THE GENETICS OF ALZHEIMER'S DISEASE, MODELLING DISABILITY AND ADVERSE SELECTION IN THE LONG-TERM CARE INSURANCE MARKET

By

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I hereby declare that the work presented in this thesis was carried out by myself at Heriot-Watt University, Edinburgh, except where due acknowledgement is made, and has not been submitted for any other degree.

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Abstract

The discovery in 1991 that a person's apolipoprotein E (APOE) genotype provides an indication of their predisposition to Alzheimer's disease (AD), which accounts for a significant proportion of long-term care costs, raises the question of adverse selection in the long-term care insurance market and about long-term care costs in general.

The aims of this thesis are: to develop multi-state models that help to quantify this potential for adverse selection; to parameterize such models using the available data; and to use these parameterized models to estimate the potential costs of adverse selection in the long-term care insurance market. This thesis is in two parts.

The first part (Chapters 1 and 2) concentrates on Alzheimer's disease (AD). In Chapter 1, I discuss the epidemiology and genetics of Alzheimer's disease and I propose a simple model for estimating the long-term care costs of Alzheimer's disease attributable to each of the APOE genotypes. I parameterize the model using data from the medical and epidemiolocal literature. Then in Chapter 2, the parameterized model is used to estimate the potential costs of adverse selection in the long-term care insurance market under the crude assumption that non-AD related care costs are a proportion of AD related care costs.

The second part (Chapters 3 to 7) concentrates on modelling the disability process in order to independently estimate the care costs arising from disability in long-term care insurance. In Chapter 3, I introduce a model of disability and discuss the datasets that I use to parameterize the model (the 1982, 1984, 1989 and 1994 National Long-Term Care Surveys) and discuss previous relevant research that

have used these datasets. Then in Chapter 4, I use maximum likelihood theory to estimate the model parameters, and compare the methodology to that of the previous research, discussed in Section 3. In Chapter 5, I discuss two methods for calculating the variance of the estimated model parameters, and use these variance estimates as weights in graduating the model parameters. I compare aggregate mortality in the disability model with a benchmark force of mortality in Chapter 6, and make some adjustments to the graduated model parameters, to make aggregate mortality in the model more consistent with the benchmark force of mortality. Then in Chapter 7, the graduated and adjusted models are used to estimate the cost of disability in a long-term care insurance contract, and these combined with the results from Chapter 2 are used to revisit the potential costs of adverse selection in the long-term care insurance market.

Introduction

Molecular genetics has far-reaching implications for all aspects of health economics, including the effectiveness, or even practicability, of insurance-based funding of all forms of care. This is a natural subject for quantitative modelling.

The insurance industry has begun to recognise the far-reaching possibilities that research into human genetics might hold for traditional insurance practice, particularly underwriting. The possibilities can be summed up, somewhat crudely, as follows:

1. if applicants for insurance have better knowledge of their medical risks than insurers, because they know the results of genetic tests, they might select against the office; but

2. it is often deemed unfair to discriminate against individuals (for example, by charging different insurance premiums) on the basis of their genetic make-up, over which they have no control.

The U.K. Government has so far avoided legislating on this very sensitive issue of the use of genetic test information by insurers, but instead has set up the Genetics and Insurance Committee (GAIC), charged with the assessment of the likely relevance and reliability of genetic test information as it relates to different kinds of insurance. Such considerations require actuarial models of the insurance process, allowing for the effects of specific genes on mortality and morbidity.

Life insurance was the first type of insurance, in the U.K., to be considered, in which context it has been suggested that the overall costs of adverse selection might be limited (Macdonald, 1997, 1999; Pritchard, 1997). These models, proposed by Macdonald (1997, 1999), were based on parameter estimates intended to be extreme,

with the aim of suggesting an upper bound on the cost of adverse selection, rather than on data-based statistical estimates. Tan (1997) applied a similar model to annuity business, with results that suggested higher costs. However, these papers acknowledge:

1. that different conclusions might hold in respect of other forms of insurance; and

2. the lack of sound epidemiological data in respect of any but a few genetic conditions.

The first study to look at specific genes is that treating breast and ovarian cancer and the BRCA1 and BRCA2 genes by Lemaire *et al.* (1999) and Subramanian & Lemaire (1999). Further research has since been done on this topic by Macdonald, Waters & Wekwete (2000), modelling breast and ovarian cancer, allowing for family history, as well as BRAC1 and BRAC2 genotype, with applications to critical illness insurance. Early-onset Alzheimer's disease and the Presenilin-1 gene have also been the subject of investigation (Gui & Macdonald, 2002).

At the time of writing, long-term care (LTC) insurance has yet to reach significant volumes in the U.K., but it is possible that it will figure in some way in any shift from public to private provision of care in old age. For that reason, it is timely to consider the problems of modelling LTC insurance. Further, one of the main reasons for requiring LTC insurance is dementia, of which Alzheimer's disease (AD) forms a significant proportion of cases. There is clear evidence that AD has a genetic component; at least one gene variant — that for the $\varepsilon 4$ allele of the apolipoprotein E (APOE) gene — has been linked to earlier onset of AD in epidemiological studies. Thus, a study of LTC insurance is also timely from the point of view of human genetics.

In Chapter 1, I propose a continuous-time Markov model for AD, and estimate its transition intensities using published medical and epidemiological studies. To allow for genetic variation arising from the $\varepsilon 4$ allele of the APOE gene, the intensity of onset of AD is modelled as a function of APOE genotype.

In Chapter 2, the model is used to estimate the costs of a single premium LTC contract for lives with each APOE genotype, and a sensitivity analysis is carried

out on the main model assumptions. These genotype specific costs are then used to estimate the potential costs of adverse selection, especially with respect to the size of the market. The model is also used to consider the costs of a comprehensive retirement package that provides both pension and LTC cover, and the effect this type of product may have on the potential for adverse selection. I provide a summary of the results and draw conclusions at the end of this chapter.

The main focus in the first two chapters is on modelling AD and genetic heterogeneity. The model described cannot be used for estimating any long-term care costs other than those arising from AD. The costs of adverse selection calculated are inititally reported as percentages of AD-related long-term care costs, which are then converted into total long-term care costs using the simple assumption that AD-related long-term care costs are a fixed proportion of total long-term care costs. The other main cause of claiming in a long-term care contract is through disability, and the rest of the thesis is concerned with modelling the disability process, with the aim of estimating the costs of disability in long-term care insurance (independently of Alzheimer's disease) — these estimates can then be combined with the results from Chapter 2 to revisit the potential costs of adverse selection in the long-term care insurance market.

It is particularly timely to consider modelling disability in ageing populations as the question of how the costs of long-term care should be apportioned between public funds and individuals is being raised in the U.K. — a Royal Commission was set up at the end of 1997 to investigate this question. The Commission finished its report in early 1999 (Sutherland *et al.*, 1999), concluding that "private sector solutions do not and in the forseeable future, will not offer a universal solution". While this may seem to be bad news for the LTC insurance business, sources in the industry believe that the Report may actually have a positive effect as:

1. the commission's recommendations about what the state would provide were not comprehensive, leaving room for insurance companies to make up any shortfall;

2. it does give greater clarity of what the state does and does not provide; and

3. it increased levels of public awareness of LTC as an issue.

The first point would mean that some LTC costs would be covered by the state. LTC insurance could then be used to cover the shortfall between actual LTC costs and what the state provides. So, as LTC insurance would then only cover part of the LTC costs, the premiums would be reduced, making LTC insurance more affordable. This combination of factors may then lead to an increase in demand for some forms of long-term care insurance.

In Chapter 3, I discuss the disability process, propose a continuous-time Markov model for disability and give details of the datasets that I use to parameterize the disability model (the 1982, 1984, 1989 and 1994 National Long-Term Care Surveys from the U.S.A.) — I also discuss previous relevant research that has used the same datasets. The model of disability proposed in this chapter is very flexible and could be applied, using relevant data, to pricing many long-term care products. Another application of this model would be to investigate trends of disability within a population to aid in public sector planning.

In Chapter 4, I estimate the model parameters, namely the transition intensities. It is not possible to estimate them in the usual way as occurrence/exposure rates since the datasets do not provide enough detail — they only provide information at discrete points in time, whereas complete lifetime history data is needed to estimate them directly. Instead, I develop maximum likelihood estimates for this restricted data and conclude by comparing the methodology from this chapter to that of the previous research done, discussed in Chapter 3.

Then in Chapter 5, I compare two methods for calculating the variance of the estimated model parameters (one which is not valid given the partial data available, even though it can be estimated), which demonstrates the effect on the variance estimates of only having partial data. The valid variance estimates are used as weights in the graduation process, where I fit parametric functions to the point estimates of the transition intensities.

I compare overall mortality in the disability models to a benchmark force of mortality in Chapter 6, and adjust the graduated model parameters in some of the disability models to make overall mortality in the models more consistent with the

benchmark force of mortality.

In Chapter 7, I use the graduated and adjusted models to estimate the costs of disability in a long-term care contract and carry out a sensitivity analysis on the main model assumptions. I then compare the overall forces of mortality in the Alzheimer's disease models with those in the disability models, to check for consistency. Using the costs of disability and the results from Chapter 2, I revisit the potential costs of adverse selection in the long-term care insurance market and provide a summary and conclusions.

Finally, in Chapter 8, I discuss areas for further research.

Chapter 1

A Simple Model of Alzheimer's Disease and the APOE Gene

1.1 Introduction

In this chapter I propose a simple Markov model for Alzheimer's disease (AD) and estimate its transition intensities from medical and epidemiological studies. Genetic variation arises because the $\varepsilon 4$ allele of the Apolipoprotein E (APOE) gene is known to indicate a predisposition to earlier onset of AD.

In this chapter and the next I concentrate on estimating the costs that arise under a long-term care insurance contract in respect of Alzheimer's disease. In Chapter 2, in order to look at the potential costs of adverse selection arising from variants of the APOE gene, the cost of other events in the ageing process (mainly disability) that trigger benefits are very simply assumed to be a multiple of those costs arising from Alzheimer's disease.

In Section 1.2, I briefly describe AD and then summarise the evidence for a genetic component of AD. In Section 1.3 the model is specified, and the transition intensities are estimated in Sections 1.4 and 1.5. Then in Section 1.6, occupancy probabilities are calculated from the model, which are then converted to prevalence rates and gene frequencies at older ages. Finally, in Section 1.7 I provide a summary and discussion. The parameterized model is applied to the question of adverse selection in the long-term care insurance market in the next chapter.

The research described in this chapter and the next is joint work with my supervisor, Professor Angus Macdonald, and formed the basis of two published papers Macdonald & Pritchard (2000) and Macdonald & Pritchard (2001). Applications of the model also include the study of long-term care costs (Warren *et al.*, 1999). Most of the research presented here was done in 1999, and is based on research papers available at that time — there is no doubt, with the great speed at which genetic knowledge is advancing, that some of the information in this chapter will, by the time of writing this thesis, have been superceded.

The processes leading to LTC insurance claims are complex, when compared with other forms of insurance, and there are no insurance data to speak of; therefore it is necessary (even for insurance companies themselves) to rely on data collected and published for a variety of reasons, mostly in the medical literature. As a result, the model proposed here is far from definitive, however the process of extracting information from the medical literature and putting it to actuarial use is very instructive, and I suggest that any shortcomings of this model shed useful light on the problems that might be faced in the future.

In a long-term care contract, claims can arise for two reasons, either: on the failure of a given number of activities of daily living (see Section 3.2 for more detail); or on reaching a certain level of cognitive impairment, resulting in a need for continual care or supervision. The term 'cognitive impairment' covers AD, which accounts for by far the greater number of cases, and other forms of mental deterioration, chiefly vascular in origin (for example, arising from strokes). Assessment is liable to be imprecise, making it difficult to decide on an exact date of inception of cognitive impairment, if such a thing exists. Moreover, although AD is the commonest form of cognitive impairment, it is hard to diagnose with certainty except by post-mortem examination. These factors introduce considerable uncertainty into epidemiological studies of AD. Breteler *et al.* (1992) noted that:

1. AD itself can have a significant vascular component;

2. some of the neuropathological symptoms of AD can also be symptoms of vascular dementia; and

3. studies by Tierney *et al.* (1988) found that post-mortem examination confirmed only 64–86% of diagnoses of AD.

1.2 Alzheimer's Disease and its Genetic Components

AD is a disease of old age; it is rare below ages 60–70. These rare cases are called 'early-onset' AD, which should not be confused with early onset of AD within the usual age range. This research is concerned only with the latter.

Families with a history of AD are sometimes observed, but AD also occurs sporadically (that is, in the absence of a family history of AD) and it is always possible that a case of AD in an affected family is, in fact, sporadic. The differences between familial and sporadic AD are not clear, although the former may be marked by earlier onset and more rapid progression.

A very few families have several cases of early-onset AD in several generations, consistent with autosomal dominant transmission (Levy-Lahad & Bird, 1996), and three genes have been found. First was the gene encoding for amyloid precursor protein (APP), involved in the production of β-amyloid. It resides on chromosome 21, which is the chromosome affected in cases of Down syndrome, sufferers of which often develop AD in middle age. Several mutations have been found, but they are rare. Later, mutations in two genes labelled presenilin-1 (PS-1) and presenilin-2 (PS-2) were identified, which appeared to be associated with AD, though the mechanisms remain unclear.

Familial AD is not restricted to early-onset cases and family history remains an important risk factor for late-onset AD (Jarvik *et al.*, 1996). Susceptibility genes have been identified, of which the most studied is that which codes for apolipoprotein E.

For a recent survey of the genetic epidemiology of AD, see Slooter & van Duijn (1997); Breteler *et al.* (1992), reviews the position before much was known about the genetic component of AD.

The pathology of AD includes:

1. senile plaques (deposits on the outside of neurones (brain cells), consisting largely of the protein β-amyloid);

2. neurofibrillary tangles (connections between neurones);

3. amyloid angiopathy (deposits of amyloid protein in the arteries of the brain)

4. loss of neurones; and

5. decreased activity of choline acetyltransferase (an enzyme).

Therefore, any gene whose expression leads to the production, or over-production, of substances associated with these changes is potentially a genetic marker for AD.

1.2.1 The Apolipoprotein E Gene

The aim of this summary is to give an impression of the progress made in understanding genetic factors of AD, as well as some of the problems and (perhaps most important) the great speed at which human genetics is advancing.

Apolipoprotein E is found in senile plaques and neurofibrillary tangles in AD patients. It has also been studied because of its role in lipid metabolism. The gene that encodes it is on chromosome 19, which was linked to families with late-onset AD by Pericak-Vance *et al.* (1991), making it a clear candidate gene for familial AD. Strittmatter *et al.* (1993) confirmed this hypothesis, which was rapidly supported by many other studies. The basic facts are as follows:

1. The APOE gene has three common alleles — $\varepsilon 2$, $\varepsilon 3$ and $\varepsilon 4$ — whose frequencies are roughly 0.09, 0.77 and 0.14 respectively.

2. Since each offspring receives one allele from each parent, there are six possible genotypes ($\varepsilon 2/\varepsilon 2$, $\varepsilon 2/\varepsilon 3$, $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 3$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$). Offspring with two copies of the same allele are called homozygotes, while those with two different alleles are called heterozygotes.

