A STOCHASTIC EVALUATION OF SOLVENCY VALUATIONS FOR LIFE OFFICES

By

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I hereby declare that the work presented in this thesis was carried out by myself at Heriot-Watt University, Edinburgh, except where due acknowledgement is made, and has not been submitted for any other degree.

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Abstract

A simple model office is used for simulation studies of U.K. with-profit life office management and solvency, in conjunction with the Wilkie model of the assets. It is assumed that the office pursues a high level of investment in equities and uses a terminal bonus system.

The circumstances leading to statutory insolvency in the model are investigated; sudden rises in dividend yields and falls in equity prices play the largest part in both distributions and sample paths. Low inflation has a minor effect.

The effects of different static and dynamic asset allocation strategies are considered. The well-known consequences of equity investment compared with fixed-interest investment — high mean returns but high variances also — are confirmed, but the higher variance is a feature of nominal rather than real accumulations. Switching strategies driven by the U.K. statutory minimum valuation basis appear to reduce the incidence of insolvency considerably, but only if unrealistically large switches are permitted. Other strategies driven by investment indices are considered.

The long-term costs of smoothing with-profit maturity values are investigated. They are found to be unstable; the measure of the relative costs which is used has a distribution whose lower quantiles are difficult to control because smoothing may be overridden by the need to meet the guarantees. Some methods of charging for the guarantees are considered.

Explicit cash-flow projections of closure and run-off at future epochs are used to measure the accuracy and timing of traditional solvency valuations; in effect the "constant interest rate" model of the traditional valuation is compared with the Wilkie asset model. Considerable differences are found between "solvency" according to a valuation and "adequacy" according to the cash-flows. Moreover, the

results of the traditional valuations are shown to be very sensitive to the criterion of insolvency which is used, in the form of an A/L ratio, leading to consideration of uniform solvency margins such as that used in the E.C..

The same traditional solvency valuations are applied to offices employing different asset allocation strategies, before or after closure, with widely differing results. Solvency valuations, by ignoring important features of individual offices, appear to give inconsistent results.

The additional assets needed to ensure cash-flow adequacy with given probability are estimated and compared for different offices, and are shown to differ greatly with the strategies used by management. As a measure of financial strength, such calculations lead to different results from the solvency valuations.

Introduction

0.1 Life office solvency

The tool which actuaries have used for 200 years to test solvency is the *prospective valuation*, although no one "correct" approach to valuation has ever been agreed. Is a gross premium or a net premium method better? Should the valuation basis be the same as the premium basis? What value should be placed on the assets? Should the valuation basis be decided by legislation?

Whatever approach is preferred, and whoever chooses the basis, a prospective valuation basis is *a model of the future*. That the interest, mortality and expense assumptions comprise a model can be disguised by their simplicity; we tend not to dignify a fixed interest rate with the name "model". They comprise a model nevertheless, and it is legitimate to ask what effect the model itself might have on the outcome of a solvency investigation, and whether other models might have advantages.

In this thesis, any set of (constant) assumptions comprising a prospective valuation basis will be referred to as a *traditional valuation model*.

To say that a life office is solvent because it has passed a solvency valuation is to say that, if the future follows the valuation model, then the office will have assets to spare after meeting all its liabilities and paying its expenses. The future is almost certain not to follow the model, however, so on some occasions a solvency valuation will close an office which is then, in the event, run off with assets to spare; at other times it will give a clean bill of health to an office which, if closed, would have been unable to meet its liabilities. We might say that the solvency valuation is doing a good job if it keeps both types of error to reasonably small proportions.

In recent years, insurance solvency has been tackled with stochastic tools. In general insurance, an analytic approach is often possible, which brings some conceptual clarity to the subject (Ruin Theory). Life assurance has proved less amenable to analysis, and there has been more reliance on simulation.

Stochastic approaches to life office solvency, although interesting, have not yet been generally accepted for practical use. Most solvency investigations are still, therefore, carried out using traditional prospective valuations. Indeed, legislation usually prescribes such a valuation; see for example [33].

0.2 The traditional model

In the U.K., life assurance practice has changed radically since 1945, posing problems for the traditional valuation model which, broadly speaking, is often more suited to the conditions prevailing before that time. In the U.S.A. and Canada too, life assurers are faced with circumstances which depart from the assumptions of the traditional valuation model. One crucial change in circumstances is *greater volatility of asset values*. In the U.K. this is due to investment in equity-type assets; in North America it is due to volatility of yield curves and, frequently, repayment options. The consequences are the same — *the traditional valuation model is less realistic than before*.

Attempts have been made to extend the traditional valuation model to suit modern conditions, mainly taking the form of accretions to the traditional model. In the U.K., a starting point is the 1952 paper by Redington [56] and the 1966 paper by Skerman [61]. After Redington, the valuation of assets and liabilities were always to be considered together, while Skerman attempted to lay down general principles for a traditional solvency valuation. Since that time changes have been within the framework of E.C. Directives.

Subsequent developments can be placed broadly into two groups.

1. Attempts have been made to define *solvency margins* — namely, amounts to be held in addition to a suitable mathematical reserve. This includes the E.C. solvency margin [33], the "Risk Based Capital" (RBC) requirements in the

U.S.A., and the "Minimum Continuing Capital and Surplus Requirements" (MCCSR) in Canada.

2. Offices can be required to show that they can still pass a given solvency test after some change in conditions. The "resilience test" in the U.K. falls into this group, as does the "Dynamic Solvency Testing" (DST) requirement in Canada.

A notable feature of this second group of approaches is that the test of solvency which offices are required to pass after the change in conditions is usually again a traditional valuation. For example, the U.K. resilience test requires offices to show that they could set up the *statutory minimum reserves* after the prescribed changes in conditions. Therefore the traditional valuation model is still the criterion of solvency.

0.3 Stochastic approaches to solvency

In more recent years, some authors have studied life office solvency stochastically. Prerequisites for such studies are broadly as follows:

1. A projection model of the cash-flows arising within an insurance company. The explicit projection of investment income and asset prices takes the place of the traditional valuation "yield" or interest rate.

2. A stochastic model for the death or survival of individual lives. A more broad-brush approach to mortality is often possible.

3. A stochastic model for inflation, asset prices and investment income.

The emphasis given by different authors to the mortality and investment elements varies. In some territories life office premiums and investments (and hence interest surplus) are closely controlled, so modelling of mortality has received most attention; see for example [48]. In some cases, the investments have additionally been modelled by simple continuous-time stochastic processes [49]. In other cases, it can be shown that mortality surplus has a small effect on solvency, compared with investment surplus, provided the number of lives assured is sufficient; see for example Frees [25].

Most solvency studies in the U.K. have used discrete-time time-series models of financial indices; one such model (the Wilkie model [68]) has frequently been adapted for use elsewhere (see for example Pentikäinen *et al* [52], Pukkila *et al* [53], Rantala *et al* [55]).

1. In 1980 the Maturity Guarantees Working Party of the Institute of Actuaries and Faculty of Actuaries studied the reserving requirements for equity-linked contracts with maturity and surrender guarantees [6].

2. In 1986 the Faculty of Actuaries Solvency Working Party studied the solvency of non-linked life assurance business [37]. In connection with this study, the Wilkie asset model was introduced [68].

3. In 1989 the Faculty of Actuaries Bonus and Valuation Research Group carried out stochastic studies of with-profit business with substantial equity backing [22].

4. In 1991, M. D. Ross and M. R. McWhirter discussed some of the problems of modelling U.K. life assurance business, with particular emphasis on the level of decision-making which ought to be modelled [57], [58].

5. In 1992, the Life Assurance Solvency Working Group in Finland reported on possible criteria for life assurance reserving, using a version of the Wilkie asset model [55].

The earliest study (that of the Maturity Guarantees Working Party) was confined to equity-linked business, over which the life office's managers have no real discretion. Later studies, particularly Ross [57], begin to treat the problems of modelling the decisions which the managers of a with-profit office have to take over asset allocation and bonus distribution. In treating such decision problems, the very concept of solvency begins to dissolve — if the managers have a large degree of discretion, then how they exercise that discretion might be the greatest single determinant of "solvency". What then does solvency mean?

0.4 Plan of this thesis

0.4.1 Survey of some previous work

The first part of this thesis is a brief survey of the recent changes in U.K. life assurance practice (Chapter 1), a survey of developments in the traditional valuation model (Chapter 2) and a brief review of the methods used in the stochastic studies listed above (Chapter 3).

0.4.2 Introduction of a simple model

In Chapter 4 a simple computer model office is described. The office transacts 10-year endowment business, and mortality, lapses and expenses are ignored in order to focus on the interaction of the *assets* and *solvency legislation*.

0.4.3 Investment strategies

In Chapter 5 some possible *asset allocation strategies* for a with-profits life office are considered. Because of certain features of the U.K. valuation regulations, described in Chapter 2, the asset allocation strategy has a direct effect on solvency and *vice versa*, leading to trade-offs between solvency and investment aims. Reversionary bonus strategies are also considered, but more briefly.

0.4.4 Maturity value smoothing

Maturity value smoothing is widely practiced in the U.K., but has not so far been studied in the literature. The question of the *cumulative cost* of smoothing, and whether or not that cost is *stable* in the long run, should be important in practice. In Chapter 6 the effect on the model of some of the smoothing methods cited by practitioners is considered.

0.4.5 Evaluating the traditional valuation model

The final aim of this thesis is to apply our simple model to evaluate the *effectiveness* of some traditional solvency valuations, and in particular the U.K. statutory minimum valuation basis. The evaluation proceeds in three steps.

Solvency valuation We use a stochastic asset model to produce a large number (say 1,000) of simulated futures ("scenarios") for inflation, fixed-interest assets and equities. The model office is subjected to these 1,000 different futures, continuing to transact new business, and it is valued every year using a traditional solvency valuation. Insolvent offices are not closed down but are allowed to carry on regardless; the object of this step is to find out *in which simulations the office fails the valuation test and at what times.*

Explicit run-offs We then take each scenario and close the office at the end of *each* future year, running off the in-force business. Thus in *each* of the 1,000 simulations we record the outcome — a surplus or a deficiency — if the office were closed and run off after 1 year, after 2 years and so on.

Comparison of solvency and run-offs Finally we compare the results of the first two steps. If we suppose that an office should be closed to new business upon failing a solvency valuation for the first time, we can see from the results of the run-offs whether or not the valuation correctly identified an office in difficulties. Moreover, because we know the results of *all* the run-offs, we can see whether the valuation missed any offices which it should have caught.

Briefly, then, we will compare the traditional valuation model with an alternative, stochastic asset model. The latter might be more realistic in an important *qualitative* sense — it embodies more than one moment — so different outcomes tell us what we lose by ignoring all but the first moment in the traditional valuation model.

Chapter 1

Background to U.K. life assurance

1.1 Investment and bonus policy since 1945

Conventional wisdom for long held that their guarantees should lead life offices to seek security of capital and steadiness of income. Surprisingly perhaps, investment in U.K. Government securities only became commonplace during the First World War; before then, mortgages, debentures and various secured loans were more usual. Subsequently, funds were invested mostly in gilts and "sound" debentures. Investment in equities before about 1945 was exceptional, apart from a few favoured sectors such as railway companies. Even after the Second World War, a significant proportion of equity assets was in the form of preferred or guaranteed stock.

In 1937, Murray detailed the investments of 10 life offices from 1871 to 1935 [46]; a series later updated by Gulland [27], [28], Williams and Elgin [19] from which the figures in Table 1.1 are extracted. (Note that these are based not on market values but on book values.)

Conventional wisdom has not stood still. Since the 1960s, the proportion of with-profits funds invested in equities and property has risen sharply, to the extent that some offices have recently claimed to be 100% invested in these sectors. Pension funds apart, life offices are probably now the main vehicle for individual investment in equities and property in the U.K.. Table 1.2, based on Forrest *et al* [23], shows the broad categories of investments — this time by market value — disclosed in "With-profits Guides" in 1989 by those offices which identified separately the assets

Asset class	1900	1925	1935	1945	1955	1961
U.K. Government securities	0.5%	39.5%	21.8%	34.8%	27.5%	26.9%
U.K. Municipal loans and securities	7.2%	2.9%	5.3%	5.8%	2.0%	2.5%
Foreign Gov't & Municipal	13.5%	12.5%	11.8%	3.6%	1.5%	1.1%
Mortgages	34.8%	11.1%	10.5%	10.2%	10.3%	11.4%
Policy loans	6.7%	8.1%	7.6%	2.5%	1.9%	2.1%
Debentures and Debenture stocks	18.1%	9.6%	15.7%	12.8%	12.1%	12.9%
Stocks inc. pref. & guaranteed	6.9%	4.2%	16.0%	20.7%	34.2%	35.2%
Others	12.3%	12.1%	11.6%	9.6%	10.5%	7.9%

Table 1.1: Asset allocation (%) of 10 offices 1900–1961

attributable to with-profits business.

Since the running yields on equities fell below those on fixed-interest securities in the 1950s — the "reverse yield gap" — equities and fixed-interest securities have yielded quite different cash-flows.

1. The income stream from equities commences at a low level, but increases in a manner loosely linked to the economic fortunes of the firm and, even more loosely, to the fortunes of the economy.

2. A substantial part of the total return on equities is in the form of capital gains.

The reverse yield gap was small at first, so the first problem to appear was that of distributing the rising dividend stream in an equitable manner. See, for example, Benz [7], and Redington's comments in the discussion of Springbett [64]. Various systems were tried; compound instead of simple bonus, special reversionary bonus, supercompound bonus and subsequently terminal bonus — in this respect the remarks of R. H. Blunt in the discussion of Benz [7] are particularly interesting.

In the 1970s there came the real changes — high inflation, high gilt yields, great volatility of share prices but overall, better real returns on equities than on gilts. Equities began to be seen as the safer *long term* investments in an inflationary economy. The expected course of dividend income moved even further away from the level or gently rising pattern which suited the reversionary bonus system; in

Office	F.I.	Property	Equity	Other
Clerical Medical	17%	16%	58%	10%
Commercial Union	10%	20%	64%	6%
Eagle Star	14%	30%	53%	3%
Equitable Life	15%	14%	64%	7%
Equity & Law	9%	26%	67%	0%
Friends' Provident	14%	17%	59%	10%
L.A.S.	19%	10%	65%	6%
London & Manchester	25%	15%	27%	32%
M.G.M. Assurance	17%	18%	63%	2%
National Mutual Life	8%	23%	58%	11%
Norwich Union	4%	33%	63%	0%
Provident Mutual	36%	20%	41%	3%
Prudential Assurance	9%	24%	61%	6%
Scottish Equitable	4%	12%	76%	8%
Scottish Provident	18%	17%	61%	4%
Scottish Widows	18%	10%	67%	5%
Standard Life	0%	24%	76%	0%
Sun Life	9%	24%	66%	1%
Wesleyan & General	15%	18%	57%	10%

Table 1.2: Asset allocation (%) of 19 offices in 1989

addition, the 1974 stock market crash emphasised the volatility of capital values and did nothing to encourage distribution of "surplus" in reversionary form. Hence life offices swung increasingly from reversionary bonus to terminal bonus.

A terminal bonus is declared only when a claim arises, and until then it is not guaranteed, so it poses less risk to solvency. Once an *ad-hoc* method of distributing unexpected surpluses, terminal bonus is now the linchpin of with-profits business. It works as follows:

1. A life office will systematically declare lower reversionary bonuses than can be supported by the emerging surplus, diverting the extra surplus into an *investment reserve*. The guarantees build up more slowly, and the investment reserve can absorb fluctuations in asset values without solvency being threatened. The investment reserve gives the office its freedom to invest in equities, and also acts as a reservoir which can be drawn upon or topped up as part of the process of smoothing policyholders' benefits. In this way, asset risks are pooled among different generations of policyholders.

2. Were the office to pay only the guaranteed benefits when a claim arose, it would normally be acting unfairly, since it would have diverted part of the earned surplus into the investment reserve. The remedy is to return the policyholder's share of the investment reserve as a terminal bonus.

The scale upon which surplus has been directed into investment reserves rather than being distributed as it emerges can be judged from recent rates of terminal bonus. After a 25 year term, terminal bonus can exceed 150% of the sum assured and reversionary bonus. This means that more than half of the policyholder's assets are in the investment reserve, and not guaranteed to be returned to the policyholder.

1.2 Asset shares

U.K. life offices, and actuaries, have acquired much more discretion over the policyholder's benefits than would be possible under a system which relied mainly on reversionary bonus. Some may regret that the discipline imposed by the reversionary bonus system has been shaken off, but most U.K. life offices appear to believe that their customers prefer the benefits of equity investment.

A fair system of determining terminal bonuses is clearly needed, which leads us to the *asset share*, namely a retrospective reserve based on the experience, not unlike the unit fund of a unit-linked policy. In a 1989 survey, Debenham *et al* [16] suggested that most U.K. offices use asset shares to some extent in setting terminal bonus rates, though there is a wide range of views about how asset shares should be calculated. An office might use a smoothed version of the experience, and there are several possible treatments of expense, mortality and surrender profits — in fact there is possibly no such thing as an "accurately" calculated raw asset share — but in essence the asset share is the policyholder's "fair share" of the office's assets. It provides a starting point for the consideration of terminal bonus rates.

1.3 Managers' discretion versus policyholders' expectations

Of the many decisions facing life office managers, four may be singled out. The first three were also discussed by Ross [57].

1. The investment strategy must be decided, bearing in mind the nature of the liabilities. As indicated above, most U.K. offices have preferred equity-type assets to fixed-interest assets, although departing from this position when short term strategy dictates. However, a reason for moving towards fixed interest investment may be the need to increase the current yield on the fund in order to meet the minimum valuation standard (see Section 2.4). We must assume that offices will do this, however reluctantly, when they would otherwise be statutorily insolvent.

2. The bonus rates must be decided. The traditional approach of analysing surplus retrospectively, while not irrelevant, is perhaps now less regarded than the desire to restrain the build-up of the guarantees and avoid constraints on the investment strategy. So we might consider what margin between the asset share and the guaranteed benefits is desirable, and declare reversionary bonuses which lead to this margin being attained. On maturity, the margin emerges as the terminal bonus, so the office is effectively aiming at a target terminal bonus.

3. The premium rates must be decided. This may be crucial for protection business, but pricing of with-profits business is often less active. In the U.K. it is not unknown for with-profits premium rates to remain unchanged for long periods — sometimes decades. The terminal bonus system affects this decision too, since premium rates and reversionary bonuses have a reduced role in achieving equity between different generations of policyholders.

4. The degree of smoothing of maturity benefits must be decided. It is usually assumed that some smoothing is needed — with-profits business is not unit-linked — but how much? The consequences of mistaking a trend for a cycle

could be expensive.

The discretion given to life offices to choose bonus and investment strategies affects both the rights of policyholders, and the measurement of solvency. If the sum assured under a 25-year with-profit endowment can be 20% or less of the maturity value, then 80% of the benefit is at the office's discretion. Further, most with-profits offices could show themselves to be solvent easily, by switching into fixed-interest securities and scrapping all future bonuses. Such actions should presumably be unacceptable, but they would not breach the valuation regulations. Something stronger is needed.

The idea of *Policyholders' Reasonable Expectations* or "PRE" appears in the 1982 Act [32] (although it had appeared before in the actuarial literature). Listing grounds for intervention by the supervisor, the 1982 Act includes the following [32, Paragraph 37(2)(a)]:

> "that the Secretary of State considers the exercise of the power to be desirable for protecting policy holders or potential policy holders of the company against the risk that the company may be unable to meet its liabilities or, in the case of long term business, to fulfil the reasonable expectations of policy holders or potential policy holders;"

PRE is not defined by the Act and has never been defined by the courts. Its meaning is clear with respect to non-profits business, less clear with respect to unit-linked business and not at all clear with respect to with-profits business. It must affect the way in which managements exercise their discretion, but how?

Brindley *et al* [11] examined PRE on behalf of the Institute of Actuaries and Faculty of Actuaries, and their conclusions, as they affect with-profits business, are summarised below :

1. PRE is virtually synonymous with equity, and is most commonly measured by asset share calculations.

2. It is not reasonable for policyholders to expect any free assets which the office may possess to be distributed.

3. If a major change takes place, such as change of ownership, it should not disadvantage existing policyholders, compared with the option of a closed fund.

4. Gradual change is acceptable in the management of with-profits business, sudden change is not.

We can ask two questions which lie at the heart of the problem.

Question 1 What constraints are placed on investment, bonus, premium rating and smoothing strategies by (i) statutory solvency and (ii) PRE?

Question 2 : If "solvency", in the sense of meeting guarantees, is too weak a test in a with-profits office, what else is needed?

These questions are linked, since the level at which the supervisor might intervene is set at *possible* failure to meet PRE, which should always be anticipated by the minimum valuation standard. Both questions will be taken up in later chapters.

1.4 Other developments

1.4.1 Statutory minimum solvency

The U.K. introduced a statutory minimum solvency standard in the 1981 Regulations [33], subsequently modified in the 1994 Regulations [34]. This did not prescribe a basis, but a *minimum* standard only; the basis used had to be at least as strong as the minimum. In part, a minimum valuation standard was needed in order to apply the E.C. solvency margins sensibly (see Sections 1.4.2 and 2.5).

The minimum solvency standard can sometimes lead to a mismatch between the valuation of assets and liabilities, for the following reasons.

1. The maximum permitted valuation interest rate is linked to current yields on the assets. In respect of equity-type assets, the relevant yield is the running yield *without any allowance for income growth*. Thus life offices with large equity holdings are restricted to low valuation interest rates.

2. The Regulations require assets to be taken at market value.

The 1981 Regulations, and the changes made in 1994, are described in Section 2.4.

1.4.2 E.C. solvency margins

The E.C. introduced solvency margins for life assurance business in the First Life Directive. For savings business, they are based largely on an unspecified mathematical reserve (assumed to be a prospective policy value). The margins vary with the type of business, and the level of reinsurance. They are described in Section 2.5.

1.4.3 The resilience test

Regulation 55 of the 1981 Regulations [33] required Appointed Actuaries to satisfy themselves that the assets held were suitable in relation to the liabilities. In 1985, the Government Actuary's Department (G.A.D.) let it be known that their "rule of thumb" test of meeting Regulation 55 was a drop of 25% in equity prices and a change of $\pm 3\%$ in gross gilt redemption yields. Although it was not part of the Regulations, nor supposed to indicate the limits of Regulation 55, it not unnaturally became an "unofficial" regulation. It was admittedly crude, and its parameters have been amended twice in recent years as changing conditions have rendered it arguably unrealistic. It is described in more detail in Section 2.6.

Chapter 2

The traditional valuation model

2.1 Introduction

The practice of valuing for solvency is almost as old as life assurance itself. At first, the results of the valuations served as much to dissuade policyholders from distributing the vast funds held by their offices as to demonstrate solvency. In due course growing surpluses led to some disbursement, controlled by the valuation, and so to bonus systems. The valuation was thus saddled with two tasks instead of one, a source of difficulty which persists to this day.

Valuations used the same tools as premium calculations, namely:

1. A mortality table.

2. A rate of interest representing the future yield on suitable assets.

3. Explicit or implicit assumed future expenses.

Uncertainty was recognized implicitly by the inclusion of margins in the assumptions. Mortality seemed to present the greatest risk, because reliable data were not available, and because the assets in which the funds could be invested were limited and, at that time, more predictable than mortality.

In modern terminology, valuations were based upon a *model* consisting of forecast or expected mortality and yields. The likelihood of error was reflected by the margins added in the forecasts. This simple, even crude, model was extraordinarily successful in allowing actuaries to control life assurance business for nearly 200 years. In retrospect, its success rested on four features:

1. The numbers of lives assured, though of different ages and dispositions, were large enough that the "Principle of Insurance" (the pooling of risks) kept aggregate mortality losses under control.

2. The assets in which funds were "prudently" invested generally yielded fairly stable rates of return. Yields were low anyway and large deviations were neither expected nor experienced.

3. The accidental (at first) generation of surpluses furnished a cushion against adverse experience which more than made up for any crudeness of the pricing and valuation models.

4. More assurance than annuity business was written, during a long period of improving mortality rates.

In order to compare this traditional valuation model with alternative models of life office operations, which *inter alia* might incorporate modern approaches to investment, surplus and regulation, we first consider the place of the traditional model in modern practice.

Some version of the traditional model is still mandatory for solvency assessment in most states, including E.C. territories. In Chapter 1 we noted that the moves in the U.K. towards equity investment and terminal bonus are not entirely in tune with the traditional model. With-profits practice has shifted to *retrospective* methods, while the traditional model is based on an entirely *prospective* approach. Consequently, attempts have been made to adapt it to the changes in practice.

1. F. M. Redington revised the principles of valuation to bring the assets and the liabilities closer together.

2. R. S. Skerman attempted to lay down a solvency standard in terms of the traditional valuation model.

3. The E.C. introduced solvency margins.

4. The U.K. introduced a minimum solvency standard.
5. In the U.K., the G.A.D. introduced a resilience (or mismatching) test.
6. In Canada and the U.S.A., the ideas underlying the E.C. solvency margins were extended to "Risk Based Capital" (RBC).
7. In Canada, "Dynamic Solvency Testing" (DST) was introduced.
8. In Australia, new solvency reporting standards based upon best estimate valuations and planned margins are due to take effect from 1995.

These developments are outlined below.

2.2 Redington's 1952 paper

F. M. Redington [56] in 1952 changed the emphasis from the valuation of liabilities alone and considered the assets and liabilities together. He introduced immunisation, new to U.K. actuaries although already known in the U.S.A. (Macauley [41]). At almost the same time, Haynes & Kirton [30] put forward a complementary cash-flow matching approach.

Neither Redington nor Haynes & Kirton extended the formal techniques of valuation, these having been discussed exhaustively in the preceding decades. Rather they were concerned with the risk of falling fixed-interest yields and the inability to invest future cash-flows at the yields assumed in premium bases. It was clear that this risk ought to be allowed for in a valuation — the question was how? Redington remarked that

> "We are less concerned about the technique of valuing at $2\frac{1}{2}\%$ than at the significance and the consequences of the $2\frac{1}{2}\%$ itself."

Although this hinted at an examination of the models which might underlie a solvency investigation, the question was still approached in terms of present values. In 1952, the calculation of present values dictated a particularly simple basis — the traditional valuation model — so the model went more or less unquestioned. In

Redington's paper and those which followed (Springbett [64], Bayley & Perks [5]) the problem of choosing "the" valuation interest rate, or an immunising portfolio of assets with respect to that rate, remained paramount. It was some time before the lead given by Haynes & Kirton was followed up.

Discussion of immunisation continued through the 1950s, but somewhat out of the mainstream. The actuarial problems presented by the assets were just beginning to be considered. Initially, thought was given to fixed-interest yields — hence immunisation — but changing attitudes towards investment soon turned attention towards equities.

2.3 Skerman's principles

In 1966 R.S. Skerman [61] put forward 5 principles for a solvency valuation, to which a 6th was added later. At the time, the E.E.C. and O.E.C.D. were contemplating standards of solvency with a view to permitting the transaction of business across national boundaries, and Skerman's paper was an attempt to codify a U.K. point of view. Skerman's principles are relevant today, because they define the usual application of the traditional valuation model, they are discernable in the U.K. minimum valuation standard, and they are still taught to actuarial students. In addition, Skerman made several significant points which were not directly embodied in the principles. The principles are:

1. *That the liabilities should be valued by a net-premium method or on some other basis producing stronger reserves.* This principle is aimed squarely at "reasonable expectations", particularly those of with-profits policyholders whose future bonus loadings should not be capitalised. Skerman said that

   > "If solvency were understood to mean no more than the fulfilment of contractual obligations, it would be appropriate to use a gross-premium method of valuation ..."

2. *That an appropriate zillmerized reserve would be acceptable in order to allow for initial expenses.* A maximum zillmer of 3% was suggested.

3. *Adequate margins over the current rate of expenses should be kept in the valuation of the liabilities, in order to provide for future renewal expenses.* Effectively, this avoids valuation of an excessive net premium — if it is invoked then a gross premium valuation results. Significantly, in view of practice elsewhere in the E.C., it allows valuation on a basis weaker than the premium basis.

4. *That appropriate recognized tables of mortality should be employed.* Skerman mentioned that the effect of mortality on endowment business is not great.

5. *That the valuation of the liabilities should be at rates of interest lower than implicit in the valuation of the assets, with due regard to the incidence of taxation.* This is examined in more detail below.

6. *The net liabilities must in aggregate exceed the surrender values if these are guaranteed.* This point was mentioned in the paper several times, and was later promoted to a 6th principle.

The general thrust of the principles was that if they were adhered to, the reserves set up would contain *implicit* margins, adequate to establish solvency without the need to hold *explicit* solvency margins.

2.3.1 The 5th principle

The operation of the 5th principle presented different problems when interest rates were very low or very high.

A large fall in interest rates would lead to a rise in the market value of assets. Common practice would prohibit the writing up of the assets in the balance sheet (even today, this would be the case in some E.C. countries). Therefore the yield implicit in the valuation of the assets would only fall gradually, and it would not be apparent at once if the liabilities could be valued satisfactorily. Skerman suggested that the progress of the valuation results at the falling rates of interest would provide satisfactory warning.

On the other hand, a large rise in interest rates would lead to a fall in the market value of the assets. Common practice would require the book value of the assets to be written down immediately, at once increasing the apparent yield on the assets. Using this yield to value the liabilities might be unsound if the assets were invested short; in that circumstance a lower interest rate for the liability valuation might be prudent. Skerman suggested a margin of the greater of 10% of the yield on the fund or $\frac{1}{2}$%, between the (gross) rate of interest implicit in the valuation of the assets and the rate of interest used to value the liabilities. As will be seen, the authorities in the U.K. arrived at a similar solution.

Two points might be made about the 5th principle.

1. It clearly envisages the continuance of historic life assurance practices — in particular, investment in fixed-interest securities for which the concept of "yield" has relevance.

2. It is dynamic in principle, because it links the valuation of liabilities to that of the assets, but by implication it does not expect the assets to be taken at market value. The possibility of valuing the assets using discounted cash-flow techniques is not ruled out.

In short, Skerman's principles encompass solvency in the conditions prevailing up to 1966, as if the problems posed by equity investment are to be regarded as a separate problem yet to be solved.

2.3.2 Other points made by Skerman

Apart from the principles, Skerman made several significant points under the heading of "Fundamentals". The first is worth quoting in full. (The emphasis is mine.)

> "*It is not practicable to assess the solvency position of an office by reference to a series of payments of income and outgo and, in order to arrive at a standard which can be used in practice, it is necessary to compress these payments into present values.* This is the concept underlying the comparison of the value of the assets and of the liabilities but, for this comparison to be meaningful, an appropriate relationship must exist between the rate of interest underlying the valuation of the assets and that

used in valuing the liabilities. Without such a relationship the results of a valuation have no mathematical meaning and can in practice be seriously misleading."

The use of present values is a matter of *practice* and not of *principle*. It follows that the very place of the traditional valuation model — a model demanded by the use of present values — in the assessment of solvency might reasonably be reconsidered as the techniques mentioned above become practical.

The other point worthy of mention is the use of the phrase "policyholders' reasonable expectations" which, with much of the substance of the principles, eventually appeared in U.K. legislation.

2.3.3 Ammeter's comments

In two papers, H. Ammeter replied to Skerman from a continental point of view. In [2], he criticized the 5th principle as too weak, compared with the usual continental practice of valuing on the premium basis. The basis of his objection, which still seems relevant today, was that current yields do not represent possible yields obtainable over the long term, so the application of the 5th principle depends on a degree of matching which it is not reasonable to expect in practice. He did however support the need for principles in aid of harmonization, and recognized one advantage of Skerman's principles, that they might avoid the need for a separate solvency margin.

In [3], he elaborated this last point, describing implicit margins within the mathematical reserves as the "natural" method, and explicit solvency margins as the "mechanical" method, of assessing solvency reserves. He emphasised the unfairness of a mechanical formula which took no account of the margins which might already exist in the mathematical reserve. For example, he said

> "It seems particularly absurd to take no account of the valuation basis used for arriving at the mathematical reserves when the "mechanical" solvency reserve is being determined, since the insurer who is more cautious in his valuation basis would have also to set up the strongest

> "mechanical" solvency reserve. It is not surprising, therefore, that private insurers throughout Europe are unanimous in refusing to accept a solution of this kind."

These remarks are particularly interesting in view of subsequent developments.

2.3.4 The adoption of the principles

The 6 principles were adopted by the Comité Européen des Assurances (C.E.A.) and proposed to the E.E.C Commission for adoption in a European Life Directive. A minor change was that the minimum margin between the rate of interest implicit in the valuation of the assets and the rate of interest to be used in the valuation of liabilities was amended to the greater of 10% of the yield on the fund or 0.8%.

2.4 The U.K. minimum solvency valuation basis

Proposals for a minimum solvency standard were made by the Department of Trade and Industry in 1974 (see [8, Appendix 1]) and were discussed by the actuarial profession [8]. The proposals were based on Skerman's principles, *via* the C.E.A.. Rather than describe the original proposals, it will be more useful to summarise the valuation regulations enacted after the discussions, which differed only in matters of detail. As well as the discussion, [8] includes a summary of the moves towards a European solvency standard which took place during the 1950s and 1960s.

In 1994 new valuation Regulations were issued [34], generally differing in detail but not in principle from the 1981 Regulations.

2.4.1 Valuation of assets

Regulations were made concerning the valuation of assets.

1. Limits were placed on the proportion of the fund which could be invested in particular categories of asset or in individual assets. Investments in excess of these proportions were deemed "inadmissible" and had to be left out of the solvency valuation.

2. Admissible assets were to be valued at market value or at an appropriate valuation should this be impossible.

The admissibility rules place relatively few restrictions on offices, given the availability of assets in the U.K.. The more important rule is the market valuation.

2.4.2 Valuation of liabilities

The regulations concerning the valuation of liabilities are more extensive. The basic requirement was for a net premium valuation [Regulation 57] with a zillmer based on the initial expenses loaded for in the premiums but of no more than 3.5% [Regulation 58] and with adequate provision for expenses and options [Regulations 61 & 62]. Rates of mortality and disability were not prescribed [Regulation 60], but surrenders must be ignored if they might reduce the liability [Regulation 64]. Future valuation strain must be avoided, and negative reserves eliminated [Regulations 56 & 63]. The matching of assets and liabilities must be considered [Regulation 55]. Of greatest importance were the limits on the valuation rate of interest [Regulation 59]. These attempted to apply Skerman's 5th principle, allowing for the caveats mentioned in Section 2.3.1.

1. The maximum interest rate which could be used was linked to the current yields on the assets [Regulation 59(2)–(5)]. Broadly speaking, up to $92\frac{1}{2}\%$ of the net redemption yield on gilts could be used, and up to $92\frac{1}{2}\%$ of the running net dividend yield on equities or similar assets with unguaranteed income.

2. In the case of fixed-interest securities, the yield must be reduced to allow for the risk of default compared with similar risk-free assets [Regulation 59(6)(a)]. Effectively this meant that fixed-interest securities should be valued at no more than the gilt yield.

3. The yield on equities or property must not exceed the yield on $2\frac{1}{2}\%$ Consols (an irredeemable gilt) [Regulation 59(6)(b)].

4. The maximum gross interest rate which could be assumed on investments to be made more than 3 years in the future is 7.2% [Regulation 59(7)].

5. The office could apportion assets to each part of the business which it wishes to value separately [Regulation 59(9)]. For example it might state that annuity business is backed by gilts and therefore use $92\frac{1}{2}\%$ of the gilt yield to value annuities.

The Appointed Actuary need not use the statutory minimum basis, but cannot use a weaker basis. In practice many companies still use stronger bases in their published returns. A life office able to satisfy the Regulations is said to be *statutorily solvent*.

2.4.3 Consequences of Regulation 59

Given the recent history of life assurance in the U.K., it is obvious that the running net dividend yield on equities is a crucial factor in the statutory solvency of with-profits business. No allowance can be made for potential growth of dividends or share prices, so when dividend yields are low, with-profits offices will be forced to use a very low valuation interest rate.

The problem of low *fixed-interest* yields was considered by Skerman (see Section 2.3.1), but possibly not with the volatility experienced from the 1970s on in mind. In current conditions, inconsistency of the values of assets and liabilities is a more serious threat. An office which might be sound if its assets and liabilities were valued consistently could suffer insolvency of a purely technical nature because of a clash between the two halves of the valuation regulations.

Bews *et al* [8] agreed that it would be inappropriate to allow for possible future increases in dividends or equity prices. Subsequently some actuaries have disagreed, see Ross [57] for example. Any assumption of future dividend growth would require great care, but it is arguable that no allowance at all is unreasonable and fails to recognize the basis of with-profits business in the U.K.. Offices might be forced to move funds from equities to gilts for reasons of statutory solvency rather than of policy; this seems to have happened to more than one office recently. If such switches are genuinely necessary to ensure solvency, then it is right that the statutory valuation should indicate when they must take place, and it is also proper that the regulations should err on the safe side. The point which troubles critics of the statutory minimum valuation such as Ross is that it has not been shown to be a satisfactory measure of solvency; one whose interventions in office management are well founded.

On a minor point, the 7.2% yield restriction on investments to be made more than 3 years in the future has led to new methods of calculating net premiums being suggested; see Bews *et al* [8] and Elliott [20].

It is important to realise that the valuation regulations were not envisaged as a test of "strict" solvency, but in keeping with Skerman's principles as a test of "adequacy" or "reasonable expectations". This was quite clear in the brief given to Bews *et al* [8], namely:

> "To consider the desirability and possibility of modification of the "six principles" ... so that
>
> (a) for the general range of long-term life assurance contracts the value of the net liabilities can be compared with the market value of the assets, even during a period of rapid change, to ensure a reasonable standard of adequacy ... rather than a mere demonstration of solvency, and
>
> (b) statutory rules for such a valuation can be designed."

2.4.4 The 1994 Regulations

New regulations were issued in 1994 [34], mainly to bring U.K. legislation into line with the E.C. Third Life Directive. The approach of the 1981 Regulations was maintained, but some changes of detail were made.

1. Derivative securities were recognised explicitly, not only as assets but as potential liabilities (Regulation 61).

2. "Reasonable expectations" were mentioned; Regulation 64 said

   > "The determination of the amount of long term liabilities ... shall be made on actuarial principles which have due regard to the reasonable expectations of policy holders ..."

Reasonable expectations had previously been mentioned in the Act but not in the Regulations.

3. Regulation 65 (following the Directive) laid down that prospective valuations were to be used on an *individual policy* basis; that methods of calculation should not be subject to arbitrary discontinuities from year to year, and that for participating policies the liabilities

   > ... "shall have regard to ... the custom and practice of the company in the manner and timing of the distribution of profits or the granting of discretionary benefits ..."

   This last provision might force U.K. life assurers to reserve for terminal bonus, which might change again the basis upon which with-profits business is conducted.

4. The maximum valuation rate of interest is based on 97.5% of running yields instead of the previous 92.5%, but for existing monies only. The maximum gross yield to be assumed for investments made 3 or more years in the future is the *minimum* of (i) the long term gilt yield on the valuation date, (ii) 6% plus 25% of any excess of the long term gilt yield over 6%, and (iii) 7.5%. For investments made during the next 3 years, the maximum yield is found by linear interpolation. (Regulation 69.)

5. The limitation of the maximum valuation interest rate to 92.5% of the yield on Consols has been removed.

6. The "mismatching" regulation, formerly Regulation 55, now Regulation 75, has been changed to

   > "The determination of the amount of the long term liabilities shall take into account the nature and term of the assets representing those liabilities and the value placed upon them and shall include prudent provision against the effects of possible future changes in the value of assets on —

(a) the ability of the company to meet its obligations arising under contracts for long term business as they arise, and

(b) the adequacy of the assets to meet the liabilities as determined in accordance with regulations 65 to 74 above."

The main effect of (b) above is to exclude Regulation 75 itself from any resilience test used to test compliance with Regulation 75.

2.5 The E.C. solvency margins

The E.E.C. First Life Directive required life assurance companies to set up explicit minimum solvency margins. For non-linked assurances these were 4% of the mathematical reserve and 0.3% of the sum at risk. Under term and group assurances, the second component could be reduced to 0.1% for terms of not more than 3 years, or to 0.15% for terms between 3 and 5 years. Some other reductions were allowed in respect of reinsurance.

The Faculty of Actuaries Solvency Working Party tried to uncover the basis of these margins [37]. Sources "on the record" included the Campagne reports [13], [14], and the Buol report [12]. The form of the margin, if not its parameters, may be due in part to these reports, but perhaps the real basis lies in the comment from an official of an E.E.C. Supervisory Authority quoted by the Working Party:

> "The rules are purely set through negotiations and are a compromise reflecting each Member State's positions and interest."

By introducing compulsory margins of this form, the E.E.C. rejected the "natural" approach of implicit margins for which Skerman, Ammeter and Buol (see below) had argued. According to G. G. Newton (in the discussion of [37]) the E.E.C. were "firmly wedded" to the idea of explicit margins in the first place. It is that very idea — that a mathematical reserve plus a margin based upon that reserve is an improvement upon a reserve alone — which is interesting here. It extends the traditional valuation model. It is unclear, however, why a margin based upon a mathematical reserve should make up for any deficiencies of the reserve or of the underlying model.

One aim of this thesis is to examine such an extension of the traditional valuation model.

2.5.1 The Campagne Reports

Few commentators doubt that the "4% of mathematical reserves" component of the margin has its origins in Campagne's reports. If that is so, then its basis is the reported profits and reserves of 10 Dutch insurance companies during 1926–1945. Campagne estimated a frequency curve of profits as a proportion of reserves; the 4% margin yielded a 5% probability of losses over 3 years exceeding the resulting free reserves.

Clearly Campagne's methods might not be appropriate in modern conditions, or in territories other than the Netherlands; its use of reported profits and reserves is a notable anachronism.

2.5.2 The Buol Report

The Buol committee reported to the O.E.C.D. in 1971 [12]. Their report might well be regarded as the European counterpart of Skerman's 1966 paper, as it also laid down principles for a solvency valuation. The main respects in which the Buol committee departed from Skerman's principles were in the association between assets and liabilities, and in the determination of the interest rate for the valuation of liabilities.

The report argued that the close link between assets and liabilities which was (by 1971) the conventional wisdom in the U.K. was only relevant in the circumstances of the U.K., with particular emphasis on the availability of long term gilts. Therefore the valuation of assets and and the valuation of liabilities were considered separately.

The report set out an algorithm for choosing the valuation interest rate:

1. The "unstrengthened" interest rate should be (i) 90% of a 20-year average of the historic yield on the assurer's assets, or (ii) $\frac{1}{3}$ of the current yield plus $\frac{2}{3}$ of the lowest yield during the previous 20 years, in either case limited to 90% of the current yield.

2. The "strengthened" interest rate should be 80% of the "unstrengthened" interest rate, giving a degree of strengthening responsive to some extent to the level of yields.

It was suggested that a life company which reserved on the strengthened basis would not have to set up explicit additional reserves. The Buol committee had the same aim as Skerman — to define a valuation standard which would *avoid* the need for explicit margins. The report said (the emphasis is theirs) [12, paragraph 64]:

> "... a distinction has to be drawn between two things: on the one hand, *the system itself*, which consists of an implicit margin obtained by strengthening the technical rate of interest, and, on the other hand, *the level of safety* which such a margin affords and which depends on the rate of strengthening employed."

Somewhat confusingly, the committee carried out some experiments to quantify the "level of safety", expressing the results in the form of the following approximate margin: 8% of the unstrengthened mathematical reserve plus 6% of the sum at risk. (The latter was in fact related to premium loadings and not mortality risk; it bears no resemblance to the 0.3% margin adopted by the E.E.C..) This result has led some commentators to assume that the Buol committee set out to determine a basis for explicit margins — the exact opposite of the truth. The remarks of G. G. Newton in the discussion of [37] are particularly relevant.

2.6 The U.K. resilience test

2.6.1 The Government Actuary's memorandum of 13 November 1985

Regulation 55 [33] requires Appointed Actuaries to ensure that the assets are suited to the liabilities, as follows :

> "The determination of the amount of long term liabilities shall take into account the nature and term of the assets representing the long term fund

and the value placed upon them and shall include appropriate provision against the effects of possible future changes in the value of the assets on their adequacy to meet the liabilities."

On 13 November 1985, the Government Actuary issued a memorandum to Appointed Actuaries which, *inter alia*, described the "Working Rule" used by G.A.D. to test compliance with Regulation 55 [54, Appendix 1]. This was to ensure that the Regulations (other than Regulation 55) could be complied with after a fall of 25% in equity prices and a rise or fall of 3% in the rate of interest. Although this test had no statutory force, and Appointed Actuaries were free to interpret Regulation 55 more or less strongly, in practice it was at once adopted as an unofficial Regulation known as the "mismatching test" or "resilience test". The memorandum was quickly followed by the issue of Temporary Practice Note No.2 by the Faculty of Actuaries and Institute of Actuaries. Some points which arose were:

1. The nature of the test — the ability to establish reserves at a single time — was quite different from cash-flow mismatching and the latter ought also to be examined.

2. The ±3% change in "interest rates" was to be interpreted as a change in gross redemption yields.

3. The fall in equity prices was *not* to be accompanied by a fall in the level of dividends. Thus the dividend yield would *rise* by 33% in the conditions envisaged by the test.

4. The reserves to be set up following the change in conditions were to include the E.C. solvency margins but not further resilience reserves — Regulation 55 itself was specifically excluded from the test. (Note that this point was clarified in the 1994 Regulations.)

5. The memorandum indicated that the parameters of the test had regard to current conditions, implying (as was generally agreed) that it would not be a suitable test in all conditions. For example, testing a further fall of 3% in interest rates after they might already have fallen to (say) 5% seemed excessive.

6. The combination of a fall in equity prices accompanied by a fall in interest rates was criticised as unrealistic.

The resilience test tested the sensitivity of the office's solvency under given investment scenarios. It did not, however, amend the concept of solvency — the solvency test within the scenario was still a traditional solvency valuation. Nor did the scenarios extend beyond a single instantaneous change in conditions. Nevertheless, within the constraints of existing Regulations it was an interesting development.

2.6.2 The Government Actuary's memorandum of 31 July 1992

Between 1985 and 1992 financial conditions changed with the result that some life assurers found increasing difficulty in meeting the requirements of the resilience test. In particular, dividend yields were at high levels, while interest rates had fallen to relatively low levels, with the result that the assumed *further* increase in dividend yields, in combination with the assumed *fall* in gilt yields, fell foul of the overall restriction of the valuation interest rate to 92.5% of the yield on Consols.

On 31 July 1992 the Government Actuary issued a memorandum to Appointed Actuaries permitting the resilience test to be weakened on a case-by-case basis, following consultation with G.A.D.. The memorandum said in respect of equities:

> "... DTI and GAD consider that it would be reasonable for appointed actuaries whose companies' equity portfolios correspond broadly to the Financial Times All-Share Index to review the parameters which they incorporate in the resilience test when the dividend yield on that index exceeds 5.25%. A gradual tapering of the 25% parameter would be envisaged but it would not be considered advisable to assume that there would be a maximum dividend yield at which no further fall in market values would be assumed."

and in respect of the restriction to the Consols yield:

> "... the limitation on the dividend yield on an equity (and the rental yield on a property) — to the yield on Consols — might be a material factor

> . . . my department would be willing to discuss with you the practical application of this aspect of your proposed resilience test, having regard to the specific assets held by your company."

and in respect of fixed interest securities:

> ". . . the absolute level of interest rates is a relevant criterion. It would be reasonable to assume that it is more likely that there could be an immediate fall of three percentage points if current yields were 15% than if they were 6%, say."

The effect of the memorandum was to allow relief to companies whose position was adversely affected by the arguably arbitrary choice of parameters made in 1985. However it was open to the criticism that if the rules might be changed whenever they proved at all onerous, they might not be serving their purpose.

Various suggestions were made for modifications to the resilience test which would automatically adjust the parameters to the prevailing conditions. For example, Purchase *et al* [54] suggested that the assumed fall in equity prices should not cause the dividend yield to move outside the range 3–7%, subject to a minimum assumed fall of $12\frac{1}{2}$%; similarly the assumed fall in fixed interest yields might be changed to the minimum of 3% and one-third of the current yield. In similar vein, Needleman [47] suggested the following changes:

1. The ±3% test to be changed to ±20% of the gross yield on Consols.

2. The fall in equity and property values to be the fall in market values corresponding to a 1.5% change in dividend or rental yield.

3. If the above changes result in the appearance of a positive yield gap (i.e. dividend yields higher than fixed interest yields) then the fall in the Consols yield and then the fall in equity prices — in that order — should be restricted to eliminate the yield gap.

2.6.3 The Government Actuary's memorandum of 30 September 1993

Further changes in financial conditions — in particular a fall in dividend yields and interest rates — led to the resilience test being revised once more. The Government Actuary's memorandum of 30 September 1993 to Appointed Actuaries made extensive changes.

1. For the non-linked liabilities of offices which wrote no with-profit business, assumed to be backed by fixed interest assets, the parameters were amended to a *rise* in interest rates of 3 percentage points and a *fall* of 20% from current levels.

2. For a with-profit office, three scenarios were to be tested: (i) a reduction in fixed interest yields by 20% combined with a fall in the value of equities of 10%; (ii) a reduction in fixed interest yields by 10% combined with a fall in the value of equities of 25%; (iii) a rise in fixed interest yields of 3 percentage points combined with a fall in the value of equities of 25%.

