single infection models, optimal 302 investment that is maximal at intermediate lifespans (Miller et al., 2007) is a hallmark of 303 innate resistance because it is characterised by the return or maintenance of individuals to a susceptible state as opposed to the conversion of them into an immune state (Donnelly 30 et al., 2015). Since susceptible individuals are vulnerable to reinfection which is likely at high 306 levels of prevalence, the benefit of innate resistance is low at high prevalence and therefore low 30 at high lifespans (in SIS systems prevalence increases with increasing host lifespan). With 30 multiple parasites and superinfection, more virulent parasites take over hosts infected with less 309 virulent parasites. When hosts live longer, the period during which these conversions occur is 310 longer and this favours the virulent parasite. However, the higher virulence of these parasites 31 also acts to reduce the infectious period and as a consequence, prevalence does not rise to the 312 high levels that are seen in equivalent single infection models. As such, optimal investment 313 increases with lifespan in the face of multiple infections and superinfection in models where it would be maximal at intermediate lifespans without the co-circulating infection because the 315 prevalence of the focal parasite is strongly limited due to the share of susceptible hosts taken 316 by the co-circulating parasite, see equation 8. 317

There is a second process that comes into play once there is permanent immunity to the parasite. In single infection models where the host is long-lived, permanent immunity leads to high host density. When host density approaches the carrying capacity there is little host turnover and prevalence levels are low (see equation 7). Therefore long-lived host populations with permanent immunity have a relatively small risk of infection and will evolve weaker resistance (Miller et al., 2007; Boots et al., 2013; Donnelly et al., 2015). For this reason a long-lived immune class can decrease the need for immunity in general at high lifespans. However, crucially, when there are multiple infections the impact of an especially long-living class arising from recovery to a permanent immune state will be substantially less because the immune individuals will be susceptible (at least to some degree) to infection by co-circulating parasites. Therefore, when acquired immunity evolves in the face of multiple parasites and superinfection, just as for innate immunity, optimal investment is higher in long-lived host populations in models where it would be maximal at intermediate lifespans without the cocirculating infection.

There is one important exception to our general prediction that investment in immunity 332 rises with host lifespan. When the co-circulating parasite is more virulent and the evolving 333 response is specific to the less virulent focal parasite, then investment decreases with increasing 334 lifespan. Two simple interactions are responsible for this result: 1 if the co-circulating parasite 335 is more virulent then it is the superinfector and it is favoured at high lifespans. Therefore the 336 benefit of specific resistance to the focal parasite, which by definition is not effective against 33 the co-circulating parasite, diminishes as lifespan increases. 2) Responding through resistance 338 to the less virulent focal parasite can actually increase the risk of infection with the more 339 virulent co-circulating parasite (since there is an increase in the availability of susceptible individuals for the co-circulating infection). Therefore, taken together, there is little fitness 34 benefit to investing resources into fighting the lesser of your enemies and specific resistance 342 to the less aggressive parasite is not favoured at high host lifespans under an assumption of 34 superinfection. We note that several of these interactions are a consequence of the interplay between strain prevalence, their relative virulence and virulence associated superinfection. For 345 this reason it is important to acknowledge that alternative mechanisms of coexistence may 346 lead to different results.

In conclusion, there are multiple factors that determine the relationship between optimal investment in immunity and host lifespan. This results in a variety of patterns for single infection models (Miller et al., 2007; Boots et al., 2013; Donnelly et al., 2015; van Boven and Weissing, 2004) but here we have shown that this intricacy can be lost when diversity in the

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parasite burden of the host population is considered. Instead it is the classic idea that long-352 lived hosts invest more in immunity that is generally supported when this key aspect of natural 353 populations is included. Our main focus has been on how multiple parasites impact on the 354 relationship of host lifespan to resistance, but more generally our inclusion of realistic ecological 35 feedbacks in evolutionary models of resistance extends results of previous multi-enemy models 356 that assumed constant rather than dynamic populations (Poitrineau et al. (2003); Jokela 357 et al. (2000)). Future work should relax the assumption that superinfection occurs and may 358 therefore involve different population feedbacks whose effects should be assessed. Such co-359 infection models would be more challenging theoretically, but the importance of including 360 ecological feedbacks is emphasized by our work. Furthermore, there is a need for a variety 361 of defence interactions against a range of enemies beyond resistance to two parasites to be 362 examined in this broader eco-evolutionary context. 363