3. The APOE $\varepsilon 4$ allele increases the risk of AD in a dose related fashion, such that $\varepsilon 4$ homozygotes ($\varepsilon 4/\varepsilon 4$) are at a greater risk then $\varepsilon 4$ heterozygotes ($\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$), who in turn are at greater risk than those without the $\varepsilon 4$ allele

(Bickeböller *et al.*, 1997; Corder *et al.*, 1994; van Duijn *et al.*, 1995; Farrer *et al.*, 1997; Jarvik *et al.*, 1996; Kuusisto *et al.*, 1994; Lehtovirta *et al.*, 1995; Mayeux *et al.*, 1993; Myers *et al.*, 1996; Poirier *et al.*, 1993; Tsai *et al.*, 1994). See Section 1.5 for risk estimates. The risk depends on age, being highest at ages 60–70, tapering off at older ages (Bickeböller *et al.*, 1997; Corder *et al.*, 1994; Farrer *et al.*, 1997).

4. It is also possible that the $\varepsilon 4$ allele is associated with earlier onset of AD (not to be confused with early-onset AD). The effect may be dose dependent (Farrer *et al.*, 1997; Frisoni *et al.*, 1995; Gomez-Isla *et al.*, 1996); or not (Corder *et al.*, 1995; Lehtovirta *et al.*, 1995; Stern *et al.*, 1997); or it may not exist at all (Liddell *et al.*, 1994; Masullo *et al.*, 1998; Norrman *et al.*, 1995).

5. Investigations into the rate of mental decline of AD patients by genotype found no evidence for any difference (Basun *et al.*, 1995; Gomez-Isla *et al.*, 1996; Masullo *et al.*, 1998; Norrman *et al.*, 1995). There is conflicting evidence about mortality. It is possible that younger age at onset should imply longer survival times, because of the usual age-related mortality differentials, and therefore that the $\varepsilon 4$ allele should be associated with longer life after onset of AD. While some studies support this (Corder *et al.*, 1995; Gomez-Isla *et al.*, 1996; Norrman *et al.*, 1995), others have found no difference (Basun *et al.*, 1995; Stern *et al.*, 1997). If $\varepsilon 4$ is associated with lighter mortality in AD patients then risk estimates from cross-sectional studies (the vast majority to date) should be interpreted with caution. An incidence study (Evans *et al.*, 1997) confirmed $\varepsilon 4$ to be a significant risk factor, but the estimated increased risk of onset was at the lower end of the reported range.

6. In contrast, the $\varepsilon 2$ allele has been found to have a protective effect against late-onset AD (Corder *et al.*, 1994; Farrer *et al.*, 1997; Gomez-Isla *et al.*, 1996; Jarvik *et al.*, 1996; Lambert *et al.*, 1998; Masullo *et al.*, 1998). However, a study of early-onset AD patients (van Duijn *et al.*, 1995), found a higher frequency of the $\varepsilon 2$ allele, and an association of $\varepsilon 2$ with a more aggressive form of AD, suggesting different rôles of APOE in early-onset and late-onset

AD. Findings relating to the $\varepsilon 2$ allele are based on the $\varepsilon 2/\varepsilon 3$ genotype, as $\varepsilon 2$ homozygotes are rare. The risk attached to the $\varepsilon 2/\varepsilon 4$ genotype is not clear, possibly because $\varepsilon 2$ and $\varepsilon 4$ have opposite effects (Jarvik *et al.*, 1996; Levy-Lahad *et al.*, 1996).

APOE $\varepsilon 4$ is the most important genetic risk factor for AD identified yet. Though it is neither necessary nor sufficient to cause AD it does increase susceptibility. Approximately 26% of Caucasians carry at least one $\varepsilon 4$ allele and it has been estimated that between 42% and 79% of AD cases are attributable to the associated excess risk (Nalbantoglu *et al.*, 1994).

1.2.2 Other Genetic Factors of Alzheimer's Disease

In 1997, a gene for the K-variant of butyrylcholinesterase (BCHE K), not a risk factor by itself, was found to act in synergy with APOE $\varepsilon 4$, such that carriers of both (an estimated 6% of Caucasians) were at over 30 times the risk of AD as a person with neither (Lehmann *et al.*, 1997). Subsequent studies (Brindle *et al.*, 1998; Singleton *et al.*, 1998) failed to reproduce the result. Although some explanations have been advanced, caution is advisable in using BCHE K as a risk factor for AD.

Payami *et al.* (1997) reported an association between AD and the A2 allele of the human leokocyte antigen (HLA); the HLA-A2 phenotype and APOE $\varepsilon 4/\varepsilon 4$ genotype had similar and additive effects on reducing age at onset of AD, at ages below 60 and above 75. Further studies would be needed to confirm these findings.

Poduslo *et al.* (1998) found the apolipoprotein CI (apo CI) gene to be a risk factor for early-onset and late-onset AD, whether sporadic or familial. Apo CI A homozygotes had 4 to 5 times the odds of developing AD, heterozygotes about twice the risk. This was not unexpected, since Apo CI is closely linked to APOE and in linkage disequilibrium with APOE and AD. Linkage disequilibrium is the non-random assortment, in a population, of two genes on the same chromosome (the strength of the linkage is inversely proportional to the distance between them). It was thought that the association of AD with APOE may be more significant.

1.3 A Model for Alzheimer's Disease

I use a continuous-time multiple state model. In this section I will discuss the reasons for this choice and how it is used to represent a (genetically) heterogenous population and I discuss the statistical framework of such a model in Section 1.3.1. For more general comments on these models, see Macdonald (1996a) or Waters (1984). Lives with each APOE genotype are assumed to form a homogeneous population, suffering the different risks of AD discussed in Sections 1.2.1 and 1.5.

An important reason for using these models is that they allow the most complete representation of the underlying process. It is then necessary to estimate a large number of transition intensities, for which adequate data do not always exist, but it is better to obtain a clear picture of the data needed than to sweep the issue under the carpet by working with some less adequate model in the first place. In particular:

1. if some simpler model is eventually recommended for use, because of missing data or for computational convenience, it is important to be able to assess its soundness in practice; and

2. if missing data become available later, for example, as the insured lives experience develops, it is a hindrance if too much has been invested in a model that cannot incorporate it.

Modern computing power is such that the computational demands of multiple state models (numerical integration of differential equations) can quite reasonably be met, for arbitrarily complex Markov models (Norberg, 1995) and for many semi-Markov models (Waters & Wilkie, 1987; Waters, 1991). The techniques can all be found in standard texts on numerical analysis, and no actuary should be prevented from choosing an adequate model by the need to use them.

Figure 1.1 shows a simple model of AD. Each genotype is represented by such a model; the transition intensities in each model will differ, representing the different genetic risks. x denotes the age at outset (for example, when insurance is purchased,) and t the elapsed duration. The choice of states is dictated entirely by the events that have been studied in the medical and epidemiological literature. For certain

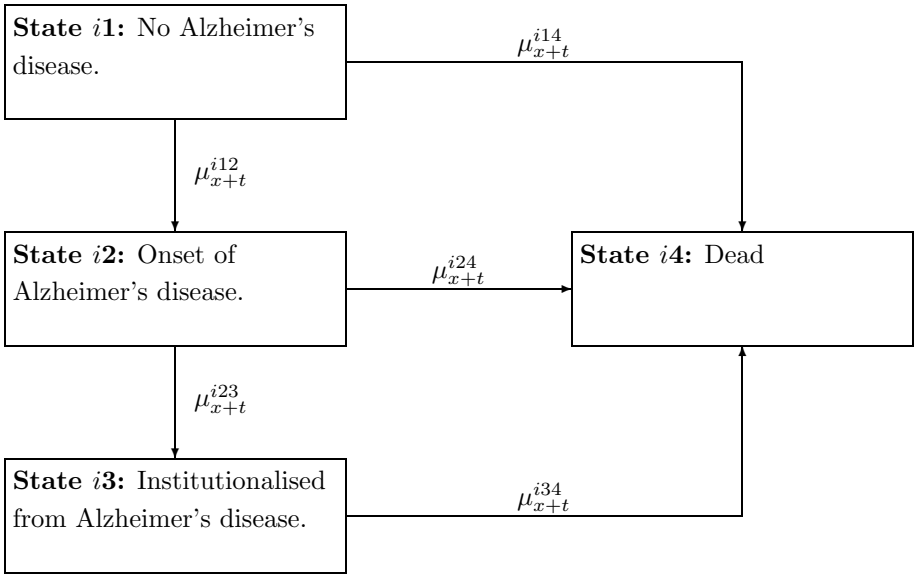

Figure 1.1: A simple model of Alzheimer's disease in the ith of M subgroups, each representing a different ApoE genotype. x is the age at outset, and t the elapsed duration.

purposes, it would be desirable to model other events, such as the start of a long-term insurance claim. No data about that event are available; however a major event that has been studied is institutionalisation. Although becoming institutionalised need not coincide with the start of an insurance claim, it is the best available proxy.

Macdonald (1999) considered frailty models as an alternative to Markov models, for genetics and insurance applications. They offer the advantage of a simple model of the genetic variability, if that is justified by the circumstances. They may be especially useful for modelling multifactorial disorders, or genes with very many alleles or mutations, but for a single gene with just a few alleles it seems reasonable to model each separately. Other possible models (such as Cox-type models) might be useful for modelling individual transitions but do not lend themselves to the inclusion of payments contingent upon complicated life histories. AD alone does not account for all long-term care costs. Broadly speaking, the need for care arises because of cognitive disorder (including AD) or loss of ability to perform Activities of Daily Living (ADLs) such as dressing, washing and feeding (see Section 3.2 for more details). A comprehensive model of long-term care costs can be specified in terms of these causes, with AD included as a component, and the impact of the

APOE gene on overall care costs can thereby be studied. However, incorporating AD explicitly in an expanded model will require data that describe, at the level of individual lives, the progress of AD and the loss of ADLs. I look at modelling the disability process in detail in Chapters 3 to 7, but even then there is no data on the loss of ADLs for a person suffering from AD — the work in these chapters is only able to look at overall care costs from disability.

1.3.1 Statistical Framework

In this section I look at the assumptions underlying the model and the equations used to calculate the occupancy probabilities. I first introduce the notation used:

1. The population is divided into M subgroups, denoted $i = 1, 2, \ldots, M$. When considering the whole population, $M = 1$. The $\varepsilon 2/\varepsilon 2$ and $\varepsilon 2/\varepsilon 3$ genotypes are combined in the model, since the former is so scarce, so when considering APOE genotypes, each is a separate subgroup, and $M = 5$.

2. Each subgroup is represented by a different model, as in Figure 1.1;

3. Lives enter the model at age x, always in the starting state (state $i1$).

4. When $M > 1$, the transition intensity between states j and k in the ith subgroup is denoted μ_{x+t}^{ijk} (when $M = 1$ the i is omitted from the superscript);

Assume that the time period of interest starts at time 0 and ends at time n and that there is a finite number of mutually exclusive states $\mathbf{S} = \{1, 2, \ldots, S\}$, 1 being the initial state at time 0.

Then, the lifetime of an individual is represented by a continuous time Markov process on the state space $\mathbf{S}$. The transition intensities (ignoring genotypes, for brevity, so that $M = 1$), μ_x^{jk} ($j \neq k$ and $j, k \in \mathbf{S}$) are assumed to exist $\forall x \in (0, n)$ and are such that:

1. The probability, at age x, of a life moving from state j to state k in the small time period δt, $P_{x\,x+\delta t}^{jk} = \mu_x^{jk}\,\delta t + \mathrm{o}(\delta t)$;

2. the probability of two or more transitions in the small period of time δt is $\mathrm{o}(\delta t)$; and

3. the Markov assumption holds, that is the transition intensities depend only on the current state and time and not on any previous history.

The Kolmogorov forward equations are then, $\forall j, k \in \mathbf{S}$:

$$\frac{\partial}{\partial t} P^{kl}_{x\,x+t} = \sum_{l \neq k} P^{jl}_{x\,x+t} \mu^{lk}_{x+t} - P^{jk}_{x\,x+t} \mu^{kl}_{x+t} \tag{1.1}$$

These equations are easily solved recursively, to give the occupancy probabilities of the model, by standard numerical methods even with a large number of states — I used a 4th order Runge-Kutta algorithm (Press *et al.*, 1993) with fixed step size 0.0005 years. They are solved forwards, the boundary condition being that the matrix of transition probabilities is simply the identity matrix for $t = 0$. For the simple calculation of occupancy probabilities, the 4th order Runge-Kutta algorithm with fixed step size proved to be sufficiently fast, however, in Chapter 4, when calculating maximum likelihood estimates many hundreds of occupancy probabilities need to be calculated. In this case, the inefficiency of using a fixed step size in the Runge-Kutta Algorithm becomes apparent and I use much more efficient algorithm with an adaptive step size, which takes the largest possible step size while keeping within a fixed accuracy. This took considerably more effort to initially set-up than the algorithm with fixed step size and I would suggest:

1. where only single or a few calculations are required, the Runge-Kutta algorithm with fixed step size is sufficient; but

2. where many calculations are required (i.e. in the intermediate process of calculating maximum likelihood estimates), the extra effort required to set-up a Runge-Kutta algorithm with adaptable step size, is justifiable (or even necessary) in terms of computer run-time.

1.4 Estimation of Transition Intensities Not Depending on APOE Genotype

In this section I estimate the transition intensities for the events: onset of AD; institutionalisation; and death. All of these must be 'estimated' from results reported

in the medical and epidemiological literature. It would be best to work with the original data, but these are almost never available. Reported results are not always ideal for the extraction of parameters for an actuarial model; often the age groups used are very wide, and different in different surveys; sometimes only graphs (such as Kaplan-Meier survival curves) are given.

A most important distinction must be drawn when estimating transition intensities from epidemiological studies (see, for example, Clayton & Hills (1993), Kahn & Sempos (1989), Lilienfeld & Hills (1993), Selvin (1996)):

1. *Prospective* studies, based on samples of the general population, ought to yield the most reliable estimates of population risk, but are expensive and time-consuming. Moreover, they are rarely even begun until substantial evidence of an effect has been accumulated from other studies.

2. *Case-based* studies, based on affected persons (and controls) often yield relative risks greatly in excess of the true population risks, precisely because the subjects are affected or at risk. However, early studies into any medical condition are almost inevitably of this type.

Current knowledge of most genetic disorders is derived from case-based studies; this is certainly true of APOE and AD (see Section 1.2.1). It is very likely that estimates of risk conferred by APOE genotype will fall as more prospective studies are carried out, but this will take time.

The approach I adopt is as follows:

1. in Section 1.4.1, I state assumptions about the general level of mortality;

2. in Section 1.4.2, I estimate the aggregate incidence of AD, denoted μ_{x+t}^{AD}, which has been investigated extensively;

3. in Section 1.4.3, I estimate the intensity of institutionalisation, following the onset of AD (that is, μ_{x+t}^{i23}) and the force of mortality following the onset of AD (that is, μ_{x+t}^{i24});

4. in Section 1.4.4, I estimate the force of mortality for lives institutionalised with AD (that is, μ_{x+t}^{i34}); and

5. in Section 1.5, I estimate the population frequencies of the APOE alleles and then estimate the incidence of AD for each genotype using odds ratios from the genetic studies: this gives estimates of μ^{i12}_{x+t}.