3. The restriction to the Consols yield was waived for the purpose of setting up minimum reserves after the assumed changes in conditions, although account had to be taken of the possible need to comply with this restriction at the next valuation.

While falling short of "mechanising" the test as suggested by Purchase *et al* and by Needleman, falls in interest rates were relative rather than absolute, which perhaps fitted the test to a wider range of financial conditions. The problem remains, however, that it is difficult to say how the test might be modified if financial conditions change significantly in future.

Note that the 1994 Regulations [34] removed the limitation of the valuation interest rate to the yield on Consols.

2.7 North American and Australian developments

The supervisors in the U.S.A., Canada and Australia have introduced (or will soon introduce) new standards of capital adequacy designed to improve the diagnostic ability of a solvency valuation.

The driving forces behind the changes have not been the same in all three territories, but they have included:

1. Concern over numbers of insolvencies in the 1970s. In the U.S.A. and Canada, insolvency is much less unusual than the U.K..

2. Recognition that the reserves published for profit-reporting purposes were not necessarily adequate for solvency purposes.

3. Moves towards unification of solvency regulation in the banking and insurance sectors.

4. Product innovation, to which traditional regulation was often poorly adapted.

5. Moves away from prescriptive solvency regulation and fixed bases towards more dynamic and responsive measurement of solvency with greater responsibility devolving on the valuation actuary.

Both Canada and the U.S.A. have adopted (i) systems of solvency reserves not unlike the E.C. solvency margins, although much more extensive, and (ii) cash-flow scenario testing. Important differences remain, however, particularly with respect to (i) the basic reserve to which the solvency margins are added, and (ii) the details of the cash-flow tests.

Australia plans to introduce a "layered" system in which the two lowest layers together form the reserve for profit reporting purposes, the next layer includes extra reserves for published solvency reporting, and the final layer, which is unpublished but which is made available to the regulators, provides for a still stronger level of solvency.

The actuarial profession in the U.K., in conjunction with the Government Actuary's Department, set up a number of Working Groups under the auspices of the Joint Actuarial Working Party (JAWP) to study these developments. Several Working Groups presented interim reports [31], [59] in November 1993 which include useful summaries, and comments on the fitness of the new standards in the different circumstances of life assurance business in the U.K..

Further useful information is given in the papers presented to the 1991 Hobart Convention of the Institute of Actuaries of Australia, in particular Brender [10] and Freeman & Vincent [24].

2.7.1 The U.S.A

In the U.S.A., two reserves are required. The GAAP reserves are published in the accounts, and separate reserves are calculated for solvency reporting. Although the states are largely autonomous, in practice they all follow more or less closely the approach of the National Association of Insurance Commissioners (NAIC). This body publishes the Standard Valuation Law (SVL). In addition, the Actuarial Standards Board (ASB) publishes guidance for actuaries which, although not statutorily binding, is professionally binding.

SVL reserves are net premium reserves, but with a valuation rate of interest determined dynamically using averages of historic bond yields. One difference between this system and that in the U.K. is that the valuation interest rate is calculated for each year's *new business* during the year preceding issue, and is then fixed and used to value that block of business in every future year. The dynamic calculation of valuation interest rates does not apply to *existing* business. This reflects the importance of matching in the U.S.A..

The SVL has been strengthened by three recent developments. First, the ASB published a standard on cash-flow testing. This did not say when or how cash-flow testing should be carried out, but it obliged actuaries to consider where its use might be necessary to establish the adequacy of the reserves.

Second, the NAIC has introduced a "Model Regulation" to the effect that, to back up his or her opinion, the actuary must carry out an "Asset Adequacy Analysis". The methods of doing so are not prescribed but since 7 interest rate scenarios

Strategy	Pattern of Interest Rates
(i)	Level
(ii)	Increases of 0.5% p.a. for 10 years, then level
(iii)	Decreases of 0.5% p.a. for 10 years, then level
(iv)	Increases of 0.5% p.a. for 5 years, then decreases of 0.5% p.a. for 5 years, then level
(v)	Decreases of 0.5% p.a. for 5 years, then increases of 0.5% p.a. for 5 years, then level
(vi)	An immediate increase of 3% then level
(vii)	An immediate decrease of 3% then level

Table 2.3: U.S.A. NAIC "Model Regulation" Asset Adequacy Analysis scenarios.

must be considered, along with any further scenarios which the actuary thinks necessary, cash-flow testing is the most natural method. The mandatory interest rate scenarios are shown in Table 2.3.

One state (New York) has made cash-flow testing of the NAIC scenarios mandatory for annuities and single premium life products; it is expected that other states will follow suit. New York Regulation 126 states that the projection must be over a sufficient term to cover the major portion of the future run-off of the cash-flows; in this it differs from the Canadian approach described in Section 2.7.2.

Third, the Society of Actuaries "Valuation Handbook" gives guidance to life office valuation actuaries. Included in this guidance is that basic mathematical reserves should be supplemented by additional reserves called "Risk Based Capital" (RBC) requirements — in effect by solvency margins. These margins are determined by applying prescribed factors to suitable measures of four sources of risk, described below, called C-1 to C-4 risk. Numerous papers in the U.S. literature try to quantify C-1, C-2 and C-3 risk; examples are Brender [9], Cody [15], Sega [60] and Vanderhoof *et al* [66].

In some respects, RBC is similar to the E.C. solvency margin, in that it prescribes a set of factors to be applied to quantities appearing in the published accounts; the insurer must possess enough capital, in addition to policy reserves, to cover the resulting margins. In covering asset risks as well as liability risks, however, it resembles the U.K. resilience test. It is also similar to the Canadian MCCSR requirements, with the important difference that the Canadians add the margins to a "best estimate" reserve rather than to a solvency reserve.

Asset class	Factor
U.S. Government issue bonds	0.0%
Bonds rated AAA – A	0.3%
Bonds rated BBB	1.0%
Bonds rated BB	4.0%
Bonds rated B	9.0%
Bonds rated C	20.0%
Bonds in default	30.0%
Residential mortgages in good standing	0.5%
Residential mortgages 90 days overdue	1.0%
Commercial mortgages in good standing	3.0%
Commercial mortgages 90 days overdue	6.0%
Mortgages in foreclosure	20.0%
Unaffiliated common stock	30.0%
Unaffiliated preferred stock	bond factor + 2.0%
Real estate (investment)	10.0%
Real estate (foreclosed)	15.0%
Cash	0.3%

Table 2.4: U.S.A. NAIC Risk Based Capital factors for C-1 risk.

In 1992, the NAIC adopted RBC for life assurers' financial reporting. The factors given below are those of the NAIC.

C-1 (Asset) Risk. C-1 risk is the risk of loss on the life assurer's assets. The importance of fixed-interest and mortgage-type assets is reflected in the level of detail at which the factors vary. For completeness, the factors are shown in Table 2.4.

C-1 factors are applied to the book values of the relevant assets, except in some of the riskiest categories such as bonds in default, for which the market value is used.

The RBC margins for bonds are adjusted to allow for the concentration of the funds in each class, and the margins for mortgages are adjusted to allow for the assurer's default and foreclosure experience relative to an industry average. The total C-1 margin is the sum of margins for each asset category, further adjusted for concentration in such a way that the margins for the ten largest asset classes are effectively doubled.

The C-1 risk factors were derived from studies of the distributions of default

Sums at risk	Factor
First \$500,000,000 at risk	0.150%
Next \$4,500,000,000 at risk	0.100%
Next \$20,000,000,000 at risk	0.075%
Sums at risk over \$25,000,000,000	0.060%

Table 2.5: U.S.A. NAIC Risk Based Capital factors for C-2 risk.

Type of portfolio	Factor
Low risk portfolio	0.5%
Medium risk portfolio	1.0%
High risk portfolio	2.0%

Table 2.6: Examples of U.S.A. NAIC Risk Based Capital factors for C-3 risk.

losses among the different asset types. For example, the bond factors are intended to cover 92% of the expected losses in each category and 96% of the expected losses for the whole bond portfolio, while the 30% factor for common stock is supposed to cover the largest loss likely over a 2-year period with 95% probability.

C-2 (Insurance) Risk. C-2 risk is the risk from excess claims. For individual life assurance the factors in Table 2.5 are applied to the sums at risk.

Clearly these factors are similar in intention to the "sum at risk" component of the E.C. solvency margin. Although the RBC factors are generally lower than the E.C. margin, the E.C. formula for the total margin is simpler and might be regarded as implicitly covering other risks for which RBC accounts explicitly.

The aim of the C-2 factors is to meet excess claims over a 5-year to 10-year period with 95% probability.

C-3 (Interest Rate) Risk. In U.K. terms C-3 risk is the mismatching risk. The factors depend on the surrender rights and guarantees granted to the policyholder — a matter of importance in the U.S.A. — and on the degree of cash-flow matching of the assets and liabilities. Table 2.6 shows some examples of the factors to be applied to the policy reserves.

A "low risk" portfolio, for example, is one in which the policyholders have no surrender guarantees and the difference between the volatility of the assets and liabilities is less than 0.125. Unless the insurer certifies that its asset and liability cash-flows are well matched, the factors are increased by 50%.

C-4 (Business) Risk. C-4 risk is a "catch-all" category covering everything from bad management to bad luck. The same factors are applied to all companies; there is no attempt to pick out particular companies at greater risk than others. The factors are 2% of life and annuity premium income and 0.5% of accident and health premium income.

The total margin allows for correlations between C-1 and C-2 risk as follows:

$$\text{Total margin} = \left((\text{C-1} + \text{C-2})^2 + \text{C-3}^2\right)^{\frac{1}{2}} + \text{C-4}$$

The office may employ statutory capital and surplus, an asset valuation reserve, voluntary investment reserves and 50% of policyholders' dividend (bonus) liabilities (called Total Adjusted Capital) to meet the RBC margin. The ratio of the Total adjusted Capital to the RBC margin must be at least 200%. (The factors given above are 50% of their original (1992) values; the change was made on cosmetic grounds, so that the published ratios should not attract adverse attention.) Below 200%, the company must file a recovery plan. Below 150%, the Insurance Commissioners office will inspect the company. Below 100%, the company *may* be put into "rehabilitation". Below 70%, and the company *must* be put into rehabilitation. In addition, if this ratio is between 200% and 250%, but on the basis of trend could fall below 190% during the year, the company must file a recovery plan.

Several factors indicate that the RBC approach is more suited to U.S. than to European conditions.

1. The basic reserving standard in the U.S.A. tends to be weaker than in E.C. territories, and the RBC requirements allow for this. The JAWP quoted an estimate by a U.S. actuary that the basic reserves might give 85% confidence of meeting liabilities, while the reserves plus RBC might increase this to 95%. Under current legislation most European insurers would have to set up stronger reserves before the addition of solvency margins.

2. Insurance business in the U.S.A. is more homogeneous than insurance business in the U.K., let alone the entire E.C.; therefore a prescriptive regime might have a better chance of success in the U.S.A..

3. The detailed nature of the RBC analysis means that RBC is very office-specific. That is, different offices might face quite different RBC requirements. (The U.K. regulations have a similar effect in practice.)

4. The RBC factors based on the assets would penalise offices with adequate free assets, especially if these were invested in equities. Some method of apportioning assets to lines of business (as in the U.K. valuation regulations), and of excluding free assets, would be required.

5. U.K. life assurers face different risks from U.S. life assurers, particularly with-profit offices with substantial equity investments. These offices are subject to a "negative risk" to the extent that they can vary future bonus; it is not clear how this should be incorporated into a system of RBC. Three possible risk factors are (i) the equity content of the relevant assets, (ii) the maturity of the portfolio, and (iii) the extent of the terminal bonus cushion.

RBC is more prescriptive and (ostensibly) more sophisticated than E.C. solvency regulation. The RBC factors were determined after detailed studies of the historic variability of the various risks, but the underlying basis of RBC is still a traditional valuation reserve. Indeed, it is acknowledged that RBC covers catastrophic risks — the relatively short-term risks of uncovering the policy reserves — rather than superceding the underlying valuation model. But broadly, it combines the aims of the U.K. resilience test and the E.C. solvency margin.

2.7.2 Canada

New Canadian regulations in 1978 replaced mandatory valuation assumptions with "appropriate" assumptions, to be decided by the Valuation Actuary. It became apparent that actuaries were using a wide range of "appropriate" assumptions, and there was concern that competitive pressure was eroding solvency standards. For these and other reasons, the Department of Insurance Canada commissioned a report from Dr. A. Brender on the establishment of a set of solvency margins (called "Minimum Continuing Capital and Surplus Requirement" (MCCSR)). The resulting recommendations encompassed a system of factors similar to (but predating) the RBC factors described in Section 2.7.1. An extended set of factors was subsequently adopted by the Canadian Life and Health Insurance Association (CLHIA) as a measure of solvency which could be used to apportion the costs of a policyholders' guarantee fund. Factors based on this extended set were then adopted by the regulators. The MCCSR is thus a static test, with statutory force.

A separate reason for change was the adoption of GAAP reporting in Canada. This led to the introduction of the "policy premium reserve" method of calculating the basic policy reserves, broadly a gross premium method on a "best estimate" basis, and likely to be inadequate by itself for solvency purposes. The Canadian Institute of Actuaries (CIA) set up a Committee on Solvency Standards for Financial Institutions to consider what additional standards might be made necessary by this move. It quickly concluded that no formula would do, and recommended that Dynamic Solvency Testing (DST) — cash-flow scenario projections — should be adopted as a standard by the CIA.

The standard as it has developed has several interesting features.

1. A number of mandatory scenarios, encompassing mortality, lapse, expense and interest rate risk, are prescribed, but the actuary must use such additional scenarios as are required. The basic idea is not to subject the financial institution to a rigid test but to explore its sensitivities.

2. Projections including 5 years' future new business are required.

3. The measure of "adequacy" in each future year is ability to meet the projected MCCSR. Instead of requiring cash-flow projections of run-offs (like New York Regulation 126), the basis of DST is a traditional solvency valuation (plus MCCSR margins). It might be described as a hybrid between the traditional valuation and true cash-flow testing.

The CIA hopes to develop DST to make it independent of arithmetic formulae.

2.7.3 The Australian valuation proposals

A useful summary of the Australian proposals is included in Scott *et al* [59]. Broadly it consists of four layers: (i) a Best Estimate Liability (BEL), (ii) Planned Margins for Profit (PMP), (iii), a Solvency Reserve (SR), and (iv) a Capital Adequacy Reserve (CAR). The first two will also form the basis of profit reporting and taxation. The main features are described below.

Best Estimate Liability. The BEL is a discounted cash-flow reserve on a best estimate basis, in the sense that best estimates of future expenses, mortality and lapses must be used. The rate of discount is also described as "best estimate", but with upper limits on the assumed yields on equities; for example, for Australian equities the limit is the yield on 10-year bonds plus 5%. The BEL must reserve for supportable bonuses, including terminal bonuses, which are defined as bonuses resulting in nil profit under the BEL calculation basis.

Planned Margins for Profit. Profit is deemed to be earned when a risk is underwritten or a service provided, and the PMP are deemed to emerge when the corresponding profit is earned. All expected future profits must be included in the PMP. In this way the capitalisation of future profits at inception or on a change of basis — often held against gross premium valuations — is avoided.

Solvency Reserve. The solvency reserve requires several contingencies to be considered. In particular, a set of factors are laid down, which must be applied to the BEL best estimate assumptions and the BEL recalculated, allowing for any discretions which the office may possess in (for example) varying charges; and a resilience reserve must be held. The parameters of the resilience test include a rise of 1.25% in gross equity yields and a change of $\pm 1.5\%$ in gross fixed-interest yields. In addition, reserves must be held in respect of inadmissible assets. The SR will be published.

Capital Adequacy Reserve. The CAR resembles the solvency margin but it is to be calculated not on a "minimum" but on an "appropriate" basis — which is partly at the actuary's discretion. For example, the parameters of the resilience test include a rise of 2.5% in gross dividend yields and a change of ±2.5% in gross fixed-interest yields. The CAR will not be published, but will be disclosed to the supervisor in confidence.

Failure to meet the CAR requirement would result in closer scrutiny by the supervisor, while failure to cover the SR would result in intervention.

In part these proposals are driven, as in Canada, by the desire to define reserves which can be used for accounting purposes, and then to base solvency reporting on the same reserves by means of additional margins. In the U.K., the Joint Actuarial Working Party favoured a similar approach, so we must turn again to the question of how effective is the combination of traditional valuation reserve plus prescribed margins

Chapter 3

Stochastic studies of life assurance solvency

3.1 Introduction

The traditional valuation model, as a determinant of solvency, has been subjected to scrutiny in recent years as stochastic models have been more widely explored.

Stochastic models of mortality and morbidity have been developed by statisticians and actuaries and used to investigate the effect on cash-flows and profits of random variation in the demographic experience. On the whole, though, such randomness makes little impact provided the number of lives assured is reasonably large (see Frees [25]).

Stochastic models have also been developed for asset yields and prices. This is different in kind from modelling mortality stochastically, since the investment risks in respect of different contracts are not independent, and might be more significant for life assurance savings business. The "Principle of Insurance" does not apply to the assets.

Since it was introduced to the profession in the U.K. by the Maturity Guarantees Working Party [6], stochastic modelling has met a mixed response. It suggests answers to important questions which are otherwise intractable, but the variability of modelled quantities can be alarmingly large. The results depend, of course, upon the model's assumptions, and some would say that the assumptions underlying the models of the assets most often used today are not sound enough to give credence to a cash-flow model which uses them. One might therefore suspend judgement until more convincing asset models are developed.

If, however, volatility is anything like that suggested by simple stochastic models, we stand to learn a good deal from such research. It might be difficult to establish results in absolute terms — for example it might be unwise to take as accurate an estimated ruin probability — but there are other, more reasonable approaches.

1. The *magnitude* of the fluctuations in key life office statistics is of interest in its own right. If it should be broadly similar when different models of the assets are used, then this would be a major qualitative result. It might be argued that *instability* rather than stability is the chief feature of modern life office finance, so actuaries should be looking out for tools with which to measure instability.

2. *Comparative* studies can be carried out, in which the effects of different actions are compared using the *same* model of the assets and (if simulation is used) the *same* set of investment scenarios. Relative results might be more credible than absolute results.

In this thesis we work along the lines of the second of these approaches.

In this section we review previous work on life office solvency based upon versions of the Wilkie model or its predecessor, and consider some questions suggested by this work. Of particular interest is the development of *strategies* to model asset allocation and bonus distribution. The papers considered are:

1. The Maturity Guarantees Working Party (1980).

2. The Faculty of Actuaries Solvency Working Party (1986).

3. The Faculty of Actuaries Bonus & Valuation Research Group (1989).

4. M. D. Ross (1991).

5. M. D. Ross & M. R. McWhirter (1991).

6. The Finnish Life Assurance Solvency Working Party (1992).

The papers listed above have used progressively more sophisticated — and possibly more realistic — strategies. It would be interesting to know if strategies which proved to be effective in controlling, for example, the solvency of an office might also appear to be realistic to a practitioner, or whether on the contrary, "realistic" strategies were ineffective.

Other papers which have explored stochastic modelling but which are not directly relevant are Purchase *et al* [54] and Hardy [29].

3.2 The Wilkie asset model

Since all but one of the papers described here use the Wilkie asset model or a variant of it, we will describe it briefly first. A full description is given in Wilkie [68].

The model consists of four discrete (annual) time series generated by four sequences of independent unit normal random variables $QZ(t), YZ(t), DZ(t)$ and $CZ(t)$. These provide annual values of the Retail Prices Index ("RPI"), an index of gross equity dividends, the current running gross dividend yield, and the gross yield on Consols, as follows:

1. $Q(t)$ — Retail Prices Index

$$\log\left(\frac{Q(t)}{Q(t-1)}\right) = 0.05 + 0.6\left\{\log\left(\frac{Q(t-1)}{Q(t-2)}\right) - 0.05\right\} + 0.05QZ(t)$$

2. $D(t)$ — An index of gross equity dividends

$$\log(D(t)/D(t-1)) = 0.8DM(t) + 0.2\log\left(\frac{Q(t)}{Q(t-1)}\right) - 0.0525YZ(t-1) + 0.1DZ(t)$$

where

$$DM(t) = 0.2\log\left(\frac{Q(t)}{Q(t-1)}\right) + 0.8DM(t-1)$$

3. $Y(t)$ — Gross dividend yield

$$\log(Y(t)) = 1.35\log\left(\frac{Q(t)}{Q(t-1)}\right) + YN(t)$$

where

$$YN(t) = \log 0.04 + 0.6\{YN(t-1) - log 0.04\} + 0.175YZ(t)$$

4. $C(t)$ — Gross yield on Consols (irredeemable fixed-interest securities)

$$C(t) = CM(t) + CN(t)$$

where

$$CM(t) = 0.05 \log \left(\frac{Q(t)}{Q(t-1)} \right) + 0.95CM(t-1)$$

and

$$\log(CN(t)) = \log 0.035 + 0.91 \log \left(\frac{CN(t-1)}{0.035} \right) + 0.165CZ(t)$$

and with the constraint that $C(t)$ can never be less than 0.05%.

The parameters given above are those of the "Reduced Standard Model".

3.3 The Maturity Guarantees Working Party

The Maturity Guarantees Working Party ("MGWP") of the Faculty of Actuaries and Institute of Actuaries was set up to study the reserving requirements of equity-backed unit-linked business with minimum maturity or surrender guarantees. Concern had arisen following the insolvency of several small life assurers in the wake of the stock market crash of 1974.

The MGWP's report [6] put forward a method of reserving rather than a suggested level of reserves, and in a break with the past the method used stochastic simulation. For this purpose a model of equity prices and dividends was developed by Wilkie [6, Appendix D] — this model was later developed into the more comprehensive model described in Section 3.2 above and in Wilkie [68].

The MGWP modelled a portfolio of regular premium business of terms from 10 years to 30 years. Premiums were invested in units whose prices were modelled by the sample paths of the share price model with dividends reinvested. The maturity value under each policy was guaranteed to be not less than the total premiums paid. The outcomes of each simulation were the undiscounted and discounted totals of the payments necessary to meet the guarantees — these represented the reserves which would be needed under that scenario. In the basic projections, no further new business was written.

The results based upon 5,000 simulations were that the distribution of maturity guarantee reserves was very long-tailed. Only 1,539 cases required a non-zero reserve, and in 1,333 of these cases the discounted reserve was under 1% of the total guaranteed amounts. However, the discounted reserve needed to ensure "solvency" with 99.9% certainty was 16% of the total guaranteed amounts, and a discounted reserve of 4% of the total guaranteed amounts was insufficient in about 98% of the simulations. These results were enough to cause life offices to stop transacting unit-linked business with maturity guarantees, at least until derivative securities became more widely available.

In addition, the MGWP tested the sensitivity of the results to changes in the the asset model and in the mix of business in the portfolio [6, Appendix E]. Two such results are summarised here, since they are to some extent also typical of projections of with-profit business.

1. If a single generation of contracts was considered (i.e. maturing in a single year) then the number of simulations in which guarantee payments were made, and the reserves required to meet any level of ruin probability, declined sharply as the policy term increased. Some examples are shown in Table 3.7.

2. Most of the guarantee claims with the standard portfolio occurred in the first few years of the projection, largely because of the much greater tendency of short-term policies to give rise to such claims.

The importance of the MGWP's report lies in its premise that solvency reserves in respect of asset risks should be established by stochastic methods, and in the example it gave of a practical method of estimating reserves, including steps towards the introduction of a suitable asset model. It was perhaps fortunate that the problem which presented itself did not require the consideration of investment and bonus strategies.

3.4 The Faculty Solvency Working Party

The Faculty of Actuaries Solvency Working Party ("FASWP") in 1986 carried out stochastic projections of non-profit and with-profit business using the Wilkie asset

Policy term	5 Yrs.	10 Yrs.	15 Yrs.	20 Yrs.	25 Yrs.	30 Yrs.
No. of simulations (of 5,000) with guarantee payments	1,127	567	297	147	54	25
Undiscounted reserve, 0.5% ruin prob.	45%	42%	42%	34%	29%	13%
Undiscounted reserve, 5% ruin prob.	32%	26%	20%	8%	0%	0%
Discounted reserve, 0.5% ruin prob.	37%	28%	23%	16%	11%	4%
Discounted reserve, 5% ruin prob.	26%	18%	11%	4%	0%	0%

Table 3.7: Maturity Guarantees Working Party — Number of simulations with guarantee claims at terms 5–30 years, and solvency reserves as % of total guaranteed amounts

model [37], [68]. Some of the features of their investigations were as follows:

1. Each set of projections followed a *single tranche* of 20-year endowment policies from inception to maturity in 1,000 scenarios.

2. The underlying assets were assumed to be invested either

   - 100% in equities, or
   - 100% in gilts, or
   - 50% in each of equities and gilts (by market value).

   In those projections in which non-profit business was invested 100% in gilts the liabilities were immunized if possible, using irredeemables or dated gilts.

3. Two bonus strategies were used for the with-profit endowments.

   - The first strategy was based upon a passive net premium "valuation for surplus". Surplus was calculated by accumulating the premiums less claims at the earned rate of return, ignoring capital appreciation or depreciation, and comparing this with a 3% net premium reserve. If the surplus was positive, it was used to provide one year's bonus; if negative, no bonus was declared.

This method led to moderate bonus rates, initially with very low standard deviations, which steadily increased during the 20 years of the projection. Only in the last years of the projections and only with 100% equity investment did the standard deviation of the bonus rate exceed half the mean bonus rate.

- The second strategy used a more realistic active bonus reserve approach. Assets were valued on a notional basis, gilts by applying a 5-year average of the net gilt yield to the future gilt income, and equities by applying a 5-year average of the dividend yields to the current level of dividend income. The interest rate used to value the liabilities was based either on the 5-year average of the net gilt yield, or on the 5-year average of the net dividend yield plus the difference between the 5-year averages of the gross and net gilt yields (the latter adjustment being to allow for the experienced growth rate in equities). A bonus rate was declared which would, if maintained until the business had run off, just exhaust the assets valued as above. (The FASWP did not state whether this strategy was modified in case the market value of the assets exceeded their notional value, particularly in the last few years of the projection.)

  The mean bonus rates given 100% gilt investment were very similar to those emerging under the passive strategy, but with a more stable standard deviation. The mean bonus rates which emerged given either 50% or 100% equity investment were much larger than those emerging under the passive strategy, and had much larger standard deviations.

Note that the active bonus strategy was regarded by the FASWP as being more realistic than the passive bonus strategy.

4. The FASWP calculated a "solvency reserve" in each year of each simulation, equal to the assets which would have to be held at the end of that year to meet the future liabilities as if the outcome were known in advance. The solvency

reserve was compared with the market value of the assets, with the statutory minimum valuation reserve and with a gross premium valuation reserve. The FASWP did not display ruin probabilities explicitly, but it was possible, from those cases in which the solvency reserve exceeded the market value of the assets, to deduce roughly where the dangers of insolvency lay. At the 5% level, only the with-profit business, invested 100% in gilts and using the active bonus strategy, came close to being solvent. If the with-profit business was invested either 50% or 100% in equities, and if the active bonus strategy was used, the *mean* margin required over the market value of the assets was positive in every year of the projection. That is, the mean position in every year under the 1,000 simulations was one of actual insolvency. The FASWP commented that

> "The reason for these disquieting results lies not so much in the bonus distribution policy actually followed, but more particularly in the assumption that the policy will be rigidly adhered to regardless of future conditions."

In other words, the bonus strategies were dynamic but not dynamic enough.

5. Other factors which ought to be taken into consideration are:

   - The projections did not allow for any additional estate which the office might possess. "Additional estate" is here taken to mean any excess of assets over policyholders' total asset shares; some authors call it the "orphan surplus".
   - There was no allowance for continuing new business. Such an approach was dictated by the aim of studying solvency assuming, as usual, an office closed to new business, but it might be expected that new business, and new business expansion, would have had a significant effect.
   - The investment strategy was entirely passive, except that non-profit business might be immunized. Otherwise, there was no attempt to relate the investment strategy to the nature or term of the guarantees.

- There was no terminal bonus strategy, except that the active bonus strategy might be described as aiming for a target terminal bonus of nil.
- The FASWP did not comment upon the individual sample paths of any of the projections

3.5 The Faculty Bonus and Valuation Research Group

The Faculty of Actuaries Bonus and Valuation Research Group ("FABVRG") in 1989 [22] modelled 20-year with-profit endowment business under conditions of declining investment returns, and *inter alia* studied the stochastic effects of the Wilkie investment model. The stochastic parts of their work fell into three parts.

The first two parts concerned the distribution of maturity values or bonus rates in the financial conditions generated by the Wilkie model, ignoring valuation considerations, and are not directly relevant here. One point of interest was the investigation of "declining Equity Backing Ratio (EBR)" investment strategies, in which the assets underlying a contract are switched progressively from equities to gilts as maturity approaches.

In the part concerned with solvency, the FABVRG first carried out deterministic projections where investment conditions were represented by the Wilkie model with the stochastic fluctuations removed. These showed the effect of assuming that the mean values of the stochastic asset model (approximately) held in future. They went on to restore the stochastic components of the Wilkie model and examined (in 10 simulations) how large the fluctuations in several key statistics, including some measures of solvency, might be. New business was allowed for, growing at 2.5% over the rate of inflation each year. The office was assumed to have an additional estate at outset of 30% of the asset shares.

Only one investment strategy was used, namely investing the fund 70% in equities and 30% in gilts every year.

A fixed bonus strategy (a single tier bonus declining from 5% to 3.5% over the first 5 years of the projection) and two dynamic bonus strategies were considered.

Valuation of	FASWP	FABVRG
gilt income	5-year average gilt yield	Fixed 7.5%
dividend income	5-year average dividend yield	Fixed 7.5% with 4% dividend increases
liabilities backed by gilts	5-year average gilt yield	Fixed 7.5%
liabilities backed by equities	5-year average dividend yield + difference between 5-year average gross and net gilt yields	Fixed 7.5%

Table 3.8: Comparison of valuation assumptions used by the Solvency Working Party and the Bonus & Valuation Research Group of the Faculty of Actuaries

1. First, the liabilities were valued at a fixed interest rate of 7.5% net, on a gross premium basis allowing for inflation of future expenses at 5%. Then the future net income on the assets was also valued at 7.5% net. Equity dividends were assumed to increase at 4% per annum (a rate broadly consistent with the trend in the Wilkie asset model). The resulting surplus was used to declare a rate of bonus which could be maintained until all the in-force business had run off. Table 3.8 compares this method with the active bonus strategy used by the FASWP.

   So although the FABVRG adopted a dynamic bonus strategy, it was based upon fixed yields for valuing both assets and liabilities, taking no account of current conditions, in contrast to the FASWP method. The strategy was dynamic, but less so than the method which the FASWP had described as too rigid.

2. Two patterns of bonus were considered. Either (i) a single tier bonus was declared, or (ii) a two-tier bonus under which the rate of bonus on sums assured was fixed at 2%, and the rate of bonus on bonus was allowed to fluctuate. The latter showed much higher fluctuations in the bonus rates which emerged; it suffered from the first tier of 2% being fixed, as there were instances of the second tier falling to nil, while none of the single tier bonuses fell to nil.

Key statistics were graphed, although with 10 simulations this served only to give an impression of the fluctuations around the deterministic trend lines. The quantities graphed were:

1. The ratio of assets to asset shares (A/AS ratio). This measured the rate at which the office added to or drew upon its free assets. Broadly speaking, low rates of expansion caused it to increase, and high rates of expansion caused it it decline towards a value of 1.0. Several projections showed it falling below 1.0.

2. The ratio of assets to the statutory minimum liabilities (A/L ratio). This represented the statutory solvency position of the office. The fluctuations in this ratio were very large. To some extent this may have been because of the uncontrolled strategies which were being pursued, but it is partly due to the differing volatilities of the assets at market value and the liabilities on a net premium basis. Note that no resilience reserve was included in the liabilities.

3. The emerging terminal bonus rates. These showed very large fluctuations. The deterministic trend of terminal bonuses was towards a rate of about 10% to 20% of the guaranteed liabilities, but even at very long durations the emerging rates varied between 0% and about 150% of the guaranteed liabilities. What was also interesting was the magnitude of the fluctuations over quite short periods within a single simulation, in the absence of any attempt to smooth maturity values.

Similar graphs showed the effect on 10 simulations of a burst of rapid growth of new business; the main effect was to reduce the A/AS ratio to 1.0 very quickly, and to make it almost impossible for the office to maintain its additional estate.

Points to consider when evaluating the methods used by the FABVRG are:

1. Investment strategies were fixed.

2. The dynamic bonus strategies were only partly dynamic in the sense that the algorithm was not a function of either historic or current conditions.

3. The dynamic bonus strategies were based on reversionary bonus only, and did not attempt to control terminal bonuses in any way. In effect, the terminal bonus target was nil as in the dynamic strategy used by the FASWP.

4. The office modelled was open to new business and had an additional estate. This was therefore a model of commercial adequacy rather than solvency, and the FABVRG introduced the A/AS and A/L ratios as appropriate indicators of the office's position.

5. Solvency was represented by the A/L ratio, and there was no attempt to relate this to solvency on a run-off basis, as had been done by the FASWP.

6. The office was assumed to pay out the full asset share where this exceeded the guaranteed benefits at maturity; there was no attempt to smooth terminal bonuses or to reserve for the cost of meeting the guaranteed benefits when the asset share was insufficient.

7. Only 10 simulations of the full office model were undertaken, and no statistical analysis was carried out.

3.6 M. D. Ross

M. D. Ross in 1989 [57] described the difficulties of modelling management strategies, in a paper dealing with his office's approach to developing a "computerized actuary" to "manage" a with-profit life office driven by a version of the Wilkie asset model.

In contrast to the simple portfolios of business considered by the FASWP and the FABVRG, the model office included a "typical" mixture of annual and single premium pensions policies. Future new business was allowed for, growing at 5% per annum over the rate of inflation. Paid-up and surrendered contracts were also allowed for. At the starting point (taken to be 1987) the office had an additional estate of 25% of the asset shares, or 29% of the statutory minimum liabilities.

The most interesting developments were the bonus and asset allocation algorithms.

1. The fundamental question of bonus policy was taken to be the relationship between guaranteed benefits and terminal bonus. Reversionary bonus was set by aiming for a *global* terminal bonus target of 40% of the projected guaranteed liabilities. That is, reversionary bonus rates were declared so that the projected

payouts in respect of all the in-force business were about 71% guaranteed benefits and 29% unguaranteed benefits in aggregate.

In order to project the benefits, the cash-flows arising in respect of (i) the assets underlying the asset shares and (ii) the liabilities were valued consistently, allowing for the then-current investment mix.

- The assets underlying each policy's asset share were split into gilts and equities by market value, in proportion to the office's aggregate asset mix.
- Income arising from the gilt component of the asset share was valued using the *lower* of (i) the long-term mean redemption yield from the Wilkie investment model, and (ii) a 5-year average of the historic gilt yields.
- Dividends on the equity component of the asset share were averaged over the previous 5 years, allowing for indexing at the *lower* of (i) the mean long-term rate of growth of dividends used in the Wilkie investment model and (ii) a 5-year average of the historic rates of growth in dividends.
- The averaged dividends were then valued at the mean long-term dividend yield used in the Wilkie model.
- Liability cashflows were valued using the *lower* of (i) the long-term yield assumptions from the Wilkie investment model, weighted by the actual asset mix on the valuation date and (ii) a 5-year average of the historic investment return.
- A single tier compound bonus rate was declared so that the value of the assets was $1.4 \times$ the value of the liabilities, but with the overriding provision that if business with only 5 years to maturity supported a lower reversionary bonus and no terminal bonus, then that lower rate of reversionary bonus was declared.

Given premium rates for policies of different terms typical of those in U.K. practice, this approach led to *achieved* terminal bonuses of more than 40% for longer term policies, and of less than 40% for shorter term policies. This was reflected in a greater tendency for the asset shares of short term policies to be lower than their guaranteed benefits at maturity.

This bonus strategy added some sophistication to the method of the FABVRG. It was no more dynamic in good times, because the rates of interest used to value both assets and liabilities were then fixed at the long-term means from the asset model — in principle no different from the FABVRG choosing a fixed rate of 7.5%. However, in more depressed conditions the valuations were based on 5-year averages of the historic yields, and this represented the influence of current conditions on the judgement of the valuation actuary. The strategy might be criticised for defaulting to the "correct" yields — the long-term mean yields used in the asset model — therefore assuming that the actuary is using a "correct" asset model. It would be preferable, if possible, to fit an appropriate model to the yields experienced in each simulation.

2. Two investment strategies were used. The first was simply to invest 80% of the fund (by market value) in equities at any given time, and the remaining 20% in gilts.

   The second was a dynamic strategy based on the fact that the maximum valuation interest rate permitted under the U.K. regulations can often be increased by switching assets from equities to gilts (see Section 2.4). If the A/L ratio (i.e. the ratio of assets to liabilities in respect of the whole office, where the liabilities were calculated on the statutory minimum basis) fell below 1.25, assets were progressively switched towards equities until, with the A/L ratio at 1.05, the fund was invested entirely in gilts. (The limit of 1.05 allowed for the fact that the E.C. solvency margins were not included in the minimum liability.)

   A similar investment strategy was used by the FABVRG in some later investigations which did not, however, employ a stochastic model (see Paul *et al* [51]).

3. Some additional features of the model included the following:

   - The premium rates were recalculated whenever the bonus earning power of the current premiums (presumably on the basis described above) fell below 1% per annum; then a premium calculated to support 1% per annum was used instead. Single premium rates were recalculated each year so that they supported the same bonuses as an annual premium policy of the same term.

- Maturity values were based on only 98.5% of the asset shares, where these exceeded the guaranteed liabilities. The remaining 1.5% represented the charge for the guarantees.
- Executive Pension policies (about 45% of the portfolio) were made paid up at a rate of about 4% per annum. Thus a significant proportion of policies becoming claims might be paid-up.
- Executive Pension policies were surrendered at a rate of 2.5% per annum, with surrender values based on 90% of the asset share, increasing to 98.5% over the last third of their term. This suggests that there was a significant contribution to the office from surrender profits; the effect of such profits was not discussed.

100 simulations were carried out. Most of the results were in the form of graphs over time of the 5%, 10%, 30%, 50%, 70%, 90% and 95% quantiles; in other words whatever quantity was being observed, the 5th worst simulation at each time was plotted, and the 10th worst, and so on. It is important to note that these did not represent the paths followed by a single simulation; an office which spends most of its time in the top decile may still visit the bottom decile occasionally.

The results for financial ratios and emerging bonus rates using the fixed investment strategy — 80% in equities — were not inconsistent with the graphs produced by the FABVRG with their fixed strategy — 70% in equities — bearing in mind that the FABVRG graphed 10 sample paths from 20-year ordinary endowments and Ross graphed percentiles from a mix of pensions business. It is hard to make direct comparisons

The comparison of the fixed and dynamic investment strategies was interesting. The upper percentiles (representing the more favourable outcomes) were affected very little; in these circumstances, the dynamic investment strategy would default to 80% equity investment, the same as the fixed strategy. The lower percentiles were generally lifted up to some degree. This was most striking in the case of the A/L ratio, which was to be expected since the level of that ratio was the criterion for changing the asset mix. Bonus rates appeared to be little changed , though the frequency of a bonus rate of nil was actually increased under the dynamic investment strategy.

The significant points brought out by this paper were:

1. This study was of an on-going office which continued to trade, like the model of the FABVRG and unlike the model of the FASWP.

2. The investment strategy was based on the statutory minimum liabilities *including* (roughly) the E.C. solvency margin but *excluding* the U.K. resilience reserve. The need to satisfy the Regulations cannot be ignored so this was a realistic choice.

3. The reversionary bonuses were driven by a *global* terminal bonus target, but constrained by maturities in the near future. This was more realistic than the FASWP in two respects: (i) the terminal bonus target was not zero, and (ii) bonuses were based on a portfolio of in-force policies rather than a single tranche of business. It also went some way to meeting a possible difficulty with the FABVRG's bonus algorithm, which was based on notional values of the assets which might be less than the market values of the assets.

4. Asset and liability cash-flows were valued, where necessary, on a fixed basis, but with a more realistic over-ride in adverse conditions. The fixed values used in normal conditions were the long-term mean yields from the asset model; this perhaps credits the actuary with too much knowledge.

5. Premium rates were dynamic, being altered if they lost too much bonus earning power.

6. The asset shares were reduced by 1.5% at maturity by way of a charge — apparently *ad hoc* — for the guarantees.

7. The model was considerably more complex than any of those used in the four papers previously reviewed, not only in respect of its management algorithms but also in respect of the business modelled. Comparison with other studies is difficult.

8. The results were based on only 100 simulations.

3.7 M. D. Ross & M. R. McWhirter

In 1991, M. D. Ross and M. R. McWhirter investigated the impact of the restrictions on equity yields in the U.K. statutory minimum valuation basis in a paper [58], which was circulated privately. This paper used a model of a with-profits office which was, in many respects, the most sophisticated yet described. First we will describe the features of their model, and then we will summarise their results.

- The office wrote non-pensions with-profit endowment business of terms 5 years to 30 years. The in-force in 1990 was built up using a comprehensive set of assumptions. Rates of return were based on the Barclays de Zoete Wedd indices; historic tax rates were used, and bonus rates and the growth of new business were modelled in detail to represent a typical U.K. experience. The office enjoyed a A/L ratio of 1.81 and a A/AS ratio of 1.20 in 1990, so was in a position of some strength.

- By default, 80% of the fund by market value was invested in equities. For purposes of comparison, a second strategy driven by statutory solvency was used, in which assets were switched from equities to gilts progressively as the ratio of assets to statutory liabilities fell from 1.30 to 1.08; the target A/L ratio was considered to be a linear function $r()$ of the proportion in equities such that $r(0.8) = 1.3$ and $r(1.08) = 0$. The lower limit of 1.08 was used because the E.C. solvency margins and the U.K. resilience reserves were ignored, and because valuations were performed at the year-end *before* new business was added on.

- The assets underlying policy asset shares were valued on a long-term basis similar to that described above in Section 3.6. Also as in Section 3.6, the yields used for discounting future cash-flows in calculating reverionary bonus were based on the assumptions in the asset model, overridden during periods of low returns.

- The two tiers of supercompound bonus were determined separately. The bonus on sums assured was the bonus earning power of the new business, allowing for a terminal bonus target of 30% over the new business as a whole. The bonus on bonus was then the bonus earning power of the whole office, allowing for the declared rate of bonus on sums assured and the 30% terminal bonus target. There were two checks; the second calculation was repeated using only those policies within 5 years of maturity and a zero terminal bonus target, and if the resulting rate of bonus on bonus was lower it was declared instead; and if the rate of bonus on bonus exceeded the rate of bonus on sums assured the bonus was recalculated on an ordinary compound basis. Changes in reversionary bonus from year to year were limited, the first tier to 0.3% and the second tier to 0.5%, though the latter was sometimes adjusted in the light of the change in the first tier. In some projections the terminal bonus target was changed from 30% to 60%.

- Maturity payments were based on 98.5% of the smoothed asset shares (with a minimum of the guarantees). The smoothing was carried out by revaluing equity assets using a weighted 4-year arithmetic moving average dividend yield. The values of gilts underlying the asset shares of maturing policies were not smoothed.

- Gilts were all of term 15 years while the office remained open to new business, reduced to 10 years and then 5 years after the office was closed. Therefore the values of the gilts underlying maturing policies, being unsmoothed, might increase the volatility of maturity payments.

- Premium rates and expenses were based on those used by Ljeskovac *et al* [39] but in the case of premiums with the same override in conditions of low bonus earning power described in Section 3.6.

- Lapse rates varied from 7.5% per annum at durations 1 – 5 years, to 1.5% per annum at durations 21 years and over. Surrender values were 95% of the asset share, increasing to 98.5% over the last third of their term, with minima of 70% and 75% of the premiums paid in years 1 and 2 respectively. Although 95% of the asset shares ought to contribute to ptofit, it is a higher proportion than that apparently used in practice by many U.K. offices.

The authors modified the parameters of the Wilkie model, most significantly to increase the force of inflation from 0.05 to 0.06 and to introduce real dividend growth of 1.5%. The latter increased the expected returns on equities relative to gilts, which some commentators have held to be too small under Wilkie's original parameterization.

The authors' purpose was to show that the U.K. statutory valuation basis acts in a manner detrimental to policyholders, by forcing life offices to switch from equities to gilts at times when the offices might in fact be solvent in cash-flow terms. Their approach was to carry out simulations under the original investment strategy (80% in equities) and to record:

- the times at which statutory insolvency occurred (i.e. the A/L ratio fell below 1.08);
- the maturity values paid each year;
- the residual assets after the office had been closed and the business run off.

The simulations were re-run assuming that the office used the asset-switching strategy dictated by statutory solvency, and the corresponding outcomes were recorded. Only 50 simulations were carried out; 49 were stochastic and 1 was deterministic, in which the variances in the Wilkie model were set to zero.

The main results of the stochastic runs were as follows:

1. Under the fixed investment strategy, 45 of the 49 simulations suffered statutory insolvency during the first 40 years. About 10% of offices were insolvent at any given time. However, only one simulation produced a deficit after 70 years.

2. Under the dynamic investment strategy the number of insolvencies fell to 11. One simulation produced a deficit after 70 years, but not the same simulation as before. The authors concluded:

   > "... this is a powerful argument for acceptance that the present valuation regulations are far too severe in terms of correctly identifying situations in which an office might become "truly" insolvent."

3. The "payout ratio" in each year in each simulation was calculated. This is the ratio of the maturity values under the fixed and dynamic investment strategies respectively; it is a measure of the extent to which the asset switching is beneficial or detrimental to policyholders — at least in retrospect. The distribution of the payout ratio was highly skewed towards values above 1.0. The median payout ratio for all policy terms was above 1.0. For example, the mean of the median payout ratios between 2011 and 2030 (20 and 40 years from outset) was 1.017 at term 10 years, 1.15 at term 20 years, 1.26 at term 30 years and 1.322 at term 40 years. Note that this result might depend significantly upon the assumption of real dividend growth. The authors concluded:

   > "In simple terms, the artificial restriction on equity yields not only seriously damages the office's health but policyholder's expectations and actual results as well."

   The payout ratio tended to revert to 1.0 after closure, because the equity proportion tended to revert to 80% under the dynamic investment strategy, indicating that the office had a considerable excess of assets when it closed.

4. The dynamic investment strategy increased the incidence of negative theoretical terminal bonus rates — that is, calculated maturity values lower than the guarantees. Thus the cash-flow solvency of individual tranches of policies deteriorated, while the statutory solvency of the whole office over the 70 years improved. The authors did not quantify the assets remaining after the run-off, so it is not possible to say to what extent the office's position was maintained on a cash-flow basis.

5. Repeating the experiment with a terminal bonus target of 60% instead of 30% cut the number of insolvencies to 14 under the fixed investment strategy and 1 under the dynamic investment strategy. The incidence of asset switching under the latter was much decreased, and not surprisingly the payout ratios were much closer to unity. However, the higher target resulted in very much lower reversionary bonuses, including 7 simulations in which there was no bonus on sums assured at all at least once in the first 40 years.

This approach applied the cash-flow solvency criterion of the FASWP to a more sophisticated model, but for understandable reasons of space and computer time it left many questions open.

1. The model office was extremely complex, being set up to reflect historical conditions up to 1990. The effects of these conditions would have a major bearing on the outcomes, at least during the first half of the projection. Of the 45 insolvencies resulting from 80% fixed in equities and a 30% terminal bonus target, 2 occurred in the first 5 years , 15 in the second 5 years, and another 13 in the third 5 years. In no case did insolvency occur for the first time after the office had been closed to new business. It is therefore difficult to draw wider conclusions about the effectiveness of the statutory minimum valuation basis.

2. The statutory minimum valuation was compared with cash-flow solvency by allowing "insolvent" offices to continue in operation for 40 years, regardless of whether statutory insolvency occurred sooner or later in the 40 years. Although statutory insolvency appeared to be a poor test of the office's ability to meet its liabilities over this long time scale, it remains possible that that an "insolvent" office would have had more difficulty in meeting its liabilities *had it been closed down at the time of insolvency.* That is, after all, what a traditional valuation attempts to determine. In this connection, the working-through of the 1990 in-force might have led to different results upon earlier closure. We will return to this point in Chapter 8.

3. The authors did not suggest a modfication of the statutory minimum valuation

basis, or an alternative to it, which would remedy the defects which they described. However, the focus of their concern was clearly the restriction on equity yields. They concluded:

> "What remains fundamental, though, is that any valuation test should not act to distort the asset mix of life offices and it is this point that lies at the heart of this paper."

3.8 The Finnish Solvency Working Group

In 1992 The Life Assurance Solvency Working Group ("LASWG") in Finland reported the results of an investigation to the Ministry of Social Affairs and Health [55]. It had the task of

> "... examining the need for the reform of the solvency regulations of life assurance companies and on the basis of the examination draw up proposals for change."

The main features of the investigation were as follows:

1. The asset model was based on a first-order autoregressive model of the rate of inflation but with parameters suited to Finnish conditions and a Gamma-distributed noise term.

   The asset categories modelled were (i) long-term loans and bonds (ii) short-term loans and bonds (iii) common stock and (iv) property. Models for both the income and price changes for these assets were based on the Wilkie model. A new development was the simulation of a *reference index* representing the growth of national product. If the index of share prices moved too far from the reference index then the mean and standard deviation of the annual change in the share price index were adjusted to bring it closer to the reference index. Details were not reported in the English summary, but Pentikäinen *et al* [52] and Pukkila *et al* [53] are useful references.