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References

- Anderson, R. M. and R. M. May, 1979. Population biology of infectious diseases: Part I.
 Nature 280:361–367.
- Antonovics, J. and P. H. Thrall, 1994. The cost of resistance and the maintenance of geneticpolymorphism in host-pathogen systems. Proceedings of the Royal Society B-Biological
 Sciences 257:105–110.
- Ardia, D., 2005. Individual quality mediates trade-offs between reproductive effort and immune
 function in tree swallows. Journal of Animal Ecology 74:517–524.
- Bonhoeffer, S. and M. A. Nowak, 1994. Mutation and the evolution of virulence. Proceedings
 of the Royal Society B-Biological Sciences 258:133–140.
- Boots, M. and M. Begon, 1993. Trade-offs with resistance to a granulosis virus in the Indian meal moth, examined by a laboratory evolution experiment. Functional Ecology 528–534.
- Boots, M. and Y. Haraguchi, 1999. The evolution of costly resistance in host-parasite systems.
 American Naturalist 153(4):359–370.
- Boots, M., A. Best, M.R. Miller and A. White, 2009. The role of ecological feedbacks in the
 evolution of host defence: what does theory tell us? Philos Trans R Soc Lond B Biol Sci.
 364(1513):27-36.
- Boots, M., R. Donnelly, and A. White, 2013. Optimal immune defence in the light of variation in lifespan. Parasite Immunology 35:331–338.
- Bowers, R. G., M. Boots, and M. Begon, 1994. Life-history trade-offs and the evolution of
 pathogen resistance: competition between host strains. Proceedings of the Royal Society
 B-Biological Sciences 257:247-253.
- Bremermann, H. J. and J. Pickering, 1983. A game-theoretical model of parasite virulence.
 Journal of Theoretical Biology 100:411-426.

- Brown, S. P., M.E. Hochberg and B.T. Grenfell, 2002. Does multiple infection select for raised
 virulence? Trends in microbiology 10:401–405.
- ³⁹² Diekmann, O., J. A. P. Heesterbeek, and J. A.J. Metz, 1990. On the definition and computation
- of the basic reproduction ratio R0 in models for infectious diseases in heterogeneous populations. J. Math. Biol. 28(4)365–382.
- Donnelly, R., A. White, and M. Boots, 2015. The epidemiological feedbacks critical to the
 evolution of host immunity. Journal of Evolutionary Biology 28(11):2042–2053.
- van den Driessche, P. and J. Watmough, 2002. Reproduction numbers and sub-threshold
 endemic equilibria for compartmental models of disease transmission. Math. Biosci.
 180(1)29–48.
- Eshel, I., 1983. Evolutionary and continuous stability. Journal of Theoretical Biology 103:99–
 111.
- Frank, S., 2002. Immunology and evolution of infectious disease. Princeton University Press,
 Princeton, USA.
- ⁴⁰⁴ Frank, S. A., 1996. Models of parasite virulence. Quarterly Review of Biology 71:37–78.
- Fuxa, J. R. and A. R. Richter, 1989. Reversion of resistance by Spodoptera frugiperda to
 nuclear polyhedrosis-virus. Journal of Invertebrate Pathology 53(1)52–56.
- Gandon, S., V. A. A. Jansen, and M. van Baalen, 2001. Host life history and the evolution of
 parasite virulence. Evolution 55:1056–1062.
- Gandon, S. and Y. Michalakis, 2002. Multiple infection and its consequences for virulence
 management. *in* pages 150-164 of. U. Dieckmann, J.A.J. Metz, M.W. Sabelis, and K.
 Sigmund, eds. Adaptive dynamics of infectious diseases: in pursuit of virulence management.
 Cambridge University Press.

