An Example of Scientific Writing

This example is based on the article *Genetics and Insurance Management* which appeared in the book published to mark the centenary of the Swedish Society of Actuaries.

It is describing a model of life histories relevant to a critical illness insurance market, and we are interested in whether or not people actually purchase critical illness insurance, and whether or not having had a genetic test affects this decision. In particular, we want to quantify the costs of adverse selection if applicants for insurance need not disclose the results of a genetic test. The following notes may help with the context.

- Critical illness (CI) insurance pays a lump sum, like life insurance, but instead of paying on death, it pays if the insured person is diagnosed with one of a list of major illnesses. The list usually includes heart attack, stroke, malignant cancer and kidney failure.
- A genetic test may reveal that someone carries a faulty copy of a particular gene that may significantly increase their risk of developing a serious illness. For example a faulty version of the APKD1 gene, carried by about 0.085% of the population, is extremely likely to lead to Adult Polycystic Kidney Disease (APKD) and to kidney failure before age 60.
- Earlier in the paper from which this example is taken, a model was introduced in which the label i was used to denote genotype: for example, we might use $i = 0$ to denote that subset of the population whose members carry a normal APKD1 gene, and $i = 1$ to denote that subset of the population whose members carry a faulty APKD1 gene.
- The model described here assumes a continuous time multiple-state framework, in which the basic model parameters are the transition intensities, and cash-flows such as premiums are payable continuously.

The author's aim is to introduce the model, and then to explain why it is inconvenient to make the usual assumption that level premiums are payable.

The Article

The size of the market is an important question, this is basically concerned with the movements into insurance states. Also we would like to know how much people are tested so we want to know about movements into the tested state too. And why are people tested for different kinds of diseases may be interesting.

To carry out the required analysis, we have to use the model in figure 1. This model is a multiple-state model with 6 states. People start healthy at a young age, and later get critical illnesses or die. Meanwhile they are all buying insurance and getting genetically tested. It is Markov because we know that intensities are always a function of age. By i I mean to indicate genotype, that is if a person has the gene or not. Transition into the state of being tested is represented by the quantity μ^{i02}_{x+t} while transition into the state of being insured is given by μ^{i01}_{x+t} unless you are tested when it is μ^{i23}_{x+t}. μ^{i04}_{x+t} to μ^{i34}_{x+t} are the transitions we are really interested in, as these are Critical Illnesses. μ^{i05}_{x+t} to μ^{i35}_{x+t} we aren't interested in as much, these are deaths and I am looking at Critical Illness insurance.

Since life offices are concerned that we may have adverse selection, which is when people who know about their future illnesses are more keen to buy insurance, this is also part of the model. That is, we have that it is possible for people who are tested to have the potential to want to buy vast amounts of insurance. And, medical scientists say that soon everyone will be tested. This is in the intensity rate μ^{i23}_{x+t}, which is between tested and insured.

Mainly the problem with this model is that it isn't possible for us to charge a level premium. Premiums can't be level in case different people are insured at different ages, when the premium they should be charged by the office isn't the same. This means the following calculation, suppose T is the future lifetime of a person age x and the sum assured is S. The equivalence principle means that

$$S\overline{A}_x = P\bar{a}_x$$

but if x is different then so is P. This means that if lots of people are in the same state but they all took out their policies at different times, we have to average over all the different values of P in the state to get a premium, and this is not Markov. It is impossible to do computations in a multiple-state model if it is not Markov, and

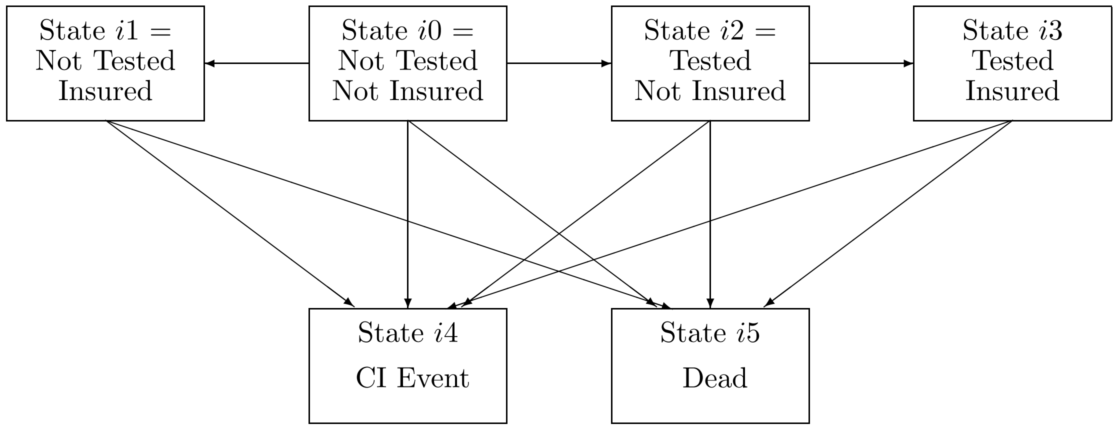

Figure 1: *The model*

all the intensities need to be constant too or we can't find μ when we come to a maximum likelihood estimate.

We can see what we mean, and what the answer is, if we use a martingale. $N(x+t)$ counts the number of deaths by that age, and $I(x+t)$ is one if the person is alive before that age and zero if they are not. It is well known that:

$$M(x+t) = N(x+t) - \int_0^t I(x+t)\mu(x+t)dt$$

is the martingale, as in the book *Statistical Models Based on Counting Processes* by P.K. Andersen, O. Borgan, R. Gill and N. Keiding. A martingale must be with respect to something, this one is with respect to what we know by watching people transition from alive to dead. The expected present value of the insurance loss is:

$$\mathrm{E}[L] = \mathrm{E}[\int_0^\infty e^{-\delta t} S \; dN(x+t) - \int_0^\infty e^{-\delta t} P \; I(x+t)dt]$$

Insurers always use the principle of equivalence, as it is the basis of all insurance and is based on the Law of Large Numbers. Premium P agrees with the principle because

$$\mathrm{E}[L] = \mathrm{E}[\int_0^\infty e^{-\delta t} S \; dN(x+t) - \int_0^\infty e^{-\delta t} P \; I(x+t)dt] = 0$$

where the force of interest is called δ.

We get better results, however, if instead of putting P as a constant level premium we make $P = S\mu(x+t)$, then

$$\begin{aligned}
\mathrm{E}[L] &= S \; \mathrm{E}[\int_0^\infty e^{-\delta t} dN(x+t) - \int_0^\infty e^{-\delta t} I(x+t)\mu(x+t)dt] \\
&= S \; \mathrm{E}[\int_0^\infty e^{-\delta t} dM(x+t)] = 0
\end{aligned}$$

because of the well-known property of a martingale which holds in this case. This P is not affected by the time when someone decides they want to buy insurance and so it is Markov, which is better in the model. If a model isn't Markov it is very difficult because the Thiele equation doesn't work. But since we have a Markov model we can use the well-known Thiele equation to find prospective policy reserves, and we have the Kolmogorov equation $\frac{d}{dt}{}_tp_x = -{}_tp_x\mu_{x+t}$ too if we need probabilities, and the Norberg equation too for moments.