The published simulations showed the effects of different initial distributions of the assets in these categories. Subsequent asset allocations were governed by an allocation rule which was not described in the English summary.

2. The bonus rule was based on the *margin* or *level of surplus*, i.e. the margin of assets over mathematical reserves (elsewhere called the "additional estate"). Four levels of surplus which acted as trigger points for the bonus rule, $U_0 \leq U_1 \leq U_2 \leq U_3$ were defined. The default for bonus rates was that they should be equal to the rate of inflation, though not less than 0% and not more than 15%. (In Finland, the sum assured *and* the premium *and* the reserve — and therefore also the loading for future expenses — are increased by the same proportion. There is no terminal bonus.)

   - U_0 was the *minimum margin* (the estimation of which was one aim of the investigation). If the actual margin U was below U_0 no bonuses were payable and it could be assumed that the supervisors were taking a close interest.
   - If the actual margin U was between U_0 and U_1 the rate of bonus was less than the rate of inflation.
   - If the actual margin U was between U_1 and U_2 the default bonus rule applied.
   - If the actual margin U was between U_2 and U_3 the rate of bonus was greater than the rate of inflation.
   - If the actual margin U was greater than U_3 a bonus intended quickly to return the margin to the U_2 level was declared.

The application of the bonus rule therefore required (i) the definition and (ii) the estimation of U_0, the minimum margin.

U_{min} was defined to be that amount which, in addition to the policy reserves and allowing for new business, would ensure that assets exceeded liabilities after 3 years with 97% probability. (This choice was influenced by the minimum margin previously adopted for general insurance business, i.e. that assets should exceed liabilities after 1 year with 99% probability.)

U_{min} was estimated as the solution of

$$(1+\alpha)U_{min} = 1.8\left(\sigma_0(V+U_{min})^2 + (\sigma_2)^2\right)^{0.5} - \beta V - Y_r$$

where α represents the expected rate of return over 3 years on the assets held; V is the mathematical reserve at outset; $\sigma_0(V+U_{min})$ is the standard deviation of the change in the office's assets over 3 years; σ_2 is the standard deviation of the non-investment profits over 3 years; β is the expected rate of investment surplus arising in respect of the mathematical reserves over 3 years, and Y_r is the expected non-investment profits arising over 3 years. The fractile value 1.8 was chosen after carrying out simulations with models of different types of life office.

Note that in the estimation of U_{min} it was assumed that *no* future bonuses would be paid — that is, in the 3-year simulations used to determine the fractile there were no bonuses.

U_{min} was generally higher than the E.C. solvency margin if the asset mix tended towards equities and real estate, and lower than the E.C. solvency margin if the asset mix tended towards bonds and loans. Therefore, for the application of the bonus rule described above (using the trigger points $U_0 \leq U_1 \leq U_2 \leq U_3$) U_0 was taken to be the maximum of U_{min} and the E.C. solvency margin. The trigger points U_1, U_2 and U_3 were defined (following some experiments) as follows:

- $U_1 = U_0 + D$
- $U_2 = U_0 + S + 2.7D$
- $U_3 = U_0 + S + 3.5D$

where S is the shareholders' equity and D is the estimated standard deviation of the investment surplus arising over three years given a fixed asset mix. Note that all 4 trigger points must be calculated as functions of current conditions each year.

Finally, the Group carried out simulations to test the effectiveness of the bonus rule in conjunction with U_0 defined as above. The main results were as follows:

1. The bonus rule was successful in steering surplus between the upper and lower limits for different types of office (risk, savings and traditional business). In none of the cases examined was there ever an insolvency.

2. In an office writing savings business, with mainly equity-type assets, surplus tended to increase so that the upper trigger points were predominant in forcing the distribution of surplus. However, in an office writing savings business, with mainly fixed-interest type assets, surplus tended to stay close to the lower limit U_0 and there were many years without bonuses. In both cases the office was assumed to expand fairly rapidly.

3. In a traditional office, invested mainly in equity-type assets, not expanding rapidly, but with moderately high inflation (mean 8% per annum) the surpluses were fairly evenly spread between the lower and upper trigger points U_0 and U_3, but tended to change rapidly. The bonus rule was nevertheless successful in ensuring solvency.

4. In a traditional office, invested mainly in equity-type assets, not expanding rapidly, but with low inflation (mean 4% per annum) and a correspondingly lower mean rate of return, the surplus tended to rise to the upper limit U_3.

The main interest in this work is the definition of a solvency margin and a bonus rule which ensure solvency in a variety of circumstances without making use of terminal bonus. However, "solvency" was still defined in terms of a traditional valuation reserve. Further, the report did not compare payouts with asset shares so it was not possible to tell whether or not the success of the system depended on retention of surpluses which would be regarded as distributable under the U.K. system of asset shares and terminal bonuses.

3.9 Conclusions

The papers reviewed above introduce a number of ideas.

1. Problems of insurance solvency related to asset risk may be modelled empirically, given a stochastic model of the assets (MGWP, FASWP).

2. In some cases, managers have considerable discretion over decision making. Ignoring that discretion in the modelling raises doubts about the results (FASWP).

3. Investment strategies have to be suited to the assets available locally, but might be driven by considerations of portfolio selection (LASWG) or solvency (Ross).

4. Bonus rules have to be suited to local practice, but might be driven by retrospective surplus (FASWP) or prospective solvency (FASWP, Ross, LASWG).

5. Explicit cash-flow projections of *closed* offices provide a standard to which other methods of assessing solvency can be compared, either in whole or in part (Ross, Ross & McWhirter).

6. The models considered include single generations of business (MGWP, FASWP), run-offs of a closed office with a realistic mix of business (MGWP, Ross) and projections allowing for future new business (MGWP, Ross & McWhirter, LASWG).

The results of each investigation differed, being specific to (i) the details of the offices being modelled, and (ii) the asset models used. This points to a possibility which could be of great significance: that it might be difficult to prescribe *detailed and uniform rules* for solvency assessment rather than *general methods*. In the U.K., this means abandoning the ideal of openness to external checks (through the data in Schedule 5 of the D.T.I. Returns), but that ideal is in practice being abandoned anyway. In other territories, it might mean less prescriptive approaches to solvency reporting, and more responsibility for individual actuaries.

The details of each model include the asset allocation and bonus rules, being themselves models of the actions of future managers. Given the discretions described in Chapters 1 and 2, it is hard to see how *uniform* rules will lead to a proper supervision of PRE, as opposed to strict solvency.

This thesis will look at some aspects of managers' discretions, mainly over asset allocation (Chapter 5) and benefit smoothing (Chapter 6). Then in Chapters 7 and 8 we will look at the effectiveness (or otherwise) of prescriptive solvency valuations in the case a particular model office. We will illustrate that in modern circumstances *the imposition of uniform standards based on the traditional model might not be an effective or a consistent measure of solvency.*

Chapter 4

A simple model office

4.1 Introduction

This chapter describes a very simple model office which will be used later to study different asset allocation rules, bonus rules and traditional valuation tests. The main simplification is to restrict the model to 10-year with-profit endowment contracts, for two reasons:

1. Simulation is computationally intensive, and shorter terms help to save computer time.

2. As a rule, short-term business is more sensitive to asset volatility than long term business — at least, given pricing assumptions typical of those in use in the U.K. — and therefore shows more clearly the impact of a stochastic asset model and the effects of asset allocation and bonus rules.

To introduce the model, we will illustrate the sample paths of simulations of various outputs, including measures of solvency, and we will investigate in more detail the circumstances which cause statutory insolvency in the model.

The model of this chapter will be called the "baseline" office in subsequent chapters. Some of the assumptions described below, and projections based upon them, have appeared in [43]. Here they are taken as starting points for the study of alternatives.

4.1.1 Timing of cash-flows in the model

The time unit used for the projection is the calendar year, and all cash-flows, bonus declarations and valuations are assumed to take place at the year end. The order of events at each year-end is:

1. Investment income and changes in the values of assets are calculated.
2. Death claims (if any) — and associated expenses — are paid.
3. Surrender values (if any) — and associated expenses — are paid.
4. Maturity values (if any) — and associated expenses — are paid.
5. Premiums are received in respect of both new and surviving business.
6. Initial and renewal expenses (if any) are paid.
7. The net assets remaining are re-invested according to the asset allocation rule in use.
8. The end-year valuations are carried out (note: the model carries out valuations on more than one basis).
9. Reversionary bonuses are calculated according to the bonus rule in use, and their cost is added to the liabilities.
10. Resilience reserves are calculated.

The last 4 steps sometimes have to be performed iteratively in order to satisfy the asset allocation rule and bonus rule simultaneously.

4.1.2 Valuation of liabilities

The basic liability valuation basis is a simplified version of the U.K. statutory minimum basis in force from 1981 – 1994, as described in Section 2.4 except that:

- The restriction of the yield assumed to be earned on investments made more than 3 years in future is assumed to be 5.5% net of tax instead of 7.2% gross.

- The restriction of the valuation rate of interest to 92.5% of the yield on consols is omitted. Note that the 1994 regulations have removed this restriction

The E.C. solvency margin is not included, but a resilience test is calculated using the original parameters suggested in 1985 by G.A.D.; namely, a fall of 25% in share prices — accompanied by a rise of 33.3% in dividend yields — and a change of ±3% in gross gilt yields (see Section 2.6).

A version of the "W2" modification of the net premium valuation method is used if the calculated valuation rate of interest exceeds 5.5% (see Elliott [20].)

4.1.3 Valuation of assets

Two sets of asset values are calculated each year — market values and actuarial values. Market values are obtained from the asset model. Actuarial values are obtained by discounting future net cashflows arising from each type of asset as follows:

1. At the end of year t the expected net interest and gross redemption proceeds under a fixed-interest security are discounted at rate g_t^* where

$$(1+g_t^*)^5 = \prod_{s=0}^{4}(1+g_{t-s})$$

   and g_i is the *net* redemption yield on gilts at the end of year i. In other words, a 5-year geometric moving average of $(1+g_t)$ is used to value future fixed interest cash-flows.

2. In respect of an equity share, a stream of *net* dividends equal to the *current* net dividend is discounted at rate d_t^* where

$$(1+d_t^*)^5 = \prod_{s=0}^{4}(1+d_{t-s})$$

   and d_i is the *net* running dividend yield at the end of year i. In other words, a 5-year geometric moving average of $(1+d_t)$ is used to value future (assumed level) dividend income.

Associated with the actuarial valuation of equities is an implicit market rate of interest. This is needed where, for example, an office is invested 100% in equities yet has to project liability cash-flows at a "realistic" rate of interest. Where such a need arises the implicit market rate of interest is taken to be g_t^*.

4.1.4 Asset allocation rule

By default, the office invests 100% of its funds in equities at the end of every year. However, if the resilience test cannot then be satisfied, the office switches out of equities and into gilts, to raise the permitted valuation interest rate and to reduce the impact of the 25% fall in share prices assumed in the resilience test. Assets are switched just up to the point at which the mismatching test can be satisfied, if that is possible. If subsequent conditions permit, the office will return to 100% equity investment.

Gilts are chosen to have terms to redemption equal to the outstanding term of the policies which they represent. That is, the gilt component of each policy's asset share is invested in gilts of the same remaining term as the policy. That part of any assets in excess of total asset shares which is to be invested in gilts is invested in gilts of various terms in the same proportions as the gilts representing the total asset shares. This crude matching means that maturity values are not affected by changes in the capital value of fixed-interest assets, which is relevant in Chapter 6 where we consider maturity value smoothing.

4.1.5 Reversionary bonus rule

The reversionary bonus rule is based upon a terminal bonus target of 25% of the guaranteed benefits at maturity. First, the reversionary bonus supported by each in-force policy is calculated, as follows:

- The assets underlying the asset share of each in-force policy are valued actuarially as described in Section 4.1.3 above.
- The resulting actuarial values and future premiums are accumulated to the maturity date at rate g_t^*, giving a projected asset share at maturity.

- The supercompound bonus which would result in a terminal bonus of 25% of the guaranteed maturity benefits is found. The rate of bonus on bonus is a fixed proportion — 5/3 — of the rate of bonus on sums assured. If the benefits already guaranteed exceed 80% of the projected guaranteed benefits at maturity the supported bonus is taken to be nil.

The bonus actually declared is a weighted mean of the bonus rates so calculated for individual policies, the weights being the projected asset shares at maturity.

The changes in the bonus rates from year to year are limited to reflect the usual practice of changing bonuses gradually; bonus rates may not increase in one year by a proportion greater than 25%, nor decrease by a proportion greater than 20% (chosen so that the maximum increase followed by the maximum decrease cancel out).

4.1.6 Terminal bonus rule

The benefit at maturity is 100% of the asset share — meaning the market value of the assets underlying the asset share — subject to a minimum of the sum assured plus reversionary bonuses on the maturity date.

4.1.7 Premium rate

The premium rate is fixed — the basis used is 5% interest and a 2.5% compound bonus loading. There are no loadings for expenses or mortality. The resulting premium rate is £96.926 per £1,000 of sum assured.

4.1.8 Tax

Interest paid on fixed-interest securities, and dividends paid on equities, are subject to tax at 25%. There is no tax on capital gains.

4.1.9 Expenses, lapses and mortality

Expenses, lapses and mortality are ignored, in order to focus on the effect of the asset model. Other authors, such as Ross [57] and Ross & McWhirter [58] have included realistic expenses in their models, and there would be no difficulty in including them here. However, we have chosen to exclude all influences except those of the asset model and the related management strategies.

4.1.10 Generation of financial scenarios

Financial scenarios are generated using the Reduced Standard version of the Wilkie asset model. This was described in Section 3.2.

4.1.11 Starting point for the projections

The model builds up an in-force over 40 years by generating a single financial scenario using the Wilkie model described above but with the variances of the white noise inputs $QZ(t), YZ(t), DZ(t)$ and $CZ(t)$ set to zero. New business is assumed to expand at the rate of Retail Price Inflation; i.e. there is zero real growth in new business. The in-force at the end of 40 years is then stable in the sense that assets and liabilities are growing alike at the rate of 5.127% per annum (the mean rate of inflation) and are identical each year apart from this scale factor.

4.2 Key financial ratios

Throughout this thesis, certain key financial ratios will be used as indicators of the office's financial health, or directly in asset allocation and bonus algorithms. The three principal ratios are defined below. In all three, "A" denotes the office's total assets at market value.

4.2.1 The ratio A/L_1

Let L_1 denote the office's aggregate policy reserves on the basis described in Section 4.1.2 *excluding* the resilience reserve (if any). Then the ratio A/L_1 measures the statutory solvency or otherwise of the office, according as it is greater than or less than 1. Notice that the E.C. solvency margin is ignored.

At the start of the projections described here, the ratio A/L_1 is 1.3844.

4.2.2 The ratio A/L_2

Let L_2 denote the sum of L_1 *including* any additional reserve which is needed to meet the resilience test. Then the ratio A/L_2 measures the ability of the office to withstand the changes in conditions embodied in the resilience test, and remain statutorily solvent, *given* the assets it currently holds. The ratio A/L_2 is used in the asset allocation rule.

At the start of the projections described here, the ratio A/L_2 is 1.0657. Therefore the resilience test is satisfied with the initial asset allocation of 100% in equities.

4.2.3 The ratio A/AS

Let AS denote the office's aggregate asset shares — that is, the asset shares underlying the office's in-force policies. By default, this is assumed to be the same as the aggregate market value of the assets underlying the asset shares of the in-force policies. The ratio A/AS measures the extent to which the office possesses assets other than those attributable to its current in-force business, which is not to say that the latter will necessarily be returned in whole to the relevant policies upon exit.

The quantity $A - AS$ is called the *additional estate* by some authors and the *orphan surplus* by others.

In a with-profits office which uses asset shares in the calculation of terminal bonus, or in which asset shares influence the idea of Policyholders' Reasonable Expectations, the ratio A/AS is particularly important. It is of less importance if the benefits are largely guaranteed in nature.

The office is given additional assets at the start of the projections so that the ratio A/AS is 1.20.

4.3 Some results using the basic model

In this section we look at some of the key features of 1,000 simulations using the baseline model described above. These are not surprising; their purpose is to allow comparisons to be drawn in later chapters, when the effects of varying the conditions or the algorithms are studied.

The projections here are over 30 years, from $t = 40$ to $t = 70$. This is quite long, considering the 10-year term of the policies, but it allows the evolution of the quantities studied to be seen, and in particular whether their distributions become stable or increasingly dispersed.

It is important to look at *both* the evolution over time of the distributions *and* the sample paths of any quantity studied. The sample paths, in particular, might suggest ways in which an algorithm could be improved, by comparison with the way in which the given quantity has been allowed to vary in practice in the past.

4.3.1 Financial conditions

The financial conditions are those generated by the Wilkie asset model with the "Reduced standard" parameters. Figures 4.1 to 4.4 show, for each of

1. The annual rate of Retail Price Inflation.
2. The net redemption yield on gilts.
3. The net dividend yield on equities.
4. The total net rate of return on equities.

over a 30 year period,

- 10 sample paths, shown as broken lines.
- The 5th, 25th, 50th, 75th and 95th quantiles, shown as solid lines.
- The deterministic "skeleton", i.e. the result of making a projection with all the stochastic inputs — the white noise terms in the asset model — given zero variance. It is shown as a line with diamond markers.

In each case here, the deterministic "skeleton" is practically co-incident with the median.

Figure 4.1 shows that there are periods of negative inflation, and that at any time about 5% of simulations show a rate of inflation of about -5% or less. This is

Figure 4.1: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the rate of Retail Price Inflation (% per annum)

not inconsistent with the data used to fit the model — Retail Price inflation in the U.K. was negative during 13 of the 73 years since 1920 — but the last such occasion was in 1942 and some authors have argued that an AR(1) model is not appropriate, see for example Kitts [36]. Others have simply truncated the values produced by the Wilkie model at some arbitrary lower bound, see for example Hardy [29].

The gilt yields shown in Figure 4.2 behave in a different manner from the dividend yields shown in Figure 4.3 and the net rates of return on equities shown in Figure 4.4. Whereas the two latter quantities almost immediately assume a stable distribution, (in the same manner as the rate of inflation), **the distribution of the gilt yield spreads out gradually**. This will have an effect on simulations carried out over longer or shorter periods; simulations over longer periods will encounter more diverse fixed-interest environments, which will (for example) make it more likely that a fixed premium or valuation basis becomes anachronistic.

Another difference between gilt and equity returns lies in the behaviour of the sample paths. Those of the gilt yield are relatively smooth, in the sense that there is no strong mean-reverting behaviour. A sample path at one extreme of the distribution is quite likely to drift along at that level for several years. The net rate of return on equities, on the other hand, shows a strong tendency to jump back quickly from very high or low values. This opens up the possibility of asset-switching algorithms

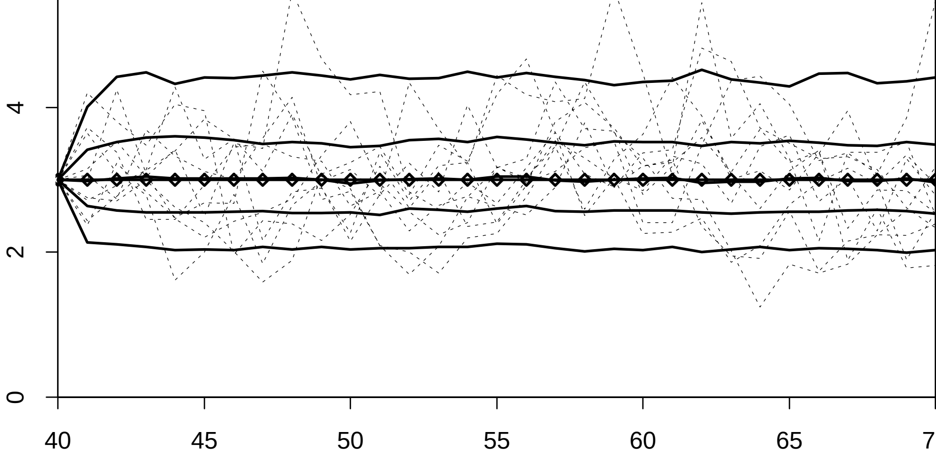


Figure 4.2: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the net redemption yield on gilts (% per annum)

Figure 4.3: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the net dividend yield (% per annum)

Figure 4.4: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the net rate of return on equities (% per annum)

which respond to high or low past rates of return on equities. Some of these will be considered in Chapter 5.

Although the dividend yield shown here is confined to a relatively narrow range of values, the dividend yield has an *inverse* effect on equity prices, so the impact of the annual changes shown in these sample paths should not be underestimated.

4.3.2 Asset allocation

Given the office's additional estate, the resilience test is satisfied at outset with the assets invested entirely in equities. Thereafter at some time, the asset allocation algorithm might have to switch part of the holding into gilts.

Figure 4.5 shows the distribution of the percentage of the fund invested in equities. The median falls from 100% to just under 90%, so only the 5th, 25th and 50th quantiles are shown.

Notice particularly that Figure 4.5 shows considerable volatility within individual simulations — indeed more volatility than would be seen in practice since in several cases the assets are switched from 100% equities to 100% gilts or *vice versa* in a single year. The effect of more realistic limits on the speed with which switching can be carried out will be considered in Chapter 5.

Figure 4.5: 5th, 25th, 50th quantiles, and 10 sample paths, of the percentage of the fund invested in equities

The presence of an additional estate has a significant impact on the asset allocation.

4.3.3 Bonuses

Figure 4.6 shows the distribution of the rate of reversionary bonus on sums assured.

The percentage limits on changes in reversionary bonus (+25% and -20%) are frequently enforced, so that the sample paths often progress geometrically. The general level of bonus rates is low, compared with recent experience, but to increase it would require rates of return higher than those assumed, or higher premium rates, or a lower terminal bonus target. Recall that the rate of bonus on sums assured is higher, by a factor of 5/3, though for a term of only 10 years this is not too important.

On the other hand, the 95th quantile, and the sample paths, show that rates of bonus which are at least as high as any seen historically can be attained, and even higher quantiles (not shown) reveal that bonus rates can greatly exceed any yet seen in practice.

Figure 4.7 shows the distribution of the rate of terminal bonus on maturing policies, expressed as a percentage of sums assured and reversionary bonus.

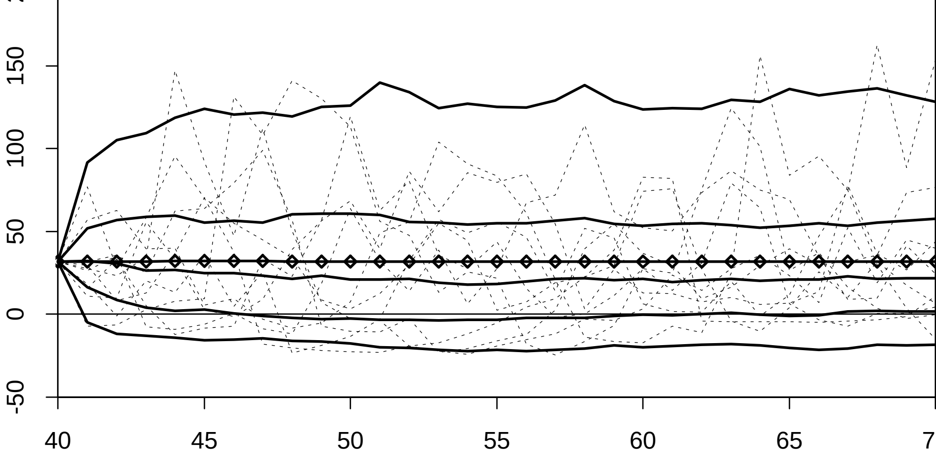


Figure 4.6: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the rate of reversionary bonus on sums assured (%)

Figure 4.7: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the rate of terminal bonus (% of sums assured + reversionary bonus)

The figure shows “negative” terminal bonuses. This is merely to indicate the magnitude of the losses in any years when the guarantees exceed the available asset shares. In this case the deficit, expressed as a percentage of the sum assured and reversionary bonuses, is treated as a “theoretical” terminal bonus, but the rate of terminal bonus actually declared is nil. It is interesting to consider the accumulated amount of these losses, and ways of defraying that cost. In Chapter 6 we will return to this question.

There is considerable volatility in terminal bonus. No attempt has been made at this stage to smooth the emerging benefits or terminal bonus rates, but some of the changes shown here are larger than past experience suggests would be acceptable. This too will be considered in Chapter 6.

The terminal bonus emerging under deterministic conditions is consistently higher than the target of 25%. This is because the reversionary bonuses are based on projections of asset shares and future premiums at the net gilt redemption yield, whereas the fund is invested in equities and earns a higher net rate of return. The median rate of terminal bonus is closer to 25% because the fund is more often invested in gilts in the stochastic projections.

The effect of the reversionary and terminal bonuses combined on maturity values is shown in Figure 4.8. This shows, not the maturity values per policy, (because the average premium is inflation-linked) but the maturity values per unit premium. This means that the figures in different years are directly comparable, either in the absence of expenses as here, or if the expenses are linked to the same inflation index as the average premium.

Apart from scale, this bears a strong resemblance to Figure 4.7. Terminal bonus plays the greater part in the volatility of the overall results.

4.3.4 Financial ratios

Figure 4.9 shows the distribution of the ratio A/L_1, measuring statutory solvency.

At any time, a small number of offices have a ratio $A/L_1 < 1$, and are therefore statutorily insolvent. Generally, recovery follows after a short period. However, the small proportion of offices for which $A/L_1 < 1$ at *a single time* hides the fact that

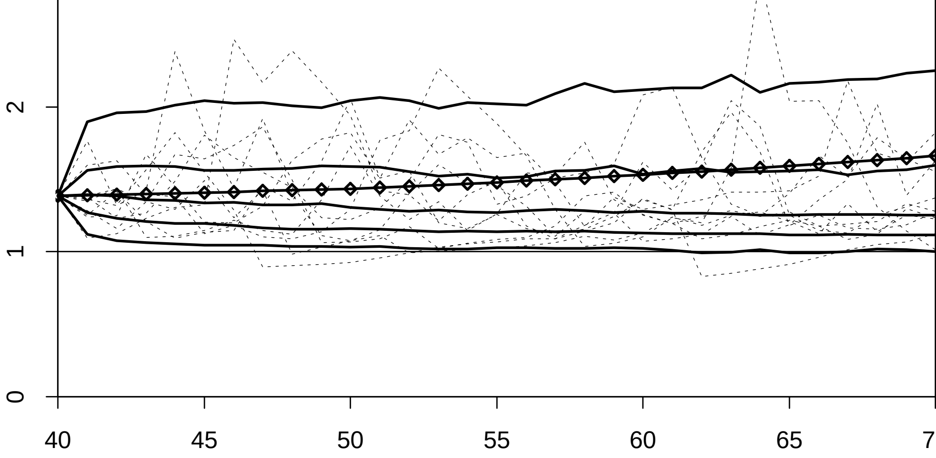


Figure 4.8: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the maturity value per unit of annual premium

Figure 4.9: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the ratio A/L_1

Figure 4.10: Cumulative proportion of simulations (of 1,000) during which the ratio A/L_1 has *ever* fallen below 1.1 (top), 1.05 (middle) or 1.0 (bottom)

different offices are statutorily insolvent at different times. Therefore, the proportion of offices which are *ever* **insolvent is much higher**. Figure 4.10 shows the *cumulative* proportion, over time, of simulations in which the ratio A/L_1 has fallen below 1.1, 1.05 and 1.0 at least once.

Of the 1,000 simulations, 276 result in statutory insolvency at some time. Moreover, 540 result in a ratio A/L_1 of 1.05 or below at some time, which is close to the level at which the E.C. solvency margins could not be covered (recall that the ratio A/L_1 excludes the E.C. solvency margin from the denominator).

Figure 4.11 shows the distribution of the ratio A/AS. The two influences on this ratio are:

1. The cost of meeting the guarantees, when guaranteed maturity benefits exceed asset shares at maturity.

2. The difference between the rate of new business growth and the rate of return earned on the additional estate, which together decide the *relative* magnitude of the additional estate, and hence the ratio A/AS. This will be touched upon in Chapter 6.

In this case, the mean rate of new business growth is below the mean rate of return on the assets, so the ratio A/AS ought to increase on the whole. Figure 4.11

Figure 4.11: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the ratio A/AS

shows that the deterministic “skeleton” rises as expected, but the median in fact *falls* (from 1.20 to about 1.09). This is an example, not uncommon, of determinism leading to a conclusion which fails to capture some important characteristic.

The scale of Figure 4.11 is perhaps not ideal for revealing the detail of the sample paths of the ratio A/AS, but it is the same scale as that of Figure 4.9, and it shows interesting differences between the behaviour of the two ratios. The ratio A/L_1 is much more volatile; indeed the ratio A/AS has extremely smooth sample paths. This difference has a bearing on their usefulness as criteria of financial strength, which will be mentioned in Chapter 7.

The ratio A/AS will also be affected by any smoothing of maturity values, or any attempt to charge for guarantees.

4.4 Discussion

4.4.1 Questions concerning strategies

The results presented here raise a number of questions, some of which will be explored in this and subsequent chapters.

1. The number of statutory insolvencies is relatively high, despite the use of

an asset allocation algorithm which is driven by statutory solvency. In part this might be due to the use of 10-year policies, but Ross & McWhirter [58] obtained comparable rates of statutory insolvency using a model with policies of terms 5 years to 40 years. Therefore we might ask what gives rise to so many insolvencies, and whether the asset allocation strategy is "shutting the door after the horse has bolted". This question will be addressed later in this chapter.

2. The solvency-driven asset allocation algorithm is needed because the office invests in equities; the office invests in equities because this gives the best long-term results for policyholders; is this reasoning sound? It is a strongly held view, especially in the U.K., but it rests partly on the assumption of an inflationary economy. Part of the high returns associated with equities — or more accurately, the inclusion of very high values in the distribution of equity returns — can be ascribed to the tendency of equity dividends to move in line with price indices over the long term. See, for example, Pentikäinen *et al* [52]. The Wilkie model, although it allows for a moderately inflationary economy in the long run, also allows for considerable periods of low or negative inflation. Under these circumstances, is there any reason to assume that equities will produce better maturity values than gilts?

   It is worth investigating the *real* maturity values, allowing for the loss (or gain!) in purchasing power during the policy term. This will be done in Chapter 5.

3. The assumption of a fixed proportion in equities, possibly subject to solvency-led constraints, is not the only plausible assumption. There might be switching opportunities within the Wilkie model. This is also investigated in Chapter 5.

4. The solvency-driven asset allocation strategy used here, which is similar to that of Ross [57], results in some reduction in the incidence of statutory insolvency (looking ahead to Chapter 5), but at the expense of very large asset switches. Assuming that asset switching is subject to more realistic market constraints, is the strategy still effective? This is investigated briefly in Chapter 5.

5. The emerging reversionary bonus rates often appear "unrealistically" low or

high, even although the changes in reversionary bonus rates are constrained. Since bonus declarations add to the guarantees, the nature of these constraints could have a considerable effect on solvency. The effect could be either way, since there might be as much reluctance to reduce bonus rates, for commercial reasons, as to increase them. Different reversionary bonus strategies and constraints are considered briefly in Chapter 5, and will also be touched upon in Chapter 6.

6. Maturity values, and terminal bonus rates, appear more volatile than past history would suggest they should be. This arises from the simple treatment of with-profits business in the model, as unit-linked business with a managed fund and a guarantee. A more realistic treatment would include the smoothing of maturity values. This is the topic of Chapter 6.

In most of these cases it is necessary to guess what actions managers might take if faced with conditions quite unlike any experienced in the past. For example, an unprecedented bull market in equities might result in (i) historically high reversionary bonus rates, or (ii) historically high terminal bonus rates, or (iii) historically high A/L ratios, or (iv) all of these. On the other hand, adverse conditions create different problems with which managements have not been faced before. It is perhaps not reasonable to reject an algorithm because it leads to an unprecedented result in unprecedented conditions. Rather such a result suggests strengths or weaknesses of the algorithms considered.

4.4.2 Questions concerning solvency

Wider questions which we will consider concern the nature of solvency, and the effectiveness of a traditional solvency valuation. These are the topics of Chapter 7 and Chapter 8. The subject area has already been introduced in Chapter 2 and Chapter 3.

1. How closely does “insolvency” under any traditional solvency valuation correspond to a deficit in the run-off?

2. What is the role of each part of the system in determining statutory solvency — valuation regulations, E.C. solvency margins, resilience test?

3. What would be the effect of different valuation regimes, such as those typical of other E.C. territories or North America?

In the remainder of this chapter we use the simple model described above to examine, as far as they can be discerned, some factors which might be associated with statutory insolvency. It is important to emphasise the strong dependence of these and subsequent explorations on the assumptions used in the model, and the structure and parameters of the underlying Wilkie asset model.

4.5 Statutory insolvency in the model

4.5.1 The general pattern of statutory insolvency

The first question to ask is whether statutory insolvency is associated with particular patterns of financial conditions or bonus declarations.

The 1,000 simulations described above included 276 in which statutory solvency ever occurred — that is, in which the ratio A/L_1 fell below 1.0 at *some* time. Figure 4.12 shows the ratio A/L_1 (sorted in ascending order) upon the *first* such failure in each of these 276 simulations.

There are comparatively few "catastrophic" failures, and many more "moderate" failures. In about half of the cases the ratio A/L_1 on first insolvency lies between 0.95 and 1.0.

Figure 4.13 shows the distribution of times at which statutory insolvency first occurs, for the 276 cases in which it does occur. There is a slight downward trend. However, if we plot the *rate* at which statutory insolvencies first occur

$$\frac{\text{No. of new insolvencies at time } t}{\text{No. of offices never insolvent before time } t}$$

(an empirical estimate of the "mortality rate" of these offices) which is shown in Figure 4.14, there is no obvious trend in the rate at which still-solvent offices fail.

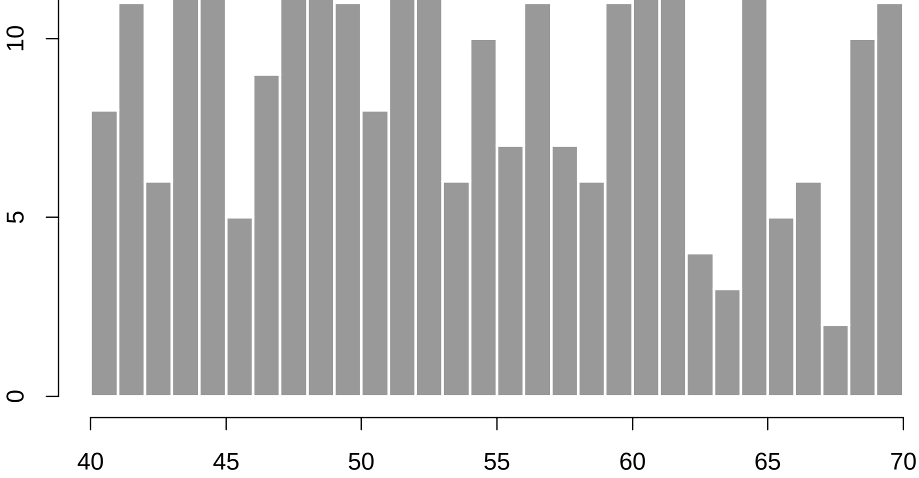


Figure 4.12: Ratio A/L_1 of the 276 statutory insolvencies on first failure (sorted)

Figure 4.13: Distribution of times at which statutory insolvency first occurs

Figure 4.14: Rate of occurrence of *new* statutory insolvencies

4.5.2 Conditions leading to statutory insolvency

Hardy [29] carried out 1,000 simulations of a model office and studied those 100 cases in which the ratio A/L_1 attained its lowest values (about 1.04 or below). An interesting feature of these simulations was a tendency to exhibit low inflation in the earlier years of the projection period. Hardy said

> "Despite the fact that expenses are linked to inflation, and that after the 5th year there is no new business, it is low inflation, not high inflation, that causes the greater insolvency risk, because of the association between inflation and equity returns."

How direct is the association between low inflation and statutory insolvency in our model? There is not likely to be low inflation throughout the 30-year projection period in any one simulation — this would be surprising given that an AR(1) process is used to model inflation — so there is little point in looking at distributions like those in Figure 4.1. Of more potential interest is the distribution of the rate of RPI *at the time when insolvency occurs*.

Figure 4.15 shows the "distribution" of the rate of Retail Price Inflation in all 276 simulations in which statutory insolvency occurred, during the 10 years before and after statutory insolvency first occurred. The timescale runs from -10 years (i.e.

Figure 4.15: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the rate of Retail Price Inflation (in 10 years around $A/L_1 < 1$)

10 years before insolvency) to +10 years (i.e. 10 years after insolvency). The word "distribution" above is placed in inverted commas because by comparing sample paths at different times, we are no longer looking at sample paths drawn from a common distribution, but simply looking for similarities between certain parts of different sample paths.

Here we are mainly interested in the conditions which *precede* statutory insolvency, but in Chapter 7 we will also be interested in the conditions *following* insolvency, so these figures show both.

Note that not all of these "sample paths" are complete, since some insolvencies occur in the first and last 10 years of the projection period. The "quantiles" are based on the number of simulations actually included at each time before or after insolvency.

There is evidence in the quantile plots of

1. a downward drift of inflation in the years preceding insolvency, and
2. a rise in inflation back to "normal" levels at the time of insolvency.

Some caution is needed in assessing these "features" — they are clear from the quantile plots, but not at all clear from the sample paths. (Only 10 paths are shown

Figure 4.16: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the ratio A/L_1 (in 10 years around RPI <-10%)

here, but it is still true of larger samples.) **Not all sample paths of inflation drift down and then rise just before insolvency; some do the opposite. Low inflation** *in terms of the distribution* **cannot be sufficient** *causal* **explanation**.

Figure 4.16 looks at this from another angle. Instead of showing the RPI in the years around a fall in the ratio A/L_1 to 1.0 or below, it shows the ratio A/L_1 in the 10 years before and after the rate of RPI falls below -10% for the first time. Inflation as low as -10% occurs in 122 of the 1,000 simulations.

There is some evidence of a reduction in the ratio A/L_1 following such severe deflation. However, only in 50 out of the 122 simulations in which the RPI reached -10% did insolvency also occur.

4.5.3 The mechanism of failure

The previous section described some evidence that low inflation influences insolvency. However this does not explain why insolvencies occur at some times and not at others. An unconvincing aspect of the "evidence" is the inconsistency of the sample paths of the rate of RPI. Individual failures are not caused by median rates of RPI. In looking for a *mechanism* of failures, the evidence of quantiles or means is insufficient — we are looking for some property of the sample paths.

Figure 4.17: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the net dividend yield (in 10 years around $A/L_1 < 1$)

Figure 4.17 shows the “distribution” of the net dividend yields, and Figure 4.18 shows the “distribution” of the net rate of return on equities, in the 10 years before and after the first occurrence of statutory insolvency.

These figures suggest a link between insolvencies and stock market crashes, which are themselves associated with increases in the dividend yield. Moreover, this is a feature displayed by the sample paths as well as the quantiles.

Figure 4.18 suggests that the net rate of return on equities is slightly low in the year preceding insolvency, but not in any earlier years. It is possible that low inflation plays a part in this. However, since we know how the Wilkie model works, we can calculate the relative effect of inflation on the factors leading to equity price falls.

The Wilkie model generates an index of dividends $D(t)$ and a dividend yield $Y(t)$. Therefore the *change* in equity prices is given by

$$\frac{D(t)}{D(t-1)} \frac{Y(t-1)}{Y(t)}$$

Inflation has only a mild effect on the index of dividends. Recall that the index of dividends is given by

Figure 4.18: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the net rate of return on equities (in 10 years around $A/L_1 < 1$)

$$\log\left(\frac{D(t)}{D(t-1)}\right) = 0.8DM(t) + 0.2\log\left(\frac{Q(t)}{Q(t-1)}\right) - 0.0525YZ(t-1) + 0.1DZ(t)$$

where

$$DM(t) = 0.2\log\left(\frac{Q(t)}{Q(t-1)}\right) + 0.8DM(t-1)$$

so that

$$\left(\frac{D(t)}{D(t-1)}\right) \propto \left(\frac{Q(t)}{Q(t-1)}\right)^{0.36}$$

That is, the *change* in dividends is in proportion to a *small* power of $\frac{Q(t)}{Q(t-1)}$, if we ignore the exponentially lagged influence of previous inflation in $DM(t)$. On the other hand

$$\left(\frac{D(t)}{D(t-1)}\right) \propto e^{0.1DZ(t)}$$

and $\{DZ(t)\}$ is a sequence of *i.i.d.* Normal(0,1) random variables. Figure 4.19 shows the relative influences of the inflation component and the white noise component on the change in the dividend index, based on a single simulation over 100 years. The solid line shows

$$e^{0.1DZ(t)} = 1.105^{DZ(t)}$$

Figure 4.19: Effect of white noise terms (solid line) and inflation terms (dotted line) on $D(t)/D(t-1)$ during 100 years

and the dotted line shows

$$\left(\frac{Q(t)}{Q(t-1)}\right)^{0.36}$$

Now recall that the dividend yield $Y(t)$ is given by

$$\log(Y(t)) = 1.35 \log\left(\frac{Q(t)}{Q(t-1)}\right) + YN(t)$$

where

$$YN(t) = \log 0.04 + 0.6\{YN(t-1) - \log 0.04\} + 0.175YZ(t)$$

so that

$$Y(t) \propto \left(\frac{Q(t)}{Q(t-1)}\right)^{1.35}$$

and

$$\left(\frac{Y(t)}{Y(t-1)}\right) \propto \left(\frac{Q(t)Q(t-2)}{Q(t-1)^2}\right)^{1.35}$$

But also

$$\left(\frac{Y(t)}{Y(t-1)}\right) \propto e^{0.175(YZ(t)-YZ(t-1))} = 1.19^{YZ(t)-YZ(t-1)}$$

Where the $\{YZ(t)\}$ are *i.i.d.* Normal(0,1) random variables.

Figure 4.20: Effect of white noise terms (solid line) and inflation terms (dotted line) on $Y(t)/Y(t-1)$ during 100 years

Figure 4.20 shows the relative influences of the inflation component and the white noise component on the change in dividend yields, based on a single simulation over 100 years. The solid line shows

$$1.19^{YZ(t)-YZ(t-1)}$$

and the dotted line shows

$$\left(\frac{Q(t)Q(t-2)}{Q(t-1)^2}\right)^{1.35}$$

The white noise terms have a greater effect than inflation on both components of the change in equity prices. This might answer the question: why is low inflation not a strong feature *of the sample paths* in the "low inflation" insolvencies?

Figure 4.21 shows the proportion of the funds invested in equities in the 10 years before and after statutory insolvency. (The bottom quantile shown before insolvency is, in fact, the 5th quantile — this might not be obvious in the Figure.) Therefore 75% of offices are invested 80% or more in equities, and almost 50% are invested 100% in equities, *at the time of insolvency*. This is higher than the average for all offices (see Figure 4.5), again consistent with a pattern of insolvencies linked to falls in equity prices.

Figure 4.21: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the % of the fund in equities (in 10 years around $A/L_1 < 1$)

For completeness, Figure 4.22 shows the "distribution" of the net gilt redemption yield in the 10 years before and after statutory insolvency. The evidence of low, or even unusually distributed, gilt yields is slight.

4.5.4 More experiments with inflation

To test further the influence of inflation on statutory insolvency, we can try the simple experiment of using constant inflation in the model. That is, the 1,000 simulations were re-run with the standard deviation QSD set to 0 in the Wilkie inflation model. The rate of inflation was therefore a constant 5.127%, with the other white noise components DSD, YSD and CSD as before.

The number of statutory insolvencies fell from 276 to 184. There were 130 scenarios in which statutory insolvency occurred in both cases. There were therefore 146 scenarios in which statutory insolvency occurred with $QSD = 0.05$, but not with $QSD = 0$. This is not evidence for the influence of *low* inflation, merely evidence that the variability induced by QSD makes insolvency more likely.

For convenience, refer to the 130 scenarios in which insolvency still occurred with $QSD = 0$ as "Group A" scenarios, and the other 146 in which insolvency only occurred when $QSD = 0.05$ as "Group B" scenarios.

Figure 4.22: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of net gilt redemption yield (in 10 years around $A/L_1 < 1$)

Consider Group A. Changing the rate of inflation will change the other elements of the asset model, and it is not obvious that the sample paths of the yields and prices should be at all similar. In particular, it is not obvious that insolvency, though it occurs in both cases, should occur at the same times in each of the 130 scenarios, and from the same causes. Figure 4.23 shows the *difference* of the times at which statutory insolvency first occurs in each scenarios, in other words

$$\text{Time of insolvency with } QSD = 0.05 - \text{Time of insolvency with } QSD = 0$$

All but a few insolvencies do in fact occur at the same times with $QSD = 0.05$ and with $QSD = 0$, suggesting that *low* inflation up to and including the time of insolvency does not play too important a part in these cases — nearly half of all the insolvencies.

Still considering the Group A simulations Figure 4.24 shows the "distribution" of the rate of RPI in the 10 years around statutory insolvency for these 130 simulations. Note carefully that these simulations are made with $QSD = 0.05$, but they are the 130 cases in which insolvency still occurred with $QSD = 0$.

This shows little evidence of low inflation before insolvency. However, Figure 4.25 shows the "distribution" of the rate of RPI in the 10 years around statutory insolvency for the 146 Group B simulations, in which insolvency did not occur with

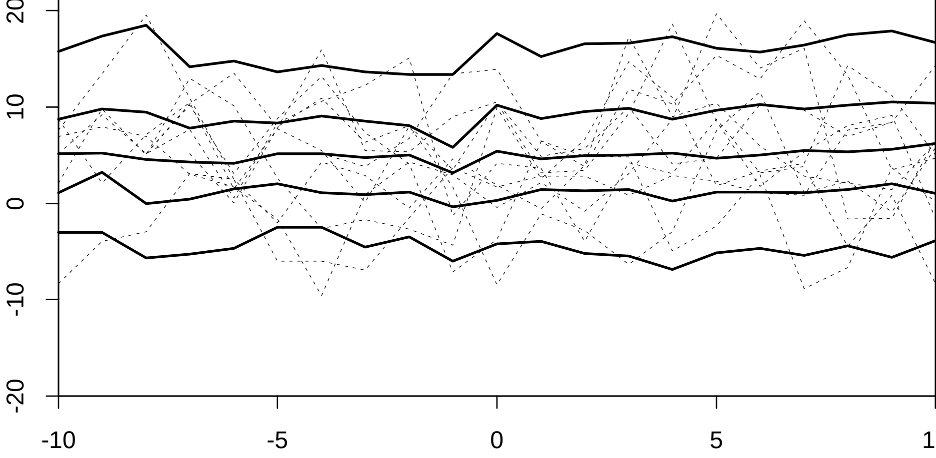


Figure 4.23: Time of insolvency with $QSD = 0.05$ *minus* time of insolvency with $QSD = 0$

Figure 4.24: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of rate of RPI (in 10 years around $A/L_1 < 1$) in those scenarios insolvent if $QSD = 0$

Figure 4.25: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of rate of RPI (in 10 years around $A/L_1 < 1$) solvent if $QSD = 0$

$QSD = 0$ but did occur with $QSD = 0.05$.

There is a more marked tendency for inflation to be low before insolvency occurs — indeed more marked even than in Figure 4.15. Once again, the rise in inflation at the time of insolvency is a feature.

Re-running the 1,000 simulations with $QSD = 0.05$, but eliminating the the white noise component of the dividend yield by setting $YSD = 0$ had the following result: the number of insolvencies fell from 276 to 11, and of these 11, 6 were also among the original 276 and 5 were not.

Re-running the 1,000 simulations with the white noise of the dividend index DSD set to zero resulted in 175 insolvencies, 113 of which were also among the original 276 simulations.

Re-running the 1,000 simulations with the white noise component of the consols yield CSD set to zero had a negligible effect, owing to (i) the predominance of equity investment, and (ii) the absence of statutory insolvencies at times when there is any degree of investment in gilts. This last point will become clearer in Chapter 5; note however that it is not to be confused with the fact that there must be 100% investment in gilts just *after* insolvency has occurred.

4.5.5 Conclusions

1. Statutory insolvency, in this model, is strongly associated with catastrophic falls in share prices. This association is a property of the sample paths.

2. Falls in share prices are influenced largely by the white noise components of the dividend yield model and dividend index models. Inflation is a lesser influence.

3. Low inflation is associated with some but not all statutory insolvencies.

4. Offices which suffer statutory insolvency tend to be slightly more heavily invested in equities than average when the insolvency occurs.

Chapter 5

The impact of different strategies

5.1 Introduction

The managers of a with-profit life office can attempt, within the constraints imposed by regulations, to control the financial position of that office by their decisions in respect of

1. asset allocation,
2. the granting of guarantees (premium setting and/or bonuses)
3. the smoothing of benefits

Each of the factors above may be both an input and an output at the same time. Management might have a preferred strategy for each, in which sense they are inputs. The interactions between them (and the regulations) might lead to departures from the preferred strategy, in which sense they are outputs.

This chapter examines the influence of the office's asset allocation and reversionary bonus strategies on statutory solvency and policyholder's benefits. The effect of a given strategy may be gauged by comparing the distributions of suitable outputs with some "baseline" set of projections, such as those illustrated in Chapter 4.