1.4.1 Baseline Mortality Tables

For convenience, I choose parametric approximations to the AM80 and AF80 Ultimate mortality tables as bases for mortality assumptions; for use in the model they are adjusted in a variety of ways. Gompertz curves were fitted to μ_{x+t} at ages 65–120, using log-linear least squares (see equation (1.2)):

$$\begin{aligned} {}^{AM80}\mu_{x+t} &= 0.000094116e^{0.084554(x+t)} \\ {}^{AF80}\mu_{x+t} &= 0.000025934e^{0.093605(x+t)}. \end{aligned} \tag{1.2}$$

Experiments with the AM80 and AF80 tables themselves showed that the Gompertz approximations had a negligible effect in long-term care applications; I use them because they are sometimes useful in numerical work. For insurance use, some allowance must be made for future improvements in mortality. No experience is available to help, but following discussion with some actuaries experienced in pricing long-term care insurance, I choose 65% of these baseline tables as the aggregate mortality assumptions.

1.4.2 The Onset of Alzheimer's Disease in the Population

AD has been the subject of some large-scale epidemiological studies, many of them pre-dating the discovery of the rôle of the APOE gene. Some of these report incidence rates, or 'occurrence/exposure' rates, which are exactly the estimates needed for transition intensities.

There is general agreement, in this literature, on the shape of the intensity μ^{AD}_{x+t} in the age range 60–85 years; it is very low at ages 60–64 (about 0 to 0.002) and increases rapidly with age, approximately doubling every 5 years. Sayetta (1986) and Hebert *et al.* (1995) found that a Gompertz curve gave the best fit, despite trying a number of more complex models.

A number of studies report the incidence of AD (that is, the intensity μ^{AD}_{x+t}) by age but not by genotype, including Copeland *et al.* (1992), Hagnell *et al.* (1992), Kokmen *et al.* (1993), Letenneur *et al.* (1994), Nilsson (1984), Ott *et al.* (1998), Rocca *et al.* (1998) and Rorsman *et al.* (1986). Of particular interest, however, is the recent meta-analysis of the incidence of AD by Jorm & Jolley (1998):

1. it draws on 23 studies world-wide, including 13 European studies;
2. the analysis is carried out separately for Europe, the U.S.A. and East Asia;
3. the incidence of AD is estimated by severity, categorised as Mild+ and Moderate+ AD, where Mild+ includes all cases classified as mild or worse; and
4. point estimates of μ^{AD}_{x+t} were obtained for 5-year age groups from 65 to 95, and no *a priori* shape of μ^{AD}_{x+t} was assumed.

I estimated μ^{AD}_{x+t} from Jorm & Jolley (1998) using the figures from the European studies and for Mild+ AD. The estimates, 95% confidence limits and the log-linear least squares Gompertz fit:

$$\mu^{AD}_{x+t} = 1.31275 \times 10^{-7} e^{0.145961(x+t)} \tag{1.3}$$

are shown in Figure 1.2. It is clear that a Gompertz curve is a very good fit.

Data on the incidence of AD among the very elderly (> 90 years) are sparse, so estimates at these ages have wide confidence intervals and the trend is uncertain. The meta-analysis by Gao *et al.* (1998) found that the rate of increase in μ^{AD}_{x+t} slowed down with age, but other studies found no evidence of this (Hebert *et al.*, 1995; Jorm & Jolley, 1998; Letenneur *et al.*, 1994). I simply extrapolated the Gompertz formula above to all ages; the effect of this assumption will depend on the particular application, or type of insurance, and this should be investigated when the model is used — in the next chapter where I apply the model, I first carry out a sensitivity analysis of this assumption.

Many studies have found men and women to be at the same risk of AD (Kokmen *et al.*, 1993; Nilsson, 1984; Ott *et al.*, 1998; Rocca *et al.*, 1998) and, when differences have been reported (Gao *et al.*, 1998; Jorm & Jolley, 1998; Letenneur *et al.*, 1994),

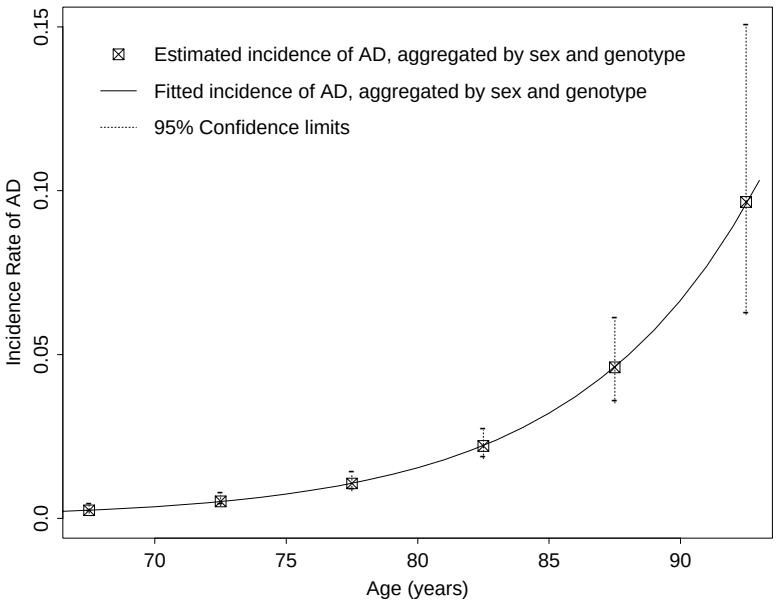

Figure 1.2: Aggregate Incidence of Alzheimer's Disease: Point Estimates and 95% Confidence Intervals. Source: Jorm & Jolley (1998).

women were found to be at greater risk only at very old ages. Figure 1.3 shows the following least-squares fits:

$$ {}^{male}\mu^{AD}_{x+t} = 1.60976 \times 10^{-7} e^{0.137301(x+t)} \tag{1.4} $$

$$ {}^{female}\mu^{AD}_{x+t} = 8.50561 \times 10^{-9} e^{0.172430(x+t)} \tag{1.5} $$

to the incidence rates found by Rocca *et al.* (1998), which were not found to be significantly different. Some experiments (described in Section 2.4) in applying the model to AD-related long-term care insurance costs using different rates of AD for men and women (equations (1.4) and (1.5)) showed that it made little difference, and in the rest of this chapter I have used the aggregate rate (equation (1.3)).

1.4.3 Time from Onset of Alzheimer's Disease to Institutionalisation or Death

The available data do not allow analysis of μ^{i23}_{x+t}, μ^{i24}_{x+t} or μ^{i34}_{x+t} by genotype.

Table 1.1 summarises the literature on time to the first of institutionalisation or death ('first event') for AD patients. Some studies give times from entry to the study

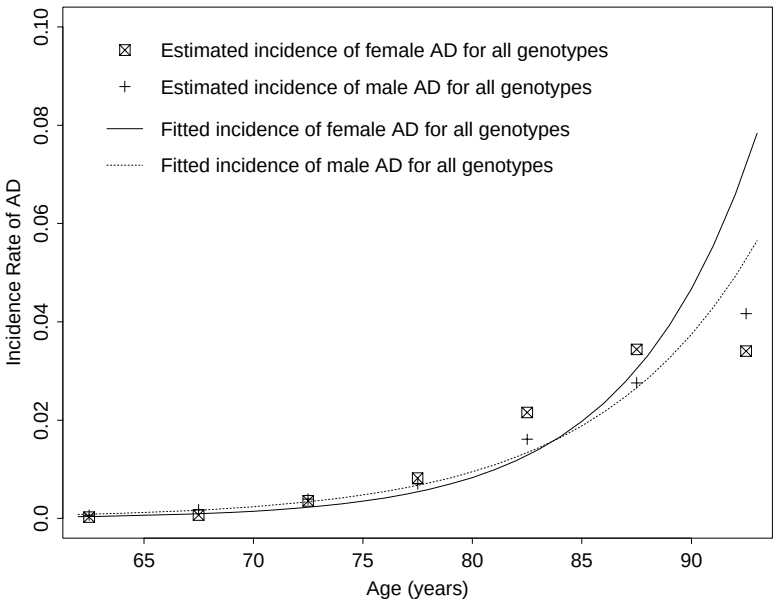

Figure 1.3: Incidence of Alzheimer's Disease by Gender: Point Estimates from Rocca *et al.*, (1998).

rather than from onset, which is usually not observed directly. A striking feature is that few lives die before becoming institutionalised. This may seem surprising as AD patients have generally been reported to suffer higher mortality than healthy lives (see Section 1.4.4). However, AD's debilitating effects are not sudden, and it may be expected that patients will be in receipt of informal care between onset and institutionalisation, which might lead μ^{i24}_{x+t} to be relatively light.

I used the data from the study by Jost & Grossberg (1995). Although it is not the largest study, it does have advantages:

1. it is a brain bank study, so all AD cases were confirmed by autopsy (the only reliable method of diagnosis);

2. there were no censored cases; and

3. the time from onset to institutionalisation is estimated.

Since genotypes cannot be distinguished here, I will just write μ^{23}_{x+t} and μ^{24}_{x+t} instead of μ^{i23}_{x+t} and μ^{i24}_{x+t}, respectively. Guided by these data, moment estimates of

Table 1.1: Mean and median times to Institutionalisation (Inst'n) or First Event for AD Patients.

	Age at		Time (years) to		% for which 1st event
Reference	Onset	Entry	Inst'n	1st Event	is death
Berg *et al.* (1988)		71.4(1)			7.1%
Heyman *et al.* (1997)		72.0(2)		3.1(2)	13.1%
Jost *et al.* (1995)	75.1(3)		4.3(1)		15.0%
Severson *et al.* (1994)		79.4(1)	2.5(2)(4)		10.0%

(1) Mean.
(2) Median.
(3) Mean age at onset of AD, if institutionalised, estimated as (mean age at institutionalisation − mean time to institutionalisation).
(4) Median time from onset estimated as 5.6 years.

μ^{23}_{x+t} (the force of institutionalisation) and μ^{24}_{x+t} (the force of mortality of an AD patient prior to institutionalisation) can be derived. The usual indicator functions (I_j) and sample path functions (N_{jk}) in respect of a single life (see Macdonald (1996b)) are defined by:

$$I_j(t) = \begin{cases} 1 & \text{if life is in state } j \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$dN_{jk}(t) = \begin{cases} 1 & \text{if life transfers from state } j \text{ to state } k \text{ at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$N_{jk}(T) = \int_0^T dN_{jk}(t) = \text{ No. of transfers from state } j \text{ to state } k$$

Also let P^{ij}_{xy} be the probability that a life in state i at age x is in state j at age y. Then equation (1.6) below is the mean age at onset of AD, given that the life was eventually institutionalised with AD (as in Jost & Grossberg (1995)):

$$\mathrm{E}\left[x + \int_x^\omega I_1(t)dt \;\middle|\; N_{23}(\omega - x) = 1 \text{ and } I_1(x) = 1\right] = x + \frac{\displaystyle\int_x^\omega (t-x)\mu^{12}_t P^{11}_{xt} \left\{\int_t^\omega \mu^{23}_s P^{22}_{ts} ds\right\} dt}{\displaystyle\int_x^\omega \mu^{12}_t P^{11}_{xt} \left\{\int_t^\omega \mu^{23}_s P^{22}_{ts} ds\right\} dt}; \quad (1.6)$$

equation (1.7) is the mean time from onset of AD to institutionalisation:

$$\mathrm{E}\left[\int_x^\omega I_2(t)dt \;\middle|\; N_{23}(\omega - x) = 1 \text{ and } I_1(x) = 1\right] = \frac{\displaystyle\int_x^\omega \mu_t^{12} P_{x\,t}^{11} \left\{\int_t^\omega (s-t)\mu_s^{23} P_{t\,s}^{22} ds\right\} dt}{\displaystyle\int_x^\omega \mu_t^{12} P_{x\,t}^{11} \left\{\int_t^\omega \mu_s^{23} P_{t\,s}^{22} ds\right\} dt} \quad (1.7)$$

and equation (1.8) is the probability that an AD patient dies before becoming institutionalised. The upper age bound, denoted ω, is taken to be 120 years:

$$\mathrm{P}\left[N_{24}(\omega - x) = 1 \mid N_{12}(\omega - x) = 1 \text{ and } I_1(x) = 1\right] = \frac{\displaystyle\int_x^\omega \mu_t^{12} P_{x\,t}^{11} \left\{\int_t^\omega \mu_s^{24} P_{t\,s}^{22} ds\right\} dt}{\displaystyle\int_x^\omega \mu_t^{12} P_{x\,t}^{11} dt} \quad (1.8)$$

Setting equations (1.6), (1.7) and (1.8) equal to their estimated values from Table 1.1, gives 3 equations, which can be solved for at most 3 unknown parameters. The parametric forms I chose were as follows:

1. $\mu_{x+t}^{12} = A + \mu_{x+t}^{AD}$, where μ_{x+t}^{AD} is given by equation (1.3). This Makeham term adjusts the incidence of AD to a level that gives the same mean age at onset (for AD patients who become institutionalised).

2. $\mu_{x+t}^{23} = D$. I felt that the data did not support anything more elaborate than a constant intensity.

3. $\mu_{x+t}^{24} = P\mu_{x+t}^{14}$. That is, the mortality of an AD patient before becoming institutionalised is a proportion of baseline mortality.

4. μ_{x+t}^{14}, baseline mortality, was taken as AM80 mortality, using the Gompertz approximation given by equation (1.2). Although it is appropriate to allow for future improvements in mortality in applications, it is not appropriate to do so in estimation based on past data. The values of D and P do not depend strongly on the baseline mortality.

Solving these equations numerically yields the solutions:

$$A = 0.02025038 \qquad D = 0.18895779 \qquad P = 0.33502488.$$

The Makeham term, A, is a nuisance parameter used to adjust the incidence of AD so that the mean age at onset in the model is the same as that in the data. Its only purpose here is to improve the estimation of the other terms, as the survival of a cohort of AD patients is strongly related to their mean age at onset. It does not furnish an estimate of the incidence of AD in the whole population, which was described in Section 1.4.2. The magnitude of D would give a mean time to institutionalisation of about 5.3 years if there were no mortality, so allowing for mortality this value would seem about right. Given the low proportion of AD lives that die before institutionalisation, P, representing 34% of baseline mortality is also as expected.

The transition intensities μ^{23}_{x+t} and μ^{24}_{x+t} are summarised in Table 1.3.

1.4.4 Mortality of Lives with Alzheimer's Disease

AD patients have been found to suffer higher mortality than the general population (Barclay *et al.*, 1985(b); Bonaiuto *et al.*, 1995; Bracco *et al.*, 1994; Burns *et al.*, 1991; van Dijk *et al.*, 1991; Evans *et al.*, 1991; Heyman *et al.*, 1996; Mölsä *et al.*, 1986; Treves *et al.*, 1986). However, there is little agreement on the magnitude of the increase, or its dependence on age at onset, duration since onset, sex, race, level of education, marital status, level of cognitive impairment, familial/non-familial AD and level of behavioural impairment. The main factors that need to be considered are:

1. *The magnitude of the increase in mortality for AD lives.* The mortality of lives with AD has been investigated using different methodologies. For example, Evans *et al.* (1991), estimated the relative risk of death for AD patients as 1.44 (95% confidence interval 1.05–1.96) times that of the unaffected. Others have suggested that AD has only a small impact on mortality: Barclay *et al.* (1985a) claimed that well-tended individuals may have life expectancy close to normal, and Sayetta *et al.* (1986) found that survival did not depend on disease acquisition.