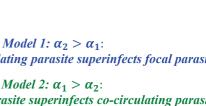
- Geritz, S., E. Kisdi, G. Meszena, and J. A.J. Metz, 1998. Evolutionarily singular strategies and the adaptive growth and branching of the evolutionary tree. Evolutionary Ecology 12(1):35–57.
- Hurford, A., D. Cownden, and T. Day, 2010. Next-generation tools for evolutionary invasion
 analyses. Journal of the Royal Society Interface 7:561–571.
- Jokela, J., P. Schmid-Hempel, and M. C. Rigby, 2000. Dr. pangloss restrained by the red queen - steps towards a unified defence theory. Oikos 89:267–274.
- Joop, G., O. Roth, P. Schmid-Hempel, and J. Kurtz, 2014. Experimental evolution of external immune defences in the red flour beetle. Journal of Evolutionary Biology 27:1562–1571.
- Kada, S. and S. Lion, 2015. Superinfection and the coevolution of parasite virulence and host
 recovery. Journal of Evolutionary Biology 28:2285-2299.
- Kermack, W. O. and A. G. McKendrick, 1927. A contribution to the mathematical theory of
 epidemics. Proceedings of the Royal Society A 115(772):700-721.
- ⁴²⁶ Kraaijeveld, A. R. and H. C. J. Godfray, 1997. Trade-off between parasitoid resistance and
 ⁴²⁷ larval competitive ability in Drosophila melanogaster. Nature 389:278–280.
- Lee, K. A., M. Wikelski, W. D. Robinson, T. R. Robinson, and K. C. Klasing, 2008.
 Constitutive immune defences correlate with life-history variables in tropical birds. Journal
 of Animal Ecology 77:356–363.
- Macdonald, G., 1957. The epidemiology and control of malaria. Oxford University Press,
 Oxford, UK.
- 433 Metz, J. A. J., S. A. H. Geritz, G. Meszena, F. J. A. Jacobs, and J. S. V. Heerwaarden, 1996.
- Adaptive dynamics: a geometrical study of the consequences of nearly faithful reproduction. *in* pages 183-231 of S. J. Van Strein and S. M. Verduyn Lunel, eds. Stochastic and spatial
- *in* pages 183-231 of S. J. Van Strein and S. M. Verduyn Lunel, eds. Stochastic and spatia
 structures of dynamical systems. Elsevier, North- Holland.

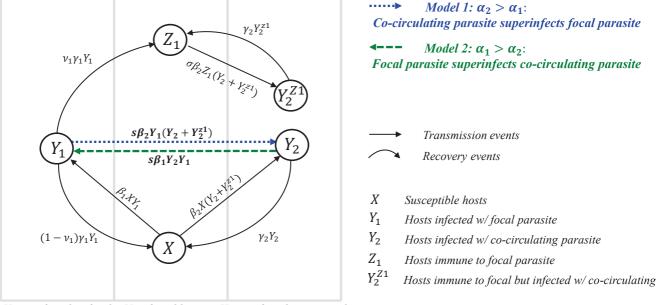
- ⁴³⁷ Miller, M. R., A. White, and M. Boots, 2007. Host life span and the evolution of resistance
- 438 characteristics. Evolution 61(1):2-14.
- Morand, S. and R. Poulin, 2000. Nematode parasite species richness and the evolution of
 spleen size in birds. Canadian Journal of Zoology 78:1356–1360.
- Mosquera, J. and F. R. Adler, 1998. Evolution of virulence: a unified framework for coinfection
 and superinfection. Journal of Theoretical Biology 195:293–313.
- Nowak, M. A. and R. M. May, 1994. Superinfection and the evolution of parasite virulence.
 Proceedings of the Royal Society B-Biological Sciences 255:81–89.
- Nunn, C. L., S. M. Altizer, K. E. Jones, and W. Sechrest, 2003. Comparative tests of parasite
 species richness in primates. American Naturalist 162:597–614.
- Nunn, C. L., S. M. Altizer, W. Sechrest, and A. A. Cunningham, 2005. Latitudinal gradients
 of parasite species richness in primates. Diversity and Distributions 11:249–256.
- Poitrineau, K., S. P. Brown, and M. E. Hochberg, 2003. Defence against multiple enemies.
 Journal of Evolutionary Biology 16:1319–1327.
- ⁴⁵¹ Ricklefs, R. E. and M. Wikelski, 2002. The physiology/life-history nexus. Trends in Ecology
 ⁴⁵² & Evolution 17:462–468.
- 453 Schmid-Hempel, P., 2002. Evolutionary Parasitology. Oxford University Press, Oxford, UK.
- Tella, J. L., A. Scheuerlein, and R. E. Ricklefs, 2002. Is cell-mediated immunity related to the
 evolution of life-history strategies in birds? Proceedings of the Royal Society B-Biological
 Sciences 269:1059–1066.
- Tieleman, B. I., J. B. Williams, R. E. Ricklefs, and K. C. Klasing, 2005. Constitutive innate
 immunity is a component of the pace-of-life syndrome in tropical birds. Proceedings of the
 Royal Society B-Biological Sciences 272:1715–1720.
- van Baalen, M. and M. W. Sabelis, 1995. The dynamics of multiple infection and the evolution
 of virulence. American Naturalist 146:881–910.