5.2 Summary statistics

While graphs help us to understand the general behaviour of the sample paths in a stochastic model office, they are less suitable for comparing large sets of alternative projections. As with single-figure mortality indices, it is useful to have some summary statistics.

We may wish to compare:

1. variability within individual scenarios, and
2. variability between different scenarios.

Given a quantity X_t^i defined at each time $t = 1, \ldots, n$ within each scenario $i = 1, \ldots, m$, define the following (sample) means:

$$\begin{aligned}
E_t(X_t^i) &= \frac{1}{n}\sum_{t=1}^{t=n} X_t^i \\
E^i(X_t^i) &= \frac{1}{m}\sum_{i=1}^{i=m} X_t^i \\
E_t^i(X_t^i) &= E^i(E_t(X_t^i)) \\
&= E_t(E^i(X_t^i))
\end{aligned}$$

Within each scenario the simplest measures of the behaviour of X_t^i are its maximum, $\max_t(X_t^i)$, minimum, $\min_t(X_t^i)$, mean, $E_t(X_t^i)$ and standard deviation, denoted $S_t(X_t^i)$ where the subscript t indicates that the extrema or moments are calculated with respect to time.

Then some suitable statistics are these:

Distribution of $E_t(X_t^i)$. We summarise the distribution of $E_t(X_t^i)$ by its mean and standard deviation over the 1,000 scenarios, denoted by $E_t^i(X_t^i)$ and $S^i(E_t(X_t^i))$ respectively.

Mean of $S_t(X_t^i)$. The quantity $S^i(E_t(X_t^i))$ sums up the range of variation *between* different scenarios. If we reverse the order of the operations, and denote by $E^i(S_t(X_t^i))$ the mean across all 1,000 scenarios of $S_t(X_t^i)$, this will measure variability *within* scenarios.

Distribution of $\max_t(X_t^i)$**.** The mean and standard deviation of $\max_t(X_t^i)$ will be denoted $E^i(\max_t(X_t^i))$ and $S^i(\max_t(X_t^i))$. A large standard deviation indicates a wide range of extremes within the 1,000 scenarios, which in turn indicates that problems, if any, will lie with a small number of "exceptional" scenarios which might repay study. A low value, on the other hand, suggests more homogeneous behaviour of the sample paths.

Distribution of $\min_t(X_t^i)$**.** The mean and standard deviation of $\min_t(X_t^i)$ will be denoted $E^i(\min_t(X_t^i))$ and $S^i(\min_t(X_t^i))$.

Correlations. Coefficients of correlation will be introduced as required, and will be denoted by $C(,)$. Note that we can either (i) calculate the correlation between two quantities X_t^i and Y_t^i within each simulation, denoted $C_t(X_t^i, Y_t^i)$, and then calculate its moments across all the scenarios, $E^i(C_t(X_t^i, Y_t^i))$ etc., or (ii) calculate moments or other statistics within each scenario, for example $\min_t(X_t^i)$ and $E_t(Y_t^i)$, and then compute the correlation $C^i(\min_t(X_t^i), E_t(Y_t^i))$. There is an example of this in Table 5.10, where X is the ratio A/L_1 and Y is the maturity value per unit premium.

5.3 Fixed asset allocation strategies

The baseline model of Chapter 4 uses a dynamic asset allocation strategy driven by statutory solvency, similar to those of Ross [57] and Ross & McWhirter [58]. These authors compared the outcomes with fixed asset allocation strategies (80% in equities) and drew unfavourable conclusions about the effects on payouts to policyholders of the dynamic strategy.

In this section we compare different fixed asset allocation strategies with the solvency-driven dynamic asset allocation strategy. Table 5.9 defines 11 fixed strategies, ranging from 100% in equities to 0% in equities. In each case the balance of the fund is invested in gilts.

Strategy	% of Assets Invested in Equities
AA No.1	100%
AA No.2	90%
AA No.3	80%
AA No.4	70%
AA No.5	60%
AA No.6	50%
AA No.7	40%
AA No.8	30%
AA No.9	20%
AA No.10	10%
AA No.11	0%

Table 5.9: Definition of Asset Allocation Strategies AA No.1 – AA No.11

5.3.1 The trade-off between solvency and high returns

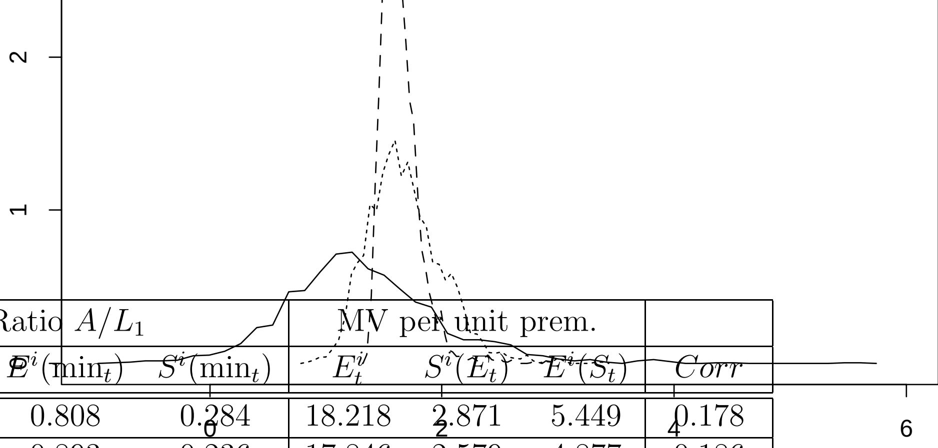
It is normally assumed that investment in equities brings the prospect of high returns (represented here by the maturity values) but at the expense of both greater variance of the returns and a greater risk of insolvency. Evidence for this has been found in simulation studies (see, for example, Forfar *et. al.* [22]). Investment in gilts, on the other hand, is held to be safer but with reduced prospects.

Table 5.10 shows, for the 11 strategies defined above, the numbers of statutory insolvencies, and some summary statistics in respect of the ratio A/L_1 and the maturity value (MV) per unit premium. The last column headed "Corr" requires explanation. It shows the correlation between $\min_t(A/L_1)$ and $E_t(MV_t^i)$; that is, both of these are calculated in respect of each scenario, and the correlation is calculated from the resulting 1,000 pairs.

The table shows the following features as the proportion invested in equities falls:

1. **The number of statutory insolvencies falls steeply**, and the distribution of the minimum A/L_1 ratio attained in each scenario becomes much more concentrated. Figure 5.26 shows estimates of the densities of the ratio A/L_1 at time $t = 70$ with 100%, 50% and 0% in equities, and Figure 5.27 shows how the distribution of the ratio A/L_1 evolves over time with 100% in equities.

2. **Equity investment results in a distribution of maturity values with**



	Ratio A/L_1			MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E_t^i	$S^i(E_t)$	$E^i(S_t)$	$Corr$
AA No.1	745	0.808	0.284	18.218	2.871	5.449	0.178
AA No.2	624	0.893	0.236	17.846	2.579	4.877	0.186
AA No.3	485	0.976	0.194	17.467	2.299	4.327	0.197
AA No.4	321	1.054	0.159	17.091	2.037	3.791	0.187
AA No.5	156	1.128	0.127	16.718	1.794	3.271	0.172
AA No.6	42	1.198	0.100	16.356	1.571	2.761	0.142
AA No.7	2	1.265	0.074	16.005	1.368	2.263	0.093
AA No.8	0	1.328	0.055	15.668	1.187	1.779	0.026
AA No.9	0	1.387	0.040	15.344	1.033	1.330	-0.063
AA No.10	0	1.442	0.031	15.020	0.916	0.977	-0.193
AA No.11	0	1.478	0.029	14.694	0.846	0.845	-0.217

Table 5.10: Comparison of ratio A/L_1 with Maturity Values, strategies AA No.1 – AA No.11 (fixed investment strategies).

Figure 5.26: Estimates of the density of the ratio A/L_1 at time $t = 70$ with 100%, 50% and 0% equity investment.

Figure 5.27: Estimates of the density of the ratio A/L_1 at times $t = 50$, $t = 60$ and $t = 70$ with 100% equity investment.

a much higher mean and standard deviation, and which is highly skewed. Changing from 100% gilts to 100% equities increases the mean maturity values by a factor of 1.24. But it also increases the standard deviation, by a factor of 3.38. This might appear unacceptably high, accompanied as it is by 745 statutory insolvencies. Figure 5.28 shows estimates of the densities of the MV per unit premium at time $t = 70$ with 100%, 50% and 0% in equities. Figure 5.29 shows how little change there is over time in the distribution of the MV per unit premium, in this case with 100% in equities.

3. Given any level of equity investment at all, the variation within scenarios is very high. Changing from 100% gilt asset allocation to 100% equity asset allocation increases $E^i(S_t(MV_t^i))$ by a factor of 6.45 (where MV_t^i denotes the maturity value per unit premium).

4. The correlations of mean maturity value per unit premium with the lowest attained A/L_1 ratio are not large but do suggest a pattern. Positive correlation suggests an association between high maturity values and high minima of the A/L_1 ratio — clearly more desirable than the opposite association. Although not large, the correlation is positive with high equity asset allocation, and is highest not at 100% in equities but at 80% in equities.

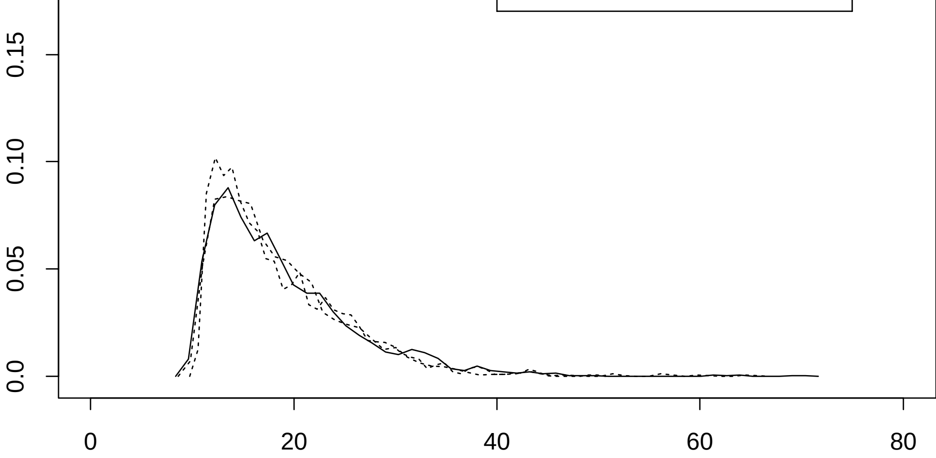


Figure 5.28: Estimates of the density of the MV per unit premium at time $t = 70$ with 100%, 50% and 0% equity investment.

Figure 5.29: Estimates of the density of the MV per unit premium at times $t = 50$, $t = 60$ and $t = 70$ with 100% equity investment.

Figure 5.30: Boxplot of ratio A/L_1 at time $t = 70$ under asset allocation strategies AA No.1 – AA No.11.

A convenient tool for comparing several distributions is the *boxplot*. For example, Figure 5.30 shows box-plots of the ratio A/L_1 at time $t = 70$.

The components of a box plot are (i) the box, (ii) the whiskers and (iii) the outliers.

1. The shaded box indicates the *quartiles* of the distribution. It is bounded above and below by the upper and lower quartiles respectively.

2. The white line in the box shows the location of the *median*.

3. The dotted lines extending above and below the box indicate, on each side, $1.5 \times$ the inter-quartile range. If, however, the furthest observation on one or other side lies closer to the nearest quartile than $1.5 \times$ the inter-quartile range, the dotted line and its terminating *whisker* will be truncated accordingly.

4. The horizontal lines beyond the whiskers indicate observations beyond the range of the whiskers. If they are clustered thickly on one side but not the other (as in this figure) this indicates skewness.

Using a box plot it is relatively easy to compare by eye the distributions under different assumptions. The disadvantage is that when comparing time-series data under different assumptions, the box plot can only show a “snapshot” at a single

Figure 5.31: Boxplot of the MV per unit premium at time $t = 70$ under asset allocation strategies AA No.1 – AA No.11.

time. In terms of the summary statistics above, it illustrates the "between scenario" variability at a given time, but not how it evolves over time.

Note that the vertical scales in this (and all other) box plots shown here are chosen to be the same as the scales used for quantile plots such as those in Chapter 4. This means that extreme outliers will sometimes be omitted from the plot. On the other hand, choosing a scale which includes *all* outliers would usually compress all but the extreme quantiles into an indistinguishable band. Using a uniform scale has the advantage that comparisons can be made by eye, and this is worth the loss of a few outliers.

Figure 5.31 shows a boxplot of the MVs per unit premium at time $t = 70$ under the strategies AA No.1 – AA No.11. Both the increased variance and skewness associated with equity investment are evident. Note also that there is much less difference between the medians and the lower quantiles.

5.3.2 The trade-off between solvency and real returns

Since rates of return in the Wilkie asset model are positively correlated with rates of inflation, high maturity values will to some extent reflect high inflation, and low maturity values, low inflation. We might obtain a more consistent comparison of

	Ratio A/L_1			Real MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
AA No.1	745	0.808	0.284	11.032	1.732	3.399	−0.109
AA No.2	624	0.893	0.236	10.840	1.645	3.167	−0.073
AA No.3	485	0.976	0.194	10.641	1.570	2.951	−0.034
AA No.4	321	1.054	0.159	10.443	1.507	2.754	0.000
AA No.5	156	1.128	0.127	10.247	1.458	2.577	0.033
AA No.6	42	1.198	0.100	10.057	1.420	2.423	0.077
AA No.7	2	1.265	0.074	9.875	1.397	2.297	0.137
AA No.8	0	1.328	0.055	9.703	1.391	2.209	0.207
AA No.9	0	1.387	0.040	9.540	1.402	2.165	0.297
AA No.10	0	1.442	0.031	9.379	1.430	2.171	0.417
AA No.11	0	1.478	0.029	9.214	1.468	2.220	0.503

Table 5.11: Comparison of ratio A/L_1 with real Maturity Values, strategies AA No.1 – AA No.11 (fixed investment strategies).

maturity values in real terms than in nominal terms.

Let RV^i_t be the real maturity value per unit premium at time t in the i^{th} scenario, defined as

$$RV^i_t = \frac{MV^i_t}{Q^i_t/Q^i_{t-10}}$$

where Q^i_t is the Retail Price Index at time t in the i^{th} scenario. Table 5.11 shows the same summary statistics as Table 5.10 in respect of RV^i_t.

There are interesting differences between Tables 5.10 and 5.11.

1. Equity investment still yields higher maturity values; changing from 100% in gilts to 100% in equities increases $E^i_t(RV^i_t)$ by a factor of 1.20, which is close to the increase of 1.24 in nominal terms.

2. **The distributions of real maturity values are quite similar, regardless of the level of equity investment**. Figure 5.32 shows estimates of the densities of the real MV per unit premium at time $t = 70$ with 100%, 50% and 0% in equities. All are highly skew.

3. **There is much less variability between scenarios**; changing from 100% in gilts to 100% in equities increases $S^i(E_t(RV^i_t))$ by a factor of only 1.18, instead of 3.38 in nominal terms.

Figure 5.32: Estimates of the density of the real MV per unit premium at time $t = 70$ with 100%, 50% and 0% equity investment.

4. $S^i(E_t(RV_t^i))$ does not vary much below about 60% equity investment (AA No.6). The minimum is at about 30% equity asset allocation (AA No.8). There is not much reason to prefer any level of equity investment between 0% and 60%, in terms of mean real maturity values.

5. The dependence of the "within scenarios" variability on the level of equity investment is much reduced; moving from 100% in gilts to 100% in equities increases $E^i(S_t(RV_t^i))$ by a factor of 1.53, compared with an increase of 6.54 in $E^i(S_t(MV_t^i))$.

6. $E^i(S_t(RV_t^i)) > E^i(S_t(MV_t^i))$ under high levels of gilt investment, despite the fact that $E_t^i(RV_t^i) < E_t^i(MV_t^i)$. This indicates that **the variability within scenarios is greater in real terms than in nominal terms**.

7. The correlations between $\min_t(A/L_1)$ and $E_t(RV_t^i)$ display a contrary trend to those between $\min_t(A/L_1)$ and $E_t(MV_t^i)$. In other words the association between higher real maturity values and higher minima of the ratio A/L_1 is strongest under investment in gilts.

Figure 5.33 shows a boxplot of the real MVs per unit premium at time $t = 70$ under strategies AA No.1 – AA No.11.

Figure 5.33: Boxplot of the real MV per unit premium at time $t = 70$ under asset allocation strategies AA No.1 – AA No.11.

The distributions of the maturity values are now all very similar, indeed more alike than the statistics in Table 5.11 might suggest. However, recall from Figures 4.2 – 4.4 that the distribution of returns on gilts disperses much more gradually than do the distributions of (i) the rate of inflation, and (ii) returns on equities, so we should expect there to be greater differences in real maturity values earlier in the term.

Figure 5.34 shows a boxplot of the real MVs per unit premium at time $t = 50$ under strategies AA No.1 – AA No.11. There is much more difference in the variability than in Figure 5.33. Since the summary statistics $E^i(S_t(RV^i_t))$ and $S^i(E_t(RV^i_t))$ take the whole projection period into account, we can see that the higher variability of the high-equity strategies which remains in the real MVs per unit premium is mainly a consequence of the earlier years of the projection.

Since the difference between nominal and real maturity values depends on the responsiveness of equity returns to inflation *versus* the "locking-in" to fixed interest yields, it is of interest to try a different level of inflation. Tables 5.12 and 5.13 show the effects on the nominal and real MV per unit premium of changing the parameter QMU in the asset model from 0.05 to 0.10 (i.e. doubling the mean force of inflation). Only 100%, 50% and 0% equity investment is shown.

Again, the range of mean maturity values is not changed much in real terms; it

Figure 5.34: Boxplot of the real MV per unit premium at time $t = 50$ under asset allocation strategies AA No.1 – AA No.11.

	Ratio A/L_1			MV per unit prem.			
Equity %	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
100%	677	0.895	0.197	22.819	4.111	8.203	0.132
50%	47	1.182	0.100	19.121	2.033	3.956	0.085
0%	8	1.340	0.090	15.987	1.091	1.53	0.283

Table 5.12: Comparison of ratio A/L_1 with Maturity Values, 100%, 50% and 0% in equities, $QMU = 0.1$ in the asset model.

	Ratio A/L_1			Real MV per unit prem.			
Equity %	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
100%	677	0.895	0.197	9.209	1.420	2.949	0.102
50%	47	1.182	0.100	7.954	1.098	2.057	0.275
0%	8	1.340	0.090	6.880	1.107	2.012	0.248

Table 5.13: Comparison of ratio A/L_1 with real Maturity Values, 100%, 50% and 0% in equities, $QMU = 0.1$ in the asset model.

falls from 1.42 to 1.33. But the variability, between and within scenarios, lies within a much smaller range in real terms. In much more inflationary conditions, therefore, the same conclusion is reached.

These results suggest that one part of the supposed penalty for investing in equities — more variable maturity values — is overstated if we look at nominal and not real maturity values. That does not affect the other part of the penalty — the very much higher incidence of insolvency. The question of whether or not that too is overstated will be discussed in Chapter 7.

5.4 Declining EBR strategies

A method often suggested of obtaining exposure to equities while protecting maturity guarantees is to employ a *declining Equity Backing Ratio* (EBR) (see, for example, Kennedy *et al* [35] and Smaller [62]). This means that the assets underlying a policy's asset share are progressively switched out of equities and into gilts as maturity approaches. Under such a strategy, the overall split between gilts and equities is not static, since it depends on the composition of the business, but the strategy itself is static.

Following Smaller [62], Forfar *et al* [22] investigated a wide range of declining EBR strategies, using a 20-year term. They said

> "It will be seen that the effect of the falling EBR has been to reduce the fluctuations of the asset share $AS(20)$, as measured by the standard deviation, but the smaller standard deviation has been achieved at the expense of a smaller mean return."

Six simple declining EBR strategies are considered here. They are described in Table 5.14. Tables 5.15 and 5.16 show the summary statistics in respect of MV_t^i and RV_t^i respectively.

These lead to very similar conclusions to those of Section 5.3. The mean maturity values, comparing one strategy with another, are in a similar relationship in both tables, but the large differences in variability as between one strategy and another shown in Table 5.15 are almost eliminated in Table 5.16.

Strategy	% of Assets Invested in Equities
AA No.12	100% until duration 5 years, 75% thereafter
AA No.13	100% until duration 5 years, 50% thereafter
AA No.14	100% until duration 5 years, 25% thereafter
AA No.15	100% until duration 5 years, 0% thereafter
AA No.16	100% until duration 5 years, falling linearly to 0% at maturity
AA No.17	100% at inception, falling linearly to 0% at maturity

Table 5.14: Definition of Asset Allocation Strategies AA No.12 – AA No.17

	Ratio A/L_1			MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
AA No.12	534	0.954	0.202	17.522	2.398	4.263	0.170
AA No.13	275	1.071	0.151	16.861	1.967	3.288	0.104
AA No.14	103	1.159	0.126	16.260	1.589	2.678	−0.024
AA No.15	39	1.212	0.119	15.738	1.295	2.611	−0.169
AA No.16	392	1.017	0.165	16.986	2.117	3.517	0.083
AA No.17	21	1.217	0.094	16.005	1.454	2.149	0.047

Table 5.15: Comparison of ratio A/L_1 with Maturity Values, strategies AA No.12 – AA No.17 (declining EBRs).

	Ratio A/L_1			Real MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
AA No.12	534	0.954	0.202	10.670	1.588	2.918	−0.047
AA No.13	275	1.071	0.151	10.330	1.487	2.597	0.004
AA No.14	103	1.159	0.126	10.033	1.440	2.507	0.019
AA No.15	39	1.212	0.119	9.789	1.465	2.673	−0.028
AA No.16	392	1.017	0.165	10.395	1.527	2.682	−0.064
AA No.17	21	1.217	0.094	9.885	1.406	2.311	0.097

Table 5.16: Comparison of ratio A/L_1 with Real Maturity Values, strategies AA No.12 – AA No.17 (declining EBRs).

Comparing the declining EBR strategies with the previous fixed strategies, the former behave broadly as we would expect given the mean proportions invested in equities during each policy's term. The numbers of statutory insolvencies are not noticeably lower.

5.5 Solvency-driven asset switching

5.5.1 Switching out of fixed and EBR strategies

The baseline model office of Chapter 4 included an asset-switching algorithm based upon statutory solvency. In this section we compare the results of this strategy with the fixed strategies of Section 5.3.

The fixed investment strategies were denoted AA No.1 (100% in equities) to AA No.11 (0% in equities). We will indicate the operation of the dynamic asset switching algorithm by an asterisk; for example strategy AA No.3* invests 80% of the funds in equities by default, switching into gilts as required to keep the ratio $A/L_2 \geq 1.0$. Strategy AA No.1* is the same as that described in Chapter 4. Table 5.17 shows summary statistics in respect of strategies AA No.1* – AA No.17*.

Comparing Table 5.17 with Table 5.10, we see that, with 50% or more in equities by default:

1. **The dynamic investment strategies reduce the numbers of statutory insolvencies considerably**. The distribution of $min_t(A/L_1)$ is much higher and much less dispersed. Figure 5.35 shows estimates of the density of the ratio A/L_1 at time $t = 70$ under Strategies AA No.1 and AA No.1*. There is a concentration just above $A/L_1 = 1.0$ (indicated by the vertical broken line) which is where the incidence of gilt investment is greatest.

2. The mean MV per unit premium ($E_t^i(MV_t^i)$) is almost the same regardless of the particular strategy, and there is no less variability between scenarios ($S^i(E_t(MV_t^i))$). In fact this statistic is slightly higher in 6 cases. Figure 5.36 shows estimates of the density of the MV per unit premium at time $t = 70$ under Strategies AA No.1 and AA No.1*.

	Ratio A/L_1			MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
Fixed defaults							
AA No.1*	276	1.040	0.095	16.896	2.812	4.779	0.358
AA No.2*	216	1.059	0.091	16.916	2.608	4.480	0.363
AA No.3*	143	1.084	0.088	16.876	2.352	4.123	0.361
AA No.4*	81	1.114	0.089	16.786	2.081	3.716	0.315
AA No.5*	35	1.152	0.092	16.596	1.823	3.253	0.261
AA No.6*	12	1.204	0.089	16.323	1.581	2.762	0.188
AA No.7*	2	1.266	0.074	16.002	1.370	2.264	0.101
AA No.8*	0	1.328	0.055	15.668	1.187	1.779	0.026
AA No.9*	0	1.387	0.040	15.344	1.033	1.330	−0.063
AA No.10*	0	1.442	0.031	15.020	0.916	0.977	−0.193
AA No.11*	0	1.478	0.029	14.694	0.846	0.845	−0.217
Declining EBR defaults							
AA No.12*	187	1.071	0.089	16.865	2.450	4.045	0.350
AA No.13*	96	1.114	0.094	16.654	1.983	3.251	0.237
AA No.14*	42	1.170	0.103	16.221	1.580	2.671	0.044
AA No.15*	28	1.215	0.110	15.727	1.288	2.610	−0.139
AA No.16*	176	1.080	0.091	16.656	2.132	3.449	0.210
AA No.17*	6	1.219	0.090	15.994	1.455	2.152	0.074

Table 5.17: Comparison of ratio A/L_1 with Maturity Values, strategies AA No.1* – AA No.17* (solvency-driven investment strategies).

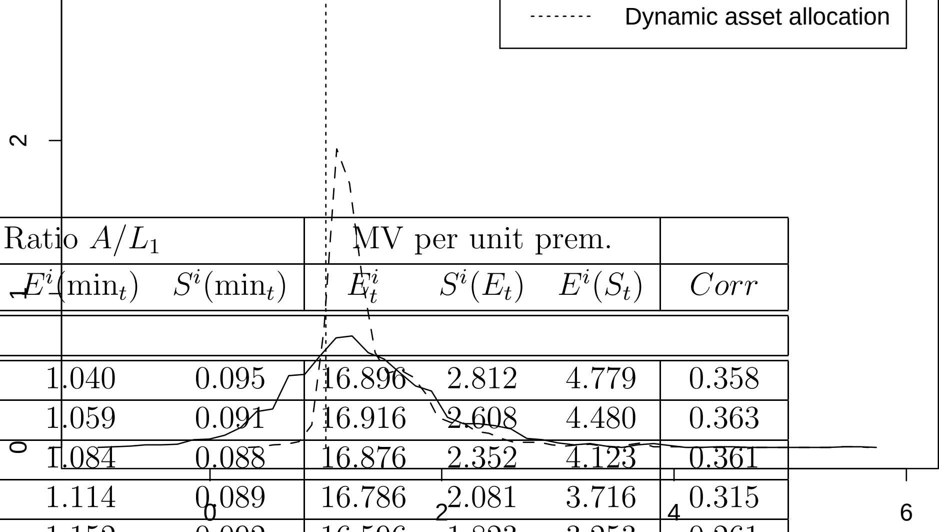


Figure 5.35: Estimates of the density of the ratio A/L_1 at time $t = 70$ with 100% equity investment and fixed or dynamic investment strategies.

Figure 5.36: Estimates of the density of the MV per unit premium at time $t = 70$ with 100% equity investment and fixed or dynamic investment strategies.

3. Variability within scenarios is lower $(E^i(S_t(MV_t^i)))$. This is what we would expect, since there is a lower level of investment in equities in some scenarios, which should therefore show less volatile maturity values.

4. The correlations of mean maturity value per unit premium with the lowest attained A/L_1 ratio are larger; there is a slightly stronger association between high maturity values and a high minimum A/L_1 ratio.

Below about 50% in equities, the dynamic strategy makes almost no difference. Figure 5.37 shows a boxplot of the ratio A/L_1 at time $t = 70$ under strategies AA No.1* – AA No.11*.

This reveals an interesting pattern; the median ratio rises to a peak as the equity proportion falls from 100% to about 40% – 50%, and then falls back as the equity proportion is further reduced. Since there are no insolvencies anyway with less than 50% in equities, these results offer no strong reasons to prefer a lower proportion in equities.

If we ask what fixed investment strategy will give a similar number of statutory insolvencies as AA No.1*, we see by inspection of Table 5.10 that the answer lies between 60% and 70% in equities. Comparing statistics at these levels, it is interesting to note that the fixed investment strategy still has a more volatile minimum

Figure 5.37: Boxplot of ratio A/L_1 at time $t = 70$ under asset allocation strategies AA No.1* – AA No.11*.

A/L_1 ratio, but that the dynamic strategy has greater variability of payouts both within and across scenarios. Similar observations may be made in respect of the other dynamic strategies AA No.2* – AA No.5*. This suggests that, for a given level of statutory solvency, a fixed investment strategy with a suitably chosen equity proportion might be preferred to a dynamic strategy with a higher default equity proportion. At this point we will not ask whether or not statutory solvency is a sensible criterion for driving decision making (other than in the obvious pragmatic sense).

The declining EBR strategies show comparable reductions in numbers of insolvencies and in maturity values.

Table 5.18 shows summary statistics of the real MV per unit premium in respect of strategies AA No.1* – AA No.17*.

5.5.2 Alternative switching thresholds

The asset switching algorithm, by switching out of equities when the ratio A/L_2 would otherwise drop below 1.0, is assuming that the parameters used in the resilience test give an appropriate level of security. The numbers of insolvencies, however, remain high under high levels of equity investment, so it is apparent that

	Ratio A/L_1			Real MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
Fixed defaults							
AA No.1*	276	1.040	0.095	10.264	1.716	3.071	0.143
AA No.2*	216	1.059	0.091	10.294	1.660	2.965	0.152
AA No.3*	143	1.084	0.088	10.289	1.583	2.837	0.175
AA No.4*	81	1.114	0.089	10.258	1.511	2.702	0.153
AA No.5*	35	1.152	0.092	10.171	1.453	2.556	0.133
AA No.6*	12	1.204	0.089	10.036	1.421	2.417	0.113
AA No.7*	2	1.266	0.074	9.873	1.397	2.297	0.140
AA No.8*	0	1.328	0.055	9.703	1.391	2.209	0.207
AA No.9*	0	1.387	0.040	9.540	1.402	2.165	0.297
AA No.10*	0	1.442	0.031	9.379	1.430	2.171	0.417
AA No.11*	0	1.478	0.029	9.214	1.468	2.220	0.503
Declining EBR defaults							
AA No.12*	187	1.071	0.089	10.283	1.610	2.805	0.168
AA No.13*	96	1.114	0.094	10.208	1.486	2.578	0.127
AA No.14*	42	1.170	0.103	10.010	1.441	2.502	0.068
AA No.15*	28	1.215	0.110	9.783	1.464	2.669	−0.014
AA No.16*	176	1.080	0.091	10.204	1.531	2.657	0.088
AA No.17*	6	1.219	0.090	9.878	1.407	2.310	0.113

Table 5.18: Comparison of ratio A/L_1 with real Maturity Values, strategies AA No.1* – AA No.17* (solvency-driven investment strategies).

Strategy	Ratio	Switching threshold
AA No.18	A/L_2	1.05
AA No.19	A/L_2	1.10
AA No.20	A/L_1	1.10
AA No.21	A/L_1	1.20

Table 5.19: Definition of Asset Allocation Strategies AA No.18 – AA No.21 (alternative switching criteria.

	Ratio A/L_1			MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
100% in equities by default							
AA No.18	162	1.079	0.089	16.679	2.743	4.647	0.329
AA No.19	89	1.120	0.086	16.471	2.655	4.488	0.297
AA No.20	513	0.986	0.128	17.022	2.924	4.884	0.418
AA No.21	293	1.048	0.128	16.601	2.866	4.668	0.368
80% in equities by default							
AA No.18	64	1.115	0.077	16.704	2.326	4.055	0.359
AA No.19	33	1.151	0.070	16.509	2.261	3.962	0.329
AA No.20	260	1.058	0.113	16.913	2.405	4.151	0.416
AA No.21	132	1.110	0.098	16.575	2.394	4.029	0.456

Table 5.20: Comparison of ratio A/L_1 with Maturity Values, strategies AA No.18 – AA No.21 (alternative solvency-driven investment strategies).

the switching criterion is not as successful as it might be.

Table 5.19 defines alternative switching strategies, based on different switching thresholds and/or different financial ratios.

Strategies AA No.18 and AA No.19 both use the same ratio, A/L_2, as the previous strategy, but begin to switch into gilts at a higher level, thus being more conservative. We will call the level at which switching takes place the "switching threshold".

Strategies AA No.20 and AA No.21 use the ratio A/L_1 (based on the statutory minimum reserve ignoring the resilience reserve). Since this is a weaker test than one using the ratio A/L_2, higher switching thresholds are used.

Table 5.20 shows summary statistics of the nominal maturity values under these switching strategies, with default investment strategies of 100% and 80% in equities.

The table shows that if the ratio A/L_1 is used as a guide to asset switching it is necessary to set a very high switching threshold; a threshold of 1.20 gives results

	Ratio A/L_1			Real MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i_t	$S^i(E_t)$	$E^i(S_t)$	$Corr$
100% in equities by default							
AA No.18	162	1.079	0.089	10.139	1.697	3.007	0.111
AA No.19	89	1.120	0.086	10.020	1.667	2.929	0.073
AA No.20	513	0.986	0.128	10.365	1.859	3.176	0.115
AA No.21	293	1.048	0.128	10.118	1.832	3.076	0.048
80% in equities by default							
AA No.18	64	1.115	0.077	10.189	1.586	2.796	0.162
AA No.19	33	1.151	0.070	10.074	1.569	2.743	0.141
AA No.20	260	1.058	0.113	10.324	1.647	2.880	0.106
AA No.21	132	1.110	0.098	10.130	1.672	2.825	0.112

Table 5.21: Comparison of ratio A/L_1 with real Maturity Values, strategies AA No.18 – AA No.21 (alternative solvency-driven investment strategies).

very similar to the use of the ratio A/L_2 with a threshold of 1.00.

Comparing Tables 5.20 and 5.21 with Tables 5.10 and 5.11, it still appears that the asset switching strategies have little advantage over a fixed strategy with a suitably chosen level of equity investment. For example, compare strategy AA No.18 above ($A/L_2 = 1.05$) with 100% in equities by default with strategy AA No.5, which has a fixed 60% in equities. AA No.18 results in slightly more insolvencies (162) than does AA No.5 (156), and the mean maturity values are similar, but the variability of the maturity values (nominal or real) within and across scenarios is about the same as that of strategy AA No.2, with a fixed 90% in equities. In other words, for a given level of statutory insolvency the fixed investment strategies give similar mean payouts with lower variability.

5.5.3 Limits on asset switching

The asset-switching algorithm permits switches of any size, so that the entire equity holding may be liquidated in a single year. In practice this would be infeasible for a large or medium sized life office. Even if its managers decided to run down its equity holdings they would have to do so gradually; some equity-type assets are inherently illiquid (such as properties) while disposal of large quantities of widely-traded stocks would move the price adversely.

It follows that an asset-switching algorithm intended to ensure solvency might

	Solvency-driven asset switching		
Default rule	None	Unlimited	25% limit
100% equities	745	276	497
80% equities	624	216	355
70% equities	485	143	236
60% equities	321	81	121
50% equities	156	35	43

Table 5.22: Comparison of the numbers of statutory insolvencies with and without an asset switching limit of 25% of the fund per year.

be too optimistic if it allowed switches of arbitrary size to take place; the protection of the solvency position might be a good deal less if there were more realistic limits on asset switches.

To investigate this, strategies AA No.1* – AA No.5* have been modified so that no more than 25% of the fund can be switched from equities to gilts or vice versa in any one year. The limit is based on the value of the assets at the end of the year, assuming that revenue cash-flows have been invested during the year in proportion to the market value of the different asset types in the fund at market value. With 50% or less of the fund in equities to start with the extent of asset switching is much reduced and we have omitted these.

Table 5.22 compares the number of statutory insolvencies with no switching (from Table 5.10), with unrestricted switching (from Table 5.17) and with switching restricted as above.

Comparing the second and third columns, the numbers of statutory insolvencies are increased, by rather more when there is a high level of equity investment. Thus much of the protection offered by the asset-switching is lost. This suggests that **the utility of solvency driven asset-switching might be considerably lower than suggested by (for example) Ross or Ross & McWhirter**. (In fairness it should be noted that these authors were not advocates of solvency-driven asset switching.)

5.6 Index-driven asset switching

Although the maintenance of solvency might occasionally impinge on asset allocation, investment policy is more usually driven by the managers' views on likely stock market behaviour. Markets are not static, and managers' views will change in line with experience, and asset allocations with them. Static strategies such as holding fixed proportions of the fund in equities, or declining EBR strategies, represent extremely long-term views. Even managers who agreed with these views might depart from the resulting allocations over short or medium terms as the market, or the liabilities, dictated. Indeed, the declining EBR strategies will force switches out of equities even when this would be far from sensible, for example when equity prices are low and dividend yields high.

In this section we investigate the effect of some investment strategies which are driven by the recent history of appropriate investment indices.

5.6.1 Cyclical and contracyclical strategies

In 1986 Waters [67] used the Wilkie asset model to explore the effect of different investment strategies on the present value of the profit emerging under a single cohort of non-profit endowment policies. Three of the strategies were:

Strategy EQ To invest each year's cashflow entirely in equities.

Strategy FI To invest each year's cashflow entirely in consols.

Strategy FE To invest each year's cashflow in a mixture of equities and consols, the proportion being weighted towards the better performing sector.

One of the interesting results was the effect of the mixed investment strategy. Investment entirely in equities gave high mean profits with a high standard deviation, and investment entirely in consols gave low mean profits with a low standard deviation, but the mixed investment strategy FE achieved a combination of mean profits only slightly lower than those uder Strategy EQ and a standard deviation only a little higher than that under Strategy FI.

This mixed strategy FE might be termed "cyclical" because the direction of investment moves in line with the market. Another set of strategies are termed "contracyclical". In these, the investor favours those sectors which have performed relatively badly, on the grounds that this increases the chances of buying at the bottom. Macdonald [42] found that contracyclical strategies outperformed cyclical strategies, under the conditions of the Wilkie model.

Define the following indices in respect of equities and consols.

Let P_t be an index of equity prices, representing the value at time t of an investment of £1 at time 0, assuming that gross dividends are reinvested at the end of every year. This is equivalent to

$$P_t = \prod_{s=1}^{s=t}(1 + e_s)$$

where e_s is the total gross rate of return on equities in year s, and we assume that $P_0 = 1$. Let C_t be an index representing the value at time t of an investment of £1 in consols at time 0, assuming that gross interest payments are reinvested at the end of every year. This is equivalent to

$$C_t = \frac{c_t}{c_0}\prod_{s=1}^{s=t}(1 + \frac{i}{c_s})$$

where i is the gross coupon rate and c_t is the price of a single bond at time t. This is because the initial investment buys $\frac{1}{c_0}$ bonds. Each year the interest at rate i on the current nominal holding is reinvested to increase the nominal holding by a factor $(1+\frac{i}{c_s})$, and at time t the value of the holding is c_t times the nominal amount.

Then a simple cyclical strategy is to switch towards that sector which has seen the greater increase in its index over the most recent period. We will look at changes in the indices over the last 1, 2 and 3 years; we will give this time period the name "index period". For example, with an index period of 2 years we would switch towards equities at time t if

$$\frac{P_t}{P_{t-2}} > \frac{C_t}{C_{t-2}}$$

and towards gilts otherwise. In other words, we would switch towards equities if

$$\frac{P_t C_{t-2}}{C_t P_{t-2}} > 1$$

Figure 5.38: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the ratio $\frac{P_t C_{t-1}}{C_t P_{t-1}}$ at $t = 70$.

To each cyclical strategy there corresponds a contracyclical strategy, under which we move the other way. The ratio $\frac{P_t C_{t-s}}{C_t P_{t-s}}$, as a function of the index period s, is of interest. Figure 5.38 shows the distribution of the index $\frac{P_t C_{t-1}}{C_t P_{t-1}}$, at index period 1 year.

The distribution is very stable, and slightly skewed towards larger values. Inspection shows that the distributions of the ratio $\frac{P_t C_{t-s}}{C_t P_{t-s}}$ with $s > 1$ are also quite stable over time, but more skewed. Figure 5.39 shows density estimates for the ratio $\frac{P_t C_{t-1}}{C_t P_{t-1}}$ at time $t = 70$, for index periods $s = 1, \ldots, 5$.

As the index period s increases, the ratio $\frac{P_t C_{t-s}}{C_t P_{t-s}}$ becomes increasingly skewed towards larger values. The extent of the skewing is shown in Table 5.23, which shows, in respect of the ratio at time $t = 70$, and for index periods $s = 1, \ldots, 10$, (i) the median and (ii) the number (out of 1,000) of values of the ratio which are not less than 1.0.

Note that the outcomes of such index-driven strategies may depend a great deal on the asset model and its parameters. In particular, the assumption of no real dividend growth in the Reduced Standard parameters ($DMU = 0$) reduces the long-term advantage of equities over gilts and some authors have preferred alternative paramterisations for this reason (see Ross [57] and Ross & McWhirter [58].

Figure 5.39: Estimates of the density of the ratio $\frac{P_t C_{t-s}}{C_t P_{t-s}}$ for $s = 1, \ldots, 5$, at time $t = 70$.

	$\frac{P_t C_{t-s}}{C_t P_{t-s}}$	
Index period s	Median	No. ≥ 1.0
1	1.014	520
2	1.022	523
3	1.036	542
4	1.054	536
5	1.081	562
6	1.098	563
7	1.093	571
8	1.099	560
9	1.120	573
10	1.139	573

Table 5.23: Median and number (out of 1,000) of positive values of $\frac{P_t C_{t-s}}{C_t P_{t-s}}$ at time $t = 70$ for $s = 1, \ldots, 10$.

Run length	P_t	C_t	$\frac{P_t C_{t-1}}{C_t P_{t-1}}$
Rises			
1	2263	651	3478
2	1686	512	2081
3	1153	426	1071
4	807	372	527
5	455	330	253
6	297	247	132
7	180	213	44
8	113	194	31
9	49	134	6
10 and over	98	921	6
Falls			
1	4478	2658	3831
2	1633	466	2036
3	468	107	956
4	145	20	434
5 and over	37	8	317

Table 5.24: Numbers of runs of rising and falling index values in 1,000 simulations over 30 years.

Cyclical strategies depend on the assumption that a rise in a given index is more likely to be followed by another rise than by a fall. Contracyclical strategies assume the opposite. It is therefore of interest to study the patterns of rises and falls of the indices. Table 5.24 shows the numbers of uninterrupted runs of rises and falls in P_t, C_t and $\frac{P_t C_{t-1}}{C_t P_{t-1}}$ in the 1,000 scenarios used here.

Note that the data upon which these run lengths are based are censored since the 30,000 values of each index were based on 1,000 simulations each of length 30 years. This should not affect the frequency with which runs of each length are observed.

Suppose that the ratio $\frac{P_t C_{t-1}}{C_t P_{t-1}}$, having risen for one or more years, then falls. Then there are two possibilities; the fall is either a run of length 1 or the start of a longer run of falls. From the table, the frequency of the first of these possibilities is $3831/7574 = 0.5058 \approx 0.51$. (The denominator is the total number of runs of falling indices.) This offers practically no guidance. Given two successive falls, then the frequency of a rise in the next year is $2036/3734 \approx 0.54$. Similarly, if we observe one or two years of rising indices, the estimated probabilities of another rise in the following year are 0.54 and 0.5 respectively. Table 5.25 shows the frequencies with

Current run length	P_t	C_t	$\frac{P_t C_{t-1}}{C_t P_{t-1}}$
Rises			
1	0.68	0.84	0.54
2	0.65	0.85	0.50
3	0.63	0.85	0.48
4	0.60	0.85	0.47
5	0.62	0.84	0.46
6	0.60	0.86	0.40
7	0.59	0.85	0.49
8	0.57	0.84	0.30
9	0.67	0.87	0.50
Falls			
1	0.66	0.82	0.51
2	0.72	0.78	0.54
3	0.72	0.79	0.56
4	0.80	0.71	0.58

Table 5.25: Frequency with which runs of rising and falling index values are followed by a *rise* in the next year.

which each index rises, following a run of given length and direction.

Both the equity index P_t and the fixed-interest index C_t are more likely to rise than to fall. The probability of a rise following a run of rises is remarkably uniform in both cases; the probability of a rise following a run of falls is less so. The fact that the fixed-interest index has a greater probability of rising than the equity index does not imply that fixed-interest assets are preferable to equity investments; the *size* of the rises and falls is also important, and the pattern of runs tells us nothing about that. Although these frequencies do support a strategy of buying after a fall in either index — preferably an extended fall — they give little support for either cyclical or contracyclical strategies.

The frequencies in respect of the ratio $\frac{P_t C_{t-1}}{C_t P_{t-1}}$ also give no clear support for dynamic strategies. There is a very marginal suggestion of falls following 3 or more rises and of rises following 1 or more falls, which supports a contracyclic strategy, but it is very marginal.

Indices tell us which way to switch, but not how much to switch. Let the proportion invested in equities at the start of the t^{th} year be E_{t-1}, and let the proportion in fixed-interest assets be $FI_{t-1} = 1 - E_{t-1}$. Then we will define a "$x\%$ switch" to be

Strategy	Index period	Switch size
Cyclical		
AA No.22	1	10%
AA No.23	1	20%
AA No.24	2	10%
AA No.25	2	20%
AA No.26	3	10%
AA No.27	3	20%
Contracyclical		
AA No.28	1	10%
AA No.29	1	20%
AA No.30	2	10%
AA No.31	2	20%
AA No.32	3	10%
AA No.33	3	20%

Table 5.26: Definition of Asset Allocation Strategies AA No.21 – AA No.33 (cyclical and contracyclical strategies based on equity and consols indices).

a switch of $x\%$ of the proportion invested in the asset class being *sold* (rather than the alternative of $x\%$ of the whole fund). For example, a 20% switch from equities to gilts at time t means that the $E_t = 0.8E_{t-1}$, and $FI_t = FI_{t-1} + 0.2E_{t-1}$. This method of switching means that if the switches are in the same direction in successive years, the *proportion* switched diminishes each year; for example if the switch is again 20%, and away from equities at time $t + 1$, then $E_{t+1} = 0.8E_t = 0.8^2E_{t-1}$, and so on.

5.6.2 Examples of strategies

Table 5.26 defines a range of suitable strategies.

In each case, the assets at time $t = 40$ are switched from 100% in equities to 50% in equities to let the strategy start from a less extreme point.

Figure 5.40 shows the distribution of the proportion invested in equities under strategy AA No.22 (i.e. switching 10% towards the best-performing sector over the previous year).

The proportion invested in equities is between 40% and 60% roughly half of the time, and excursions beyond 30% and 70% are comparatively uncommon. The "skeleton" (the solid line with diamond markers) advances steadily towards 100% in

Figure 5.40: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the proportion invested in equities under strategy AA No.22.

equities, which it will approach asymptotically. This is because under deterministic conditions the change in the equity index P_t is always greater than the change in the consols index C_t. Thus the deterministic projection displays a trend utterly at odds with the stochastic projections; not only on being compared with any sample path but also compared with any quantile.

Under the corresponding contracyclical strategy, (AA No.28) the proportion invested in equities is the mirror-image of that in Figure 5.40. This is perhaps most clearly seen in Figure 5.41, which shows estimates of the density of the proportions in equities at time $t = 70$ under each of these two strategies. The fact that the equity proportion is slightly higher under the cyclical strategy reflects the tendency of the equity index P_t to outperform the consols index C_t slightly more often than not. (The median under strategy AA No.22 is 52.2% in equities.)

Figure 5.42 shows the effect of 20% switches instead of 10% switches. The distribution is naturally more dispersed, and slightly more skewed towards higher proportions in equities.

Figure 5.43 shows the effect of index periods of 1, 2 and 3 years. Using a longer index period causes the distribution of the equity proportion to spread out almost symmetrically, though the median increases very slightly.

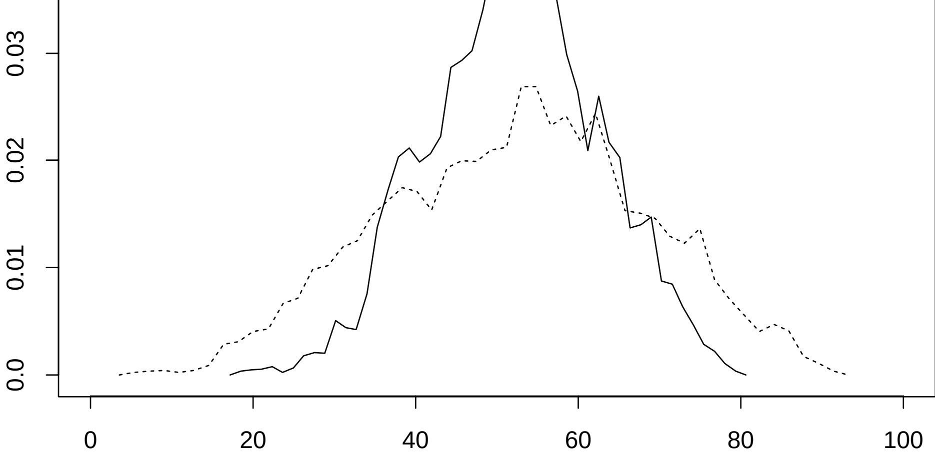


Figure 5.41: Density estimates of the proportion invested in equities at time $t = 70$ under the cyclical strategy AA No.22 and the contracyclical strategy AA No.28.