2. *The effect of age at onset on relative mortality.* The mortality of patients with AD increases with age (Bonaiuto, *et al.*, 1995; Burns *et al.*, 1991). Most studies into survival times have found no relation between age at entry into the study and relative survival (Barclay *et al.*, 1985b; Bracco *et al.*, 1994; Heyman *et al.*, 1996; Mölsä *et al.*, 1986; Stern *et al.*, 1995), except that Barclay *et al.* (1985b) found that younger lives had shorter relative survival times. Diesfeldt *et al.* (1986), investigating survival from onset of AD, found that AD patients with onset before age 76 had reduced survival times, but not those with later onset. Comparing the two methods of investigation, Walsh *et al.* (1990) found that older age at onset affected survival adversely, whereas older age at entry into the study did not; a possible explanation was that older patients have symptoms for a shorter time before presentation. Although no definitive relationship between age at onset and relative survival emerges, it is clear that:

   (a) survival with AD depends on age; and

   (b) if age at onset affects relative mortality, the relationship is only weak, but possibly stronger at younger ages.

   In terms of the model in Figure 1.1, this suggests that mortality in state $i3$ (institutionalised from AD) could be modelled by the addition of a Makeham term to the normal force of mortality; the latter is age dependent, and the Makeham term will be less significant at older ages.

3. *The effect of the duration of AD on relative survival.* Perhaps surprisingly, the duration of AD has not been found to be associated with increased mortality (Barclay *et al.*, 1985a; Bracco *et al.*, 1994; Burns *et al.*, 1991; Diesfeldt *et al.*, 1986; Heyman *et al.*, 1996; Sayetta *et al.*, 1986; Walsh *et al.*, 1990). That is, AD patients with long duration of symptoms do not suffer higher mortality than patients, of the same age, with short duration of symptoms. In terms of the model, this means that the mortality of lives in states 2 and 3 (onset of AD and institutionalised from AD) does not depend on the time spent in these states. This is especially convenient, as it allows us to work in a Markov framework.

Table 1.2: Summary Statistics on Survival Times of AD Patients.

Reference	Mean (Median) Age at Onset	Mean (Median) Survival Time	Addition to μ_{x+t}^{34}
Barclay *et al.* (1985a)	(73.3) yrs	(8.1) yrs	0.15829
Bracco *et al.* (1994)	(72.4) yrs	7.3 yrs	0.25259
Diesfeldt *et al.* (1986)	75.6 yrs	7.2 yrs	0.21056
Heyman *et al.* (1996)	(69.2) yrs	(9.7) yrs	0.10993
Jost *et al.* (1995)	75.1 yrs	8.11 yrs	0.13345
Kokmen *et al.* (1988)	80.4 yrs	6.2 yrs	0.26420
Treves *et al.* (1986)	73.9 yrs	(9.3) yrs	0.08135
Average			0.17291

4. *The effect of gender on relative survival with AD.* Many researchers have found that the differences in survival between men and women with AD can be explained by the usual mortality differential between men and women (Beard *et al.*, 1994; Bonaiuto *et al.*, 1995; Bracco *et al.*, 1994; Burns *et al.*, 1991; Heyman *et al.*, 1996; Walsh *et al.*, 1990), though Barclay *et al.* (1985a), did find greater differences. In terms of modelling, allowing for the normal differences in mortality between genders should be sufficient.

Table 1.2 summarises the literature on survival with AD. Since genotypes cannot be distinguished here, I will just write μ_{x+t}^{34} instead of μ_{x+t}^{i34}. As in the previous section, the mean age at onset (see equation (1.9)) and the mean survival time (see equation (1.10)) in the model of Figure 1.1 can be written down as:

$$\mathrm{E}\left[x + \int_x^\omega I_1(t)dt \;\middle|\; N_{12}(\omega - x) = 1 \text{ and } I_1(x) = 1\right] = \tag{1.9}$$
$$x + \frac{\displaystyle\int_x^\omega (t-x)\mu_t^{12}P_{x\,t}^{11}dt}{\displaystyle\int_x^\omega \mu_t^{12}P_{x\,t}^{11}dt}$$

$$\mathrm{E}\left[\int_x^\omega I_2(t) + I_3(t)dt \;\middle|\; N_{12}(\omega - x) = 1 \text{ and } I_1(x) = 1\right] = \tag{1.10}$$
$$\frac{\displaystyle\int_x^\omega \mu_t^{12}P_{x\,t}^{11}\left\{\int_t^\omega (s-t)(\mu_s^{23}+\mu_s^{24})P_{t\,s}^{22}ds + \int_t^\omega \mu_s^{23}P_{t\,s}^{23}\int_s^\omega (r-s)\mu_r^{34}P_{s\,r}^{33}drds\right\}dt}{\displaystyle\int_x^\omega \mu_t^{12}P_{x\,t}^{11}dt}$$

Setting equations (1.9) and (1.10) equal to their estimated values in Table 1.2, and noting the estimates of μ_{x+t}^{23} and μ_{x+t}^{24} from the previous section, there are 2

Table 1.3: Summary Of Transition Intensities for the AD Model with Baseline Mortality 100% (65%) of AM80 and AF80.

Transition	Parameter Values		B ($\times 10^{-5}$)		C ($\times 10^{-2}$)	
Intensity	A	D	Male	Female	Male	Female
μ^{24}_{x+t}	0	0.33502 (0.21776)	9.4116	2.5934	8.4554	9.3605
μ^{23}_{x+t}	0.18896	0.00				
μ^{34}_{x+t} Lower bound	0.08	1.00 (0.65)	9.4116	2.5934	8.4554	9.3605
μ^{34}_{x+t} Mean	0.17291	1.00 (0.65)	9.4116	2.5934	8.4554	9.3605
μ^{34}_{x+t} Upper bound	0.27	1.00 (0.65)	9.4116	2.5934	8.4554	9.3605

equations, which can be solved for at most 2 unknown parameters. The parametric forms I used are as follows:

1. $\mu^{12}_{x+t} = A + \mu^{AD}_{x+t}$, where μ^{AD}_{x+t} is given by equation (1.3). This is just the addition of a Makeham term to the force of incidence of AD, shifting the latter to a level that gives the estimated age at onset.

2. $\mu^{34}_{x+t} = K + {}^{AM80}\mu_{x+t}$. This is a Makeham term as discussed in (d) above.

The estimated values of K for each of the references cited are given in the last column of Table 1.2. They range from about 0.08 to 0.27, with an average of 0.173. The Makeham term A is, again, only included to improve the estimation of the other terms (see the end of the previous section).

For clarity, I summarise the transition intensities estimated here. They all have the form:

$$\mu^{ij}_{x+t} = A + D\,B\,e^{C\,(x+t)}$$

and the calculated values are given in Table 1.3. Three values are given for μ^{34}_{x+t}, an upper bound, mean value and lower bound to enable a check of how sensitive the results are, in any particular investigation, to this term.

1.5 Estimation of Transition Intensities Depending on APOE Genotype

Table 1.4 shows the population frequencies of the APOE genotypes estimated in several studies.

Some features are clear: the $\varepsilon 3/\varepsilon 4$ genotype is not uncommon (about 21%) while the $\varepsilon 2/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes are quite uncommon (about 3% and 1% respectively). It might be expected that there would be lower proportions of 'dangerous' genotypes at older ages, because these lives suffer a higher rate of AD onset, but the two age-related studies (Bickeböller *et al.* (1997) and Corder *at al.* (1995)) gave conflicting results. However, there is reasonable agreement on the gene frequencies at around ages 60–70, which is what is needed for modelling.

Farrer *et al.* (1997) is a meta-analysis, combining the results of 40 other studies, including 6,264 Caucasian subjects. As it is the largest study, and differentiates by ethnic group, gender and ascertainment methods, and as the APOE $\varepsilon 4$ allele was found with the same frequency in respect of AD diagnosed at autopsy and clinically diagnosed probable AD, I use its estimated gene frequencies, namely: $\varepsilon 2/\varepsilon 2$ 0.008; $\varepsilon 2/\varepsilon 3$ 0.127; $\varepsilon 2/\varepsilon 4$ 0.026; $\varepsilon 3/\varepsilon 3$ 0.609; $\varepsilon 3/\varepsilon 4$ 0.213; $\varepsilon 4/\varepsilon 4$ 0.018. These sum to 1.001, because of roundings used in Farrer *et al.* (1997), but I leave this small discrepancy unadjusted.

In a heterogeneous population, it is often convenient to think of a given intensity in each sub-population as a multiple (not necessarily constant) of a 'baseline' intensity, either in one of the sub-populations or in an aggregated 'average' population. Similarly, if p_1 and p_2 are the probabilities of an event in populations 1 and 2 respectively, the relative risk in population 2 (with respect to population 1) is p_2/p_1. A related quantity is the odds ratio: the odds in populations 1 and 2, respectively, are $p_1/(1-p_1)$ and $p_2/(1-p_2)$, and the odds ratio is:

$$\frac{p_2(1-p_1)}{p_1(1-p_2)}. \tag{1.12}$$

Table 1.4: Estimated Population Frequency of ApoE Genotypes

Reference	Country / Ethnicity	No. of Lives	Sex	Age Group	Allele Frequency $\varepsilon2$	$\varepsilon3$	$\varepsilon4$	Genotype Frequency $\varepsilon2/\varepsilon2$	$\varepsilon2/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon3/\varepsilon4$	$\varepsilon4/\varepsilon4$
Bickeböller *et al.* (1997)	France	1,030	M & F	All	0.085	0.770	0.145	0.01	0.12	0.03	0.59	0.24	0.01
		316	M	All	0.070	0.815	0.105	0.01	0.10	0.02	0.67	0.19	0.00
		40	M	< 60	0.050	0.800	0.150	0.00	0.08	0.02	0.62	0.28	0.00
		93	M	60–69	0.070	0.845	0.085	0.01	0.12	0.00	0.70	0.17	0.00
		80	M	70–79	0.060	0.840	0.090	0.00	0.10	0.02	0.71	0.16	0.00
		103	M	≥ 80	0.081	0.795	0.125	0.01	0.11	0.03	0.64	0.20	0.01
		714	F	All	0.090	0.750	0.170	0.01	0.13	0.03	0.55	0.27	0.02
		47	F	< 60	0.075	0.735	0.190	0.00	0.15	0.00	0.51	0.30	0.04
		75	F	60–69	0.075	0.730	0.195	0.01	0.11	0.02	0.52	0.31	0.03
		143	F	70–79	0.065	0.765	0.160	0.00	0.12	0.01	0.56	0.29	0.01
		449	F	≥ 80	0.095	0.750	0.165	0.01	0.13	0.04	0.56	0.25	0.02
Corder *et al.* (1994)	U.S.A.	243	M & F	≥ 60	0.105	0.750	0.155	0.00	0.16	0.05	0.56	0.22	0.02
		111	M	≥ 60	0.155	0.760	0.145	0.00	0.26	0.05	0.53	0.20	0.02
		132	F	≥ 60	0.085	0.755	0.160	0.01	0.12	0.05	0.58	0.23	0.02
Corder *et al.* (1995)	U.S.A.	236	M & F	All	0.095	0.755	0.150	0.00	0.16	0.03	0.56	0.23	0.02
		60	M & F	60–66	0.067	0.800	0.133		0.07	0.07	0.68	0.17	0.02
		124	M & F	67–74	0.101	0.738	0.161		0.18	0.02	0.53	0.23	0.03
		52	M & F	≥ 75	0.115	0.740	0.144		0.23	0.00	0.48	0.29	0.00
van Duijn *et al.* (1995)	Netherlands	532	M & F	< 65	0.103	0.731	0.165	0.01	0.17	0.02	0.51	0.27	0.02
		228	M	< 65	0.105	0.705	0.190	0.01	0.16	0.03	0.48	0.29	0.03
		304	F	< 65	0.100	0.745	0.155	0.01	0.17	0.01	0.53	0.26	0.02
Evans *et al.* (1997)	E. Boston	490	M & F	≥ 65	0.062	0.854	0.084	0.01	0.09	0.01	0.74	0.14	0.01
Farrer *et al.* (1997)	Caucasian	6,262	M & F	All	0.084	0.779	0.137	0.01	0.13	0.03	0.61	0.21	0.02
	Afr-Amer	240	M & F	All	0.083	0.727	0.19	0.01	0.13	0.02	0.50	0.32	0.02
	Hispanic	267	M & F	All	0.067	0.823	0.110	0.00	0.12	0.01	0.67	0.18	0.02
	Japanese	1,977	M & F	All	0.042	0.869	0.089	0.00	0.07	0.01	0.76	0.16	0.01

Table 1.4: Estimated Population Frequency of ApoE Genotypes–continued

Reference	Country / Ethnicity	No. of Lives	Sex	Age Group	Allele Frequency			Genotype Frequency					
					$\varepsilon 2$	$\varepsilon 3$	$\varepsilon 4$	$\varepsilon 2/\varepsilon 2$	$\varepsilon 2/\varepsilon 3$	$\varepsilon 2/\varepsilon 4$	$\varepsilon 3/\varepsilon 3$	$\varepsilon 3/\varepsilon 4$	$\varepsilon 4/\varepsilon 4$
Gomez-Isla *et al.* (1996)	U.S.A	129	M & F	42–102	0.063	0.810	0.130	0.00	0.11	0.02	0.64	0.23	0.01
Jarvik *et al.* (1996)	not given	310	M & F	48–98	0.098	0.750	0.132	0.01	0.12	0.05	0.59	0.20	0.01
		117	M	48–98	0.068	0.791	0.124	0.02	0.09	0.02	0.64	0.21	0.01
		193	F	48–98	0.117	0.725	0.137	0.01	0.15	0.07	0.55	0.20	0.01
Lambert *et al.* (1998)	not given	308	M & F	not given	0.081	0.805	0.114	0.00	0.13	0.02	0.65	0.18	0.01
Lehtovirta *et al.* (1995)	Finland	55	M & F	not given	0.009	0.882	0.109	0.00	0.02	0.00	0.78	0.18	0.02
Levy-Lahad *et al.* (1996)	not given	304	M & F	not given	0.100	0.765	0.135	0.01	0.13	0.05	0.60	0.21	0.01
Liddel *et al.* (1994)	U.K.	86	M & F	not given	0.110	0.766	0.123	0.01	0.17	0.03	0.60	0.17	0.03
Lopez *et al.* (1998)	Spain	45	M & F	not given	0.060	0.810	0.120	0.00	0.10	0.02	0.65	0.22	0.00
	U.S.A.	58	M & F	not given	0.045	0.760	0.195	0.00	0.09	0.00	0.55	0.33	0.03
Lucotte *et al.* (1997)	France	248	M & F	≥ 65	0.069	0.804	0.127	0.02	0.09	0.01	0.65	0.22	0.01
Nalbantoglu *et al.* (1994)	Canada	77	M & F	not given	0.050	0.900	0.050	0.00	0.10	0.00	0.82	0.06	0.01
		53	M	not given	0.060	0.930	0.020	0.00	0.11	0.00	0.85	0.04	0.00
		24	F	not given	0.040	0.860	0.100	0.00	0.08	0.00	0.76	0.12	0.04
Poirier *et al.* (1993)	Canada	77	M & F	not given	0.088	0.770	0.122	0.00	0.22	0.00	0.57	0.19	0.03
		29	M	not given	0.138	0.725	0.137	0.00	0.28	0.00	0.48	0.21	0.03
		45	F	not given	0.088	0.799	0.113	0.00	0.18	0.00	0.62	0.18	0.02
Roses *et al.* (1995)	not given	91	M & F	not given	0.104	0.731	0.165	0.01	0.11	0.08	0.57	0.21	0.02
Tsai *et al.* (1994)	U.S.A.	77	M & F	47–95	0.110	0.760	0.130	0.01	0.16	0.04	0.57	0.22	0.00

When intensities are small, the odds ratio is a good approximation to the relative risk. In many studies, the published results are either relative risks or odds ratios.

