

Annuities

ANGUS S. MACDONALD

Volume 1, pp. 73–75

In

Encyclopedia Of Actuarial Science
(ISBN 0-470-84676-3)

Edited by

Jozef L. Teugels and Bjørn Sundt

© John Wiley & Sons, Ltd, Chichester, 2004

Annuities

An annuity is a series of payments made to or from a person or an institution. The payments may be level or variable, may be due at regular or irregular intervals, and may be certain to be received or contingent upon specified events. In ordinary usage, ‘annuity’ is usually understood to mean regular payments contracted to be made to a person or persons as long as they are alive, of which a **pension** is an example. In actuarial mathematics, however, an ‘annuity’ has a much more general meaning. Its broadest definition is of a series of payments that will be made as long as any given *status* is complete or fulfilled.

The actuarial term *status* is best defined by simple examples. Consider a pension payable to someone now age x for the rest of their life. The status is ‘being alive’ and it remains complete as long as the person lives. Or, consider a pension payable from age 65 to someone who is now age 25. The status is ‘being alive and older than age 65’; it is not fulfilled until the person reaches age 65 (so the possibility exists that it might never be fulfilled) and, if they do, it then remains complete until they die. Finally, consider the coupon payments under a risk-free coupon bond with term 20 years. The status is ‘not more than 20 years from now’, and coupons will be paid as long as this condition is fulfilled.

International actuarial notation indicates statuses by subscripts to the symbols denoting expected present values, A in the case of insurance payments that will be made when a status fails, and a in the case of annuities. Assuming payments of \$1 yearly in arrears, the expected present values of the three examples above would be denoted a_x , ${}_{40|}a_{25}$ and $a_{\overline{20}|}$ respectively (the latter being an expectation only in a trivial sense).

The principle types of annuity are listed below.

- An *annuity-certain* is an annuity whose payments are certain to be made, and are not contingent on any events or subject to any risk. The coupon payments under a riskless coupon bond are an example. (See **Present Values and Accumulations** for further examples, in particular, the associated actuarial notation and formulae.)
- A *contingent annuity* is an annuity whose payments are not certain to be made, but are contingent on specified events. These can be events

that might or might not occur (such as the failure of a company that has issued a coupon bond), or events that are certain to occur but whose timing is uncertain (such as death).

- A *life annuity* is a contingent annuity depending on the survival of one or more persons.
- A *term* or *temporary* annuity is one payable for, at most, a predefined limited term. The coupons under a coupon bond provide an example, but life annuities can also be temporary; for example, an annuity could be paid to a person now age 50 until they die or until 10 years have passed, whichever happens first. This is an example of a compound status, because two basic statuses – ‘being alive’ and ‘not more than 10 years from now’ – have to be fulfilled simultaneously. The notation allows such compound statuses to be expressed simply by putting both in the subscript; assuming payment yearly in arrears, the expected present value of this annuity would be denoted $a_{50:\overline{10}|}$. ($a_{\overline{10}|:50}$ would be equally valid but unconventional.)
- A *deferred annuity* is an annuity whose payments do not commence at once, but at some time in the future. The promise to the person age 25, of a pension when they reach age 65, is an example. An annuity whose payments do commence at once is called an *immediate annuity*.
- A *reversionary annuity* is an annuity whose payments commence when a given status fails. An example is a widow’s pension, payable to a woman now age y during her lifetime, as long as her husband now age x has predeceased her; in other words, it is payable as long as the status x has failed but the status y is still complete. Assuming payment yearly in arrears, the expected present value of this annuity would be denoted $a_{x|y}$. A reversionary annuity can be regarded as a deferred annuity, in respect of which the deferred period itself is contingent upon an event.

Apart from the status that determines whether or not an annuity is in payment, the timing, frequency, and amounts of the payments may vary, and the standard actuarial notation allows for the most commonly used cases.

- By convention, the ‘basic’ annuity is one whose first payment is due at the end of the first time period; expected present values of such annuities have the unadorned symbol a (with suitable subscripts indicating the status). They are also said

2 Annuities

to be *payable in arrears*. An annuity whose first payment is due at once is called an *annuity-due* and is said to be payable in advance; the expected present values of such annuities are denoted \ddot{a} . The terminology (and notation) allows the same annuity to be expressed in different ways, for example, an immediate annuity with level yearly payments (made in arrears) is also an annuity-due, deferred for one year, and (taking a life annuity as an example) $a_x = {}_1|\ddot{a}_x$.

- Annuities payable m times a year have expected present values denoted $a^{(m)}$ or $\ddot{a}^{(m)}$, with the convention that the annual amount of annuity is always \$1, so $\ddot{a}_{40:\overline{10}|}^{(12)}$ (for example) denotes the expected present value of an annuity-due of \$1/12 per month, payable to a person now age 40 until he dies or until 10 years have elapsed, whichever period is shorter.
- A slightly theoretical abstraction is an annuity whose payments are made continuously; the expected present values of such annuities are denoted \bar{a} .
- Increasing annuities have expected present values denoted (Ia) , $(I\ddot{a})$, and so on, with many variations to account for the intervals at which the increases occur, as well as the frequency of payment. (See **Present Values and Accumulations** for examples.)
- Several types of life annuities arise in connection with the payment of regular premiums under **life insurance** policies. Note that such premiums are always payable in advance. Under policies with *installment* premiums, the premiums continued

to be paid until the end of the (policy) year of death, while policies with *apportionable* premiums refunded the balance of the last premium paid but ‘unearned’, in proportion to the period between the date of death and the next premium due date.

- Similar in concept to apportionable premiums, but obviously applicable to annuities paid to, rather than by, the individual, a *complete* or *apportionable* annuity has a final payment made on death, proportionate to the period between the last payment and the date of death. Its expected present value is denoted \dot{a} .

Standard references are [2, 3], for annuities-certain, and [1, 4], for all varieties of life annuities.

References

- [1] Bowers, N.L., Gerber, H.U., Hickman, J.C., Jones, D.A. & Nesbitt, C.J. (1986). *Actuarial Mathematics*, The Society of Actuaries, Itasca, IL.
- [2] Kellison, S.G. (1991). *The Theory of Interest*, 2nd Edition, Irwin, Burr Ridge, IL.
- [3] McCutcheon, J.J. & Scott, W.F. (1986). *An Introduction to the Mathematics of Finance*, Heinemann, London.
- [4] Neill, A. (1977). *Life Contingencies*, Heinemann, London.

(See also **International Actuarial Notation; Life Insurance Mathematics; Present Values and Accumulations**)

ANGUS S. MACDONALD