Figure 5.42: Density estimates of the proportion invested in equities at time $t = 70$ under the cyclical strategies AA No.22 (10% switches) and AA No.23 (20% switches).

Figure 5.43: Density estimates of the proportion invested in equities at time $t = 70$ under the cyclical strategy with 10% switches and an index period of 1, 2 or 3 years.

5.6.3 The effect on statutory solvency

Table 5.27 shows summary statistics in respect of the nominal MV per unit premium. For comparing these with fixed strategies it is AA No.6 (50% in equities) which gives the fairest comparison, so for convenience, the corresponding figures from Table 5.10 are also shown.

The most striking difference between cyclical and contracyclical strategies is in the pattern of insolvencies.

1. Contracyclical strategies result in more insolvencies.

2. Under the cyclical strategies, the number of insolvencies *increases* with the index period. Under the contracyclical strategies, the number of insolvencies is highest with an index period of 2 years.

3. Under the cyclical strategies, the stronger (20%) switching produces fewer insolvencies than the 10% switching, Under the contracyclical strategies the opposite is the case. Table 5.28 shows, for the contracyclical strategies, the numbers of scenarios which give rise to insolvencies under each pair of contracyclical strategies.

	Ratio A/L_1			MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E_t^i	$S^i(E_t)$	$E^i(S_t)$	$Corr$
Fixed 50% in equities							
AA No.6	42	1.198	0.1	16.356	1.571	2.761	0.142
Cyclical							
AA No.22	29	1.207	0.092	16.315	1.771	2.958	-0.113
AA No.23	20	1.224	0.089	16.283	1.873	3.107	-0.122
AA No.24	47	1.195	0.101	16.252	1.814	2.988	-0.168
AA No.25	40	1.206	0.100	16.158	1.934	3.170	-0.180
AA No.26	63	1.179	0.109	16.249	1.895	3.039	-0.213
AA No.27	58	1.183	0.110	16.140	2.053	3.251	-0.216
Contracyclical							
AA No.28	102	1.177	0.132	16.404	1.436	2.617	0.252
AA No.29	165	1.142	0.157	16.444	1.422	2.569	0.251
AA No.30	102	1.183	0.134	16.472	1.427	2.613	0.276
AA No.31	176	1.145	0.161	16.578	1.438	2.580	0.282
AA No.32	94	1.191	0.136	16.483	1.383	2.597	0.290
AA No.33	157	1.151	0.167	16.615	1.399	2.585	0.300

Table 5.27: Comparison of ratio A/L_1 with Maturity Values, strategies AA No.22 – AA No.33 (cyclical and contracyclical strategies based on equity and consols indices)

	Strategy					
Strategy	AA 28	AA 29	AA 30	AA 31	AA 32	AA 33
AA No.28	102	102	79	95	72	87
AA No.29	102	165	92	136	82	117
AA No.30	79	92	102	101	80	90
AA No.31	95	136	101	176	89	129
AA No.32	72	82	80	89	94	94
AA No.33	87	117	90	129	94	157

Table 5.28: Numbers of scenarios (out of 1,000) giving rise to statutory insolvency under each pair of contracyclical strategies AA No.28 – AA No.33.

			$\frac{MV_t^{ci}}{MV_t^{cci}}$		
Strategies	Index period	Switch size	E_t^i	$S^i(E_t)$	$E^i(S_t)$
AA No.22/28	1	10%	0.992	0.035	0.053
AA No.23/29	1	20%	0.989	0.057	0.089
AA No.24/30	2	10%	0.984	0.043	0.064
AA No.25/31	2	20%	0.975	0.069	0.106
AA No.26/32	3	10%	0.984	0.053	0.076
AA No.27/33	3	20%	0.973	0.084	0.124

Table 5.29: Comparison of ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$ of Maturity Values under cyclic strategies (MV_t^{ci}) to Maturity Values under contracyclic strategies (MV_t^{cci}).

Therefore the effect of increasing the switch size is largely to make more offices insolvent, in addition to those already insolvent. This is not always (or even usually) the case following a change in strategy; often the sets of scenarios resulting in insolvency under two different strategies have a more modest intersection.

5.6.4 The effect on maturity values

There is also a striking difference between the maturity values under the cyclical and contracyclical strategies. Each contracyclical strategy results in a higher mean MV per unit premium ($E_t^i(MV_t^i)$) than its cyclical counterpart. This makes some intuitive sense, since a contracyclical strategy aims to buy low and sell high. However, each contracyclical strategy also results in *lower* variability than its cyclical counterpart, both between scenarios and within scenarios. The difference appears to increase with the index period.

Table 5.29 shows the summary statistics of the ratio of maturity values under each cyclical strategy to those under its corresponding contracyclical strategy, calculated path-by-path. That is, let MV_t^{ci} be the MV per unit premium at time t in the i^{th} scenario under a cyclical strategy, and let MV_t^{cci} be the MV per unit premium at time t in the i^{th} scenario under the corresponding contracyclical strategy. Then form the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$, and compute summary statistics $E_t^i(\frac{MV_t^{ci}}{MV_t^{cci}})$ etc. as before.

This confirms that **the cyclical strategies give lower maturity benefits on average**, although the variability, especially within scenarios, suggests that this

Figure 5.44: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$ under strategies AA No.27 (cyclical) and AA No.33 (contracyclical).

might be far from uniformly true within each scenario. However, the distribution of the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$ is skewed: Figure 5.44 shows the distribution in the case of strategies AA No.27 (cyclical) and AA No.33 (contracyclical).

In about 75% of cases, the ratio is below 1.0, indicating that the cyclical strategy is poorer than the contracyclical strategy. The 95th quantile is very high, which means that the mean ratio $E_t^i(\frac{MV_t^{ci}}{MV_t^{cci}})$ shown in Table 5.29 understates the advantage possessed by the contracyclical strategy. Inspection shows similar skewness in the distribution of the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$ under the other strategies.

Another interesting feature of Figure 5.44 is the anomalous behaviour of the deterministic projection (the solid line with diamond markers). It shows again how deterministic projections might lead to misleading results. In this case, the question at issue is the relative effect of a pair of alternative asset allocation strategies, given two asset classes, one of which yields higher returns *in the long-run average*. Deterministic projections would lead us to prefer the wrong strategy.

Figure 5.45 shows a box plot of the MV per unit premium under the fixed (50%) strategy, and the cyclical and contracyclical strategies. It is apparent that the differences between the latter two lie mainly in the maturity values under the cyclical strategies having a much longer tail towards higher values; nevertheless each

Figure 5.45: Boxplot of the MV per unit premium at time $t = 70$ under asset allocation strategies AA No.6 and AA No.22 – AA No.33.

contracyclical strategy has a higher median than its cyclical couterpart.

Moreover, comparing these dynamic strategies with fixed 50% in equities, the cyclical strategies all result in *lower* **mean benefits with** *higher* **variability, while the contracyclical strategies all result in** *higher* **mean benefits with** *lower* **variability**. Figure 5.46 shows the distribution of the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$ in the case of strategies AA No.33 (contracyclical) and AA No.6 (fixed 50% equities).

Note that this figure is "upside-down" compared with Figure 5.44, in that the contracyclical strategy provides the numerator of the ratio. It shows that there is a similar skewness to the distribution of the ratio, so that the relative advantage of the contracyclical strategy is understated by the comparison of the means in Table 5.27.

Contracyclical strategies appear to offer the best of both worlds; higher mean benefits with lower variability. Cyclical strategies appear to be outperformed in this regard even by the fixed asset allocation strategy. However, the success of the contracyclical strategies is purchased at the price of considerably more statutory insolvencies. This leads again to the question of whether or not the statutory minimum valuation, insofar as it might drive the asset allocation strategy, is acting appropriately, to which we return in Chapter 7.

Figure 5.46: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$ under strategies AA No.33 (contracyclical) and AA No.6 (fixed 50% equities).

5.6.5 The effect on real maturity values

Table 5.30 shows summary statistics in respect of the nominal MV per unit premium. For comparing these with the fixed (50%) strategy the corresponding figures from Table 5.11 are also shown.

The contracyclical strategies still have the higher mean maturity values. The main difference between these figures, and those relating to nominal maturity values, is that the cyclical strategies now have the lower variability. Not only that, but the positions with respect to the fixed (50% in equities) strategies are reversed; all the cyclical strategies have lower variability, and all the contracyclical strategies have higher variability.

The contracyclical strategies still have better mean maturity values, but they no longer have the best of both worlds. Therefore, especially in view of their poorer solvency, it is not quite so clear that they are to be preferred.

Figure 5.47 shows a boxplot of the real MVs per unit premium at time $t = 70$, including the fixed strategy AA No.6 (50% in equities).

Compared with Figure 5.45, this Figure shows the slightly greater variability of the contracyclical strategies. It also shows that the differences in variability are less

	Ratio A/L_1			Real MV per unit prem.			
Strategy	No. < 1	$E^i(\min_t)$	$S^i(\min_t)$	E^i	$S^i(E)$	$E^i(S)$	$Corr$
Fixed 50% in equities							
AA No.6	42	1.198	0.1	10.057	1.42	2.423	0.077
Cyclical							
AA No.22	29	1.207	0.092	10.007	1.403	2.458	0.150
AA No.23	20	1.224	0.089	9.974	1.407	2.461	0.142
AA No.24	47	1.195	0.101	9.963	1.396	2.427	0.151
AA No.25	40	1.206	0.100	9.888	1.401	2.443	0.167
AA No.26	63	1.179	0.109	9.952	1.387	2.429	0.168
AA No.27	58	1.183	0.110	9.863	1.392	2.446	0.183
Contracyclical							
AA No.28	102	1.177	0.132	10.113	1.462	2.442	−0.019
AA No.29	165	1.142	0.157	10.153	1.493	2.475	−0.012
AA No.30	102	1.183	0.134	10.160	1.478	2.470	−0.030
AA No.31	176	1.145	0.161	10.245	1.523	2.533	−0.015
AA No.32	94	1.191	0.136	10.176	1.503	2.487	−0.069
AA No.33	157	1.151	0.167	10.282	1.564	2.572	−0.058

Table 5.30: Comparison of ratio A/L_1 with real Maturity Values, strategies AA No.22 – AA No.33 (cyclical and contracyclical strategies based on equity and consols indices)

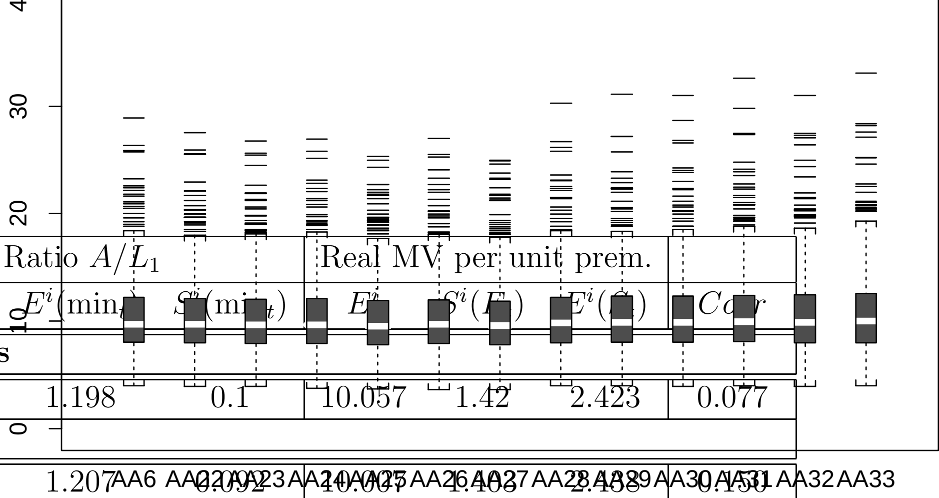


Figure 5.47: Boxplot of the real MV per unit premium at time $t = 70$ under asset allocation strategies AA No.6 and AA No.22 – AA No.33.

than seemed to be the case, looking only at nominal maturity values, largely because the distributions under the cyclical strategies are much less skewed.

We may denote the real maturity values under the cyclical strategies by RV_t^{ci}, and under the contracyclical strategies by RV_t^{cci}, and form the ratio $\frac{RV_t^{ci}}{RV_t^{cci}}$ as we did for nominal maturity values. However, this ratio is identical to the ratio $\frac{MV_t^{ci}}{MV_t^{cci}}$, since numerator and denominator are deflated by the same factor, so the comments made above regarding the latter still apply; Table 5.30 overlooks the skewness of the distribution of the ratio $\frac{RV_t^{ci}}{RV_t^{cci}}$, and understates the relative advantage of the contracyclical strategies, barring their poorer statutory solvency.

5.7 The effect of the reversionary bonus strategy

In this section we summarise the effects of changing the reversionary bonus strategy. Since the effects of many changes are less significant than changes to the asset allocation strategy we will deal with them more briefly. We consider alternative bonus rules which are (like the original bonus rule) *prospective* — that is they project asset shares and guaranteed benefits forward and set bonus rates allowing for a target terminal bonus (which may be nil). They do not make use of any retrospective assessment of surplus.

The strategies will be denoted RB No.1 – RB No.11.

Bonus strategy RB No.1 The rates of bonus supported by the prospective strategy described in Chapter 4, if rates of return follow the long-term means implied by the asset model, were 1.513% of sums assured and 2.521% of existing bonus. This strategy declares these fixed bonuses every year.

Bonus strategy RB No.2 A single-tier compound reversionary bonus was declared, using the same dynamic decision rule as before but changing the ratio of bonus on bonus to bonus on sums assured from 5/3 to 1.

Bonus strategy RB No.3 The two tiers of bonus were determined dynamically but separately. First the rate of bonus on sums assured was determined as a *simple* bonus, using a terminal bonus target of 35%; then the rate of bonus on bonus was determined, allowing for the projected rate of bonus on sums assured and using a terminal bonus target of 15%. If the rate of bonus on bonus was less than the rate of bonus on sums assured, a single tier compound bonus (as in RB No.2) was calculated instead. (This strategy is based on that of Ross & McWhirter [58]).

Bonus strategy RB No.4 The limits on annual proportionate changes in bonus rates (+25% and −20%) were removed.

Bonus strategy RB No.5 The limits on the annual proportionate changes in bonus rates were made stricter, +12.5% and −11.11% instead of +25% and −20%.

Bonus strategy RB No.6 The limits on the annual proportionate changes in bonus rates were relaxed, to +50% and −33.33% instead of +25% and −20%.

Bonus strategy RB No.7 The limits on the annual proportionate changes in bonus rates were replaced by limits on the annual absolute changes of $\pm\frac{1}{2}\%$ of the rate of bonus on sums assured.

Bonus strategy RB No.8 The limits on the annual proportionate changes in bonus rates were replaced by limits on the annual absolute changes of ±1% of the rate of bonus on sums assured.

Bonus strategy RB No.9 In addition to the limits on the annual proportionate changes in bonus rates of +25% and −20%, an absolute maximum rate of 5% and an absolute minimum rate of 1% were imposed.

Bonus strategy RB No.10 The terminal bonus target was changed from 25% to nil. This corresponds to one of the strategies used by the FASWP (see Section 3.4).

Bonus strategy RB No.11 The terminal bonus target was changed from 25% to 50%.

In view of historic patterns of bonus, only those rules with quite severe limits on changes or on absolute levels look "realistic", yet imposing such limits tends to

Figure 5.48: Boxplot of the rate of bonus on sums assured at time $t = 70$ under the original bonus strategy and strategies RB No.1 – RB No.11.

push up the terminal bonus rates which are already high in Figure 4.7. However, terminal bonus rates might be most naturally controlled by smoothing of maturity values, which is discussed in Chapter 6.

Figure 5.48 shows a boxplot of the rates of bonus on sums assured at time $t = 70$ under these strategies; note that the plot labelled "Base" at the left-hand end is the baseline office of Chapter 4 with the original strategy.

Table 5.31 shows the cumulative numbers of the 1,000 simulations under which the ratio A/L_1 ever fell below 1.0, by $t = 50$, $t = 60$ and $t = 70$. For convenience the figures in respect of the "baseline" strategy described in Chapter 4 are also shown. The details of the bonus rule have comparatively little impact on statutory insolvency, except in two cases. The exceptions are RB No.10 and RB No.11. Increasing the terminal bonus target to 50% significantly reduces the incidence of statutory insolvency, while decreasing it to nil significantly worsens statutory insolvency.

It is interesting that the number of insolvencies is lower if limits on changes in bonus rates from year to year are removed completely (RB No.4) but higher if the limits are merely relaxed (RB No.6). In addition, the imposition of absolute limits of 1% and 5% (RB No.9) increases the number of insolvencies. This suggests that the ability to reduce bonus rates quickly is a key factor in the influence of bonus policy on solvency. Overall, though, the precise form of the limits on bonus changes

Cumulative No. of statutory insolvencies			
Bonus rule	$t = 50$	$t = 60$	$t = 70$
Baseline	104	195	276
RB No.1	108	207	290
RB No.2	102	188	285
RB No.3	109	226	333
RB No.4	92	169	240
RB No.5	106	192	278
RB No.6	104	199	295
RB No.7	96	185	289
RB No.8	108	212	317
RB No.9	105	201	293
RB No.10	169	345	438
RB No.11	82	139	196

Table 5.31: Cumulative number out of 1,000 simulations ever statutorily insolvent ($A/L_1 < 1$) after 10, 20 and 30 years under prospective bonus strategies.

has a relatively small effect.

Figure 5.49 shows a boxplot of the ratio A/L_1 at time $t = 70$.

The most striking change is that of strategy RB No.10, with a terminal bonus target of nil. In this case the distribution is closely concentrated around $A/L_1 = 1.0$, with little skewness. A nil terminal bonus target is of course not sensible given high equity investment; the interesting point is that the conclusions of the FASWP (see Section 3.4) were based upon a similar bonus strategy. This result suggests that the FASWP's conclusions should perhaps not be compared with later work such as that of Ross [57] and Ross & McWhirter [58].

Apart from RB No.10, all the amended strategies show slight variations on the same theme. Compared with, for example, Figure 5.37, there is less evidence of *qualitative* differences arising from the different strategies.

Similarly, there is little difference in the distributions of maturity values, except under strategy RB No.10 where maturity values are much lower. This is because, as long as the policy asset share is higher than the guarantees at maturity, the benefit will be the same except insofar as the different build-up of liabilities has led to a different investment strategy. Details of the maturity values and terminal bonus rates are omitted.

Figure 5.49: Boxplot of the ratio A/L_1 at time $t = 70$ under the original bonus strategy and strategies RB No.1 – RB No.11.

So far as these 10-year policies are concerned, reasonable changes to the prospective reversionary bonus strategy affect the outcome less than do changes to the asset allocation strategy. It is possible that bonus strategy should have a more significant effect on longer term policies, under which the bonus additions will usually form a larger portion of the guaranteed benefits at maturity.

5.8 Conclusions

1. Equity investment results in considerably higher mean maturity values than does gilt investment, but also much greater variance of maturity values.

2. Statutory solvency is much worse under equity investment.

3. Alternative asset allocation strategies (solvency-driven switching, declining EBR) appear to offer little advantage over fixed investment strategies, for a *given* level of statutory insolvency.

4. Solvency-driven asset switching as used by Ross [57] and others is much less effective if limits exist on the speed with which assets can be switched.

5. On the basis of *nominal* maturity values, the contracyclical strategies used

here perform better than their cyclical counterparts; they result in higher mean maturity values with less variability. They are also slightly better than the corresponding fixed investment strategy.

6. **Consideration of real instead of nominal maturity values leads to different conclusions**.

    - Comparing different levels of gilt and equity investment, the difference in mean maturity values persists but the difference in volatility is much reduced. **Therefore investment in equities does not carry such a significant penalty of** *relative* **volatility as is often supposed**.

    - Comparing cyclical and contracyclical investment strategies, the *cyclical* strategies now have the less volatile (real) maturity values.

7. Changes to the reversionary bonus strategy make relatively little difference, possibly due in part to the short term of the business. The change with the greatest effect was to alter the terminal bonus target

Chapter 6

Smoothing with-profit maturity values

In this chapter we consider the smoothing of with-profit maturity values. The effect of smoothing is to pay out an amount other than the policy asset share at maturity; sometimes more and sometimes less. These differences represent the cost of smoothing. An important question is: does the cost of smoothing tend to stabilise over time, or is the long term accumulated cost of smoothing unstable?

The existence of the guarantees provides another reason for sometimes paying more than the asset share at maturity, and this too has a cost which ought to be charged for. A further question is: if smoothing takes place anyway, is the cost of meeting the guarantees absorbed into the cost of smoothing? If it is, does this reduce the need to charge for the guarantees?

To consider these questions, we will introduce some possible smoothing methods based on practitioners' responses to a survey, and then study the accumulated costs of smoothing (the "Bonus Smoothing Account") and of further meeting the guarantees (the "Guarantee Cost Account").

6.1 Introduction

Most U.K. with-profit offices claim to operate some form of smoothing of maturity benefits. In part this is seen as an intrinsic feature of with-profit business, and one which distinguishes it from unit-linked business, and as such it needs no explanation. There are other reasons for smoothing based upon the operation of the guarantees, particularly in an expanding office.

It is obvious that to pay 100% of asset shares as a *minimum* benefit when a claim arises is a one-way process. An office which pursues such a policy is in the position of providing subsidies out of its additional estate without the opportunity to replenish that resource. It might not be so obvious that under some circumstances such a policy need not prove ruinous.

It is well known that a life office whose rate of new business growth exceeds the rate of return it can earn on its funds will eventually run out of capital. See, for example, Smart [63]. The reason is that the requirement for capital grows along with the new business, while unemployed capital grows at the rate of return on the assets. However, in a with-profits office which pays a high proportion of the benefits in the form of terminal bonus this restriction might be relaxed.

Consider a simplified life office, writing endowment business of term n years. It is in a state of steady expansion, at rate e per annum, and it earns a rate of return g on its assets. It has always paid 100% of asset shares to maturing policies, so its total assets (ignoring any additional estate) are exactly equal to the total of its asset shares at any time. In year t, the cash-flows are as follows:

$$\begin{aligned} P_t &= \text{Premiums received} \\ C_t &= \text{Claims paid} \\ E_t &= \text{Expenses incurred} \\ \Delta V_t &= \text{Change in reserve during the year} \end{aligned}$$

Then the total cashflow during year t is given by

$$CF_t = gV_t + P_t - C_t - E_t - \Delta V_t$$

But because the office is expanding steadily at rate e per year,

$$V_{t+1} = V_t(1 + e)$$

so

$$CF_t = (g - e)V_t + P_t - C_t - E_t$$

Consider now the office's total asset shares at time $t + 1$, AS_{t+1}.

$$AS_{t+1} = AS_t(1 + g) + P_t - C_t - E_t$$

but because of the uniform expansion

$$AS_{t+1} = AS_t(1 + e)$$

so

$$AS_t(e - g) = P_t - C_t - E_t$$

and

$$CF_t = (g - e)(V_t - AS_t)$$

For the office to be able to finance expansion internally, the cash-flow CF_t must be non-negative. If the office made no use of terminal bonus, and attempted to pay 100% of the asset share on claims by means of reversionary bonus, then almost certainly V_t would exceed AS_t (because there would be some conservatism in the valuation basis) which tells us that (i) such a strategy would be infeasible without additional capital, and (ii) such a strategy would only be sustainable if $g > e$. This is the conventional result, that the rate of return on the assets limits the rate of new business growth in the long run.

If, however, the office made use of terminal bonus, then for some values of e in excess of g, it might be able to reduce its reserves to the point where, in aggregate, $V_t < AS_t$. Then the overall cash-flow would remain positive and expansion could proceed at a higher rate.

If $e > g$, then the office must lose any additional estate which it possesses. If we denote the total assets at time t by A_t, and the additional estate by AE_t, then

$$\begin{aligned}\frac{A_{t+1}}{AS_{t+1}} &= \frac{AS_{t+1} + AE_{t+1}}{AS_{t+1}} \\ &= 1 + \frac{(1+g)AE_t}{(1+e)AS_t}\end{aligned}$$

Therefore continued expansion at rate $e > g$ will result in the ratio A/AS tending to 1, i.e. in the loss of the additional estate, in relative terms. Note that this is true if $AE_t < 0$ as well as if $AE_t > 0$, implying that rapid expansion financed by terminal bonus is a way of escaping from a deficit in the assets — hardly to be recommended in practice, though.

Passing on from this simple model, life offices in practice are not so stable, and it is a matter of importance to maintain an adequate margin of assets over liabilities, but new business expansion is usually also sought. As a minimum, some expansion is needed to control cost inflation. Therefore offices will have regard to the maintenance of any additional estate which they possess.

Smoothing of benefits therefore appears to be desirable. In bad times, the maturity benefits will be subsidised, possibly at a level above that of the guarantees, and this will be paid for by a charge on the asset shares of policies maturing in better times. It is possible, therefore, that over the long term, smoothing will absorb the cost of meeting the guarantees.

We will consider smoothing in a wider sense here, to include any form of adjustment to the asset shares of maturing policies, including "one-way" adjustments such as the deduction of a uniform levy.

Our intuitive idea of smoothing is of a cost-neutral operation. We would like the plusses and minuses to balance out, in the long run. It is not immediately obvious that this will be the case, certainly not in all circumstances. For example, what happens to the cost of smoothing if the office is expanding? Then each generation of maturing policies will be bigger than its predecessors, and will either draw upon or contribute to the office's estate to a greater extent, depending on which way the smoothing is then working.

In this chapter we examine the long-term cost of smoothing in a simple model office.

6.2 Smoothing methods and the Bonus Smoothing Account

A survey carried out by a firm of consultants [65] revealed that, of the 38 U.K. with-profit offices which responded, 24 smoothed maturity values by limiting the percentage change from year to year, 16 made projections of asset shares in an attempt to smooth prospectively, and 11 calculated smoothed asset shares as a basis for maturity values. 10 offices claimed to limit the cost of over- or under-payment of maturity values. We will therefore use three smoothing approaches — asset smoothing and maturity value smoothing, alone or in combination.

6.2.1 Asset smoothing

Section 4.1.3 described how the assets are valued actuarially using moving average yields. This results in a smoother progression of asset values than does the market valuation, so one simple approach to smoothing benefits is to pay the smoothed value of the assets underlying the asset share of the maturing policy, instead of the market value (subject to a minimum of the guaranteed benefits. We will call this method “asset smoothing”.

The model office program uses a crude matching strategy to select dated gilts. It first determines what proportion of the assets underlying the asset share of an individual policy should be invested in gilts, and then allocates the appropriate quantity of gilts *of the same outstanding term as the policy*. In other words, the gilt component of the asset share of a policy with n years to run is always invested in gilts with n years to redemption. This contrasts with the assumptions of (for example) Ross & McWhirter, who used gilts of a fixed term to redemption.

This strategy means that only equities contribute to the smoothing of maturity values — the actuarial value of a gilt on its redemption date is the same as its market value, and the redemption date is also the maturity date of the corresponding policy. Any smoothing which occurs when assets are entirely invested in gilts must therefore be smoothing in a negative sense, due to the action of the guarantees.

Figure 6.50 shows how the ratio

Figure 6.50: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio of the actuarial value and market value of the assets in the baseline office.

$$\frac{\text{Actuarial Value of assets}}{\text{Market Value of assets}}$$

(or the ratio AV/MV for short) progresses over time in the baseline office of Chapter 4.

This ratio is slightly skewed towards values below 1.0. Inspection shows that this is due to the pattern of asset switching. Times when the AV/MV ratio might be high correspond to times of rising dividend yields and falling share prices, including those circumstances in which the office is forced to switch into gilts. Investment in gilts will cause the overall AV/MV ratio to be closer to 1.0 because the AV/MV ratio for gilts is less volatile than for equities.

In fact, the distribution of the AV/MV ratio for equities alone is almost exactly symmetrical.

6.2.2 Maturity value smoothing

An alternative, and more direct, smoothing method is to limit the change in maturity values each year. However, maturity values increase naturally because the average premium is assumed to increase with inflation, so to eliminate this effect we use the *maturity value per unit premium* instead. We often abbreviate this to "MV per unit premium" or just "MV per UP".

Using this method, the calculated maturity value per unit premium is not allowed to increase by more than 10% or to decrease by more than 9.09% in one year. These figures were chosen so that the greatest increase followed by the greatest decrease cancel out.

6.2.3 Combined smoothing

In some cases we use both smoothing methods together. The order in which the calculations are then carried out is as follows:

1. The actuarial value of the assets underlying the asset share of the maturing policy is calculated, and 100% of this is taken as the starting point.
2. The resulting maturity value per unit premium is not allowed to increase by more than 10% or to decrease by more than 9.09% in one year.
3. If the guaranteed benefits exceed the maturity value limited as above the former are paid.

6.2.4 The Bonus Smoothing Account

The Bonus Smoothing Account (BSA) was introduced by Lang & Scott [38]. It is defined, for our purposes, as the accumulation of the differences between the maturity values paid in the past, and the asset shares (at market valuation) of those maturing policies. The accumulation is at the net rate of return on the fund.

Notice that no money changes hands, and the inflows to and outflows from the BSA are notional and not physical cash-flows. The BSA is an internal book-keeping item with no separate funds. Were it to be separately accounted for it would correspond roughly to the equalisation reserve of a general insurance company.

In years when investment returns are particularly high, smoothing will tend to keep maturity values below the underlying value of the assets, so there will then be a contribution to the BSA from maturing policies. In years when investment returns are particularly low, smoothing and perhaps also the guarantees will tend to keep maturity values above the underlying value of the assets, so there will be a subsidy from the BSA to maturing policies. Note that the BSA might increase even in the latter circumstances, because of the assumed accumulation.

In this part of the work we focus on the ratio

$$\frac{\text{Bonus Smoothing Account}}{\text{Asset Shares}}$$

(or BSA/AS for short). The absolute value of the BSA tells us little. Dividing by the total asset shares gives a relative measure of the impact of the BSA on the finances of the office, from which any additional estate is removed. Changes in this ratio might, of course, be due to changes in the asset shares rather than to changes in the BSA.

6.2.5 The Guarantee Cost Account

If smoothing were so effective that the smoothed maturity values always exceeded the guaranteed benefits, then the cost of meeting the guarantees would have been absorbed completely into the cost of smoothing. If, however, the guarantees still exceed the smoothed maturity values from time to time, then there will be an additional cost in meeting those guarantees, not absorbed into the cost of smoothing. If we compare this with the cost of meeting the guarantees in the absence of smoothing, we will be able to measure the extent to which the cost of smoothing has absorbed the cost of meeting the guarantees.

Therefore define the Guarantee Cost Account (GCA) to be the accumulation of the differences between the calculated maturity value, smoothed or unsmoothed as the case may be, and the maturity benefit actually paid. Since the only time this will be non-zero is when the guarantees are effective, the GCA measures the cost of the guarantees, given the method used for calculating maturity values. Note that the GCA as defined here is always zero or negative. We usually show the ratio GCA/AS, for the same reasons as given above in respect of the BSA.

	Asset share			Cost of		End-year	
Year	Unsmoothed	Smoothed	Guarantee	Smoothing	Guarantee	BSA	GCA
Without smoothing							
1	100	n/a	90	n/a	0	n/a	0
2	95	n/a	92	n/a	0	n/a	0
3	104	n/a	94	n/a	0	n/a	0
4	92	n/a	95	n/a	3	n/a	-3
5	90	n/a	96	n/a	6	n/a	-9
With smoothing							
1	100	97	90	-3	0	3	0
2	95	99	92	4	0	-1	0
3	104	101	94	-3	0	2	0
4	92	99	95	7	0	-5	0
5	90	92	96	2	4	-11	-4

Table 6.32: Example of the operation of the BSA and GCA, ignoring the effect of interest.

6.2.6 An example of the BSA and GCA

Table 6.32 sets out a simplified example of the operation of the BSA and the GCA. In this example, the accumulation of both the BSA and the GCA at the fund rate of return is ignored, and the table only shows the contributions each year to and from the accounts.

First the operation of the GCA in the absence of smoothing is shown. In years 1, 2 and 3, the asset shares of maturing policies exceed the guaranteed benefits, so there is no cost associated with the guarantees. In years 4 and 5, however, the guarantees must be met, and the GCA is debited with the cost.

In the bottom half of the table, it is assumed that the asset shares of maturing policies are smoothed as shown in the third column. In each of years 1, 2 and 3, the difference between the smoothed and unsmoothed asset shares represents the cost of smoothing, and this is debited from or credited to the BSA. In year 4, the guaranteed benefits exceed the *unsmoothed* asset share but not the *smoothed* asset share. Since the latter is paid, there is a cost associated with the smoothing but *no cost associated with the guarantees.* In year 5 the guaranteed benefits exceed *both* the smoothed and unsmoothed asset shares. The total cost of £6 is debited to the BSA, but of this £4 represents the cost of meeting the guarantees *after* smoothing, so is also debited to the GCA.

So with smoothing, the *entire* cost of meeting the guarantees is absorbed by the normal operation of smoothing in Year 4, while only part of the cost is absorbed in Year 5. It is seen that the GCA is part of the BSA — that part which represents the "residual" cost of the guarantees after smoothing has absorbed what it can.

Alternative methods of meeting the cost of the guarantees have been explored, for example by using options (see Wilkie [69]).

6.3 The effect of smoothing on benefits

6.3.1 Changes in maturity values

The effect of smoothing on maturity values can be studied by comparing the maturity values paid with and without smoothing, within each simulation. We would expect smoothing to reduce the *differences* between the maturity payouts in successive years. Define

$$\begin{aligned} MV_t &= \text{Unsmoothed MV per unit premium} \\ MV_t^a &= \text{MV per unit premium with asset smoothing} \\ MV_t^m &= \text{MV per unit premium with maturity value smoothing} \\ MV_t^c &= \text{MV per unit premium with combined smoothing} \end{aligned}$$

(See Figure 4.8 for the distribution of MV_t.) Then let ΔMV_t be the difference $MV_t - MV_{t-1}$, and define ΔMV_t^a, ΔMV_t^m and ΔMV_t^c similarly. Figures 6.51 – 6.54 show the distributions of ΔMV_t, ΔMV_t^a, ΔMV_t^m and ΔMV_t^c.

These figures show that asset smoothing on its own brings a significant reduction in the changes of maturity values from year to year, but (comparing Figures 6.52 and 6.53) that changes in excess of 10% are still frequent. The maturity value smoothing on its own or with asset smoothing has a much stronger effect than asset smoothing alone on the more extreme quantiles, though the quartiles are very similar under any of the smoothing methods.

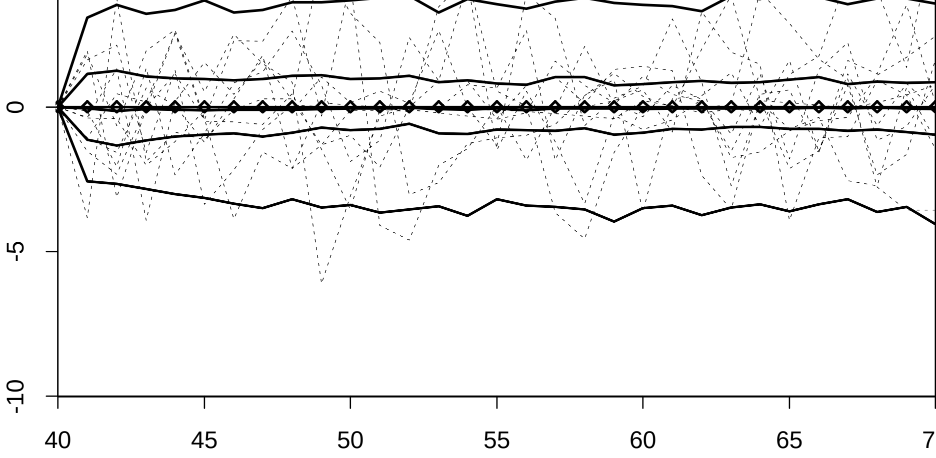


Figure 6.51: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the difference ΔMV_t with no smoothing.

Figure 6.52: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the difference ΔMV_t^a with asset smoothing only.

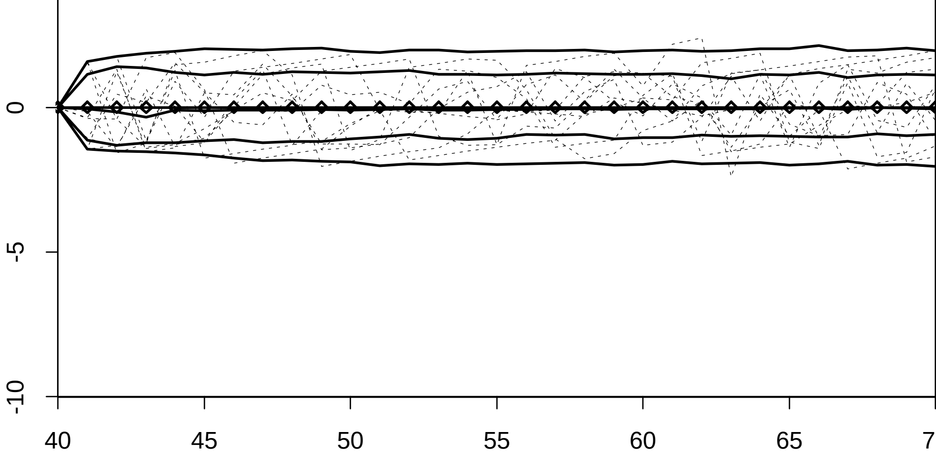


Figure 6.53: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the difference ΔMV_t^m with maturity value smoothing only.

Figure 6.54: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the difference ΔMV_t^c with asset and maturity value smoothing.

Figure 6.55: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio $\frac{MV_t^a}{MV_t}$ with asset smoothing only.

6.3.2 The effect on individual policyholders

Figures 6.55 and 6.56 compare the smoothed with the unsmoothed maturity values. There, the ratio $\frac{MV_t^x}{MV_t}$, where $x = a$ or m within each simulation indicates the difference which smoothing makes to the individual policyholder.

These Figures suggest that smoothing might have a very large effect on the benefits of individual policyholders. Figure 6.55 naturally resembles Figure 6.50. It might seem as if asset smoothing and maturity value smoothing had similar effects, with the latter allowing slightly greater divergence from the unsmoothed values (as befits the stronger smoother) but Figure 6.57 shows that this is not so. This shows that the distribution of the ratio $\frac{MV_t^a}{MV_t^m}$ is quite similar to those of the ratios $\frac{MV_t^a}{MV_t}$ and $\frac{MV_t^m}{MV_t}$. The choice of smoothing method also makes a great difference to individual payouts.

A feature of these figures is the small inter-quartile range. Although most sample paths include some extreme values, the distribution is in fact quite tightly concentrated.

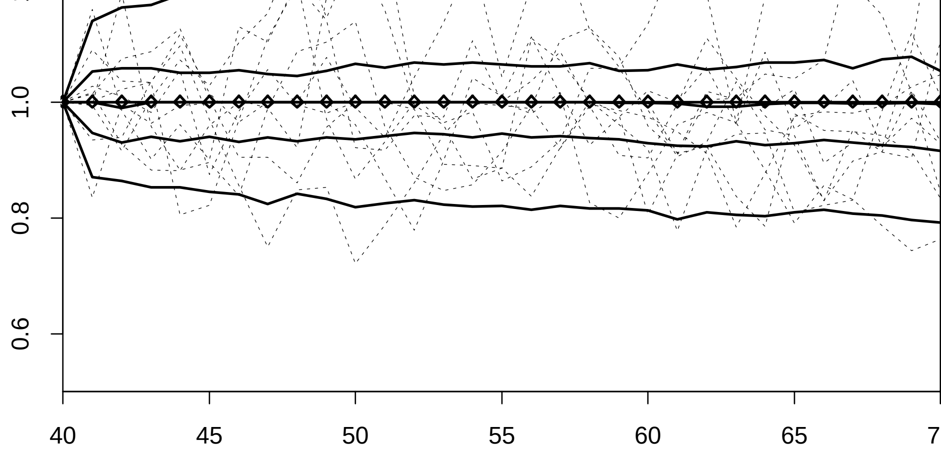


Figure 6.56: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio $\frac{MV_t^m}{MV_t}$ with maturity value smoothing only.

Figure 6.57: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio $\frac{MV_t^a}{MV_t^m}$.

6.3.3 Summary measures of smoothness

In this section we consider summary measures of the effect of smoothing on maturity benefits. As in Chapter 5, the differences caused by smoothing are observed within each of the 1,000 scenarios, so we first summarise the differences within each scenario, and then across all 1,000 scenarios.

Given a quantity X_t^i defined at each time $t = 1, \ldots, n$ within each scenario $i = 1, \ldots, m$, define $E_t(X_t^i)$, $E^i(X_t^i)$, $E_t^i(X_t^i)$ etc. as in Section 5.2.

In this section let MV_t^i denote the MV per unit premium at time t in the i^{th} scenario. Here we use the superscript i to denote the i^{th} scenario and not, as before, the smoothing method, since the latter will always be clear from the context. We base our measures not on the MV per unit premium MV_t^i but on a *normalised* MV per unit premium NMV_t^i. This makes comparison between different policies, and between real and nominal maturity values, much easier. NMV_t^i is defined as

$$NMV_t^i = \frac{MV_t^i}{E_t^i(MV_t^i)}$$

The measures which we show in Table 6.33 are the following:

Distribution of $E_t(|\Delta NMV_t^i|)$. We have shown in Figures 6.51 – 6.54 the distributions of the differences in the MV per unit premium from year to year. It is more relevant to consider the absolute differences in a summary figure, since the mean difference will clearly be close to 0. Within each scenario this can be summarised by its (sample) mean and standard deviation. In respect of the normalised MV per unit premium in the i^{th} scenario denote these by $E_t(|\Delta NMV_t^i|)$ and $S_t(|\Delta NMV_t^i|)$ respectively, where the subscript t on SD_t indicates that the moments are calculated with respect to time. Then we summarise the distribution of $E_t(|\Delta NMV_t^i|)$ by its mean and standard deviation over the 1,000 scenarios, denoted by $E_t^i(|\Delta NMV_t^i|)$ and $S^i(E_t(|\Delta NMV_t^i|))$ respectively.

Mean of the s.d. of $|\Delta NMV_t^i|$. The quantity $S^i(E_t(|\Delta NMV_t^i|))$ sums up the range of variation *between* different scenarios. If we reverse the order of the operations, and denote by $E^i(S_t(|\Delta NMV_t^i|))$ the mean across all 1,000 scenarios of $S_t(|\Delta NMV_t^i|)$, this will sum up the range of variation *within* scenarios.

Normalised MV per UP			
Smoothing	$E^i_t(\|\Delta NMV^i_t\|)$	$S^i(E_t(\|\Delta NMV^i_t\|))$	$E^i(S_t(\|\Delta NMV^i_t\|))$
None (baseline)	0.136	0.049	0.145
Asset smoothing only	0.087	0.032	0.087
MV smoothing only	0.073	0.019	0.038
Asset and MV smoothing	0.065	0.019	0.040
Non-normalised MV per UP			
Smoothing	$E^i_t(\|\Delta MV^i_t\|)$	$S^i(E_t(\|\Delta MV^i_t\|))$	$E^i(S_t(\|\Delta MV^i_t\|))$
None (baseline)	2.306	0.826	2.445
Asset smoothing only	1.427	0.517	1.425
MV smoothing only	1.175	0.305	0.622
Asset and MV smoothing	1.044	0.298	0.633

Table 6.33: Summary of distributions of $|\Delta NMV^i_t|$ and $|\Delta MV^i_t|$ with and without smoothing.

In addition to the summary statistics based upon the *normalised* MV per unit premium, Table 6.33 also shows the corresponding statistics based upon the *non-normalised* MV per unit premium. These will be shown in this and the following table only, for comparison with the normalised figures.

The features of Figures 6.51 – 6.54 are clearly shown in these summary figures. Note in particular the differences in $E^i(S_t(|\Delta NMV^i_t|))$; maturity value smoothing reduces the "within scenario" changes in maturity values by roughly 75%, compared with no smoothing, and by roughly 50%, compared with asset smoothing.

However, large changes in maturity values are not necessarily to be avoided, especially where they might accompany large changes in the inflation index. It is also important to consider the real value of the MV per unit premium. Following Chapter 5, denote by NRV^i_t the normalised real MV per unit premium, at time t in the i^{th} scenario, defined as

$$NRV^i_t = \frac{RV^i_t}{E^i_t(RV^i_t)}$$

where

$$RV^i_t = \frac{MV^i_t}{Q^i_t/Q^i_{t-10}}$$

and Q^i_t is the Retail Price Index at time t in the i^{th} scenario.

Normalised MV per UP			
Smoothing	$E_t^i(\|\Delta NRV_t^i\|)$	$S^i(E_t(\|\Delta NRV_t^i\|))$	$E^i(S_t(\|\Delta NRV_t^i\|))$
None (baseline)	0.149	0.046	0.141
Asset smoothing only	0.102	0.028	0.088
MV smoothing only	0.091	0.023	0.069
Asset and MV smoothing	0.081	0.022	0.063
Non-normalised MV per UP			
Smoothing	$E_t^i(\|\Delta RV_t^i\|)$	$S^i(E_t(\|\Delta RV_t^i\|))$	$E^i(S_t(\|\Delta RV_t^i\|))$
None (baseline)	1.531	0.468	1.447
Asset smoothing only	1.013	0.283	0.879
MV smoothing only	0.901	0.232	0.683
Asset and MV smoothing	0.799	0.218	0.622

Table 6.34: Summary of distributions of $|\Delta NRV_t^i|$ and $|\Delta RV_t^i|$ with and without smoothing.

Then define $E_t^i(|\Delta NRV_t^i|)$, $S^i(E_t(|\Delta NRV_t^i|))$ and $E^i(S_t(|\Delta NRV_t^i|))$ as above. Table 6.34 shows the results.

Comparing Tables 6.33 and 6.34, $E_t^i(|\Delta NRV_t^i|)$ appears to be consistent with $E_t^i(|\Delta RMV_t^i|)$, and $S^i(E_t(|\Delta NRV_t^i|))$ with $S^i(E_t(|\Delta NMV_t^i|))$. However, there are two interesting points.

1. **The real MV per unit premium is less smooth, when scale is allowed for, than the nominal MV per unit premium**. This can be seen by comparing $E_t^i(|\Delta NMV_t^i|)$ with $E_t^i(|\Delta NRV_t^i|)$, but it is not apparent from a comparison of the corresponding non-normalised figures.

2. **The impact of maturity value smoothing on $E^i(S_t(|\Delta NRV_t^i|))$ is much less than on $E^i(S_t(|\Delta NMV_t^i|))$**. Instead of reducing within-scenario variation by 75% as described above, it only reduces it by 50%, and that is not very much more than the asset smoothing on its own. This tends to confirm that part of the variability — perceived as lack of smoothness — of the unsmoothed maturity values is related to inflation, and that maturity value smoothing cuts across that.

So the conclusion that maturity value smoothing is very much stronger than asset smoothing is clearly true in nominal terms, and still true but less decisively so in real terms. It is probably true, however, that a life office in the position of

Figure 6.58: Cumulative proportion of statutory insolvencies under different smoothing methods.

setting out its "bonus philosophy" would have difficulty in persuading the public to pay attention to smoothness in real terms.

6.4 The effect of smoothing on statutory solvency

Figure 6.58 shows the cumulative proportion of simulations in which statutory insolvency has ever occurred, from time $t = 40$ to $t = 70$, without smoothing and under the three smoothing methods.

The asset smoothing *appears* to make practically no difference to the incidence of insolvency, while the maturity value smoothing does make a small difference. The total number of insolvencies over the 30 years, which was 276 without smoothing, *falls* to 266 with asset smoothing only, and rises to 320 with maturity value smoothing only and to 322 with the combined smoothing.

The most interesting feature is **the fall in the number of insolvencies with asset smoothing**. Although slight overall, the effect is not simply due to a different experience in 10 simulations; inspection shows that only 223 simulations result in statutory insolvency both without smoothing and with asset smoothing. This is an example of behaviour which is very similar in its aggregate effects, but shows greater differences on a scenario-by-scenario basis.

6.5 The behaviour of the Bonus Smoothing Account

In this section we examine the behaviour of the BSA under different smoothing schemes. It is helpful to start by setting out the properties which we would like the BSA to possess.

1. The long-term cost of smoothing should be capable of being financed by the normal operation of the smoothing method. That is, the BSA should ideally tend to a stationary distribution.

2. The long term cost of smoothing should be reasonable in relation to the office's resources. We will be satisfied if the ratio BSA/AS is relatively small with high probability.

3. Smoothing should contribute to the control of the cost of meeting the guarantees, by absorbing as much as possible of that cost into the cost of smoothing. This will be achieved if the ratio GCA/AS is significantly reduced by the action of smoothing.

In addition, we should require of the smoothing method itself that it should result in satisfactorily smoothed maturity values — a subjective matter — and that it should not impair solvency.

6.5.1 The cost of guarantees without smoothing

In the absence of smoothing, and given that the maturity benefit is at least 100% of the asset share, the BSA will be the same as the GCA and will represent the accumulated cost of meeting the guarantees; that is, the accumulation of the differences between the asset shares of maturing policies and their guaranteed benefits, in those years when the latter exceeded the former.

Figure 6.59 shows the distribution of the ratio BSA/AS in the absence of smoothing. What is most interesting is how large (in magnitude) it can be; even at the median level it approaches -0.2 — an accumulated cost of about 20% of the

Figure 6.59: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS without smoothing.

asset shares, in other words — while the 5th quantile approaches -0.6. The need to control this cost was discussed in Section 6.1 above.