- van Baalen, M., 1998. Coevolution of recovery ability and virulence. Proceedings of the Royal
- 463 Society B-Biological Sciences 265:317–325
- van Boven, M. and F. J. Weissing, 2004. The evolutionary economics of immunity. American
 Naturalist 163:277–294.
- 466 Versteegh, M. A., I. Schwabl, S. Jaquier, and B. I. Tieleman, 2012. Do immunological,
- 467 endocrine and metabolic traits fall on a single pace-of-life axis? Covariation and constraints
- among physiological systems. Journal of Evolutionary Biology 25:1864–1876.

Case I	Resistance type under evolution	Infection hierarchy	Specificity	Figure
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	SIS avoidance $(\beta_1(a), \nu = 0)$ SIS recovery $(\gamma_1(a), \nu = 0)$ SIR avoidance $(\beta_1(a), \nu > 0)$ SIR recovery $(\gamma_1(a), \nu > 0)$ SIR acquired immunity $(\nu_1(a))$ SIR acquired immunity $(\nu_1(a))$	acute co-circulating $(\alpha_1 < \alpha_2)$ acute co-circulating $(\alpha_1 < \alpha_2)$ acute focal $(\alpha_1 > \alpha_2)$ acute focal $(\alpha_1 > \alpha_2)$ acute co-circulating $(\alpha_1 < \alpha_2)$ acute co-circulating $(\alpha_1 < \alpha_2)$ acute focal $(\alpha_1 > \alpha_2)$ acute focal $(\alpha_1 > \alpha_2)$ acute co-circulating $(\alpha_1 < \alpha_2)$	specific, $c = 0$ general, $c > 0$ specific, $c = 0$ general, $c > 0$ specific, $c = 0$ general, $c > 0$ specific, $c = 0$ general, $c > 0$ general, $c > 0$ general, $c > 0$ specific, $c = 0$ general, $c > 0$ specific, $c = 0$ general, $c > 0$ specific, $c = 0$ general, $\sigma < 1$ specific, $\sigma = 1$	see figure $2a$ see figure $2a$ see figure $2b$ see figure $2b$ see figure $2c$ see figure $2c$ see figure $2d$ see figure $2d$ see figure $3a$ see figure $3b$ see figure $3c$ see figure $3d$ see figure $4a$

Table 1. Table of evolving resistance scenarios detailing the infection framework and type of resistance to the focal parasite that can evolve.





Hosts infected w. focal Uninfected hosts Hosts infected w. co-circulating

Figure 1. Flow chart showing epidemiological transitions for a situation where a host can recover to immunity against a focal parasite but where there is in addition a second parsite co-circulating in the host population (for simplicity there is no immunity to the co-circulating parasite). Parasite coexistence in the host population (and not within individual hosts) is facilitated by virulence associated superinfection. In model 1 the cocirculating parasite (represented by the density of hosts infected with that parasite, Y_2) is more virulent than the focal parasite (represented by Y_1) and therefore individuals move from the focal infection class Y_1 to the co-circulating infection class Y_2 when the co-circulating infection is transmitted to an individual infected with the focal infection. In model 2 the focal parasite is more virulent than the co-circulating parasite and therefore individuals move from the co-circulating infected class Y_2 to the focal infected class Y_1 when the focal infection is transmitted to an individual already infected with the co-circulating infection. Birth and death of hosts also occur but are omitted here for simplicity.

