Appendix S1: ***Steady state and stability analysis of parasite-grass-forb model***

The two-step model outlined in the main text (equations 1 and the subsequent normalisation to ensure total frequency equals 1) can be written as single-step discrete-time system of equations.

$$
\begin{aligned}
P_{t+1} &= P_t\left(w_{PP}P_t + w_{PG}G_t + w_{PF}F_t\right)/N(P_t,G_t,F_t),\\
G_{t+1} &= G_t\left(w_{GP}P_t + w_{GG}G_t + w_{GF}F_t\right)/N(P_t,G_t,F_t),\\
F_{t+1} &= F_t\left(w_{FP}P_t + w_{FG}G_t + w_{FF}F_t\right)/N(P_t,G_t,F_t),
\end{aligned}
\tag{A1}
$$

where the function in equation (A1) is defined as

$$
\begin{aligned}
N(P_t,G_t,F_t) &= P_t\left(w_{PP}P_t + w_{PG}G_t + w_{PF}F_t\right)\\
&\quad + G_t\left(w_{GP}P_t + w_{GG}G_t + w_{GF}F_t\right) + F_t\left(w_{FP}P_t + w_{FG}G_t + w_{FF}F_t\right)
\end{aligned}
\tag{A2}
$$

To find the steady states we simultaneously solve the system of equations (A1) with the condition that $P_{t+1} = P_t = P,\ G_{t+1} = G_t = G$ and $F_{t+1} = F_t = F$. We can determine the stability of each steady state by evaluating the eigenvalues, λ_i, of the Jacobian of (A1) and a steady state is stable if $|\lambda_i| < 1$.

There are three equilibria where only one species exists $(P,G,F) = (1,0,0), (0,1,0)$ and $(0,0,1)$. Below we show the results for one equilibrium (the others are analogous due to the symmetry: i.e. the results for $(P,G,F) = (0,1,0)$ can be determined by interchanging the subscripts *P* and *G*).

$(P,G,F) = (1,0,0)$ has associated eigenvalues, $\lambda_i = 0, \dfrac{w_{GP}}{w_{PP}}, \dfrac{w_{FP}}{w_{pp}}$.

There are three equilibria where two species coexist (again we show the results for one equilibrium but the others can be determined by interchanging the appropriate subscripts).

$$
(P,G,F) = \left(\frac{w_{PG} - w_{GG}}{w_{PG} + w_{GP} - w_{GG} - w_{PP}}, \frac{w_{GP} - w_{PP}}{w_{PG} + w_{GP} - w_{GG} - w_{PP}}, 0\right)
\tag{A3}
$$

with associated eigenvalues

$$
\lambda_1 = 0,\ \lambda_2 = \frac{w_{GG}(w_{GP} - w_{PP}) + w_{PP}(w_{PG} - w_{GG})}{w_{GP}w_{PG} - w_{PP}w_{GG}},\ \lambda_3 = \frac{w_{FG}(w_{GP} - w_{PP}) + w_{FP}(w_{PG} - w_{GG})}{w_{GP}w_{PG} - w_{PP}w_{GG}}
$$

It can be shown that whenever the equilibrium in (A3) has positive values then $0 < \lambda_1, \lambda_2 < 1$ and $0 < \lambda_3$. Stability of the equilibrium therefore requires that $\lambda_3 < 1$ which is true if the following two conditions hold:

$$(w_{FP}-w_{GP})(w_{PG}-w_{GG})+(w_{FG}-w_{GG})(w_{GP}-w_{PP})<0$$
$$(w_{FP}-w_{PP})(w_{PG}-w_{GG})+(w_{FG}-w_{PG})(w_{GP}-w_{PP})<0$$

The three species coexistence equilibrium is

$$(P,G,F)=(P^*,G^*,F^*)\,, \tag{A4}$$

this expression can be determined algebraically and it can be shown that the equilibrium has positive values when all one species and two species equilibria are unstable. To determine the stability of equilibrium (A4) we consider the following system two difference equations (which is analogous to equations (A1) and uses the condition $F_t=1-P_t-G_t$):

$$\begin{aligned} P_{t+1}&=P_t\,NP(P_t,G_t)/N(P_t,G_t)\,,\\ G_{t+1}&=G_t\,NG(P_t,G_t)/N(P_t,G_t)\,, \end{aligned} \tag{A5}$$

where the functions in equation (A5) are defined as

$$\begin{aligned} NP(P_t,G_t)&=w_{PP}P_t+w_{PG}G_t+w_{PF}(1-P_t-G_t),\\ NG(P_t,G_t)&=w_{GP}P_t+w_{GG}G_t+w_{GF}(1-P_t-G_t),\\ NF(P_t,G_t)&=w_{FP}P_t+w_{FG}G_t+w_{FF}(1-P_t-G_t),\\ N(P_t,G_t)&=P_t\,NP(P_t,G_t)+G_t\,NG(P_t,G_t)+(1-P_t-G_t)_t\,NF(P_t,G_t). \end{aligned} \tag{A6}$$

Stability of (P^*,G^*,F^*) in (A5) can then be determined by ensuring that the Jacobian, *J*, of equations (A5) at the coexistence equilibrium satisfies the Jury conditions which state that $2>1+\det(J)>|\mathrm{trace}(J)|$. Algebraic conditions that ensure that the Jury conditions are satisfied can be determined (but are complicated). These conditions are used to produce figures 2 and 3 in the main text. For the special case where $w_{PG}=w_{FP}=w_{GF}=a$, $w_{GP}=w_{PF}=w_{FG}=b$ and $w_{PP}=w_{GG}=w_{FF}=c$ it can be shown that (P^*,G^*,F^*) is stable provided $ab>c^2$ and unstable (with oscillatory dynamics) otherwise.