6.5.2 The BSA with smoothing

Figures 6.60 – 6.62 show the distribution of the ratio BSA/AS under the three different smoothing methods.

To what extent do these results satisfy our requirements?

1. **The quantiles from the median downwards are all drifting slowly down**. For example, with asset smoothing only, there is a distinct downward drift, more marked at the 5th quantile. In the later part of the period, the 75th quantile is approximately 0, so about 75% of offices have a negative BSA at any time.

2. **The ratio BSA/AS shows a marked tendency to spread out**, with the possible exception of positive ratios with asset smoothing only. The fact that the 5th quantile is falling in all cases indicates a lack of long-term stability.

3. **The ratio BSA/AS reaches large magnitudes with reasonably high probability**, especially in the presence of maturity value smoothing. From

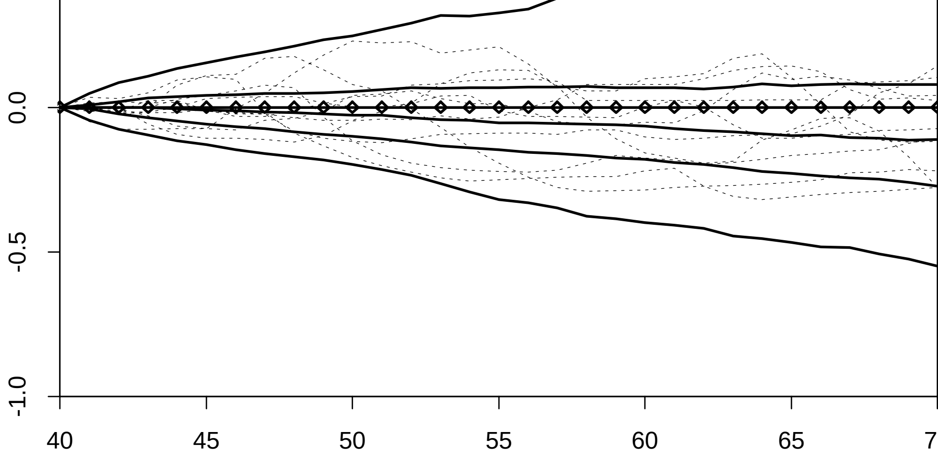


Figure 6.60: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with asset smoothing only.

Figure 6.61: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with MV smoothing only.

Figure 6.62: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with combined asset and MV smoothing.

	GCA/AS		Quantiles of GCA/AS				
Smoothing	Mean	s.d.	5th	25th	50th	75th	95th
None (baseline)	−0.23	0.18	−0.58	−0.30	−0.19	−0.11	−0.02
Asset smoothing only	−0.19	0.19	−0.55	−0.27	−0.15	−0.06	0.00
MV smoothing only	−0.08	0.09	−0.26	−0.12	−0.05	−0.02	0.00
Asset and MV smoothing	−0.09	0.10	−0.29	−0.12	−0.06	−0.02	0.00

Table 6.35: Mean, standard deviation and quantiles of the ratio GCA/AS at time $t = 70$, with and without smoothing.

Figures 6.61 and 6.62, we see that eventually about 10% of offices might have a BSA equal in magnitude to about 50% of the asset shares. This might be felt to represent inadequate control of the cost of smoothing. Acceptable limits are a subjective matter, but most actuaries would probably not be comfortable if the accumulated cost of smoothing represented more than a small part of the total policyholders' assets; certainly in the case of subsidising maturity values. In this regard, even the quartiles in Figures 6.60 – 6.62 might be beyond acceptable limits.

4. **The smoothing has only absorbed part of the cost of the guarantees**. Table 6.35 shows the distribution of the ratio GCA/AS at time $t = 70$.

The part played by the guarantees is worth considering in more detail.

The asset smoothing on its own absorbs only a small part of the cost of the guarantees, and the standard deviation of the ratio GCA/AS is slightly higher. The maturity value smoothing absorbs a much larger part of the cost of guarantees, and reduces considerable the dispersion of the GCA. This explains the greater downward drift of the 5th quantile when there is maturity value smoothing. Intuitively, on those occasions when a large fall in the value of the assets reduces the value of the asset shares suddenly, the asset smoothing on its own, being the weaker method, will more often allow the calculated maturity value to fall below the level of the guarantees; when this happens, the cost of meeting the guarantee falls on the GCA. However, the stronger maturity value smoothing may prevent or delay the smoothed maturity values from falling below the level of the guarantees, and in this case so the cost of subsidising maturity values falls on the BSA rather than on the GCA. The example in Table 6.32 shows what is happening.

This feature is illustrated by Figure 6.63, which shows the number of simulations at each time during the projection period for which the calculated rate of terminal bonus was negative. This is the same as the number of simulations in which the guarantees exceeded the calculated maturity value. The fact that over 25% of the offices are meeting guarantees at any time is itself interesting. The figure shows that asset smoothing has little effect, but that maturity value smoothing reduces by about one-third the incidence of negative terminal bonus.

Maturity value smoothing is much more effective than asset smoothing. It also absorbs a much larger part of the cost of the guarantees, which would be desirable if the cost of smoothing itself was bounded, but this does not appear to be the case. None of the smoothing methods meet our criteria for the reasonable behaviour of the BSA. Asset smoothing on its own is perhaps closest.

In the next sections we will consider ways in which the instability of the long term cost of smoothing might be controlled, with emphasis on the probability of large negative values of the BSA.

Figure 6.63: No. of simulations with negative theoretical terminal bonus, with and without smoothing.

6.5.3 The effect of new business growth

Section 6.1 described how a life office's additional estate — even if negative — might be eliminated in relative terms by new business growth. The same is true of the BSA. The higher the rate of new business growth, the smaller the increments to the BSA relative to the in-force asset shares, and the smaller the changes in the ratio BSA/AS.

In the baseline office the rate of new business growth is equal to the rate of price inflation. Figures 6.64 and 6.65 show the effect on the distribution of the ratio BSA/AS of new business growth 5% and 10% greater than the rate of price inflation, respectively. In both cases the combined smoothing algorithm is used, so these should be compared with Figure 6.62.

Given the parameters we have used for the Wilkie model, 5% real new business growth represents a mean rate a little above the mean rate of return on the assets.

A high rate of new business growth quite effectively limits the growth of the BSA/AS ratio. However, new business growth cannot sensibly be viewed as a means towards that end. Growth, if attainable, would be aimed at for other reasons; it is not likely that containment of the cost of smoothing would or should dictate new business policy. In particular, the consequences of suffering lower rates of new

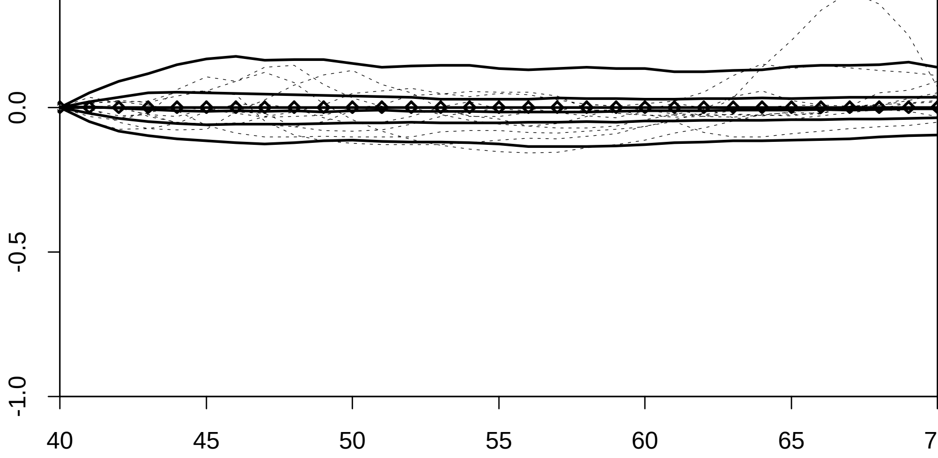


Figure 6.64: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with 5% real new business growth.

Figure 6.65: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with 10% real new business growth.

Figure 6.66: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with -5% real new business growth.

business growth, or even closure, might be serious. Figure 6.66 shows the effect of -5% real new business growth.

The benefits of high new business growth can quickly be lost if that rate of growth cannot be sustained. The simple forms of smoothing which we have used are not sufficiently self-regulating in these circumstances. Therefore other means of controlling the ratio BSA/AS should be investigated.

6.5.4 Feedback from the BSA

A possible approach to the control of the BSA/AS ratio is to bring it into the smoothing algorithm. If the ratio stands at a high level, maturity values could be increased to bring it down — or at least to prevent it from rising further — while if it reaches a low level, maturity values could be cut to bring it back up. A simple method is to multiply the calculated maturity value by the factor (called the "feedback factor")

$$1 + \frac{BSA}{\mathrm{AS}}$$

at some stage in the smoothing process. In the model, this was carried out before the limits on the annual changes in maturity values were applied, and naturally

Figure 6.67: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with feedback and asset smoothing only.

before the calculated maturity value was checked against the guarantees. In other words, feedback was applied between the steps 1 and 2 described in Section 6.2.1. It is arguable that feedback should come in after step 2, since otherwise the effect might be to put feedback into the maturity value and then, by applying the smoothing rule 2, to take it out again. On the other hand, to bring in feedback after all the smoothing rules have been applied might be felt to negate the very purpose of smoothing.

Figures 6.67 — 6.69 show the effect of this feedback in conjunction with asset smoothing, maturity value smoothing and combined smoothing respectively.

1. Comparing Figure 6.67 with Figure 6.60 (asset smoothing only), it is evident that the feedback is only partially effective. It reduces the ratio BSA/AS acceptably for positive values, but it has little effect on the downward drift of the 5th quantile. It is interesting to note, again, that the number of statutory insolvencies falls slightly further, to 263 (from 266 with no feedback), but that only 232 of these cases also give rise to insolvency if there is no feedback, and only 226 if there is no smoothing at all. In all 332 scenarios give rise to insolvency under at least one of these three strategies. It is misleading to assume that a modest change in the number of insolvencies implies modest

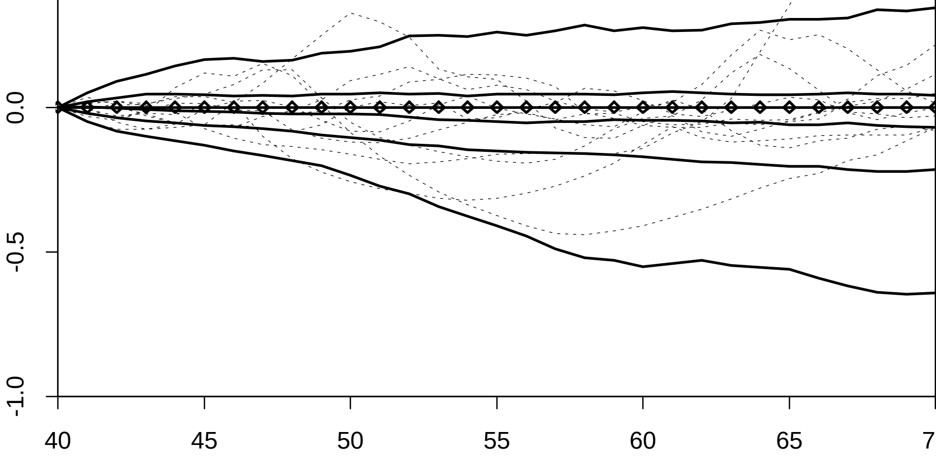


Figure 6.68: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with feedback and MV smoothing only.

Figure 6.69: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with feedback and combined asset and MV smoothing.

	GCA/AS		Quantiles of GCA/AS				
Smoothing	Mean	s.d.	5th	25th	50th	75th	95th
None (baseline)	−0.23	0.18	−0.58	−0.30	−0.19	−0.11	−0.02
Without feedback							
Asset smoothing only	−0.19	0.19	−0.55	−0.27	−0.15	−0.06	0.00
MV smoothing only	−0.08	0.09	−0.26	−0.12	−0.05	−0.02	0.00
Asset and MV smoothing	−0.09	0.10	−0.29	−0.12	−0.06	−0.02	0.00
With feedback							
Asset smoothing only	−0.43	0.47	-1.23	−0.58	−0.30	−0.13	−0.01
MV smoothing only	−0.26	−0.37	−0.77	−0.34	−0.16	−0.05	0.00
Asset and MV smoothing	−0.27	0.38	−0.80	−0.35	−0.17	−0.05	0.00

Table 6.36: Mean, standard deviation and quantiles of the ratio GCA/AS at time $t = 70$, with and without feedback from the ratio BSA/AS.

changes within each scenario.

2. Comparing Figure 6.68 with Figure 6.61 (maturity value smoothing only), again the distribution is compressed for positive values of the ratio, but not for negative values.

3. Comparing Figure 6.69 with Figure 6.62 (combined smoothing), the ratio is slightly *more* dispersed than before below the median.

4. It is also noticeable — especially in Figures 6.69 and 6.62 — that the sample paths are not always closer to zero in the presence of feedback.

The explanation for the failure to control negative values of the ratio BSA/AS lies in the operation of the guarantees. If the guarantees are called upon when the BSA is negative, then the office will be unable to reduce the maturity value by applying the feedback factor. No such restriction will apply when the BSA is positive, so the feedback will be slightly out of balance.

Table 6.36 shows the distribution of the ratio GCA/AS, without smoothing, with smoothing but without feedback, and with both smoothing and feedback (the first two are reproduced from Table 6.35 for convenience).

Feedback has not only increased the cost of the guarantees, compared with the smoothing methods without feedback: it has also increased it compared with the unsmoothed case. More precisely, the distribution of the

Figure 6.70: No. of simulations with negative theoretical terminal bonus, with and without smoothing, in the presence of feedback.

ratio GCA/AS is more dispersed in the presence of feedback, and the lower quantiles are significantly reduced, while the upper quantiles are only slightly increased. In other words, the "worst cases" are worse with feedback than without feedback, or even without smoothing. Figure 6.70 shows the incidence of negative terminal bonus rates when feedback is used. They are larger even than those arising with no smoothing.

It is worth looking in more detail at the ratio BSA/AS with and without feedback. Figure 6.71 shows the distribution, during the whole period $t = 40$ to $t = 70$, of those 100 sample paths which comprised the bottom decile of the distribution of the ratio BSA/AS at time $t = 55$, assuming combined smoothing and no feedback. Figure 6.72 shows the corresponding distribution of those sample paths comprising the top decile at $t = 55$.

There is a qualitative difference between the behaviour of the ratio BSA/AS above and below zero. There is much greater volatility of positive values of the ratio. In neither case is there any strong tendency for the ratio to revert to zero. This does not satisfy our intuitive wish for smoothing to be cost-neutral over time.

Figures 6.73 and 6.74 show the corresponding distributions with combined smoothing and feedback.

The effect of the feedback on positive values of the BSA is clear. In fact by time

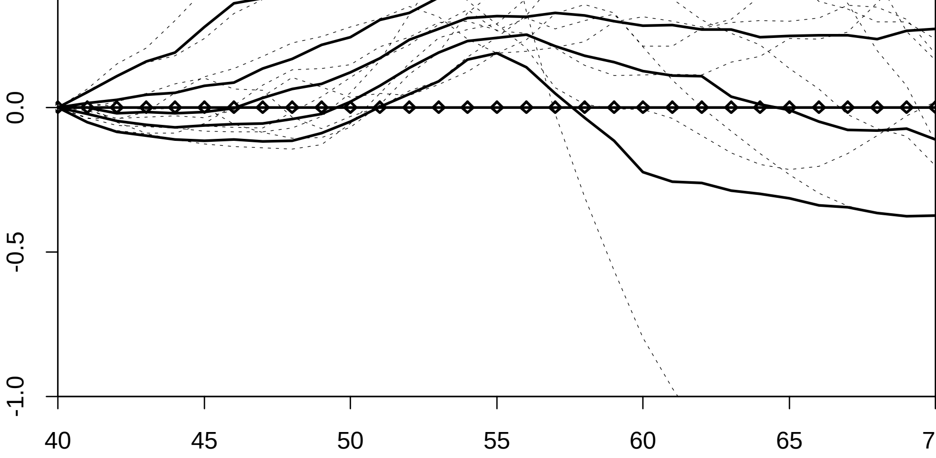


Figure 6.71: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS *given* in the bottom decile at $t = 55$.

Figure 6.72: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS *given* in the top decile at $t = 55$.

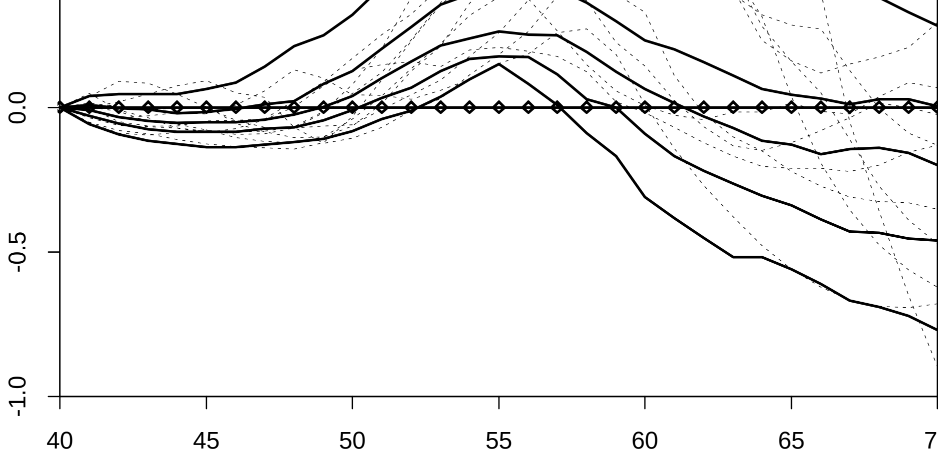


Figure 6.73: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with feedback *given* in the bottom decile at $t = 55$.

Figure 6.74: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with feedback *given* in the top decile at $t = 55$.

Figure 6.75: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with smoothing *given* negative terminal bonus at $t = 55$.

$t = 70$ more than 75% of these cases have negative BSAs, which certainly indicates a reversion towards zero. The same tendency exists, but much less strongly, when the BSA is negative. The reason is, as mentioned above, that the full feedback factor cannot be applied to reduce the maturity payment if the guarantees have been uncovered. The force of this can be gauged from Figure 6.75, which shows the distribution of the ratio BSA/AS for those simulations in which the guarantees were uncovered at time $t = 55$, assuming combined smoothing and feedback. (There were 310 such cases.)

In other words, in 310 simulations the calculated maturity values (after smoothing) of policies maturing at $t = 55$ were less than the guaranteed benefits. This is indicated by a negative theoretical terminal bonus at that time, which is how it is described in the figure. The figure shows that, at time $t = 54$, the BSA in these simulations was almost always negative. Therefore the BSA at time $t = 55$ will also be negative *before* the calculation of maturity values (BSAs shown here are after the payment of claims) and the feedback factor will be less than 1. However, the imposition of the guarantees means that the feedback factor is wholly or partly ineffective.

Table 6.37 summarises the effect on benefits of smoothing with and without feedback. (The figures without smoothing and without feedback are reproduced

Normalised MV per UP			
Smoothing	$E_t^i(\|\Delta NMV_t^i\|)$	$S^i(E_t(\|\Delta NMV_t^i\|))$	$E^i(S_t(\|\Delta NMV_t^i\|))$
No smoothing			
None (baseline)	0.136	0.049	0.145
Without feedback			
Asset smoothing only	0.087	0.032	0.087
MV smoothing only	0.073	0.019	0.038
Asset and MV smoothing	0.065	0.019	0.040
With feedback			
Asset smoothing only	0.095	0.037	0.102
MV smoothing only	0.071	0.023	0.043
Asset and MV smoothing	0.067	0.023	0.043
Normalised real MV per UP			
Smoothing	$E_t^i(\|\Delta NRV_t^i\|)$	$S^i(E_t(\|\Delta NRV_t^i\|))$	$E^i(S_t(\|\Delta NRV_t^i\|))$
No smoothing			
None (baseline)	0.149	0.046	0.141
Without feedback			
Asset smoothing only	0.102	0.028	0.088
MV smoothing only	0.091	0.023	0.069
Asset and MV smoothing	0.081	0.022	0.063
With feedback			
Asset smoothing only	0.111	0.033	0.102
MV smoothing only	0.091	0.027	0.071
Asset and MV smoothing	0.085	0.026	0.068

Table 6.37: Summary of distributions of $|\Delta NMV_t^i|$ and $|\Delta NRV_t^i|$ with and without feedback.

from Tables 6.33 and 6.34 for convenience.)

Feedback has little effect on the overall smoothness if maturity value smoothing is used, not surprisingly since the smoothing is applied after the feedback. It has a greater effect under asset smoothing alone, increasing both $E_t^i(|\Delta NMV_t^i|)$ and $E^i(S_t(|\Delta NMV_t^i|))$ and the corresponding "real" measures. It is intuitively clear that if feedback counters the effect of smoothing on the BSA, it should reduce the smoothness. Table 6.37 suggests that it does so moderately. Figure 6.76 (compare with Figure 6.52) confirms that the smoothing is not much less effective than it is without feedback.

The effect of feedback on statutory solvency is shown in Table 6.38.

The most interesting aspect of this qualitative discussion of the behaviour of the ratio BSA/AS is the apparent difficulty of applying maturity value smoothing in a

Figure 6.76: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the difference ΔMV_t^a with asset smoothing only and feedback.

	No. of statutory insolvencies	
	No feedback	With feedback
Asset smoothing only	266	263
MV smoothing only	320	389
Asset and MV smoothing	322	423

Table 6.38: Effect of feedback on statutory insolvency between $t = 40$ and $t = 55$.

		Quantiles of BSA/AS at $t = 70$				
Charge on asset share	No. Insolv.	5th	25th	50th	75th	95th
0%	276	−0.58	−0.30	−0.19	−0.11	−0.02
1%	261	−0.54	−0.26	−0.15	−0.06	0.07
2%	243	−0.50	−0.23	−0.11	0.00	0.19
3%	232	−0.46	−0.20	−0.08	0.07	0.31
4%	223	−0.43	−0.17	−0.05	0.14	0.43
5%	211	−0.40	−0.14	0.00	0.22	0.56

Table 6.39: Effect of a uniform charge on the asset shares of maturing policies.

way which is effective but not otherwise detrimental. Recall that this method was the commonest of those cited by the respondants to the Tillinghast survey [65].

6.6 Charging the asset shares of maturing policies

Smoothing the maturity values does not, in this simple model, fully absorb the cost of meeting the guarantees. Therefore an explicit charge might be considered for this purpose. A simple form of such a charge is to aim to pay out less than 100% of the asset shares of maturing policies, as assumed by Ross [57], Ross & MacWhirter [58] and others. In this section we consider the effect of such a charge.

6.6.1 Without smoothing

Table 6.39 shows the effect of a levy of between 1% and 5% of the asset shares of maturing policies, in the absence of smoothing. The numbers of statutory insolvencies arising between $t = 40$ and $t = 70$ are shown, and the 5th, 25th, 50th, 75th and 95th quantiles of the ratio BSA/AS at the end of the period.

Note that the charge will not always be levied, because before or after it is applied the calculated maturity value might be less than the guaranteed benefits.

Roughly speaking, a charge of 2% of asset shares results in about 25% of offices having a positive BSA/AS ratio at any time, while a charge of 5% results in about half of offices having a positive BSA/AS ratio at any time. Figure 6.77 shows the distribution of the ratio with a charge of 5%.

The distribution is fairly symmetrical, and it resembles the distribution of the

Figure 6.77: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio $BSA/AS$with a 5% charge on asset shares and no smoothing.

ratio when smoothing is applied. In particular, it is expanding steadily, and the individual sample paths show a fairly stable pattern of remaining positive or negative. Thus on its own, levying a charge on the asset shares seems little better than benefit smoothing.

The measures of smoothness, $E_t^i(|\Delta NMV_t^i|)$ etc., are almost identical to those of the baseline office and are not shown.

The appropriate level of a charge on the asset shares, measured by some criterion such as the median level of the ratio BSA/AS, will vary considerably with the policy term and the office's strategies.

6.6.2 With smoothing

Table 6.40 shows the distribution of the ratio BSA/AS at time $t = 70$ when combined smoothing is used (no feedback) and charges of 2.5% or 5% are levied on the asset shares of maturing policies.

Compared with Table 6.39, the effect of smoothing is to raise the distribution of the ratio BSA/AS, due to the absorption of some, at least, of the cost of meeting the guarantees by the cost of smoothing. **Thus a charge on the asset shares is slightly more effective with smoothing than without**. For example, if the

		Quantiles of BSA/AS at $t = 70$				
Charge on asset share	No. Insolv.	5th	25th	50th	75th	95th
0%	322	-0.50	-0.23	-0.10	0.08	0.56
2.5%	267	-0.40	-0.14	0.03	0.29	0.85
5%	232	-0.33	-0.06	0.17	0.49	1.17

Table 6.40: Effect on the ratio BSA/AS at time $t = 70$ of a uniform charge on the asset shares of maturing policies in conjunction with combined smoothing.

Figure 6.78: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with a 2.5% charge on asset shares and combined smoothing.

aim were to peg the median value of the ratio BSA/AS at 0.0, a charge of 5% would be needed if there were no smoothing, but no more than 2.5% if there were smoothing. For comparison with Figure 6.77, Figure 6.78 shows the distribution of the ratio with combined smoothing and a 2.5% charge on the asset shares.

Again, the measures of smoothness, $E_t^i(|\Delta NMV_t^i|)$ etc., are almost identical to those of the corresponding offices with no charges on the asset shares, and are omitted.

6.6.3 Retrospective feedback from charges on asset shares

Section 6.39 suggested that levying charges on asset shares might improve the distribution of the ratio BSA/AS, but only in the sense of lifting it into a more positive

Figure 6.79: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS with a 10% charge on asset shares, feedback and no smoothing.

region. The dispersion of the ratio was little changed, which meant that the possibility of large negative values was only avoided by accepting large positive values. This was true even in the presence of feedback from the ratio BSA/AS. This seems to be rather a blunt instrument.

An alternative is to levy a large enough charge on the asset shares to satisfy the needs of solvency and stability below the line (if possible), and to apply feedback so that the excess charges (which will be the rule) are returned retrospectively to the policyholders. Not, admittedly, to the same policyholders who provided them, but it is arguable that this is an inevitable consequence of pooling investment risks.

Figure 6.79 shows the effect of a charge of 10% of the asset shares of maturing policies, combined with feedback but with no smoothing. There are several interesting points.

1. The figure shows satisfactory stability in its upper range, appearing to level out and not spreading noticeably.

2. In its lower range the distribution continues to disperse.

However, the combination of charges plus feedback results in maturity values which progress no more smoothly than those with no smoothing or charges at all.

		Quantiles of BSA/AS at $t = 70$				
Charge on asset shares	No. Insolv.	5th	25th	50th	75th	95th
No smoothing						
5% charge	222	−0.37	−0.10	0.00	0.05	0.07
10% charge	205	−0.33	−0.06	0.05	0.12	0.17
Combined smoothing						
5% charge	366	−0.59	−0.18	−0.02	0.11	0.45
10% charge	309	−0.54	−0.13	0.05	0.20	0.58

Table 6.41: Effect on statutory solvency and on the ratio BSA/AS at time $t = 70$ of feedback in conjunction with charges on asset shares.

The various measures of smoothness $E_t^i(|\Delta NMV_t^i|)$ etc., are almost identical to those of the corresponding offices without charges on the asset shares and are not shown.

For comparison, Table 6.41 also shows the effect of using combined smoothing in conjunction with charges on the asset shares and feedback.

6.7 Restrictions on feedback

The examples of feedback in Section 6.5.4 assumed that the benefits were to be enhanced whenever the ratio BSA/AS was positive, and reduced whenever it was negative. This was in accord with the idea of "cost neutral" smoothing. However, the results were less than satisfactory both as regards statutory solvency and the stability of the cost of smoothing.

It might be considered unreasonable, in view of the potential instability of the BSA, to attempt to control it around a mean level of $BSA = 0$. It might instead be reasonable to allow the ratio BSA/AS to rise to some positive level before applying feedback. In this section we consider that and one other possibility.

Delayed feedback The feedback algorithm applies the feedback factor

$$1 + \frac{BSA}{AS}$$

to the calculated maturity value, before applying the limits on the changes in maturity values. It does so whatever the value of the ratio BSA/AS. Thus, benefits are enhanced when the ratio is positive, no matter how small. We will call this “full feedback”.

Instead we might delay the application of feedback until the ratio had reached some positive level, so that the BSA might have a more favourable distribution, especially at the bottom end. We will call this “delayed feedback”, although the “delay” involved is not temporal. Of course, feedback is still applied whenever the ratio is negative. For example, with a threshold of 0.1, we would apply feedback when the ratio BSA/AS was below 0 or above 0.1, but not in between.

Offset feedback We might expect to be too late if we wait for the BSA to become negative before reducing the calculated benefits; that could mean that too often the reduction cannot be effected because of the guarantees. In addition, a threshold of 0.1 or 0.2 (for example) means that in some circumstances a small change in conditions between one year and the next will result in a jump of 10% or 20% in the benefits, which is possibly inequitable (and certainly not smooth).

Therefore we might consider moderating feedback by *offsetting* the ratio used to calculate the feedback factor. For example, with an offset of 0.1, we would apply the factor

$$0.9 + \frac{BSA}{AS}$$

so that calculated benefits would be reduced when the ratio BSA/AS was below 1.1, and enhanced above that level.

Note that offsetting the feedback factor is effectively imposing a charge on the asset shares of maturing policies, so it has a lot in common with the charging strategies of Section 6.6.3.

Inspection shows that the visible effect on smoothing is small (and the measures $E_t^i(|\Delta NMV_t^i|)$ etc. are almost identical to those of the corresponding office with full feedback) so we will not show any figures, but Table 6.42 shows the effect on the distribution of the ratio BSA/AS at time $t = 70$ of:

		Quantiles of BSA/AS at $t = 70$				
Limits on feedback	No. Insolv.	5th	25th	50th	75th	95th
Full feedback						
$BSA/AS < 0 \quad \text{or} > 0.0$	423	−0.64	−0.22	−0.07	0.04	0.34
Modified feedback						
$BSA/AS < 0 \quad \text{or} > 0.1$	405	−0.64	−0.21	−0.05	0.06	0.37
$BSA/AS < 0 \quad \text{or} > 0.2$	361	−0.59	−0.17	−0.03	0.10	0.41
BSA/AS offset by 0.1	319	−0.57	−0.15	0.04	0.18	0.56
BSA/AS offset by 0.1 and offset $BSA/AS < 0 \quad \text{or} > 0.1$	304	−0.55	−0.13	0.05	0.19	0.57

Table 6.42: Effect on statutory solvency and on the ratio BSA/AS at time $t = 70$ of modified feedback, with combined smoothing.

	GCA/AS		Quantiles of GCA/AS				
Limits on feedback	Mean	s.d.	5th	25th	50th	75th	95th
Full feedback							
None (baseline)	−0.27	0.38	−0.80	−0.35	−0.17	−0.05	0.00
Modified feedback							
$BSA/AS < 0 \quad \text{or} > 0.1$	−0.26	0.38	−0.78	−0.33	−0.16	−0.05	0.00
$BSA/AS < 0 \quad \text{or} > 0.2$	−0.26	0.39	−0.78	−0.33	−0.16	−0.04	0.00
BSA/AS offset by 0.1	−0.33	0.41	−1.00	−0.47	−0.23	−0.06	0.00
BSA/AS offset by 0.1 and offset $BSA/AS < 0 \quad \text{or} > 0.1$	−0.33	0.41	−0.97	−0.47	−0.23	−0.06	0.00

Table 6.43: Mean, standard deviation and quantiles of the ratio GCA/AS at time $t = 70$, with combined smoothing and modified feedback.

1. delaying feedback until the ratio reaches 0.1;
2. delaying feedback until the ratio reaches 0.2;
3. offsetting the ratio BSA/AS by 0.1 as described above;
4. offsetting the ratio BSA/AS by 0.1 as described above *and* applying feedback only when the *offset* ratio is less than 0 or greater than 0.1 (a combination of delayed and offset feedback).

Table 6.43 shows the distribution of the ratio GCA/AS at time $t = 70$.

Comparing these four strategies, we see that they have similar effects but to different degrees.

1. They all improve statutory solvency. All these strategies retain assets which are used to subsidise maturity payments under the full feedback. Therefore

the statutory solvency should be improved, although this is not certain because there might be second order effects arising from the asset allocation. Offset feedback retains more assets in the BSA than delayed feedback, with a correspondingly larger improvement in solvency.

2. The distribution of the ratio BSA/AS is improved. Offset feedback is more effective than delayed feedback, for the same reasons as above, but what is most interesting is that the improvements are small at the 5th and 25th quantiles.

3. Under delayed feedback, the distribution of the ratio GCA/AS is improved, but by a negligible amount. Under offset feedback, the distribution of the ratio GCA/AS is *lower* than it is with full feedback. At first sight this is surprising, but it has a simple explanation. Applying a feedback factor of less than 1.0 increases the chances of uncovering the guarantees; whenever this happens, the GCA is called upon to subsidise the benefits. Offset feedback decreases all feedback factors, while delayed feedback only decreases those between the specified limits, so the depletion of the GCA will be greater under offset feedback — in effect it is absorbed less into the cost of smoothing.

   The extent to which guarantees are uncovered more often with modified feedback can be gauged from Table 6.44. This shows the numbers of simulations with full feedback and with modified feedback under which the stated negative theoretical terminal bonus rates were ever attained. (Recall that a negative theoretical terminal bonus rate is equivalent to the uncovering of the guarantees.) The table shows that offset feedback encounters theoretical terminal bonus rates in the range $-20\% - -30\%$ much more frequently. Inspection also shows that at most times more offices have negative terminal bonus under offset feedback than under full feedback.

None of these modifications of full feedback have remedied, to any extent, the defects of smoothing with or without feedback. However, Table 6.43 in particular shows where the problem lies; attempts to reduce maturity values to control the cost of smoothing simply transfer the adverse experience from the BSA to the GCA.

A further observation is that such methods as we have studied make the processes

	Theoretical Terminal Bonus					
Limits on feedback	0%	-10%	-20%	-30%	-40%	-50%
Full feedback						
None (baseline)	933	777	488	178	71	37
Modified feedback						
$BSA/AS < 0$ or > 0.1	920	764	461	165	67	38
$BSA/AS < 0$ or > 0.2	911	748	447	159	59	33
BSA/AS offset by 0.1	924	790	593	281	87	42
BSA/AS offset by 0.1 and offset $BSA/AS < 0$ or > 0.1	919	778	585	273	87	42

Table 6.44: Numbers of simulations in which the theoretical terminal bonus rates ever fell below the levels shown.

of *granting* guarantees — that is, setting premium rates and declaring reversionary bonuses — and the process of *charging for* guarantees quite separate. Guarantees are granted prospectively, to be paid for retrospectively. Such an approach is consistent with recent practice in the U.K. (see Chapter 1) but here it seems to lead to difficulties.

6.8 How robust is the cost of smoothing?

The whole idea of with-profits business smoothing out the peaks and troughs of the stock market depends on the stock market behaving in that way. Any smoothing strategy is vulnerable to changes in conditions which do not conform to the expected pattern. For example, what happens if a life office subsidises maturity payments during a downswing which turns out to be a permanent reduction in rates of return? In this section we look at a few possibilities.

In addition to perverse behaviour of the investment markets (or perhaps behaviour not in line with perverse expectations!) a smoothing strategy might not give exactly the expected results because of the practicalities of implementing it. For example, if assets are smoothed, what happens if the smoothed values are biased? We also consider some of these aspects.

In this section, the combined smoothing algorithm with no feedback was used unless otherwise stated.

6.8.1 Robustness to changes in financial conditions

The results given so far show that the Wilkie asset model provides financial conditions which permit asset smoothing and maturity value smoothing which, if not stable, are at least not biased to any significant extent. However, the Wilkie model posits a stable underlying structure of first and second moments of the quantities modelled. This means, for example, that the force of inflation always reverts to the same mean, and always suffers random shocks with the same variance.

It is arguable that large-scale or persistent changes in financial conditions do not fit into this framework, and that they might instead be modelled by assuming that the underlying structure of (say) first and second moments can change from time to time. In between such changes, a Wilkie-type model might be assumed to be adequate. Although such time series models are well known (see, for example, Geoghegan *et al* [26]) they have not been applied for actuarial use.

They do, however, suggest a simple method of investigating a given shift in financial conditions, by changing parameters in the Wilkie model. A simple example would be to change the parameter QMU, which represents the mean force of inflation, from 0.05 to 0.25 (say) to represent a move to a more inflationary economy. Three questions about such changes are:

1. How long does it take the smoothing algorithm to catch up with a permanent shift in conditions?
2. How long does it take the smoothing algorithm to bounce back from a temporary but large scale shift in conditions?
3. What is the potential for losses (or profits) before the smoothing algorithm does catch up?

Some more simple minded smoothing algorithms — for example, smoothing the assets by discounting future income at a fixed yield — would never catch up. On this ground alone, any such method is bound to up-date the assumptions dynamically in some way. The 5-year moving average yields which we have used, even if they ought not to be regarded as estimates of mean future yields, do at least keep the office in touch with changes in the general level of yields.

Figure 6.80: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS, $YMU = 0.05$ from $t = 40$.

We experimented with permanent shifts in the model parameters representing mean values, namely

QMU	the mean force of inflation
YMU	the mean gross dividend yield
DMU	the mean real rate of dividend growth
CMU	the mean real gross gilt yield

Of these, only changes in YMU had significant effects. The pattern of BSA/AS ratios was little affected by (moderate) changes in the others. This is interesting in the case of inflation, since that influences all the other elements of the Wilkie model, but perhaps should not be surprising. The influence of inflation on other variables is exponentially lagged in the Wilkie model, which itself imparts an element of smoothing, and the influence of the limits on annual changes in relative maturity values might still predominate in the smoothing algorithm itself. CMU had limited influence anyway, since the level of gilt investment in the model was low.

Figure 6.80 shows the effect on the BSA/AS ratio of a change in the mean gross dividend yield YMU at time $t = 40$, from 0.04 to 0.05.

Figure 6.81: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS, $QMU = 0.25$ from $t = 50$ to $t = 54$

The immediate effect was to cause equities to be significantly overvalued by the office for a short time, while the moving average dividend yield caught up. The mean reduction in the BSA during the 30 years after the change, was about 5% of total asset shares. Clearly the valuation of equities using a low dividend yield leaves the office vulnerable to comparatively small shifts in that yield.

It is noticeable in this figure, as in some others, that once the median BSA/AS ratio fell below zero it tended to drift further down.

While a permanent shift in the level of inflation had a comparatively mild effect, a burst of very high inflation or deflation had a noticeable impact. Figure 6.81 shows the effects on the BSA/AS ratio of a change in the mean force of inflation QMU from 0.05 to 0.25 between times $t = 45$ and $t = 50$, reverting to 0.05 at $t = 50$. The most striking effect is the great increase in the dispersion of the ratio, particularly upwards. This might be explained by the imposition of limits on the annual changes in maturity values; as one effect of high inflation in the Wilkie model is to drive up equity prices, these limits might result in considerable increases in the BSA.

A short period of negative inflation has a similar but opposite effect.

These experiments are not extensive enough to lead to general conclusions, but they do suggest that the stability of quantities such as the ratio BSA/AS might depend on the structural stability of the asset model being used. If the market is

Figure 6.82: 5th, 25th, 50th, 75th, 95th percentiles, and 10 sample paths, of the ratio BSA/AS, net dividend yield 0.1% too low.

more realistically modelled by less stable models, it is not clear that our smoothing algorithms will be suitable.

6.8.2 Robustness to errors in the asset valuation

The 5-year moving average component of the smoothing algorithm represents, perhaps, a courageous approach to bonus policy, at least when actuarial values exceed market values. It is plausible that such a strategy depends on an unbiased actuarial valuation of the assets. Where the main parameter in the valuation is a dividend yield, the risk from small errors might be considerable.

Figure 6.82 shows the effect on the ratio BSA/AS of using a net dividend yield 0.1% too low in the cash-flow valuation of equity assets. That is, a dividend yield 0.1% below the 5-year moving average used before. Such a small error might be difficult to detect in practice, especially since there might well be commercial pressure on the actuary to find reasons for placing a slightly higher value on the assets — 0.1% sounds very trivial!

Even so small an error or bias has a very significant effect on the office's strength. This is also reflected in the number of statutory insolvencies — up from 276 to 432 during the 30 years — but not significantly in the relative payouts. Table 6.45

	Mean MV per UP deflated		Mean payout ratio	
Smoothing	Mean	s.d.	Mean	s.d.
None (baseline)	10.264	1.716	1.000	0.000
Asset valn. yield 0.1% too low	9.910	1.665	0.990	0.040
Asset valn. yield 0.2% too low	9.973	1.681	0.998	0.050

Table 6.45: Comparison of maturity values deflated by Retail Price Inflation, and payout ratios with respect to the baseline projection.

summarises the effect on benefits of the perturbations described here. The first two columns show the means and standard deviations of the mean maturity value per unit premium in each simulation, deflated to allow for inflation. The last pair of columns show the mean and standard deviation of the mean values of the ratios $\frac{MV_t^c}{MV_t}$ in each simulation.

These figures suggest that smoothing methods might sometimes be sensitive to the assumptions, and lead to errors whose detection would not be easy.

6.9 Conclusions

1. The relative cost of smoothing (measured by the ratio BSA/AS) is not necessarily stable over time; the accumulated historic costs of smoothing can be considerable.

2. The smoothing of maturity values appeared to be more effective than the smoothing of asset values, and absorbed a larger part of the cost of the guarantees, but the accumulated cost of smoothing was considerably less stable.

3. The rate of new business growth has a considerable impact on the relative cost of smoothing; an office with declining new business might be limited in its ability to smooth maturity values.

4. Applying a simple form of feedback to control the level of the ratio BSA/AS was not successful, because the cost of the guarantees prevented the feedback from being applied in adverse circumstances.

5. Combinations of explicit charges on the asset shares, smoothing and feedback were only partially successful in controlling the ratio BSA/AS. Any method which limits the *growth* of the ratio BSA/AS appears to be vulnerable to the action of the guarantees.

6. The cost of smoothing depends on the stability of the asset model, particularly if the assets are revalued. The sensitivity of equity values to dividend yields is a particular risk.

Chapter 7

Adequacy *versus* solvency

In this chapter we will use cash-flow projections to study the effectiveness of traditional solvency valuations. The asset model provides 1,000 scenarios, under each of which the future cash-flows of the model office are projected for 20 years. In each future year we can carry out a solvency valuation (as in Chapter 4) and so study that measure of financial strength. Then we ask the question: given that the office was insolvent at time t in the i^{th} scenario, what happens if we actually run off the in-force business from that time, under that scenario? Implicitly we compare the outcomes under (i) the model underlying the solvency valuation, and (ii) *one* scenario generated by the Wilkie asset model. Over a large number of scenarios, this gives a picture of the effectiveness of the solvency valuation.

Further, given the projections of cash-flows in each scenario, we can see whether or not *different* valuation methods and bases give different results. In particular, we can compare prospective policy values and asset shares as measures of financial strength.

7.1 Introduction to "adequacy"

7.1.1 PRE and modelling

In Section 1.2 "Policyholders' Reasonable Expectations" (PRE) was discussed. Although not as well defined as other factors which influence life office management, it cannot be left out of modelling entirely.

The essence of PRE is in the constraints which it places upon management. Explicitly or implicitly, life offices give their policyholders some expectations about how they will conduct their business. They should therefore, as far as possible, conduct their business accordingly.

The words "as far as possible" matter here, especially as they affect modelling. There might be circumstances under which some departure from expected conduct would be consistent with PRE.

1. The environment (economic, political or otherwise) in which the life office operates might change for reasons outside its control. For example, if the Government decided to raise taxes from long term savers by increasing the taxation of life assurance funds, reasonable expectations would surely change.

2. The life office might be forced into a position where it could not satisfy the reasonable expectations of all its policyholders. In the absence of rescue by Government or the industry, PRE as it was *a priori* would be rendered meaningless, but the office would have to conduct its subsequent affairs (most likely following closure) in accordance with some notion of PRE *a posteriori*. In this context, it is reasonable to assume that the policyholders expect the office to remain statutorily solvent. Therefore statutory insolvency is itself an example of departure from PRE.

PRE could be interpreted to mean that the office implements strategies of which policyholders are made reasonably aware. The strategies might be rigid ("we will always invest 80% of our fund in equities") or flexible ("we will invest our fund as we see fit"). It is reasonable for the office to do anything consistent with the strategies which its policyholders believe it to be following. For example, the office with the rigid investment rule and the office with investment freedom might both, at some time, have 80% of their funds in equities. All else being equal, they would be indistinguishable under any statutory valuation test; both would be solvent, or both insolvent. However, the second office could move away from this investment position when necessary, while the first office could not. The supervisor, who might have views on the volatility of equities, might not think that both offices were

Figure 7.83: Comparison of adequacy and solvency — Example 1.

equally adequate. The above is an extreme example — no office would hobble itself so completely as the first office above — but it is a useful point of view because it can be translated into modelling assumptions.

A test of adequacy would be to ask whether or not the office could be closed to new business, and could continue to implement its intended strategies while its existing business was run off. A closed fund lacks the resilience of a continuing fund, so it would be fair to adopt more cautious strategies after closure — PRE *a posteriori* — but it would not be fair to abandon them completely; this would be a change which policyholders would not reasonably expect.

For example, consider Figure 7.83. The top plot represents an A/L ratio for a life office writing 10-year business. The progress of the ratio is shown over 20 years, and it falls below 1.0 in years 8 – 10. Therefore the office is briefly insolvent.

The bottom plot represents the assets remaining at the end of a run-off period (suitably scaled to appear on the same figure as the A/L ratios). That is, the value plotted at time $t = 10$ represents the assets at time $t = 10$ after closing the office at time $t = 0$; the value plotted at time $t = 11$ represents the assets at time $t = 11$ after closing the office at time $t = 1$, and so on. The residual assets are negative if the run-off ends at times $t = 19$ or $t = 20$, or in other words if the office is closed at times $t = 9$ or $t = 10$.

Figure 7.84: Comparison of adequacy and solvency — Example 2.

In this example, there is close agreement between the outcomes of the run-offs and the results of the valuation. The A/L ratio is a reasonably good test of when the office would fail to meet its current liabilities. The agreement is not perfect, but it is reasonable.

Now consider Figure 7.84. This time the pattern of residual assets after the run-off is different. There is a deficit in nearly every year, the only exceptions being upon closure in two of the years when the A/L ratio indicated insolvency. This time there is very poor agreement between the outcomes of the run-offs and the results of the valuation. In nearly every case in which the valuation indicated failure, the run-off proceeded satisfactorily.

7.1.2 A definition of adequacy

Define an office to be "adequate" *at a given time* if, upon being closed *at that time*, and not thereafter departing too far from its intended strategies, it has surplus assets after the last policy has expired. If instead there is a deficit, the office is "inadequate" *at the given time of closure*.

Thus in the example of Figure 7.83, the office was found to be adequate at times 0 – 8 and 11 – 20. In the example of Figure 7.84, the office was found to be adequate only at times 9 and 10.

Adequacy as defined here depends on the projected future, and on the time of closure. Given the same projection of future conditions, an office may be adequate at some times and inadequate at other times.

For testing adequacy by modelling, it is necessary to decide how the asset allocation, bonus, benefit smoothing and possibly other strategies might be modified after closure, while meeting PRE *a posteriori*. These decisions are subjective, but the effect of different strategies on adequacy and solvency might be revealing.

Our first approach will be to take the baseline model office of Chapter 4, and to modify the asset allocation and bonus strategies after closure. Then we will use this to compare adequacy and a number of traditional solvency valuations. Then in Chapter 8 we will look at the effect of some changes in strategy. The computer time needed to carry out a single investigation of adequacy is very large, since the program must be run allowing for the office to close to new business in each year of some chosen time horizon, so we will not be able to consider such an extensive set of strategies as we did in Chapter 5.

7.1.3 Post-closure strategies in the baseline model

The nature of the bonus and asset allocation strategies does not change after closure, but the office implements them with more caution.

Bonus strategy

Recall from Chapter 4 that reversionary bonuses were set by projecting asset shares and future premiums at (effectively) a 5-year moving average gilt yield, aiming for a terminal bonus rate of 25% of the projected guaranteed benefits at maturity. This was done for each generation of policies separately, and the bonus declared was a weighted average of the results. Changes in the rates of bonus were limited to $+25\%$ or -20%.

After closure, the bonus strategy is modified in two ways; the terminal bonus target is increased to 50% instead of 25%, and bonus rates are allowed to fall by up to 40% in a year. Thus bonus declarations become more cautious, and larger bonus cuts are tolerated in the absence of marketing considerations.

Asset allocation strategy

Recall that the baseline asset allocation strategy is to invest 100% of the fund in equities, unless the ratio A/L_2 would fall below 1.0 in which case sufficient assets are switched into gilts to avoid this happening, if that is possible.

After closure, the investment strategy is modified by investing a maximum of 50% of the fund in equities, subject to the same switching algorithm should the ratio A/L_2 fall below 1.0.

7.2 Adequacy of the baseline office

In previous Chapters, projections were made over a 30-year period, from $t = 40$ to $t = 70$. In computational terms, 30,000 model-years of computer time was needed for each set of simulations.