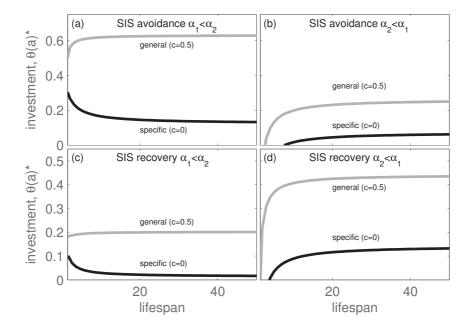


Figure 2. Optimal investment in specific and non-specific resistance in an *SIS* structured host population. In (a) and (b) the function of the resistance is avoidance. In (c) and (d) the function of the resistance is recovery (i.e. increased rate of disease clearance). In (a) and (c) resistance is evolved to counter the relatively avirulent infection while in (b) and (d) resistance is evolved to counter the relatively virulent infection. In all cases both infections will be equally countered when resistance is completely general (c = 1). Parameters were: $q = 0.1 \beta_1 = 2 \beta_2 = 4 \alpha_1 = 2 \alpha_2 = 8$ with s = 0.45, in the case of evolving avoidance $\hat{\beta}_i = \beta_i (1 - 0.5\theta)$ with $\gamma_1 = \gamma_2 = 0.35$ and in the case of evolving recovery $\hat{\gamma}_i = \gamma_i (1 + 2.5\theta)$ with $\beta_1 = \beta_2 = 1$. In all cases investment in resistance relates to reproduction according to $\theta(a) = 1 - (a^{\mu})/(a_{max}^{\mu})$ with $a_{max} = 1.9$ and $\mu = 12$.



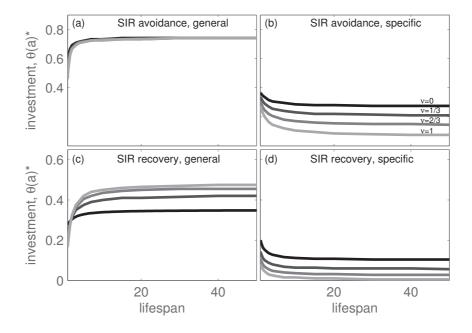


Figure 3. Optimal investment in specific and non-specific resistance in an *SIR* structured population developed to counter the relatively avirulent infection. In (*a*) and (*b*) resistance is through avoidance while in (*c*) and (*d*) resistance is through increased recovery. The proportion of recovered individuals entering the immune class is ν while the proportion returning to a susceptible state is $1 - \nu$. As ν increases above 0 towards 1 the population becomes *SIR* (dark grey through to light grey curves). In (a) and (c) c = 0.5 while in (b) and (c) c = 0. In (a) and (b) the trade-off exponent is $\mu = 18$, in (c) and (d) $\mu = 24$. Note, $\alpha_1 < \alpha_2$ and $\sigma = 1$ throughout, for other parameter values see caption of figure 2.

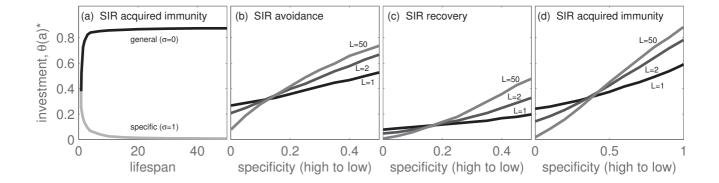


Figure 4. Optimal investment in specific (grey curve) and non-specific (black curve) acquired immunity developed to counter the relatively avirulent infection is given in (a). In (b), (c) and (d) optimal investment for a range of values of specificity is given for avoidance, recovery and acquired immunity respectively in an SIR structured population. In each case three separate curves are displayed for the following values of host lifespan, 1/b = 1 (black curve), 1/b = 2 (dark grey curve) and 1/b = 50 (grey curve). (b), (c) and (d) indicate that there is a critical value of specificity below which, where resistance is general, high lifespans are associated with higher investment than low lifespans. On the other hand, beyond this critical value, where resistance is specific to the relatively avirulent infection, low lifespans are associated with higher investment than high lifespans. In (a), (c) and (d) the trade-off exponent is $\mu = 24$ and in (b) $\mu = 18$. In (b) and (c) $\sigma = 1$ and $\nu = 1$. Note, $\alpha_1 < \alpha_2$ and for other parameter values see caption of figure 2.