Few studies report prospectively the incidence of AD by genotype. Two that do are Evans *et al.*, (1997) and Slooter *et al.*, (1998). Both have quite small study populations, and neither provides age specific estimates of AD risk, so they are not appropriate for modelling purposes.

Table 1.5 gives the Odds Ratios (ORs) of AD and 95% confidence intervals from a number of genetic studies. The 'reference' populations (also shown in the table) were either the $\varepsilon 3/\varepsilon 3$ genotype or the three non-$\varepsilon 4$ genotypes combined. The estimated ORs vary considerably across studies. For example, estimates of the OR for the $\varepsilon 3/\varepsilon 4$ genotype (relative to the $\varepsilon 3/\varepsilon 3$ genotype) range from 1.8% to 3.7%, and for the $\varepsilon 4/\varepsilon 4$ genotype, from 6.2% to 30.7%. Some of the variation may be explained by the differences between the studies themselves. In particular, differences may arise from: the method of ascertainment of patients, the countries of study, the method of diagnosis of AD, the age structure of the samples, the reference/risk genotypes, and whether they are cross-sectional or prospective studies. I make the following observations:

1. The study by Lopez *et al.* (1998) suggests that the risk of AD associated with the APOE $\varepsilon 4$ allele may be different in different countries.

2. In support of the above, Mayeux *et al.* (1993) found that the association between APOE and AD may depend on ethnic group and, in particular, may not be present in black populations.

3. Despite the differences between studies: the presence of one or two $\varepsilon 4$ alleles is consistently reported to be significantly associated with AD; and homozygotes are generally reported to have higher risk of onset of AD than heterozygotes.

4. The weakest associations between APOE and AD were reported in the two prospective studies, those by Evans *et al.* (1997) and Slooter *et al.* (1998). This is as expected for the reasons given in Section 1.4.

For modelling purposes, the genetic risk of AD at different ages is important. Few studies have considered this; the odds ratios from two that have are given in

Table 1.5: Aggregated Odds Ratios of AD for the ApoE $\varepsilon 4$ Allele.

Reference	Study Type[1]	Reference Genotype[2]	Genotype[2]	Odds Ratio Mean	95% CI
Evans *et al.* (1997)	P	$\varepsilon 3/\varepsilon 3$	$\varepsilon 3/\varepsilon 4$ & $\varepsilon 4/\varepsilon 4$	2.27	1.1–4.9
Frisoni *et al.* (1995)	C	-/-	$\varepsilon 4$/-	6.6	2.2–19.5
			$\varepsilon 4/\varepsilon 4$	17.9	4.5–70.5
Jarvik *et al.* (1996)	C	$\varepsilon 3/\varepsilon 3$	$\varepsilon 2/\varepsilon 3$	0.4	0.2–0.96
			$\varepsilon 2/\varepsilon 4$	1.4	0.6–3
			$\varepsilon 3/\varepsilon 4$	3.1	2–4.7
			$\varepsilon 4/\varepsilon 4$	30.7	7–131
Kuusisto *et al.* (1994)	P	-/-	$\varepsilon 4$/-	2.7	1.4–5.2
			$\varepsilon 4/\varepsilon 4$	9.1	3.5–23.4
Lambert *et al.* (1998)	C	-/-	$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	4.66	3.14–6.93
Lehtovirta *et al.* (1995)	C	-/-	$\varepsilon 4$/-	5.1	2.4–11.1
			$\varepsilon 4/\varepsilon 4$	21.4	2.8–166.3
Liddell *et al.* (1994)	C	-/-	$\varepsilon 4$/-	2.2	1.1–4.7
			$\varepsilon 4/\varepsilon 4$	10.7	2.3–48.8
Lopez *et al.* (1998)	C	-/-	$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	2.34[3]	1.03–5.55
			$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	3.64[4]	1.78–7.69
Mayeux *et al.* (1993)	P	-/-	$\varepsilon 4$/-	4.2[5]	1.8–9.5
			$\varepsilon 4/\varepsilon 4$	17.9[5]	4.6–69.8
			$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	15.3[6]	3.0–78.1
			$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	0.7[7]	0.1–6.4
			$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	4.5[8]	0.7–27.7
Myers *et al.* (1996)	P	$\varepsilon 3/\varepsilon 3$	$\varepsilon 3/\varepsilon 4$	3.7	1.9–7.5
			$\varepsilon 4/\varepsilon 4$	30.1	10.7–84.4
Nalbantoglu *et al.* (1994)	A	-/-	$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	15.5	6.2–38.5
Slooter *et al.* (1998)	P	$\varepsilon 3/\varepsilon 3$	$\varepsilon 2/\varepsilon 3$	0.4	0.1–1.0
			$\varepsilon 2/\varepsilon 4$	1.3	0.2–8.5
			$\varepsilon 3/\varepsilon 4$	1.8	1.0–3.1
			$\varepsilon 4/\varepsilon 4$	6.2	1.4–28.2
Tsai *et al.* (1994)	C	-/-	$\varepsilon 4$/- & $\varepsilon 4/\varepsilon 4$	4.6	1.9–12.3
			$\varepsilon 4$/-	3.6	1.5–9.8

(1) Study Type: C indicates clinic/hospital; P, population/community; and A, autopsy/brainbank.
(2) Dash (-) represents $\varepsilon 2$ or $\varepsilon 3$ alleles.
(3) Study population–Gerona, Spain.
(4) Study population–Pittsburgh, USA.
(5) Mixture of White, Black and Hispanic ethnic groups.
(6) White ethnic group only.
(7) Black ethnic group only.
(8) Hispanic ethnic group only.

Table 1.6: Odds Ratios of AD by Genotype and Age.

Bickeböller *et al.* (1997)				Corder *et al.* (1994)			
Age		Odds Ratio		Age		Odds Ratio	
Group	Genotype	Mean	95% CI	Group	Genotype	Mean	95% CI
60–69	$\varepsilon2/\varepsilon3$	0.3	0.0–2.3	60–66	$\varepsilon2/\varepsilon3$	0.1	-
	$\varepsilon2/\varepsilon4$	-	-		$\varepsilon2/\varepsilon4$	1.2	-
	$\varepsilon3/\varepsilon4$	3.1	1.4–6.9		$\varepsilon3/\varepsilon4$	11.1	-
	$\varepsilon4/\varepsilon4$	29.1	3.6–239.5		$\varepsilon4/\varepsilon4$	123.8	-
70–79	$\varepsilon2/\varepsilon3$	0.4	0.1-2.3	67–74	$\varepsilon2/\varepsilon3$	0.3	-
	$\varepsilon2/\varepsilon4$	-	-		$\varepsilon2/\varepsilon4$	1.1	-
	$\varepsilon3/\varepsilon4$	3.2	1.5–6.6		$\varepsilon3/\varepsilon4$	4.6	-
	$\varepsilon4/\varepsilon4$	-	-		$\varepsilon4/\varepsilon4$	20.8	-
80+	$\varepsilon2/\varepsilon3$	0.3	0.0–2.6	75–92	$\varepsilon2/\varepsilon3$	0.5	-
	$\varepsilon2/\varepsilon4$	-	-		$\varepsilon2/\varepsilon4$	1.6	-
	$\varepsilon3/\varepsilon4$	1.3	0.5–3.4		$\varepsilon3/\varepsilon4$	3.2	-
	$\varepsilon4/\varepsilon4$	-	-		$\varepsilon4/\varepsilon4$	10.0	-
60+	$\varepsilon2/\varepsilon3$	0.4	0.1–0.9	60+	$\varepsilon2/\varepsilon3$	0.3	-
	$\varepsilon2/\varepsilon4$	1.6	0.5–5.5		$\varepsilon2/\varepsilon4$	1.1	-
	$\varepsilon3/\varepsilon4$	2.2	1.5–3.5		$\varepsilon3/\varepsilon4$	4.4	-
	$\varepsilon4/\varepsilon4$	11.2	4.0–31.6		$\varepsilon4/\varepsilon4$	19.3	-

Table 1.6. Bickeböller *et al.* (1997) is based on hospital admissions, and Corder *et al.* (1994) on autopsy cases; both use $\varepsilon3/\varepsilon3$ as the reference population. Although some ORs are missing, because of small sample sizes, the trends are fairly clear:

1. The odds of AD among the higher risk genotypes ($\varepsilon3/\varepsilon4$ and $\varepsilon4/\varepsilon4$) fall with age. This may be expected as higher risk genotypes will succumb to AD more rapidly, reducing the proportion of such genotypes within the population.

2. Conversely, the protection conferred by the lower risk genotype ($\varepsilon2/\varepsilon3$) seems to weaken with age, possibly as this genotype becomes more common in the remaining population.

These trends are supported by the meta-analysis by Farrer *et al.* (1997), and as it is from this study that I take estimates of the APOE genotype risks, I cite some relevant details:

1. The aggregate relative odds from Farrer *et al.* (1997) (relative to the $\varepsilon3/\varepsilon3$ genotype) are shown in Figure 1.4.

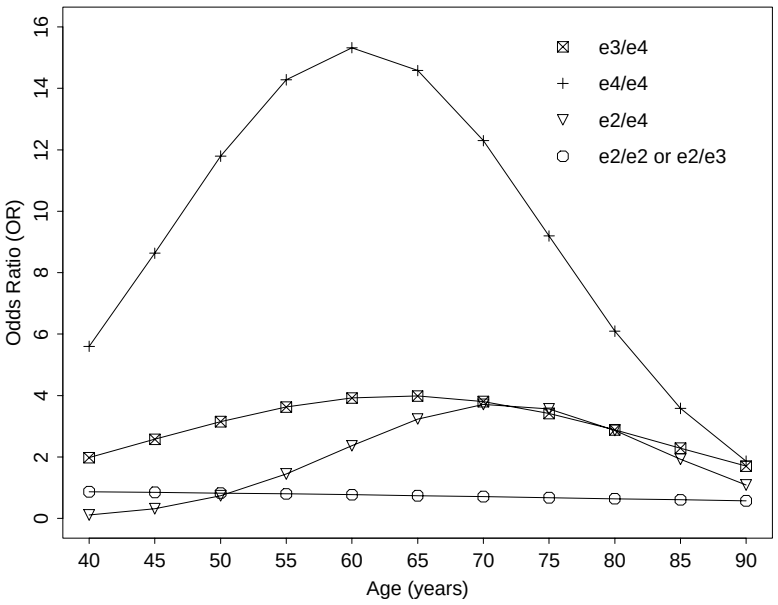

Figure 1.4: Odds ratios (ORs) of AD relative to $\varepsilon 3/\varepsilon 3$ genotype for males and females combined. Source: Farrer *et al.* (1997).

2. The genotype risks of AD were not significantly different in respect of Caucasian males and females, except in the case of the $\varepsilon 3/\varepsilon 4$ genotype, for which women had a significantly higher risk of AD. The relative odds of AD by APOE genotype for Caucasian men and women are shown in Figures 1.5 and 1.6. (The authors kindly provided me with the numerical values of the odds ratios; confidence intervals were not available.)

3. The genotypes $\varepsilon 2/\varepsilon 2$ and $\varepsilon 2/\varepsilon 3$ were combined as there were very few $\varepsilon 2/\varepsilon 2$ genotypes, and the risks associated with the two genotypes appeared to be similar.

4. Note that the risks associated with the APOE $\varepsilon 4$ allele were considerably higher than those found in the two population-based studies by Evans *et al.*, (1997) and Slooter *et al.*, (1998).

For use in the AD model, these odds ratios have to be converted into relative risks. More precisely, it is necessary to find a plausible set of age- and genotype-dependent transition intensities that are consistent with the odds ratios and together

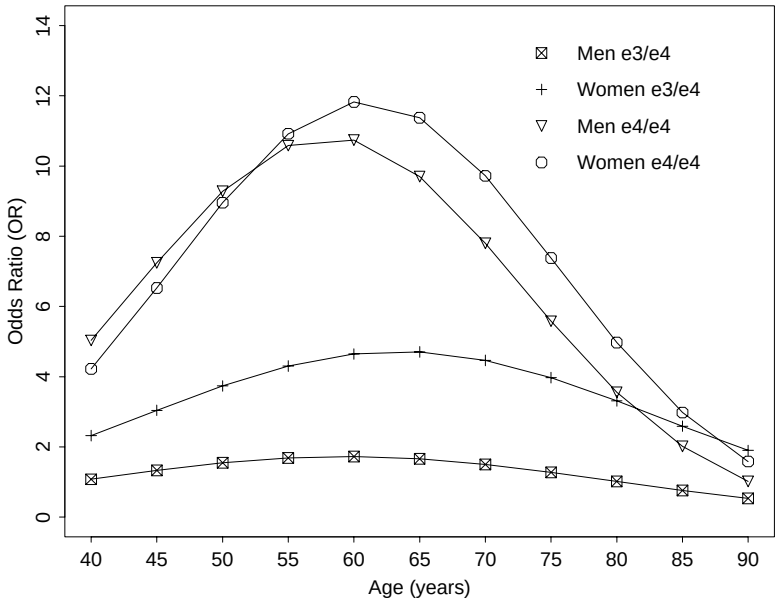

Figure 1.5: Odds ratios (ORs) of AD relative to $\varepsilon 3/\varepsilon 3$ genotype for $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes. Source: Farrer *et al.* (1997).

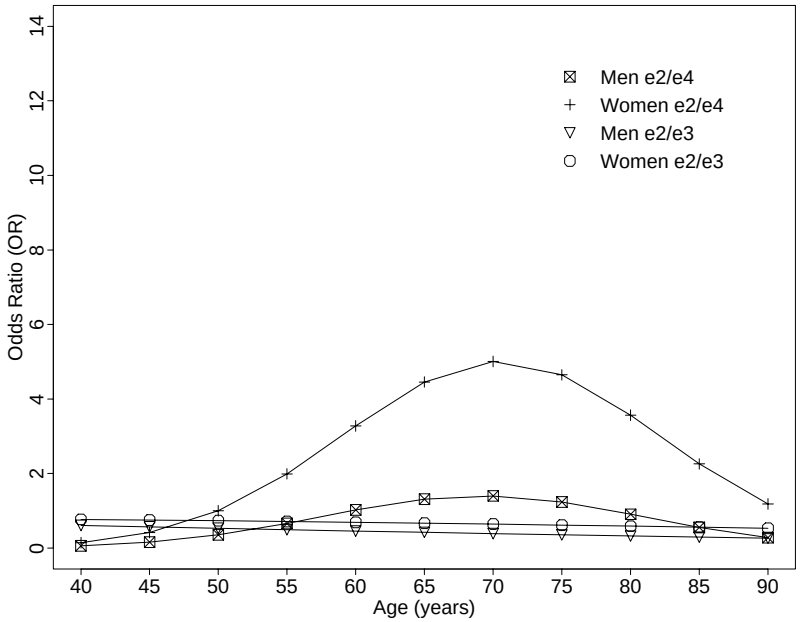

Figure 1.6: Odds ratios (ORs) of AD relative to $\varepsilon 3/\varepsilon 3$ genotype for $\varepsilon 2/\varepsilon 2$ or $\varepsilon 2/\varepsilon 3$ and $\varepsilon 2/\varepsilon 4$ genotypes. Source: Farrer *et al.* (1997).

are consistent with the aggregate incidence of AD. There is no unique solution to this problem. The method I used was to model the incidence of AD for the ith genotype as:

$$\mu^{i12}_{x+t} = r_1 f^i_{x+t}\, \mu^{AD}_{x+t} \tag{1.13}$$

where:

1. μ^{AD}_{x+t} is the aggregate incidence rate of AD (from Section 1.4.2);
2. f^i_{x+t} is a parametric function representing the risk relative to the aggregate incidence rate , where $f^i_{x+t} = 1$ in the case of the $\varepsilon 3/\varepsilon 3$ genotype; and
3. r_1 is a constant chosen so that the aggregate incidence of AD based on the modelled intensities is consistent with the aggregate incidence μ^{AD}_{x+t}.