In order to test adequacy over a time horizon of n years (on the basis of 1,000 stochastic simulations),

$$1000(\frac{1}{2}n(n+1) + rn)$$

model-years of computer time are needed, where r is the number of years required to run off the liabilities after closure — in other words run-time increases with the square of the time horizon. This is because one set of simulations must be made assuming closure at the end of the first year, which takes $1000(r+1)$ model-years; another set assuming closure at the end of the second year, which takes $1000(r+2)$ model-years, …, finally a set assuming closure after n years which takes $1000(r+n)$ model-years. Therefore a 30-year time horizon would entail 765,000 model-years of computation.

Using a 20-year time horizon cuts the computation to 410,000 model-years per 1,000 simulations. In practice this allows a reasonable number of possibilities to be explored. Therefore, in this section, adequacy is investigated from time $t = 41$ to $t = 60$.

Figure 7.85 shows the number of offices inadequate each year from $t = 41$ to $t = 60$ (dotted line) and the cumulative number out of 1,000 which have ever been inadequate. These are shown also in Table 7.46

Figure 7.85: Inadequacy in the baseline office, from $t = 41$ to $t = 60$.

No. of inadequate offices					
Time	Current	Cumulative	Time	Current	Cumulative
41	8	8	51	76	80
42	8	8	52	84	91
43	10	10	53	89	96
44	20	20	54	96	104
45	24	24	55	103	112
46	30	31	56	116	125
47	36	38	57	126	136
48	50	53	58	130	143
49	53	58	59	137	150
50	61	65	60	143	157

Table 7.46: Totals and cumulative totals of inadequacies in the baseline office.

There is a steady increase in the number of inadequate offices. Some offices suffer a temporary spell of inadequacy, and therefore the cumulative number inadequate usually exceeds the number inadequate in any given year, but in the the majority of cases an office which becomes inadequate remains inadequate.

Recall that Ross & McWhirter [58] compared statutory solvency with cash-flow adequacy by allowing an office to remain open for 40 years and then running off its liabilities over a further 30 years. In terms of adequacy as defined here, they investigated its adequacy at time 40 only. An interesting question, especially in view of the computational burden of testing adequacy at every time point, is whether some such "sampling" approach — testing adequacy only at a limited number of time points — might give satisfactory results. In the example here, the tentative conclusion might be "yes", since so few offices ever recover from an inadequate position. However, this feature is influenced by the payment of a minimum of 100% of unsmoothed asset shares to maturing policies, so such a conclusion would be premature.

We now consider how adequacy might be used to test the effectiveness of a traditional valuation. In the first instance we will use the U.K. statutory minimum valuation.

7.3 Adequacy *versus* statutory solvency

Given two different models of asset income, what happens if we *compare* the effects of these different models on our assessment of an office's strength? To consider two extreme outcomes:

1. We might find out that the two models led to similar results, and usually agreed on whether or not an office was capable of meeting its liabilities. Then the choice between the models might rest mainly on practicalities.

2. Or, we might find out that the two models led to different outcomes, and often disagreed on whether or not an office was capable of meeting its liabilities. Then we might be forced to consider whether or not one of the models was significantly less realistic than the other.

Figure 7.86: Inadequacy and statutory insolvency in the baseline office, from $t = 41$ to $t = 60$.

We might at least learn from the comparison if the model we choose to use has inherent limitations.

A second way in which this comparison might be illuminating is in comparing different assumptions within the framework of a traditional valuation model. For example, consider the historic debate in the U.K. between net premium and gross premium valuations, or the practice in most E.C. territories other than the U.K. of valuing on the premium basis. Do these different approaches yield different results when measured against an alternative model?

Similarly, it is of interest to observe how the comparison between different models is affected by different management strategies. We will take up these other questions in Chapter 8. First we compare adequacy in the baseline office with statutory solvency.

7.3.1 Incidence of inadequacy and statutory insolvency

Figure 7.86 shows the numbers of simulations in which the baseline office was inadequate and statutorily insolvent in each year, and also the cumulative totals. For insolvencies, these are also given in Table 7.47.

This comparison has some striking features.

No. of statutorily insolvent offices					
Time	Current	Cumulative	Time	Current	Cumulative
41	8	8	51	34	112
42	16	19	52	38	124
43	17	25	53	42	137
44	20	37	54	38	143
45	25	52	55	37	153
46	21	57	56	34	160
47	23	66	57	39	171
48	29	81	58	36	178
49	28	93	59	37	184
50	32	104	60	39	195

Table 7.47: Totals and cumulative totals of statutory insolvencies in baseline office.

1. The cumulative total of offices statutorily insolvent during the 20 years is 195, compared with 157 which are inadequate at some time. This difference is not particularly large.

2. **The number of statutorily insolvent offices in each year is, however, very much lower than the number of inadequate offices**. The latter is close to the cumulative total of inadequate offices, as observed above. Statutory insolvency appears to be much less persistent than inadequacy.

A pragmatic view might be that the difference between 195 statutory insolvencies and 157 inadequacies is not so great, given 1,000 simulations of 10-year policies over 20 years, and so the two different tests are giving similar answers. However, the results above do not show whether or not the statutory insolvencies and inadequacies coincide.

7.3.2 Coincidence of inadequacy and statutory insolvency

Figure 7.87 shows the numbers of offices at each time during the 20 years which were inadequate, or statutorily insolvent, or both. The same results are shown in Table 7.48.

Several features emerge from this comparison.

1. During the first few years, the number of statutory insolvencies is higher than the number of inadequacies, although both are small.

Figure 7.87: Coincidence of inadequacy and statutory insolvency in the baseline office, from $t = 41$ to $t = 60$.

No. of statutorily insolvent and/or inadequate offices			
Time	Insolvent and Inadequate	Solvent but Inadequate	Insolvent but Adequate
41	4	4	4
42	8	0	8
43	8	2	9
44	8	12	12
45	15	9	10
46	16	14	5
47	17	19	6
48	19	31	10
49	22	31	6
50	23	38	9
51	26	50	8
52	30	54	8
53	35	54	7
54	33	63	5
55	30	73	7
56	30	86	4
57	31	95	8
58	30	100	6
59	34	103	3
60	30	113	9

Table 7.48: Coincidence of inadequacy and statutory insolvency in the baseline office, from $t = 41$ to $t = 60$.

2. In later years, the majority of statutorily insolvent offices are also inadequate (roughly 70% – 80%).

3. **In later years, the majority of inadequate offices are solvent, and the proportion is steadily increasing**.

Of the 157 offices which were inadequate at some time, 131 were also statutorily insolvent at some time. In this sense the statutory valuation test made only 26 "Type II" errors in total, by incorrectly failing an adequate office. On the other hand, it made 64 "Type I" errors, by passing an inadequate office. However, Table 7.47 shows that it would be incorrect to ascribe even this level of efficiency to the statutory valuation as a test of adequacy, since there is much less coincidence between adequacy and statutory solvency *at any particular time*.

7.3.3 The timing of closure

If we take as a working assumption that an office will be closed to new business on the first occurrence of statutory insolvency, we can look at when such closures occur, compared with times of adequacy or inadequacy. Of course this assumption is simplistic, since alternatives to closure might be available, but the threat of closure is not a remote one to an office whose solvency is in doubt.

Confining our attention to the 131 cases in which both inadequacy and statutory insolvency occurred, how close were the times at which each office first became insolvent, and first became inadequate? We might regard statutory solvency as a good test of adequacy if insolvency tended to occur just before inadequacy, so providing a timely warning. Failing that, we might be satisfied if insolvency tended to occur within one or two years of inadequacy.

Table 7.49 shows that in more than half of the cases, insolvency *followed* inadequacy by one year; that in 93 of the 131 cases, insolvency occurred within one year either way of inadequacy, and in 102 cases within 2 years.

It must be borne in mind that these results are incomplete, because some of those offices which were closed despite being adequate during the 20 years of investigation might have become inadequate had the investigation been extended beyond 20 years.

Time first inadequate *minus* time first insolvent	Number of offices
5 years or more	13
4 years	1
3 years	6
2 years	7
1 year	7
0 year	16
-1 year	70
-2 years	2
-3 years	2
-4 years	0
-5 years or less	7

Table 7.49: Comparison of times by which insolvency preceded inadequacy in the baseline office.

			Quantiles				
	Mean	s.d.	5th	25th	50th	75th	95th
All simulations at time $t = 50$	1.400	0.333	1.030	1.159	1.305	1.589	2.044
When first inadequate	1.292	0.237	0.978	1.129	1.253	1.418	1.708
1 year after first inadequate	1.213	0.187	0.943	1.062	1.242	1.304	1.524
2 years after first inadequate	0.981	0.141	0.809	0.896	0.962	1.041	1.197

Table 7.50: Distribution of ratio A/L_1 following the first occurrence of inadequacy, compared with distribution in all 1,000 scenarios at time $t = 50$.

If there were any such offices, the time to their first inadequacy has been censored, and they have not been counted in Table 7.49.

Figure 7.88 shows the distribution of the ratio A/L_1 at the time of inadequacy and in the 10 years following inadequacy, for the 157 offices which were ever inadequate. This can be compared with Figure 4.9. For convenience, some statistics comparing the distribution of the ratio A/L_1 within 2 years of inadequacy with the distribution in all 1,000 scenarios at time $t = 50$ (arbitrarily chosen) are set out in Table 7.50.

The lower quantiles of the ratio A/L_1 at the onset of inadequacy are slightly below those of all the simulations at time $t = 50$, but only slightly. It is not surprising that they should be lower, since the smaller the amount of assets, the

Figure 7.88: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the ratio A/L_1 following the first occurrence of inadequacy.

more vulnerable an office should be to inadequacy, all else being equal. But in practical terms the difference is small. The most striking feature is the fall in the ratio A/L_1 during the 2 years following the onset of inadequacy. The distribution of the ratio A/L_1 does seem to be linked to the onset of inadequacy

The conclusion is that, for this particular office, statutory insolvency is moderately effective at detecting the onset of inadequacy — 102 out of 195 first-time insolvencies were within 2 years of the onset of inadequacy — but that inadequacy and statutory insolvency otherwise behave in different ways.

7.3.4 Varying the A/L_1 ratio in the solvency valuation

Under the U.K. statutory minimum valuation basis, the link between the value of assets and value of liabilities might be somewhat tenuous, as described in Section 2.4. It follows that there is no particular logic in setting the test at the "$A/L_1 < 1.0$" level - that is, closing the office when the market value of the assets is less than the value of the liabilities. Indeed, the E.C. solvency margins represent an adjustment to the level of the A/L_1 ratio at which action will be taken.

Table 7.51 shows the effectiveness of the valuation test applied at different levels. (The last column in the table is described in Section 7.3.5.)

	Closure if A/L_1 falls below					
	0.90	0.95	1.00	1.05	1.10	0.981
Inadequate offices	157	157	157	157	157	157
Total closures	46	96	195	424	677	157
Correct closures	46	89	131	151	156	118
Incorrect closures	0	7	64	273	521	39
Missed closures	111	68	26	6	1	39

Table 7.51: Comparison of adequacy and solvency over 20 years if the statutory minimum valuation is applied at different levels.

1. A "correct" closure is defined as an office closed by the valuation test which was inadequate at some time, though not necessarily when closed.

2. An "incorrect" closure is any other office closed by the valuation test.

3. A "missed" closure is an office which was inadequate at least once but which never failed the valuation test.

The most striking feature of Table 7.51 is the extreme sensitivity of the results to the level of the A/L_1 ratio which triggers closure. Bearing in mind that the E.C. solvency margin — not included in the A/L_1 ratio here — is 4% of the mathematical reserve (since mortality is ignored in the model) it can be seen that its impact is larger than might have been expected.

Changes of $\pm 10\%$ in the level of A/L_1 ratio used in the test are sufficient to swing from all "Type I" errors (missed closures) to almost all "Type II" errors (incorrect closures). Such changes are not large, and we might expect different methods or bases of valuation to have effects at least as great.

To see this in more detail, Figure 7.89 shows the numbers of correct, incorrect and missed closures for values of the A/L_1 ratio between 0.90 and 1.10 in steps of 0.01.

This illustrates the sensitivity clearly. In addition, there appears to be a marked upward swing in the total number of closures (almost all "incorrect") for A/L_1 ratios above about 1.04 — the level of the E.C. solvency margin.

Again it must be borne in mind that these results have been censored by the time horizon of 20 years; an incorrect closure might be a correct closure given a longer time horizon, and so on.

Figure 7.89: Accuracy of statutory minimum valuation basis using A/L_1 ratios between 0.9 and 1.1, in the baseline office.

7.3.5 A closure criterion — "equal errors"

The last column in Table 7.51 shows that applying the solvency test at the "$A/L_1 = 0.981$" level gives equal numbers of "Type I" and "Type II" errors, and equal numbers of inadequacies and closures. These equalities necessarily coincide, since if we define

$$
\begin{aligned}
CC &= \text{No. of correct closures.} \\
IC &= \text{No. of incorrect closures.} \\
MC &= \text{No. of missed closures.} \\
TC &= \text{Total No. of closures.} \\
IA &= \text{Total No. of inadequacies.}
\end{aligned}
$$

then $CC+IC = TC$, and $CC+MC = IA$, so if $IC = MC$ it follows that $TC = IA$ as well. In a sense this is a neutral, if not an optimal, choice of solvency criterion, and one which allows different valuation methods and bases to be compared. We will use this as a basis for comparison in the later sections.

7.4 Aspects of the U.K. Regulations

A major theme of solvency regulation in the E.C., North America and Australia has been the establishment of "solvency" margins in addition to mathematical reserves. Almost the only break away from this approach has been the cash-flow tests in New York Regulation 126 (see Section 2.7.1). In this section we consider the effect of the two additional layers imposed under U.K. regulations; the E.C. solvency margin and the resilience reserve.

7.4.1 The effect of the E.C. solvency margin

It is worth looking in more detail at the effect of allowing for the E.C. solvency margin. That is, we count an office as "insolvent" if the ratio A/L_1 falls below 1.04 (in the absence of mortality).

Note that the E.C. solvency margin is only taken into account for determining solvency and not in the asset switching algorithm. This ensures that the asset mix each year and the rate of return on the fund are the same as in the baseline office, so the cash-flows are unchanged and hence so is the adequacy in each simulation. Including the E.C. solvency margin in the ratio A/L_2 for the purposes of asset switching would tend to decrease the proportion in equities.

Figure 7.90 (corresponding to Figure 7.86 but to a larger scale) shows the numbers insolvent and inadequate in each year, and also the cumulative totals.

Clearly the incidence of insolvency is considerably greater, so that the number of offices insolvent in any year more nearly approaches the number inadequate. There are 353 insolvencies, of which 207 are suffered by offices which are adequate throughout.

Figure 7.91 shows the numbers of offices at each time during the 20 years which were inadequate, or statutorily insolvent, or both (corresponding to Figure 7.87 and to the same scale). It is now the case that an office which is insolvent at the beginning of the period is slightly less likely to be inadequate than adequate (a majority of "Type II" errors) while the opposite is the case in the later years.

Table 7.52 compares the times at which inadequacy and insolvency first occur,

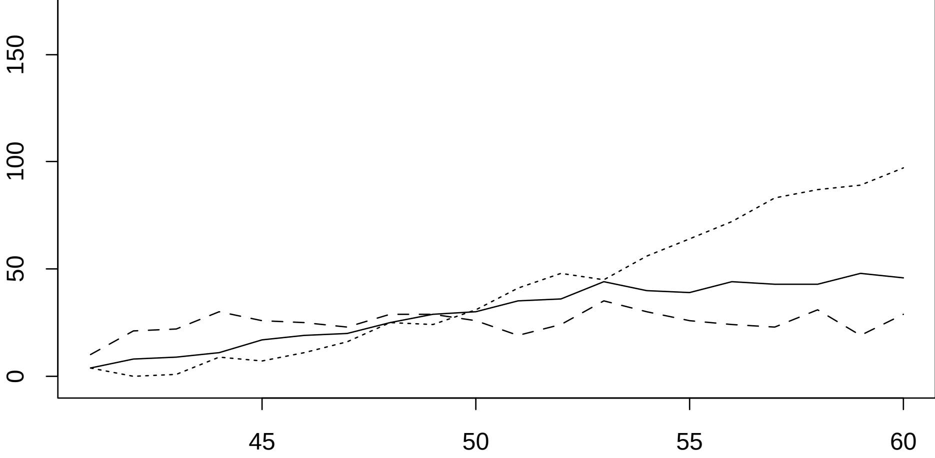


Figure 7.90: Inadequacy and statutory insolvency at the "$A/L_1 < 1.04$" level in the baseline office, from $t = 41$ to $t = 60$.

Figure 7.91: Coincidence of inadequacy and statutory insolvency at the "$A/L_1 < 1.04$" level in the baseline office, from $t = 41$ to $t = 60$.

Time first inadequate *minus* time first insolvent	Number of offices
5 years or more	26
4 years	1
3 years	8
2 years	11
1 year	7
0 year	16
-1 year	64
-2 years	5
-3 years	1
-4 years	1
-5 years or less	6

Table 7.52: Comparison of times by which insolvency at the "$A/L_1 < 1.04$" level preceded inadequacy in the baseline office.

in the 146 cases in which both do occur.

87 of the insolvencies occur within 1 year of inadequacy, and 103 within 2 years. Thus the number within 1 year of inadequacy is *less* than before (there were 93 cases at the "$A/L_1 < 1.0$" level), while the number within 2 years is only 1 more than before (there were 102 cases at the "$A/L_1 < 1.0$" level).

Expressed as a proportion of the 146 "correct" closures, these are 59.6% within 1 year of inadequacy and 70.5% within 2 years, which may be compared with 71.0% and 77.9% respectively at the "$A/L_1 < 1.0$" level.

Expressed as a proportion of the total number of insolvencies, these are 24.6% within 1 year of inadequacy and 29.2% within 2 years, compared with 47.7% and 65.0% respectively at the "$A/L_1 < 1.0$" level.

It is worth looking at the timing of the insolvencies detected by the statutory minimum valuation applied at different levels of the ratio A/L_1, since the observations above suggest that increasing the ratio merely counts more offices to be insolvent at *some* time, and not necessarily at any time close to inadequacy. Of course, using a higher A/L_1 ratio must result in some insolvent offices becoming insolvent sooner.

Table 7.51 above showed that the accuracy of the statutory solvency valuation is very sensitive to the A/L_1 ratio which is used. Table 7.53 below shows that the timing of closure, even of inadequate offices, is also sensitive to the A/L_1 ratio.

	Level of A/L_1 ratio				
	0.90	0.95	1.00	1.05	1.10
No. within 1 year	41	76	93	84	63
% of "correct" closures	89.1%	85.4%	71.0%	55.6%	40.4%
% of all insolvencies	89.1%	64.4%	47.7%	19.8%	9.3%
No. within 2 years	42	77	102	101	80
% of "correct" closures	91.3%	86.5%	77.9%	66.9%	51.3%
% of all insolvencies	91.3%	65.3%	52.3%	23.8%	11.8%

Table 7.53: Numbers of insolvencies within 1 or 2 years of inadequacy at various levels of A/L_1.

Even so small a change as the addition of a 4% solvency margin alters considerably the accuracy and timing of the statutory minimum valuation. The fact that it was moderately effective in this model at the "$A/L_1 < 1.0$" level should not at this stage be taken as anything but coincidence, but the sensitivity of the results to a small change in the reserving requirements is of some interest.

The effect of requirements along the lines of the C-1 Risk Based Capital factors in the U.S.A. would be even more severe. For example, the current RBC requirements would require extra capital of at least 30% of the value of equities to be held. While this is not unlike the U.K. resilience test as far as the assets are concerned, an important difference is that the resilience test takes into account the effect of falls in asset values on yields, and hence on the valuation of liabilities.

In the remainder of this section we consider the statutory minimum reserve and resilience test together as a test of solvency.

7.4.2 The effect of the resilience reserve

It is probably unfair to regard the U.K. resilience test as a test of insolvency, in the sense of leading to an immediate threat of closure or other serious action. It functions as a warning of potential mismatching risks, and barring catastrophes should allow remedial action to be taken by the office — hence its use in our asset switching algorithm.

Should the ratio A/L_2 ever fall below 1.0, however, at a time when a solvency-driven asset allocation strategy is in use, this would indicate that the office would

	Closure if A/L_2 falls below			
	0.90	0.95	0.99	0.9526
Inadequate offices	157	157	157	157
Total closures	71	151	250	157
Correct closures	71	114	139	117
Incorrect closures	0	37	111	40
Missed closures	86	43	18	40
Correct to 1 year	58	90	93	
Correct to 2 years	62	97	105	

Table 7.54: Comparison of adequacy and solvency over 20 years using the ratio A/L_2 as the solvency criterion.

be unable to set up the statutory minimum reserve after the change in conditions supposed in the mismatching test, despite 100% investment in gilts, and this might be a good enough reason for intervention to occur. Therefore we might look at the effectiveness of a solvency test of the form "$A/L_2 < x$", where $x < 1.0$, even in the presence of the asset switching driven by the ratio A/L_2 itself. This must be a stronger test than the statutory minimum on its own, since $A/L_2 \leq A/L_1$.

Table 7.54 shows the numbers of correct and incorrect closures, where closure takes place if the ratio A/L_2 falls below the levels shown. Like Table 7.51, the level at which the number of insolvencies equals the number of inadequacies is shown (in this case, $A/L_2 = 0.9526$). In addition, the numbers of closures which are within 1 year or 2 years of the first occurrence of inadequacy are shown.

This test is certainly not better than the statutory minimum test on its own. Indeed at the "$A/L_2 < 0.99$" level, there are 250 closures of which 139 are correct, 93 to within 1 year, while using the "$A/L_1 < 1.00$" test there are 195 closures, of which 131 are correct, 93 to within 1 year. Inspection shows that 91 of the closures within 1 year of inadequacy are the same in both cases. Increasing the number of incorrect closures from 64 to 111 for the sake of another 8 correct closures, but with no significant improvement in timing, might not be thought worthwhile.

Figure 7.92 shows the numbers of closures of each type for A/L_2 ratios between 0.9 and 1.0.

Figure 7.92: Accuracy of statutory minimum valuation including resilience reserve using A/L_2 ratios between 0.9 and 1.1.

7.5 Alternative valuation methods

This section describes alternative valuations which will be compared with the statutory minimum valuation. The basis of the comparison is the effectiveness of each valuation at indicating inadequacy, in respect of accuracy and timing.

7.5.1 Smoothed asset values

A feature of the U.K. regulations much commented upon is the association of a net premium valuation of the liabilities with a market valuation of the assets. See, for example, Bews *et al* [8] and Ross [57]. Given high levels of equity investment, it is supposed that the value placed upon the assets will often be more volatile than the value placed upon the liabilities.

One way to test this supposition is to replace the raw market value of the assets with a smoothed market value. To be consistent, the liabilities should then be valued at rates of interest consistent with the smoothed asset values. Not all of the potential mismatch will be removed if we continue to use the net premium method; hence some have favoured gross premium valuations with smoothed asset values (see Springbett [64]).

In Section 4.1.3 we defined "actuarial" values of the assets based on cash-flow

Basis 2 - Smoothed asset values						
	Closure if A/L falls below					
	0.90	0.95	1.00	1.05	1.10	0.979
Inadequate offices	157	157	157	157	157	157
Total closures	55	117	245	402	517	157
Correct closures	50	95	138	149	154	116
Incorrect closures	5	22	107	253	363	41
Missed closures	107	62	19	8	3	41
Correct to 1 year	42	77	90	86	79	
Correct to 2 years	44	82	102	103	95	

Table 7.55: Comparison of adequacy and solvency over 20 years using the smoothed valuation basis No.2 as the solvency criterion.

valuations at geometric moving average yields. Here we will take these to be our smoothed asset values. Correspondingly we will value the liabilities at a rate of interest which is a weighted average of the moving average yields used to value the assets; the weights are taken to be the market values of each type of asset in the fund.

Note that equities are valued by applying smoothed *dividend* yields to current dividends. Therefore investment in equities results in a low valuation rate of interest, just as in the statutory minimum basis. This makes comparison with the statutory minimum basis itself easier.

We will call the statutory minimum basis "Basis 1", and the smoothed version described here "Basis 2". Table 7.55 shows the accuracy of the smoothed valuation with various levels of A/L ratio as the closure criterion.

Figure 7.93 shows the numbers of correct and incorrect closures etc. under the smoothed valuation basis No.2.

Comparing these results with Table 7.51 and Figure 7.89 shows that the use of smoothed yields for both asset and liability valuations reduces slightly the sensitivity of the number of closures to the A/L ratio which is used, and in particular appears to remove the marked increase which was apparent above $A/L_1 = 1.04$ in Figure 7.89. The statutory minumum basis (unsmoothed) is slightly better at the "$A/L_1 < 1.00$" level, closing 195 (131 correct) compared with 245 (138 correct) but both valuation bases show such sensitivity to the A/L ratio that this is not remarkable.

Comparison with Table 7.49 suggests also that Basis No.2 using smoothed yields

Figure 7.93: Accuracy of valuation Basis 2 (smoothed asset values) using A/L ratios between 0.9 and 1.1.

and asset values is slightly less sensitive to larger values of the A/L ratio. Although the difference is not great, this is of some interest when the addition of a solvency margin to a mathematical reserve is considered.

7.5.2 Static valuation bases

A feature of the U.K. statutory minimum basis is that the maximum interest rate is calculated dynamically. To a more limited extent the same is true in the U.S.A., but there the interest rate so calculated is applied only to new business. In other E.C. territories the valuation basis is static, and almost always the same as the premium basis. In this section we consider the effect of static valuation bases.

Basis 3 This is a gross premium basis, using the same interest rate of 5% and bonus loading of 2.5% as does the premium basis. This represents, loosely, the most common practice in E.C. territories of valuing on the premium basis, but adapted to the method of premium calculation common in the U.K..

Basis 4 This is a net premium basis, using an interest rate of 3%. No Zillmer is used, since there are no expenses. 3% is broadly typical of the interest rates used in net premium valuations of taxed with-profits contracts in the U.K.. It is rather weaker than the interest rate implied by the premiums; the annual

Basis 3 - Valuation on the premium basis						
	Closure if A/L falls below					
	0.90	0.95	1.00	1.05	1.10	0.935
Inadequate offices	157	157	157	157	157	157
Total closures	86	188	357	583	754	157
Correct closures	71	110	140	150	156	103
Incorrect closures	15	78	217	433	598	54
Missed closures	86	47	17	7	1	54
Correct to 1 year	58	82	85	73	60	
Correct to 2 years	60	89	98	89	74	
Basis 4 - 3% net premium basis						
	Closure if A/L falls below					
	0.90	0.95	1.00	1.05	1.10	0.902
Inadequate offices	157	157	157	157	157	157
Total closures	151	305	534	726	849	157
Correct closures	100	131	148	155	157	103
Incorrect closures	51	174	386	571	692	54
Missed closures	57	26	9	2	0	54
Correct to 1 year	80	83	75	62	54	
Correct to 2 years	85	95	91	76	67	

Table 7.56: Comparison of adequacy and solvency over 20 years using valuation bases 3 and 4 as the solvency criterion.

premium is £92.926 per £1,000 of sum assured, which is equivalent to a rate of interest of about 0.6% with no bonus loading.

Table 7.56 shows

1. the results of these 2 valuation bases, using A/L ratios of 0.90, 0.95, 1.00, 1.05 and 1.10 as the criterion of solvency;
2. the results for that level of the A/L ratio which results in an equal number of incorrect closures and missed closures;
3. the number of closures within 1 and 2 years of the first occurrence of inadequacy.

For comparison with Figures 7.89 and 7.93, Figures 7.94 and 7.95 show the numbers of correct and incorrect closures under valuation bases No.3 and No.4 using A/L ratios from 0.90 to 1.10, in steps of 0.1, as the solvency criterion.

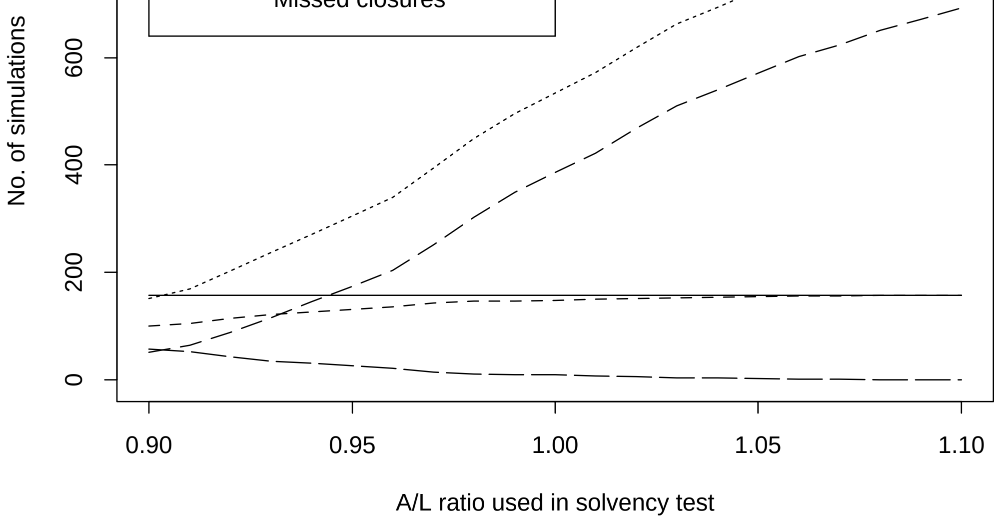


Figure 7.94: Accuracy of valuation Basis 3 (gross premium) using A/L ratios between 0.9 and 1.1.

Figure 7.95: Accuracy of valuation Basis 4 (3% net premium) using A/L ratios between 0.9 and 1.1.

Figure 7.96: Accuracy of valuation Basis 5 (net premium, 92.5% of gilt yield) using A/L ratios between 0.9 and 1.1.

These figures, compared with Figures 7.89 and 7.93 show some interesting features. However, we will introduce two further bases before drawing our conclusions.

7.5.3 Dynamic valuation methods

Basis 5 This is a net premium basis, in which the interest rate is 92.5% of the current net redemption yield on gilts. There is no further restriction on the yields assumed in respect of future investments. This is broadly similar to the U.K. statutory valuation basis, with potential dividend growth allowed for but adjusted to eliminate the risk premium relative to gilts.

Basis 6 This is a net premium basis, in which the interest rate is 63% of a 10-year (geometric) moving average of the net redemption yield on gilts. This approximates the "strengthened" basis proposed by the Buol committee (see Section 2.5.2).

Table 7.57 and Figures 7.96 and 7.97 show the results.

Again these figures, compared with Figures 7.89, 7.93, 7.94 and 7.95, show some interesting features.

1. The six bases surveyed fall into two broad categories, within each of which

Basis 5 - Net premium, 92.5% of gilt yield						
	Closure if A/L falls below					
	0.90	0.95	1.00	1.05	1.10	0.979
Inadequate offices	157	157	157	157	157	157
Total closures	47	97	200	403	601	157
Correct closures	47	89	132	151	155	117
Incorrect closures	0	8	68	252	446	40
Missed closures	110	68	25	6	2	40
Correct to 1 year	41	76	93	84	75	
Correct to 2 years	42	77	103	101	92	

Basis 6 - Net premium, 63% of 10-yr average gilt yield						
	Closure if A/L falls below					
	0.90	0.95	1.00	1.05	1.10	0.935
Inadequate offices	157	157	157	157	157	157
Total closures	92	190	402	618	764	157
Correct closures	80	122	146	153	157	112
Incorrect closures	12	68	256	465	607	45
Missed closures	77	35	11	4	0	45
Correct to 1 year	63	90	84	71	59	
Correct to 2 years	67	100	97	87	73	

Table 7.57: Comparison of adequacy and solvency over 20 years using valuation bases 5 and 6 as the solvency criterion.

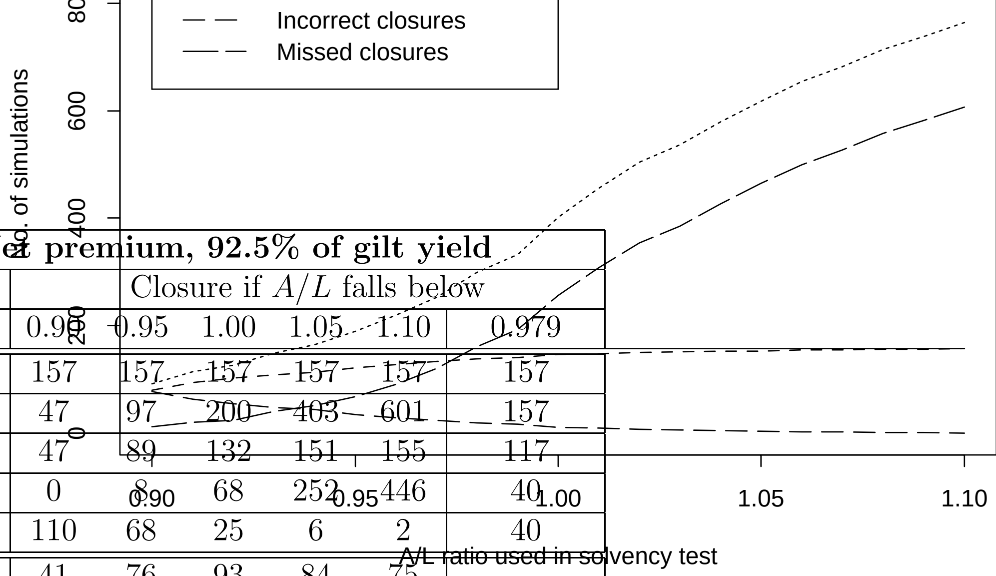


Figure 7.97: Accuracy of valuation Basis 6 (net premium, 63% of 10-yr average gilt yield) using A/L ratios between 0.9 and 1.1.

the results are similar. Bases No.1, No.2 and No.5 are similar; call this the "dynamic" group; and Bases No.3, No.4 and No.6 are similar; call this the "static" group. (Basis No.6 is not by definition static but belongs among the static bases by virtue of its behaviour.)

2. Up to the "$A/L < 1.0$" level, the static bases all close about twice as many offices as the dynamic bases. In this region, they behave rather like the dynamic bases do with a higher A/L ratio.

3. Above the "$A/L < 1.0$" level, the numbers of closures under the dynamic bases increases rapidly, closing the gap with the static bases. The most satisfactory in this respect is Basis No.2, which smooths both asset and liability valuation bases.

4. The level of A/L ratio at which exactly the same number of offices were closed as were inadequate gives some indication of the optimum performance of a valuation basis to an observer with no preference for "Type I" or "Type II" errors. These "minimum" numbers of errors in every case were quite close — between 39 and 54 — but were attained at very different levels of A/L ratio. Under the dynamic bases, the minimum errors were attained with closure around the "$A/L < 0.97$" level; under all the static bases the corresponding A/L ratio was much lower.

What all these valuation bases have in *common* is the sensitivity of the results — their ability to close the right offices in approximately good time — to the level of the A/L ratio which is used to trigger closure. Where they *differ* is in how good the "obvious" "$A/L < 1.0$" criterion is compared with alternatives.

The important corollaries are (i) that the imposition of solvency margins which effectively increase the level of the A/L ratio used as a closure criterion is a very blunt instrument; (ii) the use of a uniform solvency margin across a variety of valuation regimes is potentially iniquitous. Following from (ii) above is the conclusion that, *if* traditional solvency valuations must be used, then the whole solvency system — mathematical reserves, solvency margins, resilience tests etc. — should be designed as a whole, in particular to avoid the duplication of margins. This the Australians and Canadians have done; the E.C. approach is less satisfactory.

7.5.4 The A/AS ratio as a solvency criterion

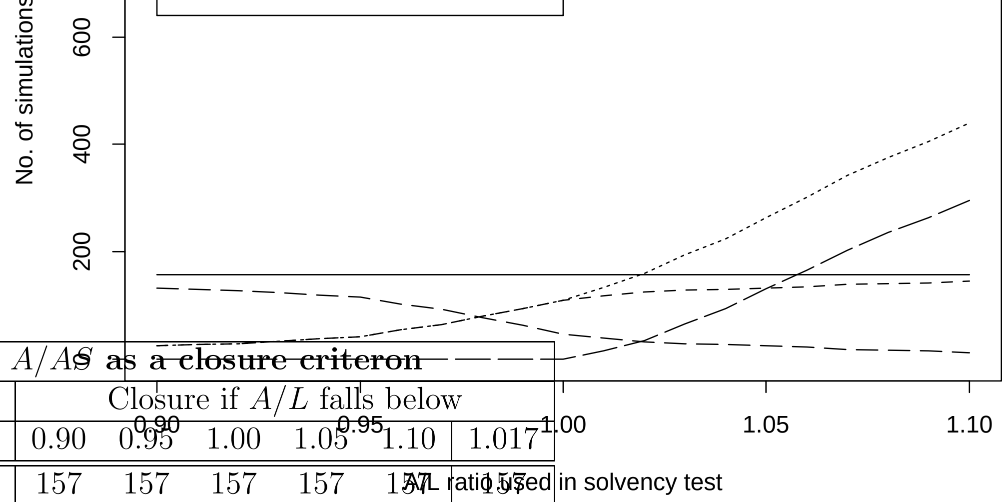
As described in Chapter 1, with-profits business in the U.K. relies on the terminal bonus system, not only to maintain solvency with high levels of equity investment, but also for the capital with which to write new business. This is particularly true of mutual offices.

As terminal bonuses began to form a greater part of the maturity benefit, particularly in the 1980s, some actuaries drew attention to the resulting dilution of solvency standards, where solvency was based on a valuation of the guaranteed benefits alone. It has been suggested that solvency reserves ought to allow for future terminal bonus; see in particular Lyon [40]. Since terminal bonuses are not guaranteed it might not be appropriate to reserve for them in full measure, but not to reserve at all is possibly to neglect reasonable expectations.

Where the policy asset share plays a part in the determination of terminal bonus — upon which there is widespread though not uniform agreement in the U.K. — then the current asset share is a natural measure of the quantum of assets needed to meet maturity benefits. It is not sufficient on its own since it disregards the guarantees, and it is not suitable for non-profit business, but where the terminal bonus system is the major determinant of policyholders' reasonable expectations it is arguably a more logical choice than a policy value based on the guaranteed benefits.

Consequently we will consider the A/AS ratio as a measure of "solvency" for with-profits endowments. Table 7.58 and Figure 7.98 show the numbers of closures for selected values of the ratio A/AS.

In two senses the A/AS ratio seems far better than any of the A/L ratios as a solvency criterion. At the "$A/AS < 1.0$" level it closes only 110 offices, all of them correctly. And at the "$A/AS < 1.017$" level it makes only 33 errors each of "Type I" and "Type II". However, relatively few of the offices closed are closed within 1 or 2 years of the first occurrence of inadequacy; the timing of the ratio A/AS seems



The ratio A/AS as a closure criterion						
	Closure if A/L falls below					
	0.90	0.95	1.00	1.05	1.10	1.017
Inadequate offices	157	157	157	157	157	157
Total closures	25	42	110	263	440	157
Correct closures	25	42	110	132	145	124
Incorrect closures	0	0	0	131	295	33
Missed closures	132	115	47	25	12	33
Correct to 1 year	0	1	11	24	31	
Correct to 2 years	1	3	22	48	72	

Table 7.58: Comparison of adequacy and solvency over 20 years using the ratio A/AS as the solvency criterion.

Figure 7.98: Accuracy of the ratioA/AS as a solvency criterion.

Time first inadequate *minus* time first insolvent	Number of offices
5 years or more	0
4 years	0
3 years	0
2 years	0
1 year	0
0 year	0
-1 year	11
-2 years	11
-3 years	20
-4 years	25
-5 years or less	43

Table 7.59: Comparison of times by which insolvency at the "$A/AS < 1.0$" level preceded inadequacy in the baseline office.

rather poorer than that of any of the A/L ratios.

Table 7.59 shows that the timing of the A/AS ratio is, in fact, completely different from the examples given previously of the timing of an A/L ratio. The A/AS ratio at the "$A/AS < 1.0$" level does not close a single office before the occurrence of inadequacy. Moreover, it tends to close offices long afterwards, not immediately afterwards.

No closures occur before the first inadequacy because payment of 100% of the asset shares as a minimum ensures that only cash-flows *out* of the additional estate are possible; further, if the ratio A/AS ever falls below 1.0 a deficit is certain following closure and run-off at *any future time*.

The reason closures, though all "correct", are so late is that in most cases inadequacy is caused by a financial crisis during the run-off. The same financial crisis might cause the A/AS ratio to fall below 1.0, if the office is allowed to remain open to new business, and will trigger closure at that time. Compare Figure 7.99 with Figure 7.88. Figure 7.99 shows the distribution of the ratio A/AS at the time of inadequacy, and in the 10 years afterwards, for the 157 inadequate offices. Figure 7.88 showed the corresponding distribution of the ratio A/L_1. It is clear that the latter is far more responsive to inadequacy than is the A/AS ratio. A comparison of Figures 4.9 and 4.11 confirms that the ratio A/AS is much less volatile than the ratio A/L_1, the reason being that changes in the value of assets affect numerator

Figure 7.99: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the ratio A/AS following the first occurrence of inadequacy.

and denominator equally in the ratio A/AS. Therefore inadequacy related to such changes in value might not be detected by the ratio A/AS.

7.6 Conclusions

This chapter has thrown some light on the operation of the traditional solvency valuation. Although the model is a very simple one, some clear conclusions have emerged.

1. There is little coincidence, from year to year, between offices which are *insolvent* and offices which are *inadequate*.

2. Given the *same* valuation basis, the number of closures (insolvencies) is sensitive to the level of A/L ratio which is used as a criterion of closure.

3. Comparing *different* valuation bases, there are large differences between the numbers of closures, given the same A/L ratio as a closure criterion.

4. The effect of a uniform solvency margin, even one so small as 4% of mathematical reserves, can differ markedly depending on the underlying valuation regime.

5. The ratio A/AS is not a satisfactory alternative to the ratio A/L as a measure of solvency, even under the circumstances of this model which reflects somewhat retrospective management practices.

The comparison of adequacy and solvency suggests some interesting topics worth further study.

1. The pattern of adequacy itself — constantly increasing numbers inadequate with few recoveries from inadequacy — probably depends on the details of the simple baseline office. In other circumstances, offices might recover from inadequacy more frequently. We wish to investigate this in order to see how reasonable it might be to estimate adequacy by checking run-offs at just a few times within that time horizon (to make the computation more feasible).

2. The baseline office showed a significant but not overwhelming incidence of inadequacy. Do we obtain a similar relationship (or lack of it) between insolvency and inadequacy in an office which suffers much higher levels of inadequacy?

3. The accuracy of the various solvency valuations tested above showed great sensitivity to the level of A/L ratio which was used as a closure criterion. Is this also the case in different circumstances?

4. The A/AS ratio appeared to be a more accurate indicator of inadequacy than any A/L ratio, but its timing was so much worse that it is hardly to be preferred. It is probably wrong to regard the A/L ratios examined here as having better timing because of any predictive ability; the question really is how many adequate offices have to be closed in order to catch enough of the inadequate ones?

In the next chapter we investigate these questions.

Chapter 8

Further aspects of adequacy

8.1 Variations on the traditional valuation

In Chapter 7 we introduced adequacy and compared it with solvency, both statutory (in the U.K.) and according to other measures. The circumstances were those of the model described in Chapter 4; in particular the investment strategy was driven by the ratio A/L_2.

The aim of this chapter is to compare solvency and adequacy as in Chapter 7 under some different conditions, and to see if the behaviour of a particular valuation method is consistent, or not, when applied in a range of circumstances.

The range of examples is necessarily limited, because of the time-consumimg nature of adequacy testing. They are as follows; unless otherwise stated the modifications described are made to the office of Chapter 7, which we denote here by "Office A":

Office B We first look at an office with 100% in equities before closure, reduced to 50% after closure. Although extreme, this allows direct comparison to be made with Office A.

Office C Next we look at an office which pursues the same investment and bonus strategies as Office A before closure, but continues to employ them without moderation after closure. This tests the "cost" of Policyholders' Reasonable Expectations, insofar as they may be represented by the continuance of given strategies.

Both of the above offices have this in common; that they result in a higher number of inadequacies than does Office A. They therefore allow us to compare adequacy and solvency under *more* extreme conditions.

Office D An office with 70% in equities before closure, 35% after closure. These levels are fixed; the office does not employ the solvency-driven asset switching algorithm of Office A. This level of equity investment is typical of practice in the U.K..

Office E An office with 100% of its funds in gilts and a terminal bonus target of nil (i.e. aiming to pay all its benefits in guaranteed form). This approximates to the practice in some territories outside the U.K., notably Germany.

These last two offices result in lower numbers of inadequacies than in Office A. They therefore allow us to compare adequacy and solvency under *less* extreme conditions. Finally, we consider two offices with some of the features considered in earlier chapters.

Office F An office with the "combined smoothing" algorithm of Chapter 6. This results in the additional estate being credited with increments to the Bonus Smoothing Account, as well as being debited under the operation of the guarantees.

Office G An office in which the resilience test is modified along the lines of the Government Actuary's Memorandum of 30 September 1993, by testing a rise of 1.5% in the dividend yield and a proportionate change of ±20% in the gilt yield. The U.K. resilience test continues to develop, with the evident aim of making it suited to a wider range of circumstances; this is worth testing.

In Chapter 7, we considered the effect of 6 valuation bases in some detail, including the use of different A/L ratios as a solvency criterion. In this chapter, we will show less detail to aid comparison. For each office we will show:

Figure 8.100: Inadequacy and statutory insolvency in Office A, from $t = 41$ to $t = 60$.

1. The numbers inadequate and statutorily insolvent each year, and the corresponding cumulative totals.

2. The effect of varying the A/L ratio used as the closure criterion under the statutory minimum solvency test only.

3. The effect of using the valuation bases No.2 – No.6 defined in Section 7.5, and the ratio A/AS, each at the same ($A/L < 1.0$) level.

4. The A/L ratio at which each of these solvency tests achieves equal numbers of "Type I" and "Type II" errors.

In what follows, figures comparable to Figure 7.86 will be shown, so that the build-up of inadequacy and insolvency can be compared with that in Office A. However, in some cases the numbers of inadequacies or insolvencies are too great for the scale used in Figure 7.86. For convenience, therefore, Figure 8.100 reproduces Figure 7.86 to the same scale as will be used in this chapter.

Figure 8.101: Inadequacy and statutory insolvency in Office B (fixed 100% in equities), from $t = 41$ to $t = 60$.

8.2 Offices with higher levels of inadequacy

8.2.1 Office B: 100% in equities

In Office B the investment strategy is fixed, at 100% in equities before closure and 50% in equities after closure. It is thus the same as Office A except for the removal of the dynamic investment strategy. Figure 8.101 shows the numbers of simulations resulting in statutory insolvency and inadequacy each year, and the cumulative totals of the same.

The total number of inadequacies is 316. As in Office A, recovery from inadequacy is infrequent; at time $t = 70$ the number then inadequate is 294. The total number of statutory insolvencies is 645, so while the number of inadequacies has doubled, the number of insolvencies has trebled.

Figure 8.102 shows the numbers of correct and incorrect closures etc. under the statutory minimum valuation basis. It shows graphically the effect of a harsh valuation; even at the "$A/L < 0.90$" level the test is far too strong, and increasing the A/L ratio simply catches more adequate offices. It would be wrong to say that a weak test is wanted here, since 316 inadequacies out of 1,000 is hardly impressive, but this test seems to deploy its strength in the wrong places.

Figure 8.102: Accuracy of statutory minimum valuation basis in Office B using A/L ratios between 0.9 and 1.1.

Office B: 100% in equities							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
Inadequate offices	316	316	316	316	316	316	316
Total closures	645	293	609	688	530	625	246
Correct closures	314	257	313	313	309	314	246
Incorrect closures	331	36	296	375	221	311	0
Missed closures	2	59	3	3	7	2	70
Correct to 1 year	201	44	194	190	197	198	10
Correct to 2 years	231	144	231	221	239	232	81

Table 8.60: Comparison of adequacy and solvency, in Office B under valuation bases No.1 – No.6, using $A/L < 1.0$ as the closure criterion, and using the A/AS ratio.

Table 8.60 shows the numbers of correct, incorrect and missed closures in Office B, applying the valuation bases No.1 – No.6 of Section 7.5 at the "$A/L < 1.0$" level, and also using the ratio A/AS at the "$A/AS < 1.0$" level. Also shown are the number of *correct* closures which are within 1 and 2 years of the first occurrence of inadequacy.

Table 8.60 has two outstanding features.

1. Basis No.2 seems to be much better than anything else, except perhaps the A/AS ratio. Its timing looks worse, until we note that it lands about the same proportion of its *total* closures within 2 years of inadequacy, while closing about

Figure 8.103: Accuracy of valuation Basis No.2 (smoothed asset and liability valuation bases) in Office B using A/L ratios between 0.9 and 1.1.

200 fewer adequate offices.

2. Using the A/AS ratio as a solvency criterion also looks to be more effective, except that as in Chapter 7 its timing is very poor.

Basis No.2 looks *so* much better, it is worth looking at how sensitive it is to changes in the A/L ratio used as the closure criterion. This is shown in Figure 8.103.

Considering the extreme investment strategy, the shortness of the policy term and the contrast with Figure 8.102, Basis No.2 seems to be remarkably insensitive to the level of the A/L ratio which is used. This was remarked upon also in Section 7.5.1, but it is much more striking here.