Supporting Information S1 Next Generation Matrix

Invasion fitness (Metz et al., 1996; Geritz et al., 1998) can be derived through a linear stability 1 analysis of a mutant ecological model in a population consisting of residents at their population 2 attractor (usually a stable point equilibrium). If the steady state corresponding to no mutants but positive residents is unstable then the mutant can invade. Hence, eigenvalues (of the coefficient matrix, A, of the linearised system, $\dot{x} = Ax$) determine the invasion potential of the mutant and in particular the dominant eigenvalue is a measure of invasion fitness. 6 When a mutant host invades a resident population that is challenged by multiple infections, high dimensionality prevents direct derivation of invasion fitness. Instead, following the next 8 generation method (Diekmann et al., 1990), the linearised system can be decomposed into a two matrices, A = F - V. If the largest absolute value of the eigenvalues of the matrix FV^{-1} 10 is greater (smaller) than 1, then by the next generation theorem (van den Driessche and 11 Watmough, 2002; Hurford et al., 2010) the invasion fitness is positive (negative), but note 12 that conditions on the matrices F and V apply, see van den Driessche and Watmough (2002). 13 For general resistance as described in the main text, i.e. allowing for the possibility 14 of evolving avoidance $(\beta_1(a^m), \beta_2(a^m))$, or evolving recovery $(\gamma_1(a^m), \gamma_2(a^m))$ or evolving 15 acquired immunity $(\nu(a^m))$, the corresponding birth and death matrices are: 16

17	F=	$\begin{bmatrix} a^m - qH^r \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	$a^m - qH^r$ 0 0 0 0	$a^m - qH^r$ 0 0 0 0 0	$a^m - qH^r$ 0 0 0 0 0	$a^m - qH^r$ 0 0 0 0 0	,			
18	V=									
	$\beta_1(a)$	$a^m)Y_1^r + \beta_2$	$(a^m)(Y_2^r + Y$	$\binom{r}{21} + b$	-(1 -	$\nu_1(a^m))\gamma_1(a^m)$	<i>m</i>)	$-\gamma_2(a^m)$	0	0
		$-\beta_1$	$(a^m)Y_1^r$	α_1	$+b+\gamma_1(a^m)$	$)+s\beta_2(a^m)$	$(Y_2^r + Y_{21}^r)$	0	0	0
19		$-\beta_2(a^m)(Y_2^r + Y_{21}^r)$			$-s\beta_2(a^m)(Y_2^r+Y_{21}^r)$		$\alpha_2 + b + \gamma_2(a^m)$	0	0	
		0			$- u_1(a^m)\gamma_1(a^m)$		0	$b + \sigma\beta_2(a^m)(Y_2^r + Y_{21}^r)$	$-\gamma_2(a^m)$	
		0			0		0	$-\sigma\beta_2(a^m)(Y_2^r+Y_{21}^r)$	$\alpha_2 + b + \gamma_2(a^m)$	

References

- Diekmann, O., J. A. P. Heesterbeek, and M. J.A.J., 1990. On the definition and computation of the basic reproduction ratio R0 in models for infectious diseases in heterogeneous populations. J. Math. Biol. 28(4)365–382.
- van den Driessche, P. and J. Watmough, 2002. Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Math. Biosci. 180(1)29–48.
- Geritz, S., E. Kisdi, G. Meszena, and J. Metz, 1998. Evolutionarily singular strategies and the adaptive growth and branching of the evolutionary tree. Evolutionary Ecology 12(1):35-57.
- Hurford, A., D. Cownden, and T. Day, 2010. Next-generation tools for evolutionary invasion analyses. Journal of the Royal Society Interface 7:561–571.
- Metz, J. A. J., S. A. H. Geritz, G. Meszena, F. J. A. Jacobs, and J. S. V. Heerwaarden, 1996. Adaptive dynamics: a geometrical study of the consequences of nearly faithful reproduction. *in* pages 183-231 of S. J. Van Strein and S. M. Verduyn Lunel, eds. Stochastic and spatial structures of dynamical systems. Elsevier, North- Holland.