I did this for for males and females separately and combined and only looked at ages 60 and over, in order to get a better fit in the age range of interest in applications. The form of the ORs, either rising to a peak and then falling, or gently declining, suggested a similar pattern of relative risks, and the following family of functions provided a satisfactory fit (note that constant relative risks, or proportional hazards, result in odds ratios with exponential growth):

$$f^i_{x+t} = Ee^{-F((x+t)-k_1)^2 - G((x+t)-k_2)} + H. \tag{1.14}$$

In actuarial notation this is a GM(1,3) function, familiar in the graduation of life tables (Forfar, McCutcheon & Wilkie, 1988), although as described below either F or G is set to zero. This is flexible enough to give a good approximation to the ORs, and is also suitable for extrapolating beyond age 90. The fitting procedure was as follows:

1. by considering the form of the OR, set either $F = 0$ (giving an exponential function) or $G = 0$ (giving a bell-curve function), and set H equal to 0 or 1;
2. the best value of k_1 or k_2 was found, to the nearest integer, by inspection;
3. the resulting ORs were calculated from the model in Figure 1.1 using the intensities from previous sections; and

Table 1.7: Parameters for the Relative Risk of AD for Males, Females and in Aggregate, by Genotype.

		Parameter Values						
Gender	Genotype	E	F	G	H	k_1	k_2	r_1
Both	$\varepsilon4/\varepsilon4$	13.5	0.00529	0	1	60	–	0.93
	$\varepsilon3/\varepsilon4$	2.98	0.00312	0	1	62	–	
	$\varepsilon2/\varepsilon4$	2.87	0.00938	0	1	68	–	
	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	0.754	0	0.00859	0	–	60	
Female	$\varepsilon4/\varepsilon4$	10.4	0.00504	0	1	60	–	0.88
	$\varepsilon3/\varepsilon4$	3.68	0.00319	0	1	62	–	
	$\varepsilon2/\varepsilon4$	4.21	0.0102	0	1	68	–	
	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	0.675	0	0.00692	0	–	60	
Male	$\varepsilon4/\varepsilon4$	8.94	0.00656	0	1	60	–	1.27
	$\varepsilon3/\varepsilon4$	1.92	0.00103	0	0	51	–	
	$\varepsilon2/\varepsilon4$	1.42	0.00506	0	0	67	–	
	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	0.434	0	0.0160	0	–	60	

4. the remaining coefficients (either E and F, or E and G) were fitted by least squares.

For the calculations in point 3 above the following parameters were used:

1. $\mu^{14}_{x+t} = 0.65 \times {}^{AM80}\mu_{x+t}$ for males and $\mu^{14}_{x+t} = 0.65 \times {}^{AF80}\mu_{x+t}$ for females and in aggregate, where ${}^{AM80}\mu_{x+t}$ and ${}^{AF80}\mu_{x+t}$ are given by equation (1.2).

2. $\mu^{23}_{x+t} = 0.189$, calculated in Section 1.4.3.

3. $\mu^{24}_{x+t} = 0.335 \times \mu^{14}_{x+t}$, calculated in Section 1.4.3.

4. $\mu^{34}_{x+t} = 0.173 + \mu^{14}_{x+t}$, the mean value calculated in Section 1.4.4.

The fitted parameters are given in Table 1.7. Figure 1.7 shows that the modelled ORs closely reproduce the estimates from Farrer *et al.* (1997) (see Figure 1.4; only the aggregate ORs are shown, and the modelled ORs start at age 61 because at age 60 all lives are assumed to be unaffected).

To determine the parameter r_1, the aggregate incidence of AD in the whole model was calculated, and this fitted to μ^{AD}_{x+t} by least squares. If ${}^{i}P^{11}_{x\,x+t}$ is the probability that a life with genotype i, healthy at age x, is unaffected by AD at age $x+t$, and if p^i_x is the population frequency of the ith genotype at age x then the aggregate

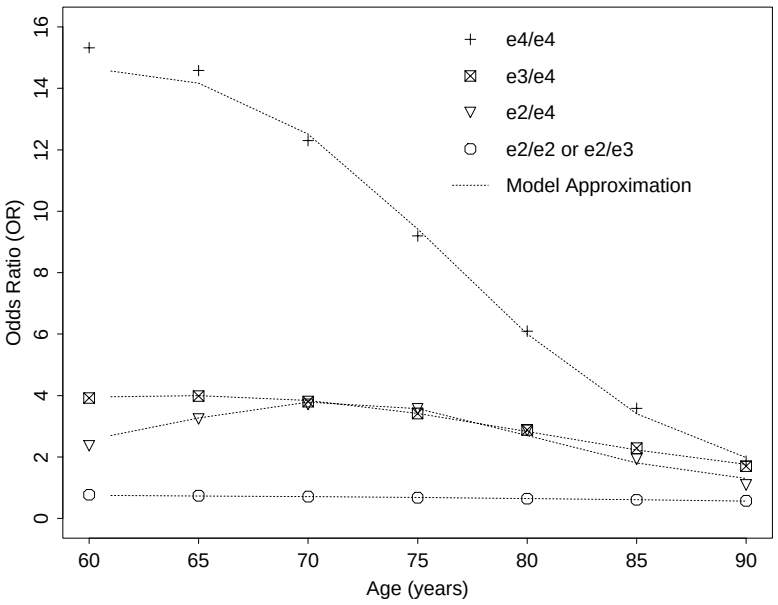

Figure 1.7: Odds ratios (ORs) of AD relative to $\varepsilon 3/\varepsilon 3$ genotype from Farrer *et al.* (1997), compared with ORs computed using modelled relative risk functions.

incidence of AD is:

$$\text{Aggregate incidence of AD at age } (x+t) = \frac{r_1 \left\{\sum_{i=1}^{5} p_x^i \, {}^iP_{x\,x+t}^{11} \, f_{x+t}^i\right\} \mu_{x+t}^{AD}}{\sum_{i=1}^{5} p_x^i \, {}^iP_{x\,x+t}^{11}} \quad (1.15)$$

I took $x = 60$, and for the p_x^i used the gene frequencies of the Caucasian control populations in Farrer *et al.* (1997), which were given in Table 1.4. The incidence of AD, μ_{x+t}^{AD}, was taken as that estimated in equation (1.3) and the occupancy probabilities, ${}^iP_{x\,x+t}^{11}$, were calculated by solving Kolmogorov's forward equations numerically (see Section 2.3).

The values of r_1 are given in Table 1.7. The adjustment to the overall level only had a marginal effect on the modelled ORs for the individual genotypes.

The relative risk functions for males and females are given in Figures 1.8 and 1.9. For females, the $\varepsilon 4/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 2/\varepsilon 4$ genotypes are unambiguously high-risk; the relative risks exceed 1.0 at all ages. For males, only the $\varepsilon 4/\varepsilon 4$ genotype confers higher risks at all ages. The $\varepsilon 2$ allele appears to be protective, so the $\varepsilon 2/\varepsilon 2$ and $\varepsilon 2/\varepsilon 3$ genotypes are low-risk, while the $\varepsilon 3/\varepsilon 4$ and $\varepsilon 2/\varepsilon 4$ genotypes are initially at higher risk but are at lower risk from about age 75. The protection apparently given

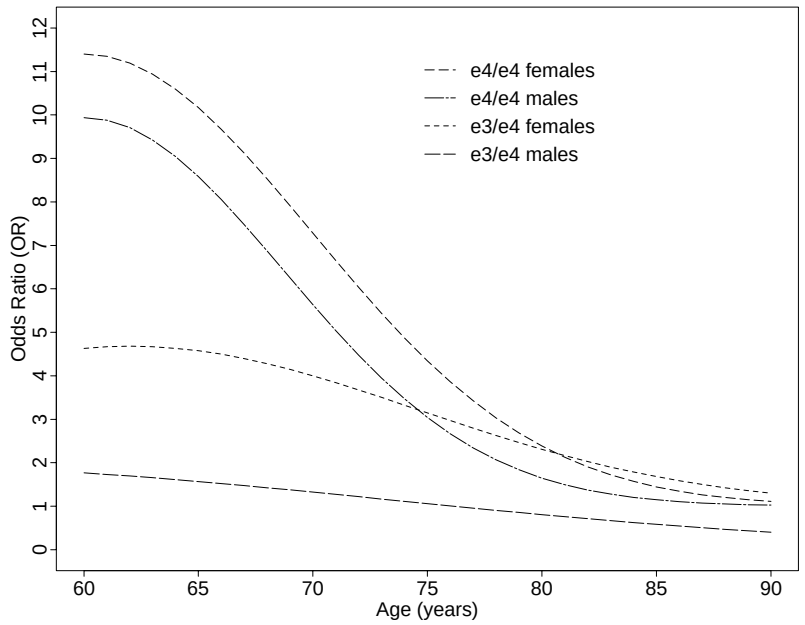

Figure 1.8: Modelled risk of AD, relative to the $\varepsilon3/\varepsilon3$ genotype, for $\varepsilon4/\varepsilon4$ and $\varepsilon3/\varepsilon4$ genotypes. Based on odds ratios from Farrer *et al.* (1997).

by the $\varepsilon2$ allele in males means that the $\varepsilon3/\varepsilon3$ genotype confers slightly higher than average risk; this is why, in Table 1.7, $r_1 > 1$ for males. It must be remembered that data in respect of males are relatively sparse, and data in respect of very old males even sparser, so these effects should be treated with caution; more confidence should be placed in the relative risks in respect of females.

These risk estimates probably overstate the true population risks, perhaps quite substantially, as they are from clinic- and autopsy-based studies, which investigate precisely the subjects that are affected or already known to be at risk. To allow for this possibility I will also consider models assuming that the true relative risks are a proportion $m < 1$ of those estimated above. This can be done by adjusting equation (1.13) so that for genotype i:

$$\mu^{i12}_{x+t} = r_m \left\{ (f^i_{x+t} - 1)\, m + 1 \right\} \mu^{AD}_{x+t} \tag{1.16}$$

where f^i_{x+t} is as above, and r_m is chosen as above so that the aggregate incidence of AD in the model is consistent with μ^{AD}_{x+t}. Values of $r_{0.5}$ and $r_{0.25}$ are shown in Table 1.8.

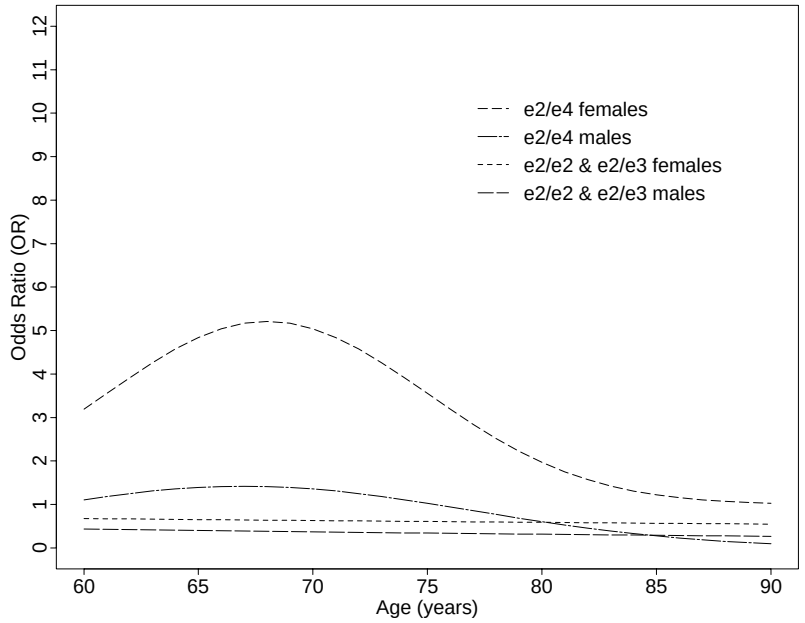

Figure 1.9: Modelled risk of AD, relative to the $\varepsilon 3/\varepsilon 3$ genotype, for $\varepsilon 2/\varepsilon 4$ and $\varepsilon 2/\varepsilon 2$ & $\varepsilon 2/\varepsilon 3$ genotypes. Based on odds ratios from Farrer *et al.* (1997).

Table 1.8: r_m for $m = 1, 0.5$ and 0.25.

Gender	r_1	$r_{0.5}$	$r_{0.25}$
Both	0.93	0.96	0.97
Female	0.88	0.94	0.97
Male	1.27	1.11	1.05

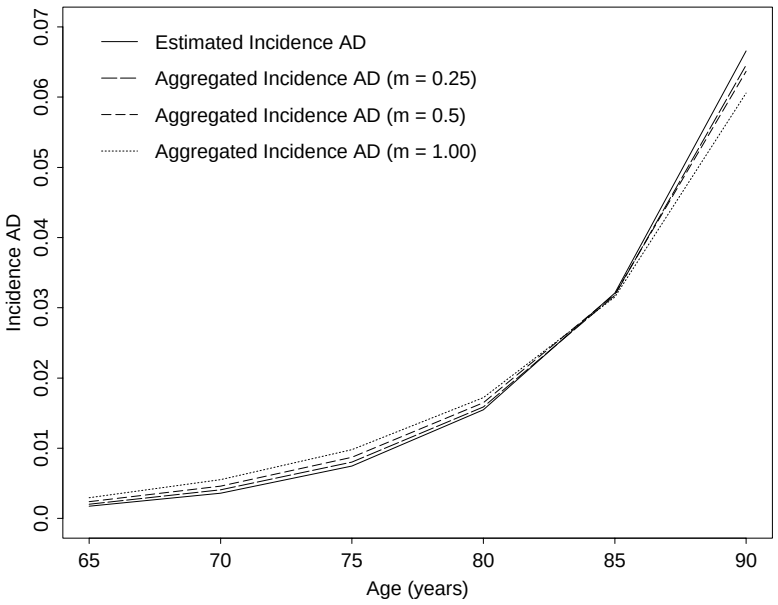

Figure 1.10: Comparison of the estimated population incidence of AD μ_{x+t}^{AD} with the aggregated incidence of AD for different levels of relative risk, males and females combined.

Figure 1.10 shows that the aggregate incidence of AD in the genetic model for both sexes combined is quite close to μ_{x+t}^{AD}. It also shows that increasing the level of relative risk tends to overstate the incidence of AD at younger ages, and to understate it at older ages; the reason is that higher relative risks deplete the high-risk groups more quickly, leaving a relatively healthier population at older ages.