Table 8.61 shows the "optimum" performance of each solvency test, in the sense of closing exactly the same number of offices as are inadequate (equivalently, making equal numbers of "Type I" and "Type II" errors).

The most obvious feature in Table 8.61, this time *including* Basis No.2, is how similar the "best" efforts of each basis are. Given 1,000 offices, 316 inadequacies, and upon being invited to choose 316 offices to close, Bases No.1 – No.6 get between 267 and 278 correct. In addition, the "optimum" A/L ratios are all similar apart from Basis No.2.

Office B: 100% in equities							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
"Optimum" A/L ratio	0.830	1.022	0.831	0.801	0.876	0.833	1.054
Inadequate offices	316	316	316	316	316	316	316
Total closures	316	316	316	316	316	316	316
Correct closures	278	267	272	273	277	274	281
Incorrect closures	38	49	44	43	39	42	35
Missed closures	38	49	44	43	39	42	35

Table 8.61: Comparison of adequacy and solvency in Office B, at the A/L ratios yielding "equal errors" under under valuation bases No.1 – No.6, and using the A/AS ratio.

This table, along with the tables in Section 7.5, suggest that all valuation bases will do a similar job provided we can find the appropriate level of A/L ratio to use as a solvency criterion. Unfortunately, that level is not the intuitively appealing "$A/L < 1.0$". Even less is it "$A/L < 1.04$".

8.2.2 Office C: same strategies after closure

In Office A three strategies are amended after closure; the upper limit on equity investment is cut to 50%, the terminal bonus target is increased to 50%, and the allowable drop in reversionary bonus rates is doubled to 40%. In Office C these changes are not made, so no concession is made to the changed circumstances. Such a test represents a rigid interpretation of policyholders' reasonable expectations.

Figure 8.104 shows the numbers of simulations resulting in statutory insolvency and inadequacy each year, and the cumulative totals of the same. (Note again that this is not to the same scale as Figure 7.86.)

The number of inadequacies has more than doubled; in total there are 368. Of these, only 215 are inadequate at time $t = 70$, so there is a much higher incidence of *recovery* from inadequacy. However, the numbers of statutory insolvencies are exactly the same as before (because the strategies are identical *before* closure). This changes the relative patterns of inadequacy and insolvency. There is now a considerable number of inadequacies at outset (69), and where before the total of insolvencies always outran the total of inadequacies, now it is the other way round.

Figure 8.104: Inadequacy and statutory insolvency in Office C (same strategies after closure), from $t = 41$ to $t = 60$.

So much so, that the *cumulative* number of insolvencies never exceeds the *current* number of inadequacies.

This is an interesting observation; **the actions of the managers** *after* **closure have a significant effect on adequacy, not just in the long term but on immediate closure too. Statutory minimum solvency makes no such distinctions**.

Figure 8.105 shows the numbers of correct and incorrect closures etc. under the statutory minimum valuation basis.

This is quite different to the corresponding figures in respect of Offices A and B. The point emerging here is that the *same* valuation basis, applied to offices using different strategies, can give quite different results. Note that (i) Offices A, B and C are identical at the starting point, time $t = 40$, and (ii) Offices B and C differ only in the degree of caution which might be employed during a run-off.

Table 8.62 shows the effect of the valuation bases No.1 – No.6 of Section 7.5 at the "$A/L < 1.0$" level.

The numbers of closures in this table are the same as under Office A in Chapter 7. Again the harshness of the static valuation bases No.2 (premium basis) and No.3 (3% net premium) is apparent. In addition, such small proportions of the closures are within 2 years of inadequacy that none of these bases can be described

Figure 8.105: Accuracy of statutory minimum valuation basis in Office C using A/L ratios between 0.9 and 1.1.

Office C: Same strategies before and after closure							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
Inadequate offices	368	368	368	368	368	368	368
Total closures	195	245	357	534	200	402	110
Correct closures	168	196	238	303	171	269	110
Incorrect closures	27	49	119	231	29	133	0
Missed closures	200	172	130	65	197	99	258
Correct to 1 year	16	18	24	31	16	24	1
Correct to 2 years	20	25	29	44	20	31	1

Table 8.62: Comparison of adequacy and solvency, in Office C under valuation bases No.1 – No.6, using $A/L < 1.0$ as the closure criterion, and using the A/AS ratio.

Office C: Same strategies before and after closure							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
"Optimum" A/L ratio	1.043	1.041	1.002	0.965	1.045	0.996	1.077
Inadequate offices	368	368	368	368	368	368	368
Total closures	368	368	368	368	368	368	368
Correct closures	267	267	242	240	268	254	245
Incorrect closures	101	101	126	127	100	113	123
Missed closures	101	101	126	127	100	113	123

Table 8.63: Comparison of adequacy and solvency in Office C, at the A/L ratios yielding "equal errors" under under valuation bases No.1 – No.6, and using the A/AS ratio.

as particularly accurate; but here again, the A/AS ratio is by far the poorest.

Table 8.63 shows the "optimum" (equal errors) performance of each solvency test.

8.3 Offices with lower levels of inadequacy

8.3.1 Office D: 70% in equities

In Chapter 5, fixed asset allocation strategies were compared with solvency-driven strategies, over a 30-year time horizon, with the result that 100% investment in equities *with* solvency-driven asset switching gave rise to the same number of statutory insolvencies as 60% – 70% investment in equities *without* the asset switching.

In Office D, the proportions in equities are fixed as 70% before closure and 35% after closure. Figure 8.106 shows the numbers and cumulative totals of inadequacies and statutory insolvencies.

The total number of inadequacies is almost halved compared with Office A (85 instead of 157). However, the number of statutory insolvencies has *increased* (232 instead of 195). This confirms the outcome of Office B; that solvency is more sensitive than adequacy to the difference between a fixed and a solvency-driven asset allocation strategy. It also agrees with the conclusion of Ross & McWhirter [58].

Figure 8.107 and Table 8.64 show the effects of various valuation bases.

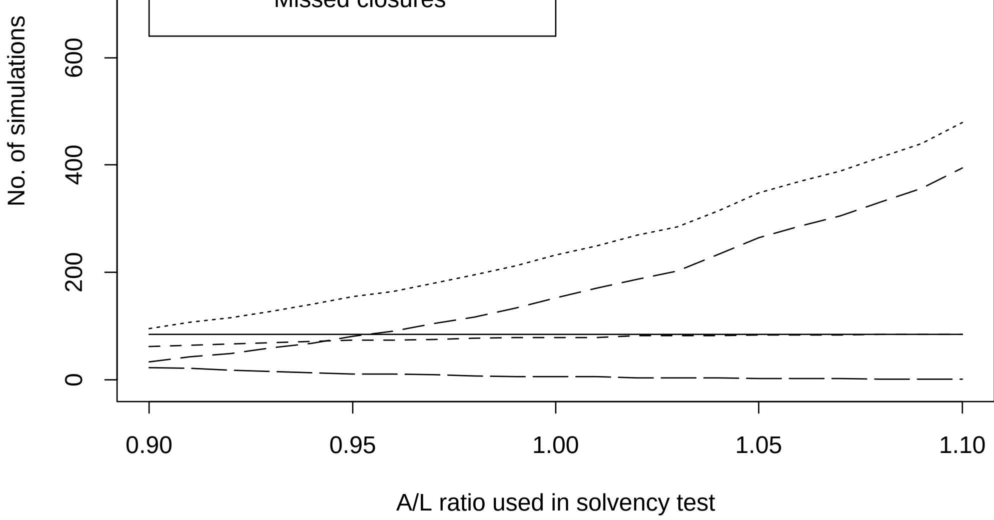


Figure 8.106: Inadequacy and statutory insolvency in Office D (70% in equities), from $t = 41$ to $t = 60$.

Figure 8.107: Accuracy of statutory minimum valuation basis in Office D using A/L ratios between 0.9 and 1.1.

Office D: 70% in equities							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
Inadequate offices	85	85	85	85	85	85	85
Total closures	232	67	241	338	185	256	46
Correct closures	79	55	78	81	76	79	46
Incorrect closures	153	12	163	257	109	177	0
Missed closures	6	30	7	4	9	6	39
Correct to 1 year	60	11	59	54	58	60	1
Correct to 2 years	65	35	64	60	68	64	8

Table 8.64: Comparison of adequacy and solvency, in Office D under valuation bases No.1 – No.6, using $A/L < 1.0$ as the closure criterion, and using the A/AS ratio.

Office D: 70% in equities							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
"Optimum" A/L ratio	0.889	1.022	0.884	0.850	0.910	0.869	1.048
Inadequate offices	85	85	85	85	85	85	85
Total closures	85	85	85	85	85	85	85
Correct closures	57	61	53	53	59	57	66
Incorrect closures	28	24	32	32	26	28	19
Missed closures	28	24	32	32	26	28	19

Table 8.65: Comparison of adequacy and solvency in Office D, at the A/L ratios yielding "equal errors" under under valuation bases No.1 – No.6, and using the A/AS ratio.

Again, Basis No.2 stands out, especially compared with No.1, the statutory minimum basis. Table 8.65 shows the "optimum" performance of each solvency test.

The pattern of previous examples is repeated; the range of A/L ratios at which equal numbers of errors are attained is quite wide, and the numbers of errors are very similar for under all the tests except the A/AS ratio.

8.3.2 Office E: A traditional "fixed-interest" office

Office E represents a "traditional" fixed-interest, reversionary bonus office. The starting point at time $t = 40$ is the same as in Office A, but then (i) the assets are switched to 100% in gilts, and (b) the terminal bonus target is changed from 25% to 0%. This does not mean that the terminal bonus *paid* is always nil — if the asset

Office E: "Traditional" fixed interest							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
"Optimum" A/L ratio	0.889	1.022	0.884	0.850	0.910	0.869	1.048
Inadequate offices	0	0	0	0	0	0	0
Total closures	0	0	29	78	0	0	0
Correct closures	0	0	0	0	0	0	0
Incorrect closures	0	0	29	78	0	0	0
Missed closures	0	0	0	0	0	0	0

Table 8.66: Comparison of adequacy and solvency in Office E, at the A/L ratios yielding "equal errors" under under valuation bases No.1 – No.6, and using the A/AS ratio.

shares at maturity exceed the guaranteed benefits then a terminal bonus is paid as usual — but that the office does not use terminal bonus in a strategic way. Table 8.66 shows the effect of the valuation bases No.1 – No.6.

There are now no inadequacies. (The question of "how close" the office might be to inadequacy will be dealt with in Section 8.6.) More interesting, there are no insolvencies either under any of the dynamic valuation bases. In fact, there are no insolvencies under the statutory minimum basis (No.1) even if the solvency criterion is increased to "$A/L < 1.10$". There are, however, insolvencies under the static valuation bases. This confirms their relative harshness.

8.4 Other changes

8.4.1 Office F: Introducing smoothing

In Chapter 6 several methods of smoothing maturity benefits were introduced. Here we examine the effect of one of them on adequacy and solvency. Office F is the same as Office A except that the "combined" smoothing defined in Section 6.2.3 is used; in this, the assets underlying the maturing policy's asset shares are smoothed first, and then limits are aplied to the year-on-year changes in maturity values. Figure 8.108 shows the numbers and cumulative totals of inadequacies and statutory insolvencies.

The total number of inadequacies has increased from 157 to 233, and the total number of statutory insolvencies has also increased from 195 to 248.

Figure 8.108: Inadequacy and statutory insolvency in Office F ("combined smoothing" from Chapter 7), from $t = 41$ to $t = 60$.

Office F: "Combined" smoothing							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
Inadequate offices	233	233	233	233	233	233	233
Total closures	248	292	442	597	252	469	211
Correct closures	185	197	212	223	186	220	173
Incorrect closures	63	95	230	374	66	249	38
Missed closures	48	36	21	10	47	13	60
Correct to 1 year	115	121	125	119	116	131	8
Correct to 2 years	143	146	146	139	145	153	40

Table 8.67: Comparison of adequacy and solvency, in Office F under valuation bases No.1 – No.6, using $A/L < 1.0$ as the closure criterion, and using the A/AS ratio.

Figure 8.109 and Table 8.67 show the results under the various valuation bases.

The main difference between this and previous tables is the A/AS ratio; it is much more comparable with the dynamic valuation bases (which in turn are much less harsh than the static bases). This is a consequence of the flows to and from the additional estate resulting from smoothing; previously, when all the cash-flows were *out* of the additional estate, the A/AS ratio falling below 1.0 made a deficit inevitable in *any* subsequent run-off, so no "incorrect closures" were possible with the "$A/AS < 1.0$" criterion. Its timing is still markedly poorer than the prospective solvency tests, however.

Figure 8.109: Accuracy of statutory minimum valuation basis in Office F using A/L ratios between 0.9 and 1.1.

Office F: "Combined" smoothing							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
"Optimum" A/L ratio	0.993	0.981	0.943	0.909	0.991	0.941	1.006
Inadequate offices	233	233	233	233	233	233	233
Total closures	233	233	233	233	233	233	233
Correct closures	180	181	165	166	178	180	178
Incorrect closures	53	52	68	67	55	53	55
Missed closures	53	52	68	67	55	53	55

Table 8.68: Comparison of adequacy and solvency in Office F, at the A/L ratios yielding "equal errors" under under valuation bases No.1 – No.6, and using the A/AS ratio.

Table 8.68 shows the "optimum" performance of each solvency test. The A/AS ratio now joins the other solvency tests, which again have similar numbers of "equal errors".

8.4.2 Office G : An amended resilience test

In 1993 the resilience test used by the G.A.D. in the U.K. was amended as described in Section 2.6. The purpose of the change was to make the test more appropriate in a wider range of circumstances, the fixed parameters of the original (1985) test having caused problems when yields had been at extremely high or low levels.

Figure 8.110: Inadequacy and statutory insolvency in Office G (amended resilience test), from $t = 41$ to $t = 60$.

In Office G the parameters of the resilience test are changed from their original values of a 25% fall in equity prices, and a ±3% change in gross fixed-interest yields, to a rise of 1.5% in the dividend yield and a proportionate change of ±20% in the fixed-interest yield. This is therefore a similar, though not identical, test to that now used by the G.A.D..

Note that the asset allocation is driven by the same criterion as in Office A — 100% in equities, but switch into gilts if necessary to keep the ratio A/L_2 above 1.0 — but that the denominator L_2 of this ratio now allows for the amended resilience test. Therefore when comparing this with Office A, we will be interested in whether or not the amended resilience test (i) improves statutory solvency (i.e. switches into gilts at more appropriate times and/or in more appropriate quantities); (ii) improves adequacy; (iii) improves the accuracy of the statutory minimum valuation. Figure 8.110 shows the numbers and cumulative totals of inadequacies and statutory insolvencies.

The total number of inadequacies is hardly changed from Office A; up from 157 to 165. Of these, 127 are in common (i.e. in 127 scenarios inadequacy occurs in both Office A and Office G). The total number of insolvencies is also nearly the same; down from 195 to 191. Of these, 165 are in common.

Figure 8.111 and Table 8.69 show the effects of the various valuation bases.

Figure 8.111: Accuracy of statutory minimum valuation basis in Office G using A/L ratios between 0.9 and 1.1.

Office G: 1993 resilience test							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
Inadequate offices	165	165	165	165	165	165	165
Total closures	191	227	382	554	197	431	111
Correct closures	126	136	140	155	126	153	111
Incorrect closures	65	91	242	399	71	278	0
Missed closures	39	29	25	10	39	12	54
Correct to 1 year	94	98	94	84	93	95	7
Correct to 2 years	101	106	103	96	100	104	20

Table 8.69: Comparison of adequacy and solvency, in Office G under valuation bases No.1 – No.6, using $A/L < 1.0$ as the closure criterion, and using the A/AS ratio.

Office G: 1993 resilience test							
	Solvency valuation basis						
	No.1	No.2	No.3	No.4	No.5	No.6	A/AS
"Optimum" A/L ratio	0.992	0.979	0.941	0.908	0.991	0.941	1.021
Inadequate offices	165	165	165	165	165	165	165
Total closures	165	165	165	165	165	165	165
Correct closures	118	122	101	102	118	111	124
Incorrect closures	47	43	64	63	47	55	41
Missed closures	47	43	64	63	47	55	41

Table 8.70: Comparison of adequacy and solvency in Office G, at the A/L ratios yielding "equal errors" under under valuation bases No.1 – No.6, and using the A/AS ratio.

Table 8.69, compared with the corresponding tables in Chapter 7, shows that **the amended resilience test makes very little difference to this model**. Table 8.70 shows the "optimum" performance of each solvency test.

Again, the amended resilience test makes little difference.

8.5 Discussion

The results from Offices B – G above, simple though the models are, suggest that the conclusions of Chapter 7 are not entirely peculiar to the circumstances of Office A which was used there. We can say that:

1. **The same valuation basis may behave quite differently in the circumstances of different offices**.

2. **One office can suffer more inadequacies than another, but fewer insolvencies**.

3. Different management of the *same* office in future can radically alter both adequacy and solvency, not necessarily in the same direction.

4. There is no consistent pattern to the strength or weakness of a given valuation basis in different circumstances, except perhaps the harshness of the static bases compared with the dynamic bases.

5. **The addition of a uniform solvency margin to a traditional reserve not only gives inconsistent results if different valuation bases are used, but also between one office and another**.

6. In the different circumstances of these offices, there is great similarity (within each office) between the numbers of "equal errors" attained by the prospective valuations at the "optimum" level of the A/L ratio.

7. The A/AS ratio has not been shown to be any better a measure of adequacy than any of the dynamic prospective bases.

8. The recent amendments to the U.K. resilience test have only a small effect on this model.

These conclusions do not give any encouragement to the employment of solvency margins on top of traditional valuation reserves, or at least not uniform solvency margins. The systems of RBC margins in the U.S.A., and MCCSR margins in Canada, are more office-specific than the E.C. margins; moreover they are used in conjunction with more uniform reserving bases. All the evidence, however, from the work described in Chapter 3, the moves towards cash-flow testing in North America, and the simple models described here, points away from prescribed margins and towards cash-flow analysis.

This does not solve the regulator's problems; on the contrary it creates a problem. If traditional valuations are not a reliable measure of strength — though it might be fairer to actuaries past to say that they have been rendered unreliable by the evolution of modern practices — what to use instead? There must, for pragmatic reasons, be some test of strength.

Following on from the above, we suggest such a test which is based on adequacy. As a result, it is completely office-specific, and has nothing to do with prescribed bases or margins (unless the regulator specifies the asset model and its parameters).

The suggested measure is the *estimated additional assets needed to avoid inadequacy*. The projections of run-offs, as well as revealing *which* scenarios give rise to deficits (inadequacies) also reveal *how bad* is the inadequacy. The amount of any deficit after the run-off can be discounted back to the starting point, using the earned rates of return in the particular scenario; the result is a measure of the additional assets which would have extinguished the deficit. (Under dynamic investment strategies, this might only be approximate.) Given a large enough number of scenarios, suitable quantiles of these discounted deficits indicate the additional assets needed to avoid inadequacy with given probability.

Essentially this measure is the same as was used by the FASWP (Chapter 3), but applied to an ongoing life office instead of to a single tranche of business.

In the remainder of this chapter, we investigate (i) the additional assets needed to avoid inadequacy, in the examples of Offices A – G above; and (ii) whether or not, in these simple models, adequacy can reasonably be measured using fewer run-offs than we have used.

8.6 The adequacy margin

In this section we define an adequacy margin which measures the magnitude of an office's inadequacy.

The definition follows the Faculty of Actuaries Solvency Working Party's approach to the calculation of adequacy margins in [37]. Given the projected cashflows assuming that the office is closed at time t, the assets remaining after the last policy has matured can be discounted back to the starting point of the projection. Let the result be X. If X is positive, there was a surplus after the run-off; if X is negative there was a deficit. Since we think naturally of holding extra assets to meet a deficit, we define the *adequacy margin for closure at time t* to be

$$\frac{-X}{\text{Asset shares at the starting point of the projection}}$$

We will usually express adequacy margins as a *percentage* of the asset shares at outset.

The adequacy margin is — sometimes exactly and sometimes approximately — the answer to the question "what extra assets (as a proportion of the asset shares) should be held *now* in order to close the office in t years' time and run off the liabilities with zero surplus?" A positive adequacy margin means that extra assets are needed; a negative adequacy margin means that the assets are more than sufficient.

In the case of the offices with solvency-driven asset allocation strategies, the adequacy margin does not give *exactly* the assets needed to eliminate the run-off surplus. This is because the asset allocation depends on the ratio A/L_2, and if assets equal in value to the adequacy margin were actually added to the office's assets this would, in some cases, alter the asset allocation and hence the rates of return. If the asset allocation strategy is independent of the office's total assets, then the adequacy margin does give exactly the assets needed to eliminate the run-off surplus.

In terms of adequacy, a positive adequacy margin for closure at time t indicates inadequacy at time t, and a negative margin indicates adequacy.

The discounting of the run-off surplus or deficit is perhaps most naturally carried out at the rates of return earned on the fund, although there are other possibilities. For example, solvency margins identified as such might be invested in fixed-interest assets. Or, adequacy margins which formed part of an office's free assets might be invested in equities. In this section, the rate of return on the fund will be used.

First we investigate the adequacy margins arising in the baseline office, Office A, before comparing these with the corresponding margins in Offices B – G.

8.6.1 Adequacy margins in Office A

Figure 8.112 shows the 5th, 25th, 50th, 75th, and 95th quantiles, and 10 sample paths, of the adequacy margin for closure at times $t = 41$ to $t = 60$, as a percentage of the asset shares at outset.

Some features of this figure require explanation.

1. Since the office pays 100% of the asset share on maturity, or the guaranteed benefits if these are greater, the adequacy margin is -20% in those simulations in which the guarantees are never called upon. In such cases, the benefits are met entirely out of the policies' own resources (the policy asset shares) and the additional estate of 20% of the asset shares at time $t = 40$ falls into surplus after the run-off. This explains why the 5th percentile is constant at -20%.

2. , For the same reason, the 25th and 50th quantiles are -20% of asset shares at outset. The 75th and 95th quantiles are higher than -20% at outset,

Figure 8.112: 5th, 25th, 50th, 75th, 95th quantiles, and 10 sample paths, of the adequacy margin (as % of asset shares at outset).

indicating that the proportion of simulations in which that the guarantees exceed the asset shares of maturing policies at least once during the run-off, following closure at time $t = 41$, lies between 50% and 75%.

3. The adequacy margins tend to be non-decreasing, again because at least 100% of asset shares are paid to maturing policies. Decreases are still possible, however, because the run-off surpluses are not necessarily discounted at the same rates of return (Closing the office at different times might alter the ratio A/L_2 after closure and hence the asset mix.) But many of the sample paths are level for considerable periods; these indicate simulations in which the office was able to maintain 50% in equities during the whole run-off, following closure during these periods.

Given a time horizon, the size of the adequacy margin needed to survive closure at any time during that time horizon can be calculated for each simulation. Let $AM_{i,t}$ be the adequacy margin for closure at time t in the i^{th} simulation. Then a margin

$$AM_i = \max_t AM_{i,t}$$

is needed to survive closure at any time t during the time horizon in the i^{th}

Figure 8.113: Maximum margins (sorted) in 1,000 simulations over time horizons of 5,10,15 and 20 years, in the baseline office (Office A).

simulation. The distribution of the 1,000 values of AM_i will indicate the adequacy margins which might be needed to achieve adequacy with any given probability.

In what follows the *maximum margin* in the i^{th} simulation means the maximum adequacy margin AM_i needed to ensure adequacy in the i^{th} simulation over some given time horizon.

Some caution is needed here. Given the simplifications made in the model, and the model's strategies, little weight should be attached to the absolute values of the adequacy margins in the tails of their distributions. However, the development of the maximum margins as the time horizon is increased, and the comparison of maximum margins under different conditions, will be of interest.

Figure 8.113 shows the maximum margins over four time horizons, of length 5, 10, 15 and 20 years. In each case the 1,000 maximum margins for that time horizon have been sorted in ascending order to give an idea of their distribution. The highest margin of all is almost 45% of the asset shares at outset.

Obviously, the maximum margins are non-decreasing functions of the time horizon. Figure 8.113 does not show much of the detail for those maximum margins which are positive, so Figure 8.114 shows just the positive maximum margins for the same four time horizons on a logarithmic scale.

These figures suggest that the distribution of maximum margins for the longer

Figure 8.114: Maximum margins (sorted) in 1,000 simulations over time horizons of 5,10,15 and 20 years, in the baseline office (Office A) on a log scale.

Moments and quantiles of maximum margins					
			Quantiles		
Time horizon	Mean	s.d.	90th	95th	99th
41 to 45	-16.24	6.17	-7.38	-3.01	5.06
41 to 50	-12.99	7.96	-1.38	1.57	13.31
41 to 55	-10.44	8.73	0.43	4.28	17.65
41 to 60	-8.35	8.91	1.63	5.69	19.48

Table 8.71: Moments and quantiles of maximum margins (expressed as % of asset shares at $t = 40$) in the baseline office (Office A) over different time horizons.

time horizons is approximately linear below zero and approximately exponential above zero

Figure 8.114 also shows the 95% and 99% points of the maximum margins. These, along with the 90% points and the first two moments, are shown in Table 8.71.

The 90%, 95% and 99% points suggest what additional resources would be needed at outset, as a percentage of asset shares at outset, to ensure adequacy with these given probabilities. However, as mentioned above the absolute values ought not to be given too much weight.

It is worth pointing out, however, that the ratio A/L_1 at outset is 1.38, which appears to indicate that the office could reduce its assets by 33.3% of the asset shares at outset, and remain solvent. This ignores any requirement to set up an E.C. solvency margin; allowing for that the office could reduce its assets by about 29% of the asset shares at outset, and satisfy the E.C. requirements. Yet such a deficiency in the assets would fall very far short of adequacy. This is, of course, largely a reflection of the U.K. practice of accruing terminal bonuses outside the reserves.

Note that the figures in Table 8.71, although quoted as percentages of the asset shares, are in fact the assets needed in addition to the *total* assets. An A/AS ratio of 1.20 at outset does not, for example, suggest that the office could lose assets worth 19.48% of its asset shares and remain adequate over a 20-year time horizon with 99% probability. It means that the office would have to *acquire* further assets worth 19.48% of its asset shares to put itself in that position. It is interesting that an additional estate of 20% of the asset shares does not confer a higher probability of adequacy.

8.6.2 Adequacy margins in Offices B – H

The adequacy margins in Offices B – G are distributed after the pattern of those in Office A. They are most easily summarised and compared in a table such as Table 8.71 above. Tables 8.72 – 8.75 show the same moments and quantiles of the maximum margins over time horizons from 5 years to 20 years.

It is interesting to note what happens if we alter the assets possessed at outset by one of the offices with a solvency driven investment strategy. As mentioned above, this will alter the asset allocation and hence the solvency and adequacy experience. We would expect a *reduction* in the assets held at outset, which of itself must worsen adequacy, to result in more frequent switches towards gilts, which tends to improve adequacy. This will be reflected in the difference between the adequacy margins of the two offices.

For example, suppose we took away from Office B, at time $t = 40$, assets worth 10% of its asset shares, leaving it with an A/AS ratio of 1.10 instead of 1.20. Then its adequacy margins would simply increase by 10%, in every scenario. But if we took the same amount away from Office A, at time $t = 40$, we should expect the

Time horizon 5 years ($t = 41 - t = 45$)					
Moments and quantiles of maximum margins					
				Quantiles	
Office	Mean	s.d.	90th	95th	99th
A	-16.241	6.167	-7.380	-3.010	5.056
B	-15.128	9.551	-5.067	3.210	23.316
C	-10.566	9.429	1.649	5.704	18.837
D	-18.720	3.456	-16.005	-12.379	-3.600
E	-19.744	0.192	-19.530	-19.477	-19.343
F	-15.655	8.367	-4.953	-0.279	12.173
G	-16.210	6.198	-7.236	-2.694	6.180
H	-7.718	4.101	-3.072	-0.058	9.057

Table 8.72: Moments and quantiles of maximum margins (expressed as % of asset shares at $t = 40$) in Offices A – H over a 5-year time horizon.

Time horizon 10 years ($t = 41 - t = 50$)					
Moments and quantiles of maximum margins					
				Quantiles	
Office	Mean	s.d.	90th	95th	99th
A	-12.995	7.957	-1.376	1.570	13.312
B	-10.158	14.863	6.185	21.439	55.697
C	-7.588	9.961	4.170	8.494	22.709
D	-16.892	6.086	-10.239	-3.744	9.029
E	-18.769	0.600	-18.215	-17.999	-17.074
F	-12.002	10.001	1.769	6.642	18.921
G	-13.133	7.671	-1.488	1.418	8.925
H	-5.881	5.073	0.416	3.178	12.525

Table 8.73: Moments and quantiles of maximum margins (expressed as % of asset shares at $t = 40$) in Offices A – H over a 10-year time horizon.

adequacy margins to increase by less than 10%, because of the compensating shifts towards gilts. We therefore define Office H as follows:

Office H Office H is identical to Office A, except that we have reduced its additional estate at time $t = 40$ to 10% of the asset shares instead of 20%.

These tables show some very interesting points.

1. These margins are additions to the *total* assets, though expressed as percentages of the asset shares. Therefore the existence of a 20% additional estate (10% for Office H) does *not* mean that margins up to +20% are acceptable. Only negative margins are acceptable.

Time horizon 15 years ($t = 41 - t = 55$)					
Moments and quantiles of maximum margins					
			Quantiles		
Office	Mean	s.d.	90th	95th	99th
A	−10.443	8.730	0.433	4.278	17.649
B	−5.574	18.380	19.151	32.564	64.375
C	−5.371	9.987	5.312	9.138	27.520
D	−15.119	7.693	−4.474	0.544	17.680
E	−18.593	0.636	−18.045	−17.814	−16.771
F	−9.275	11.649	4.676	10.781	28.240
G	−10.618	8.353	0.387	3.790	12.266
H	−4.519	5.537	1.679	4.493	15.453

Table 8.74: Moments and quantiles of maximum margins (expressed as % of asset shares at $t = 40$) in Offices A – H over a 15-year time horizon.

Time horizon 20 years ($t = 41 - t = 60$)					
Moments and quantiles of maximum margins					
			Quantiles		
Office	Mean	s.d.	90th	95th	99th
A	−8.351	8.914	1.630	5.691	19.476
B	−1.264	21.818	27.486	41.593	79.337
C	−3.722	9.765	6.253	9.625	28.855
D	−13.457	9.050	−1.275	6.062	18.459
E	−18.365	0.684	−17.667	−17.463	−16.478
F	−7.450	11.906	6.814	11.853	29.817
G	−8.630	8.478	1.561	5.134	15.194
H	−3.289	5.640	3.001	6.601	15.668

Table 8.75: Moments and quantiles of maximum margins (expressed as % of asset shares at $t = 40$) in Offices A – H over a 20-year time horizon.

2. The margins in Office E — the "traditional" office — are all quite close to -17%, and do not vary much with the time horizon. This shows that the cost of meeting the guarantees is easily contained by the 20% additional estate, despite paying a minimum of 100% of the asset shares on maturity.

3. All the offices except Office E require additional assets at the 95% level over the longer time horizons. Even Office D does not escape.

4. **Offices A, B, C, F and G are *identical* at time $t = 40$ — the point at which adequacy is being measured**. They differ only in the strategies which are employed *after* time $t = 40$. Offices B and C differ only in the strategies which will be employed during a run-off. Offices A and F differ only in the smoothing of benefits. Yet the maximum margins are different, each reflecting the future strategies. Traditional valuations, based entirely on an office's *current* state, are completely blind to this source of variation. Dynamic Solvency Testing in Canada, whatever its faults in falling back on the MCCSR test, has at least this strength: the actuary has to have regard to the likely actions of management, given the circumstances of each scenario tested (hence the "Dynamic").

5. We saw, in Chapter 7, and again in this chapter, that a uniform solvency margin could have different effects depending on the circumstances of the office, and on the valuation method in use. The maximum margins shown here are estimates of additional margins needed to avoid ruin with given probability — "true" solvency margins in one sense — and **the range of different answers is another point against uniform solvency margins**. Note that every office *including* Office H has the same total of asset shares at outset, so these maximum margins are comparable.

6. Office H, despite having lost assets worth 10% of its asset shares compared with Office A, does *not* have maximum margins 10% higher (as a percentage of asset shares at $t = 40$).

   - The *mean* maximum margins are about 8.5% higher over a 5-year time horizon, but only about 5% higher over a 20-year time horizon.

- The *standard deviations* of the maximum margins are lower over all time horizons.
- The upper quantiles of the maximum margin in Office H never exceed those in Office A by more than 5%, and over longer time horizons the 95th and 99th quantiles are even sl lower than those in Office A.

This example is worth looking at from the opposite point of view: suppose we started with Office H, and carried out projections of adequacy margins (as we have done). Suppose our (or the government's) criterion was "no more than 5% chance of ruin over a 10-year time horizon" (this does not seem excessive). Then Table 8.73 shows that additional assets equal to 3.178% of asset shares are required. Now suppose that, with proper actuarial prudence, we said "3.178% good — 10% better!". We have now transformed Office H into Office A, and note that *the investment strategy is identical in both offices.* But the improvement in the additional estate simply allows management to invest more in equities, more often, with the result that if we re-do the calculation of adequacy margins, Table 8.73 tells us that we need *another* 1.57% of asset shares *on top of* the extra-prudent 10% we had already added.

In this example, an asset allocation strategy driven by solvency had this unfortunate side-effect. But clearly *any* strategy which regards "free" assets, however defined, as available for investment in riskier assets will do the same. Such approaches to "free" assets are extremely common in the U.K..

8.7 Testing adequacy at less frequent intervals

In Office A and some others, very few offices ever recovered from adequacy. In such cases, it might be reasonable to test for adequacy over any given time horizon by testing adequacy at one time only — at the end of the time horizon. For example, in Office A there were 157 cases of inadequacy during 20 years, and of these 143 were inadequate *at* time $t = 60$. Therefore, had we tested adequacy *only* at time $t = 60$, instead of at all times $t = 41, 42, \ldots, 60$, we would only have missed 14 cases, or about 10% of the total. This might be considered a reasonable price to pay, given

	Office C		Office F	
Time t	Number	Cum. tot.	Number	Cum. tot.
$t = 45$	100	155	41	47
$t = 50$	141	232	94	117
$t = 55$	183	310	139	183
$t = 60$	215	368	155	233

Table 8.76: Numbers and cumulative totals inadequate at 5-yearly intervals in Offices C and F.

the enormous reduction in computing time.

Ross & McWhirter [58] effectively do this, using a time horizon of 40 years. They found that only 1 simulation (out of 49) resulted in a deficit after closure at the end of 40 years, or in our terms only one inadequacy out of 49 at time 40 years after the start. If their office, like Office A, was such that few inadequate offices recover, then their result is a fair indication of the ability of that office to transact business over the next 40 years. If, however, their office is such that temporary spells of inadequacy are more common, then the test they applied is too weak. In particular, it is then inconsistent to compare statutory insolvency occurring in *any* of the 40 years with inadequacy only at the *end* of 40 years.

Two of the offices above display reasonable rates of recovery from adequacy; Office C (with the same strategies before and after closure) and office F (with maturity benefit smoothing). Table 8.76 shows the numbers inadequate, and the cumulative totals of inadequacies, at 5-yearly intervals in these offices.

It is obvious that if we were to test the adequacy of either of these offices only at time $t = 60$, we would form a much more favourable view of their adequacy than would be justified by the facts. Indeed, if we compared office F with Office A on this basis, (155 inadequacies instead of 143) we would conclude that smoothing had a much milder effect than the cumulative totals suggest (233 inadequacies instead of 157).

Note that Ross & McWhirters' model included two features — benefit smoothing and a charge on the asset shares — which make it likely that it has some of the properties of Office F.

However, testing adequacy in every future year is laborious, so it is still worth knowing if sampling inadequacy in just a few future years gives reasonable results.

	Office C			Office F		
Time horizon t	Number	Cum. tot.	Detected	Number	Cum. tot.	Detected
5 years	100	155	100	41	47	41
10 years	141	232	162	94	117	101
15 years	183	310	225	139	183	161
20 years	215	368	271	155	233	204

Table 8.77: Numbers and cumulative totals inadequate at 5-yearly intervals in Offices C and F, and the numbers of inadequacies detected by testing only at 5-yearly intervals.

Here we consider testing adequacy at 5-yearly intervals. For example, with a time horizon of 5 years we would test adequacy at time $t = 5$ only; With a time horizon of 10 years we would test adequacy at times $t = 5$ and $t = 10$, and so on. (It is clear already from Table 8.76 that the first of these examples will not be satisfactory in Office C.) We wish to see *how many* inadequacies are counted if we cut down the test points. We also wish to know how the *estimated maximum margins* are affected if we base them on the reduced data.

8.7.1 Numbers of inadequacies

Table 8.77 shows the numbers of inadequacies detected by testing for inadequacy at 5-yearly intervals during the given time horizons (in the columns headed "Detected". For convenience, the figures in Table 8.76 are reproduced also.

In the case of Office F, about 87% of the cumulative number of inadequacies is detected, which might be a reasonable level in practical circumstances. In Office C rather fewer are detected — about 74% over the 20-year time horizon. This is an improvement on testing adequacy only at the end of the time horizon, but if we note that the additional samples at $t = 45$, $t = 50$ and $t = 55$ caught an extra 56 inadequacies but did *not* catch 97 more, it is clear that reducing the number of times at which adequacy is tested can open up considerable gaps in our knowledge.

8.7.2 Maximum margins

Although testing adequacy only at a sample of times in the future causes us to miss some inadequacies, it might not affect the upper quantiles of the maximum margins.

Time horizon 20 years ($t = 41 - t = 60$)					
Moments and quantiles of maximum margins					
			Quantiles		
Time horizon	Mean	s.d.	90th	95th	99th
Office C					
5 years	−11.526	8.625	−0.043	3.793	17.270
10 years	−8.789	9.293	2.437	6.207	18.868
15 years	−6.627	9.373	3.643	7.098	22.192
20 years	−4.997	9.154	4.765	8.099	23.607
Office F					
5 years	−18.458	9.458	−5.786	−0.841	12.173
10 years	−14.157	10.823	0.023	6.294	18.921
15 years	−11.3	12.328	4.346	9.969	25.721
20 years	−9.308	12.557	6.298	11.061	26.516

Table 8.78: Moments and quantiles of maximum margins (expressed as % of asset shares at $t = 40$) in Offices C and F, with adequacy tested at 5-yearly intervals only.

Intuitively, those simulations which produce the worst results are more likely to be inadequate for longer, and hence have a higher chance of being detected. They are also more likely to be represented in the upper quantiles of the distribution of the maximum margin.

We will widen the definition of a maximum margin from that above,

$$AM_i = \max_t AM_{i,t}$$

so that the maximum is taken over the set of times t at which adequacy is tested, which now need not be every year during the time horizon. Table 8.78 shows the moments and quantiles of the resulting distributions of maximum margins for time horizons of 5, 10, 15 and 20 years, with adequacy tested at 5-yearly intervals.

Comparing this with Tables 8.72 – 8.75, we see that the upper quantiles of the maximum margins are reduced by comparatively little; in practical terms the figures in Table 8.78 might be quite reasonable lower bounds to the more accurately — and laboriously — computed figures.

8.8 Conclusions

1. The same valuation basis, applied to different offices, varies in its effectiveness compared with the run-off cash-flows.

2. The strategies used by the office's managers have a very great effect on adequacy, including strategies *after closure* which do not affect the assessment of solvency.

3. The effect of a uniform solvency margin on the accuracy of a traditional valuation also depends very much on the individual circumstances.

4. Adequacy margins based on the distributions of projected run-off deficits are also very dependent on future strategies, again including strategies after closure.

5. Estimating maximum margins by projecting closure and run-off at a smaller set of future times underestimates the *numbers* of future inadequacies but (in this model) yields quite good lower bounds for maximum margins.

Chapter 9

Summary

In this chapter we summarise the conclusions of the preceding chapters, but first we will state our main conclusion. **The wide range of strategies available to life office managers, even allowing for constraints such as PRE, make prescriptive or mechanical measurement of financial strength unreliable. Traditional tools such as solvency valuations and uniform solvency margins give inconsistent results in different circumstances. It is necessary to allow for the features of individual offices and their managements in assessing financial strength.**

9.1 The development of solvency assessment

Until the development of computers, the measurement of solvency relied upon present values based upon particularly simple models of future economic conditions. The assets in which funds were invested, however, were such that the simple models gave workable tools for measuring solvency and surplus. The outcomes in respect of Office E in Chapter 8 suggest that fixed-interest investment combined with suitable bonus rules would would allow a simple office to be managed tolerably well with traditional solvency valuations.

The volatility of inflation, yields and equity prices since the 1950s has made the traditional "constant interest rate" model look unrealistic. Moreover, with-profits funds have invested heavily in equities and properties during that time. Therefore the model underlying solvency valuations is, qualitatively, unlike the experience of the corresponding assets.

Concerns about the effectiveness of solvency valuations have been pursued in three main directions (see Chapter 2).

1. Mandatory or minimum valuation bases have been prescribed in territories where there is no premium tariff and no practice of valuing on the premium basis — principally in the U.K., the U.S.A. and Canada.

2. Additional margins have been prescribed to strengthen these mathematical reserves — the E.C. solvency margins, RBC requirements in the U.S.A., MCCSR in Canada and the U.K. resilience test.

3. Tests which are less prescriptive and more specific to the features of individual companies have been developed, notably Dynamic Solvency Testing in Canada but also in the U.S.A. and Australia.

Some of the RBC and MCCSR margins have been based on tail probabilities in stochastic studies, but in the main the details of these rules have been determined in *ad hoc* ways or by sensitivity analyses within the traditional model. A particular criticism of the E.C. solvency margins is their application to a broad range of valuation regimes and methods. Even Canadian DST is based on projections of the results of traditional solvency valuations.

An alternative approach to solvency is to find stochastic replacements for the traditional valuation models, and to examine the distribution of projected outcomes, in particular estimated ruin probabilities. Solvency reserves are then determined as the funds needed to limit ruin probabilities to some small value (see Chapter 3).

9.2 Life office modelling in the U.K.

Models of U.K. with-profits life offices must allow for the considerable degree of discretion given to managers to control asset allocation, bonus and smoothing strategy, and perhaps other factors too (see Chapters 3 and 4). The strategies which are chosen have a considerable effect on the outcomes, including solvency. Among the questions which arise are those considered in this thesis:

1. How much of the fund should be invested in equities?

2. What should be the balance between reversionary and terminal bonuses?

3. How should investment returns be smoothed?

4. How should financial strength be measured?

A simple model office, writing 10-year with-profit endowment business, was set up. In order to focus on asset allocation and bonus strategies, (i) mortality, lapses and expenses were excluded and (ii) the office was taken to be in a state of stable and sustainable growth as a starting point. The model was the basis for a series of simulation studies, using the Wilkie Reduced Standard model of inflation, fixed-interest yields and equity returns.

9.3 Equity investment and statutory insolvency

In Chapter 4 the circumstances leading to statutory insolvency in the model were investigated, particularly in the light of the suggestion made in a previous study that low inflation has an adverse effect on solvency. The main conclusions were:

1. There was some indication that the *distribution* of the rate of inflation was low in the years preceding insolvency, but the *sample paths* did not show the same feature. Thus low inflation — or its consequences — could not provide a clear mechanism of failure.

2. Statutory insolvency was strongly associated with catastrophic falls in share prices. This association was a property of the sample paths.

3. Falls in share prices were influenced largely by the white noise components of the dividend yield model and dividend index models. Inflation was a lesser influence.

9.4 Asset allocation strategies

Previous studies using the Wilkie asset model have compared equity and gilt (fixed-interest) investment and concluded that the former achieves much higher mean returns but with a much higher variance of returns; therefore equity investment is "riskier". In Chapter 5 a range of asset allocation strategies were studied, particularly their effects on (i) maturity values and (ii) statutory solvency.

The conclusion above was confirmed in terms of *nominal* maturity values, but examination of *real* maturity values suggested that the relative safety of gilts may be overstated; the variance of real maturity values under gilt investment, relative to that under equity investment, was much less than the variance of nominal maturity values. On the other hand, investment in equities led to much higher numbers of statutory insolvencies.

The asset allocation strategy was modified along the lines suggested by Ross [57] to switch from equities to gilts when necessary in order to maintain statutory solvency; this did cut the numbers of insolvencies considerably, but only by allowing asset switches of up to 100% of the fund in a single year. Restricting the proportion of the fund switched in a single year to a lower level (25%) removed much of the advantage of the solvency-driven asset switching.

A range of dynamic asset allocation strategies was studied, including declining EBR, cyclical and contracyclical strategies. On the basis of *nominal* maturity values, the contracyclical strategies performed better than their cyclical counterparts, giving higher mean maturity values with less variability, and slightly better than the corresponding fixed investment strategy. On the basis of *real* maturity values, cyclical strategies displayed the lower variability.

Changes to the reversionary bonus strategy made relatively little difference, possibly due in part to the short term of the business.

9.5 Maturity value smoothing

In Chapter 6 the effect of maturity value smoothing was investigated. Smoothing is often cited as one of the "hallmarks" of with-profits business, particularly in conjunction with equity investment. It is usually assumed that the long-term costs of smoothing will balance out. However, in this model the cost of smoothing (measured by the ratio BSA/AS) was not necessarily stable over time; the accumulated historic costs of smoothing were considerable in certain cases.

1. Two methods of smoothing cited by respondents to an industry survey were studied. Smoothing of maturity values appeared to be more effective than smoothing of asset values, and it absorbed a larger part of the cost of the guarantees, but at the expense of stability of the accumulated cost of smoothing.

2. The relative cost of smoothing was sensitive to the rate of new business growth; high growth contained the cost but there appeared to be considerable danger in smoothing while new business was declining.

3. Applying a simple form of feedback to control the level of the ratio BSA/AS was not successful, because the need to meet the guarantees prevented the feedback from being applied in adverse circumstances. In fact, any method which limited the volatility of the ratio BSA/AS appeared to be vulnerable to the incidence of the guarantees.

4. The cost of smoothing depends on the stability of the asset model, particularly if the assets are revalued. The sensitivity of equity values to dividend yields is a particular risk.

9.6 Inadequacy *versus* insolvency

In Chapters 7 and 8 the accuracy and timing of traditional solvency valuations were measured by comparing the valuation results with subsequent run-offs in each of 1,000 scenarios. The term "adequacy" was used to indicate a surplus after a run-off.

1. There was little coincidence, from year to year, between offices which were statutorily insolvent and offices which were inadequate. Over a long time, however, the statutory valuation succeeded in detecting a fairly high proportion of inadequate offices.

2. The number of insolvencies was very sensitive to the level of A/L ratio which was used as a criterion of closure. Therefore the imposition of a uniform solvency margin, even one as small as 4% of mathematical reserves, can have a considerable and to a large extent arbitrary effect.

3. Different valuation methods and bases resulted in large differences between the numbers of insolvencies. In particular, fixed (static) bases seemed to be markedly harsher than dynamic bases including the U.K. statutory minimum basis. It follows that the effect of a uniform solvency margin, depends very strongly on the underlying valuation regime.

4. The ratio A/AS was not a satisfactory alternative to the ratio A/L as a measure of solvency.

As well as the effect of different valuation bases on the *same* office, the effect of the same valuation bases on *different* offices was considered. The broad conclusion was that both (i) the outcomes and (ii) the effectiveness of a traditional valuation varied markedly from office to office. The strategies used by the office's managers had a very great effect on adequacy, including strategies *after closure* which had no effect on the assessment of solvency. A uniform solvency margin in these circumstances does not lead to any specific improvement in the solvency valuation.

The simulated distribution of the discounted surpluses and deficits after run-offs allows adequacy margins to be estimated, namely additional assets sufficient to avoid inadequacy with some given probability. These margins are also very dependent on the strategies used by the managers; in particular high equity investment requires high margins. There was no link between the level of adequacy margins and any traditional valuation reserve.

In some cases offices were found to recover from adequacy with reasonably high frequency, making it necessary to test run-offs from every possible time of closure in order to measure adequacy within any given time horizon. Some limited trials suggested that reasonable lower bounds of adequacy margins for given time horizons might nevertheless be found by projecting run-offs following closure at a limited set of future times.

9.7 Further questions

The questions which we have explored here, in an empirical way, raise a host of further questions, many concerned with relaxing the strong simplifying assumptions made here. We can only indicate some possible lines for further research.

1. What is the effect of a mixture of business of different terms on the cost of smoothing and the comparison between solvency and adequacy?

2. It is unlikely that the premium rate would remain fixed in the face of the very low rates of return which occur in some scenarios. What is the effect of a premium-setting strategy?

3. The traditional with-profits endowment is being replaced by the unitised with-profits contract in some market sectors. Many aspects of the management of unitised with-profits business remain to be decided; for example there is no agreement on valuation methods. What is the effect of this new bonus system on adequacy?

4. How might option pricing or hedging strategies be incorporated into asset allocation and bonus strategies? Do such approaches offer a stronger basis for the measurement and control of surplus than a traditional valuation?

5. What effect does the pattern of new business, and surrenders, in the past and in the future, have on the working out of the in-force?

6. How do different starting conditions — for example, an additional estate — affect the outcomes?

7. What is the effect of changing the asset model? Alternatives to Wilkie's AR(1) model of inflation have been suggested, for example ARCH models: are solvency and adequacy sensitive to such assumptions?

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