Decreasing the level of relative risks for high-risk genotypes means increasing the relative risks for low-risk genotypes. Using a lower value of r_m will diminish any effects of the (possibly anomalous) feature, noted above, that the $\varepsilon 3/\varepsilon 4$ genotype is low-risk for males.

I chose a simple model for the relative risks (equations (1.13) and (1.14)). Alternatives were considered, in particular cubic polynomials and Gamma functions, but these gave poorer fits, and were less suitable for extrapolation (cubics to older ages and Gamma functions to younger ages). Also, it is easily seen that if an OR is specified as a function of time, and the transition intensity in the reference population is

given, the transition intensity in the second population is determined (as the solution to an ODE); Figure 1.7, therefore, gives good support for the choice of model. Further refinement seems somewhat spurious, given the type of data used, and in view of the major sensitivity analysis needed in respect of the dominant parameter m.

1.6 Occupancy Probabilities

Figures 1.11 and 1.12 for females and Figures 1.13 and 1.14 for males, show occupancy probabilities (see Section 1.3.1 for more detail) up to age 90 for lives healthy at age 60, with high ($m = 1$) and low ($m = 0.25$) relative risks, respectively. Each shows:

1. Occupancy probabilities in respect of each genotype (with $\varepsilon 2/\varepsilon 2$ and $\varepsilon 2/\varepsilon 3$ combined).

2. Occupancy probabilities calculated by aggregating all the genotypes in the model. In the notation of equation (1.15) the probability of being in state j ($j = 1, 2, 3, 4$) at age $60 + t$ is $\sum_{i=1}^{i=5} p_{60}^{i}\, {}^{i}P_{60\,60+t}^{1j}$, where the sum is over all genotypes. These are labelled 'Aggregated Genotypes'.

3. Occupancy probabilities based on the aggregate incidence of AD, μ_{x+t}^{AD}. These are labelled 'Aggregate Model'.

With high relative risks ($m = 1$), the effect of the $\varepsilon 4/\varepsilon 4$ allele is clear; AD cases rise to a peak in the early 70s, by which time over 10% of the original cohort are in one of the AD states, and then fall away. A similar but smaller effect can be seen for the $\varepsilon 3/\varepsilon 4$ genotype. With low relative risks ($m = 0.25$) these features are all diminished; in particular the peaks in the early 70s disappear.

For males and females with low relative risks (Figures 1.12 and 1.14) the aggregated results from the genetic model are very close to the results from the aggregate (population) model. For females with high relative risks, the rate of onset of AD seems to be too low at younger ages and too high at older ages.

Also of interest are prevalence rates, namely the proportion of those alive at every age who are in each of the three live states. Figures 1.15 and 1.16 show these for females (for $m = 1$ and $m = 0.25$, respectively), and Figures 1.17 and 1.18 show them for males (for $m = 1$ and $m = 0.25$, respectively), including, for convenience, the two AD states combined.

The most striking feature is the prevalence of AD in respect of the $\varepsilon 4/\varepsilon 4$ genotype under high relative risks (Figure 1.15); it increases almost linearly. Again, for males and females the aggregated results from the genetic model are quite close to those from the aggregate model. Moreover, they fall within the range of prevalence rates actually observed. Breteler *et al.* (1992) cite the following rates: 47.2% at ages 85 and over (Evans *et al.*, (1989)); 31.7% at ages 85 and over (Pfeffer *et al.*, 1987); and 28.0% at ages 90 and over (O'Connor *et al.*, 1989); some other studies gave lower figures.

So far, it has been assumed that the gene frequencies given by Farrer *et al.* (1997) are appropriate for age 60. They will change with age, as higher-risk genotypes die more quickly. In order to consider entrants (to a study, or into insurance) at ages over 60, gene frequencies at older ages need to be estimated. Table 1.9 shows estimates of the gene frequencies in respect of lives unaffected by AD at ages 65, 70 and 75. Using the notation of equation (1.15), these are given by:

$$p^i_{60+t} = \frac{p^i_{60}\, {}^iP^{11}_{60\,60+t}}{\sum_{j=1}^{j=5} p^j_{60}\, {}^jP^{11}_{60\,60+t}}. \tag{1.17}$$

These are not the gene frequencies in respect of the whole population; lives alive but who have AD are omitted (as is appropriate for insurance applications). Nor are they the gene frequencies in respect of the healthy population; lives with disabilities other than AD are included.

Gene frequencies in the whole population at older ages can also be estimated, as:

$$\frac{p^i_{60}({}^iP^{11}_{60\,60+t} + {}^iP^{12}_{60\,60+t} + {}^iP^{13}_{60\,60+t})}{\sum_{j=1}^{j=5} p^j_{60}({}^jP^{11}_{60\,60+t} + {}^jP^{12}_{60\,60+t} + {}^jP^{13}_{60\,60+t})} \tag{1.18}$$

but these are not so relevant for insurance applications.

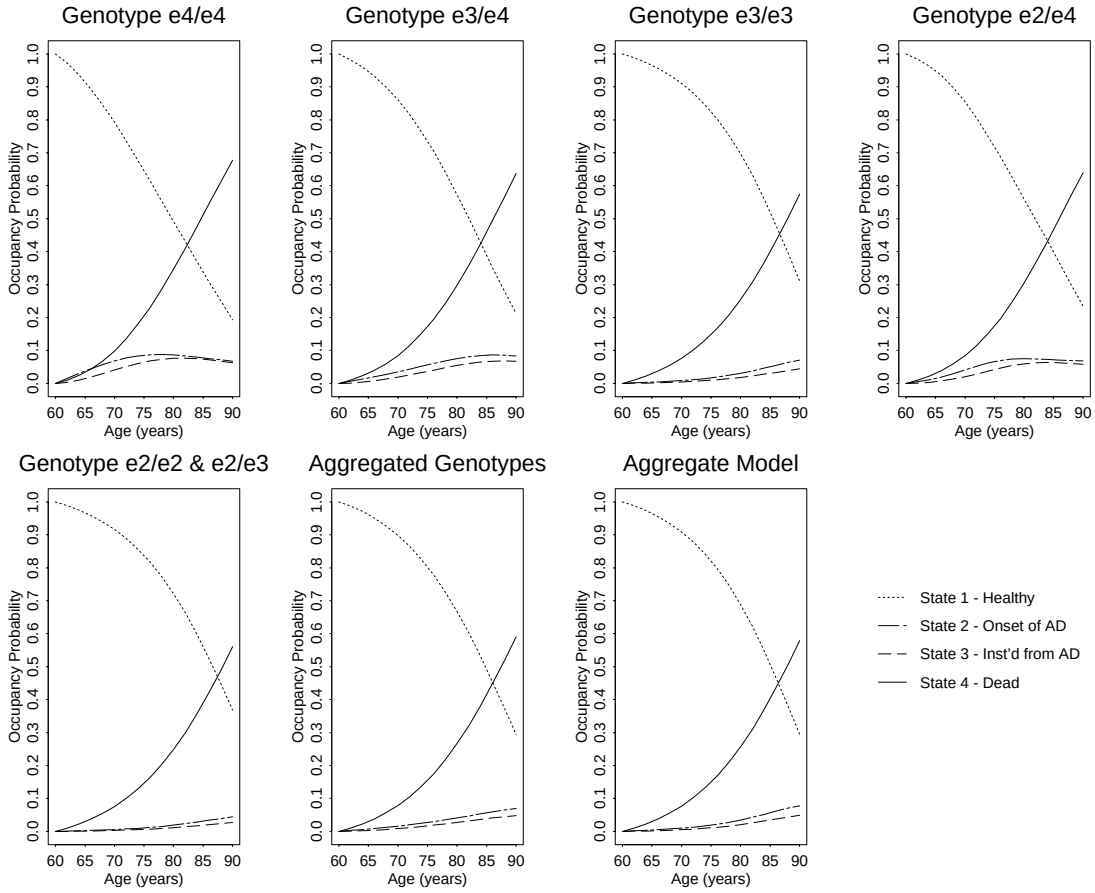

Figure 1.11: Occupancy probabilities for females, healthy at age 60, high relative risks ($m = 1$).

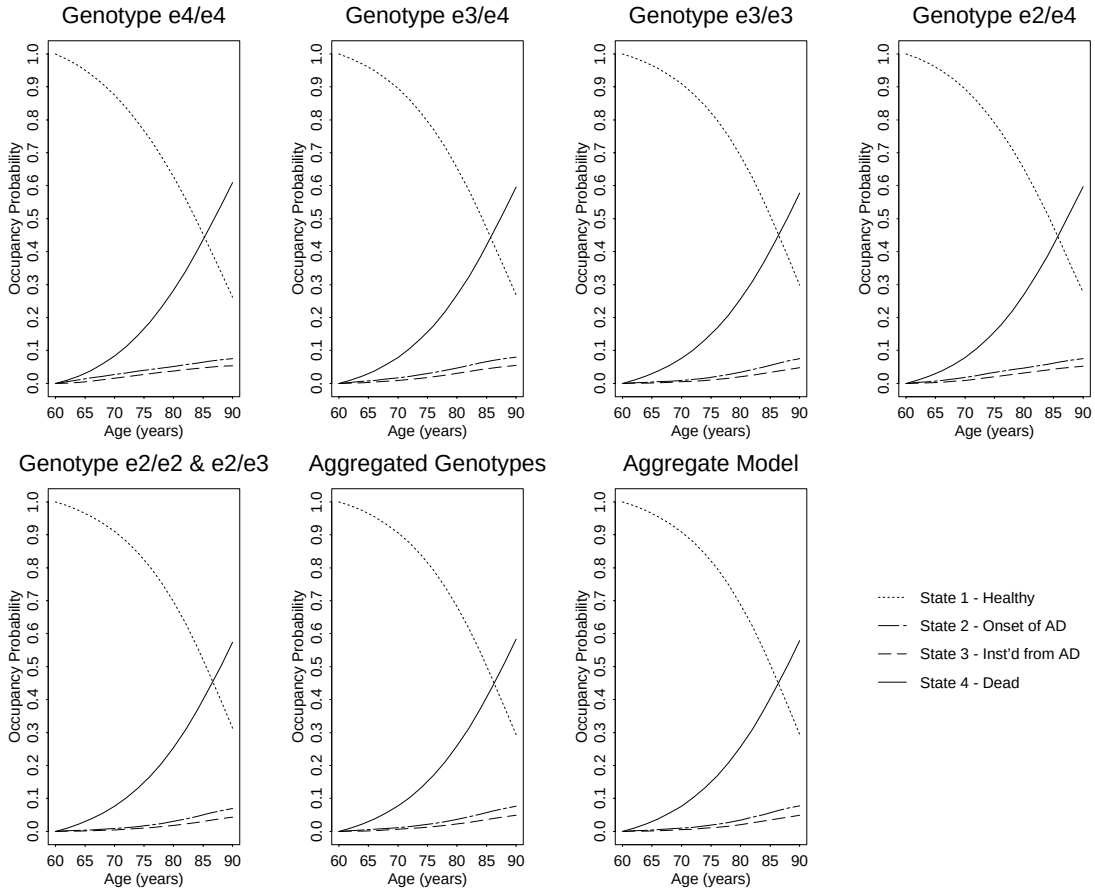

Figure 1.12: Occupancy probabilities for females, healthy at age 60, low relative risks ($m = 0.25$).

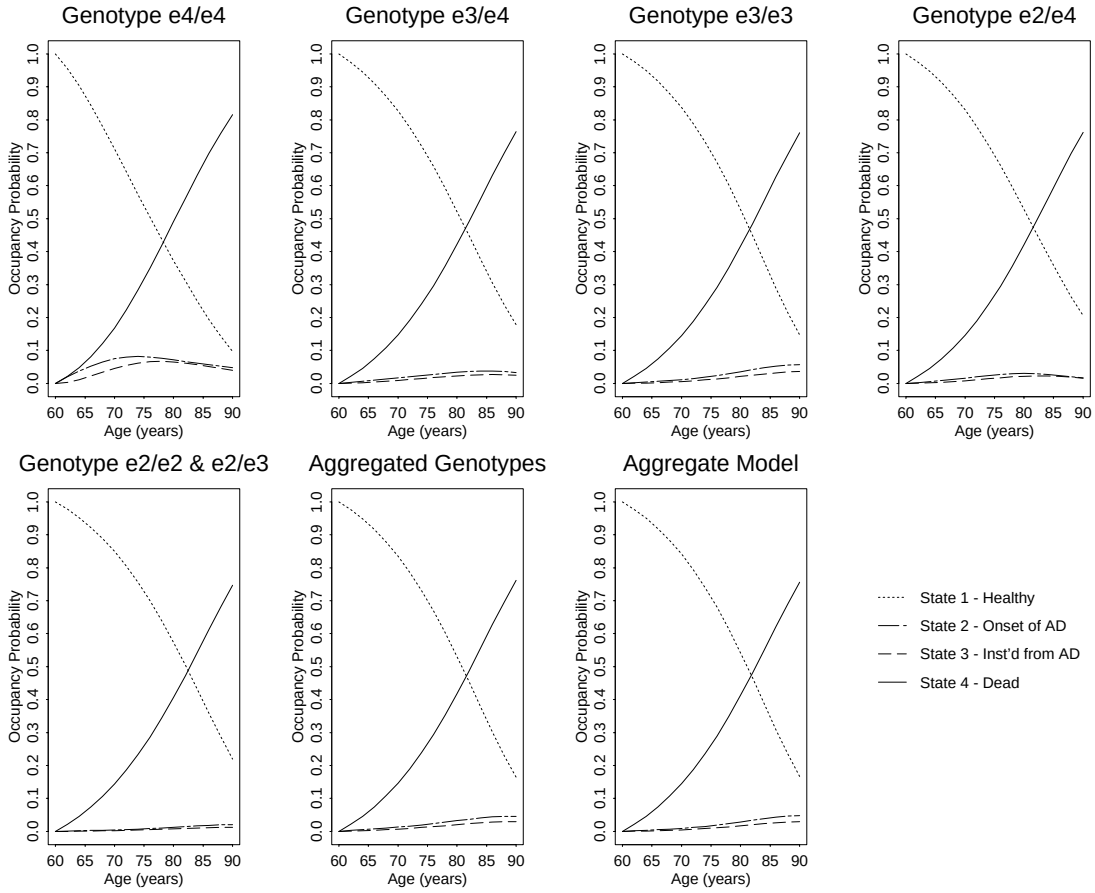

Figure 1.13: Occupancy probabilities for males, healthy at age 60, high relative risks ($m = 1$).

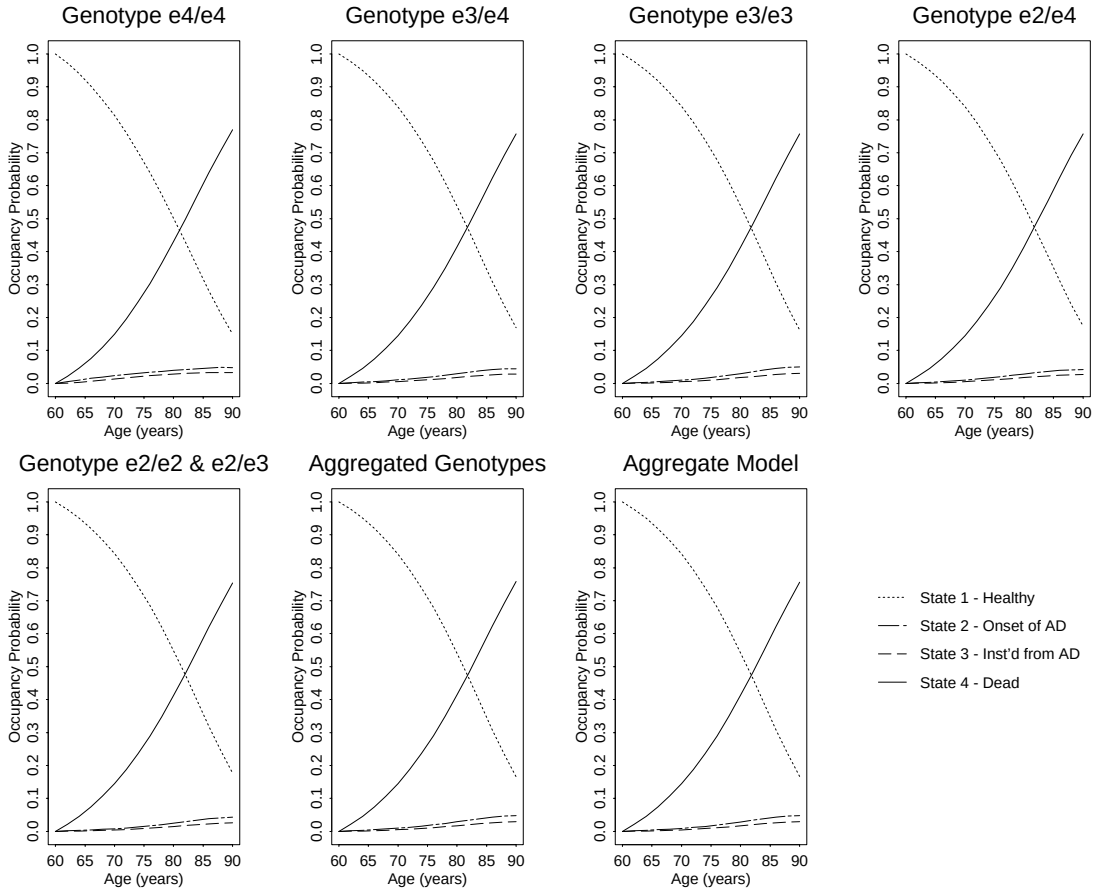

Figure 1.14: Occupancy probabilities for males, healthy at age 60, low relative risks ($m = 0.25$).

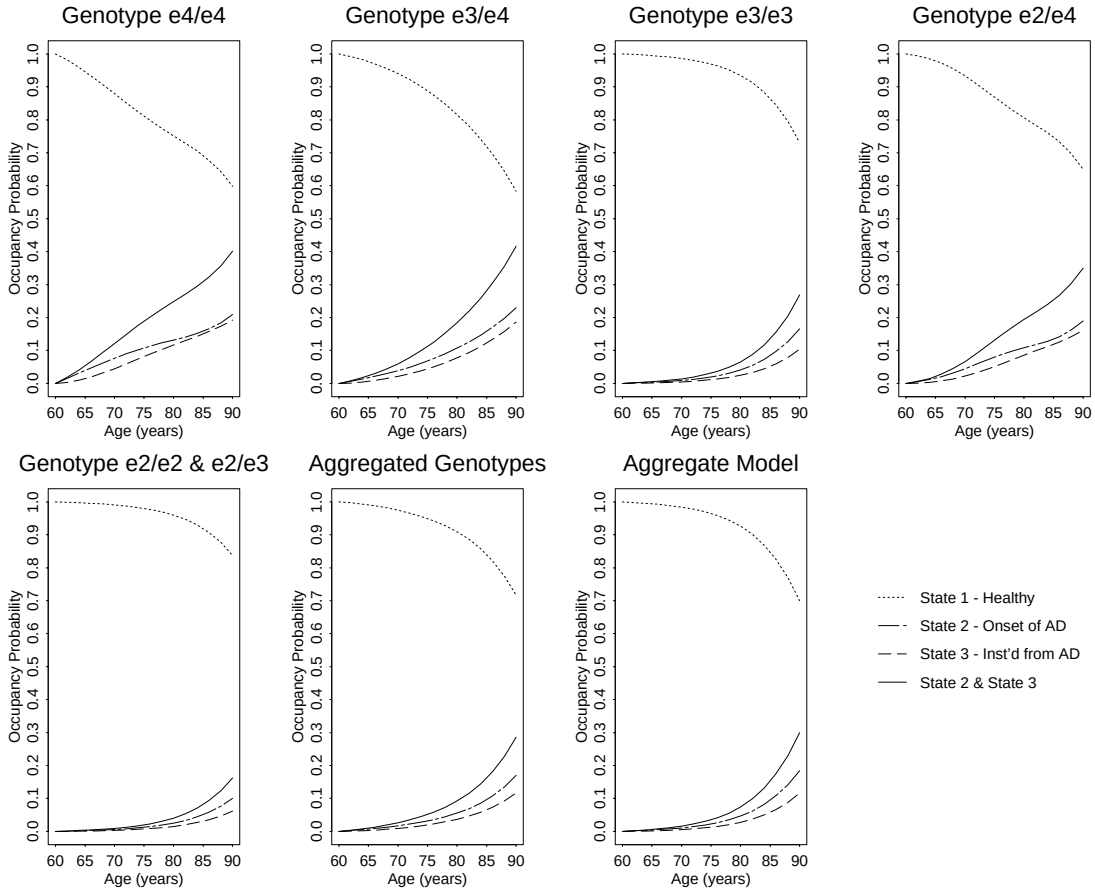

Figure 1.15: Prevalence rate of Alzheimer's disease for females healthy at age 60, high relative risks ($m = 1$).

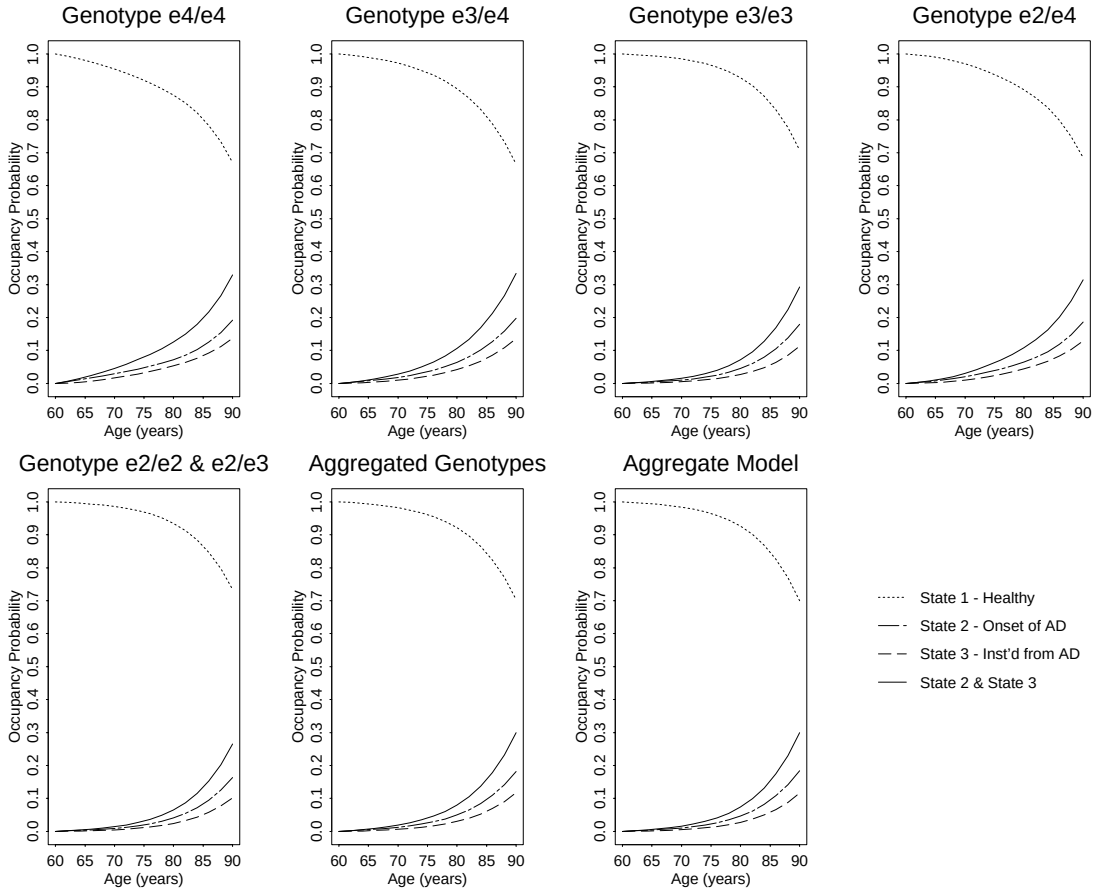

Figure 1.16: Prevalence rate of Alzheimer's disease for females healthy at age 60, low relative risks ($m = 0.25$).

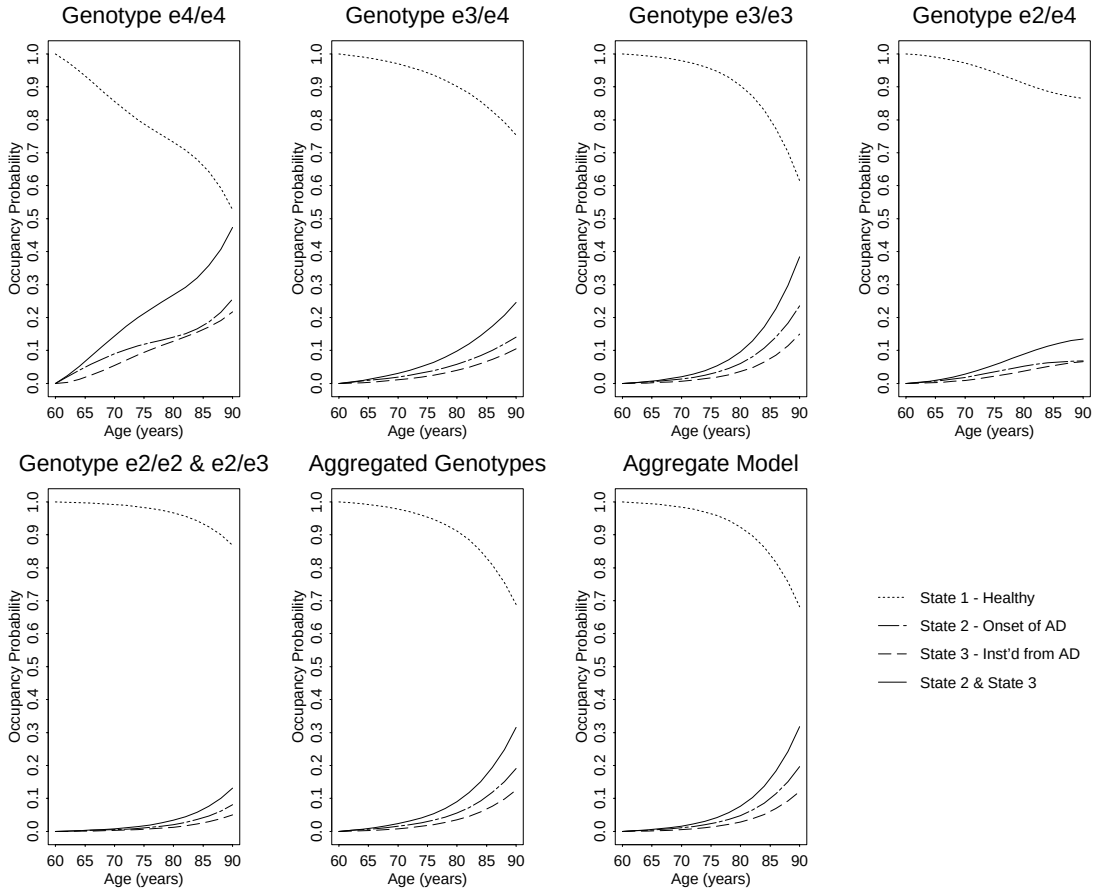

Figure 1.17: Prevalence rate of Alzheimer's disease for males healthy at age 60, high relative risks ($m = 1$).

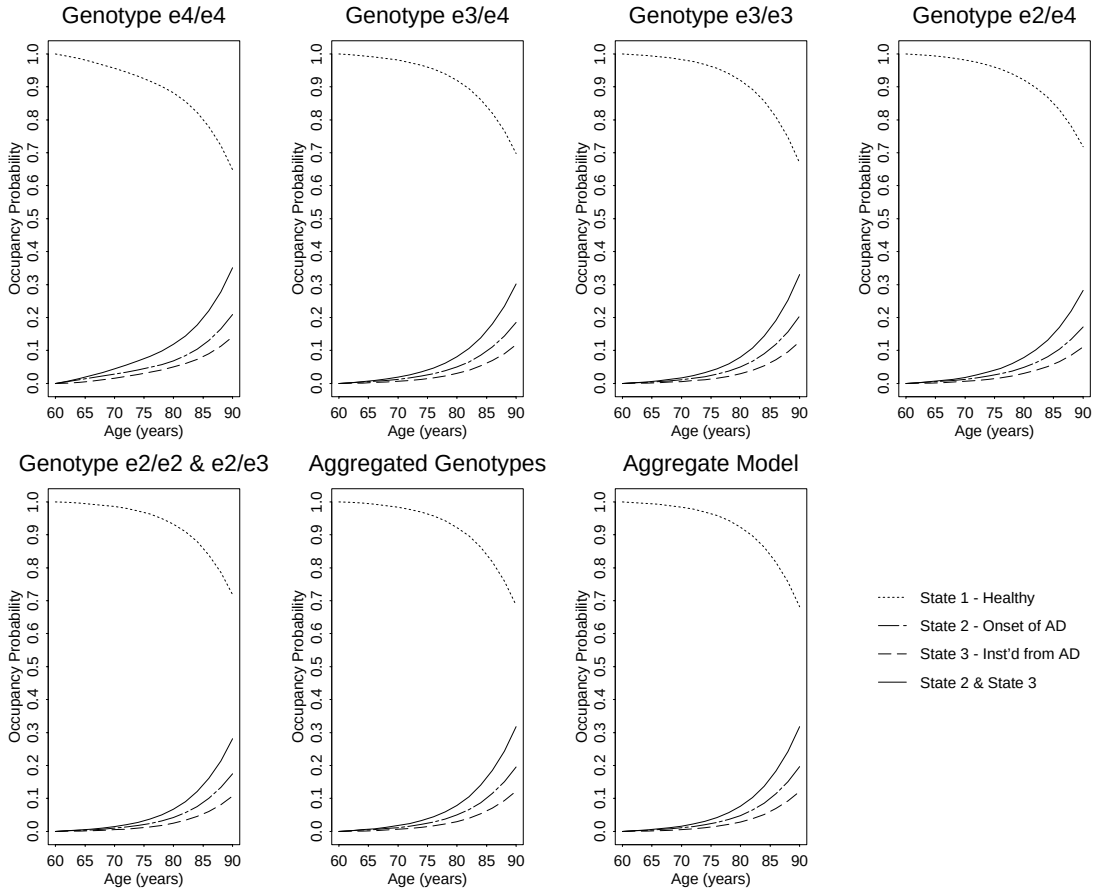

Figure 1.18: Prevalence rate of Alzheimer's disease for males healthy at age 60, low relative risks ($m = 0.25$).

1.7 Summary and Discussion

In this chapter I have specified and calibrated a simple continuous-time Markov model of AD allowing for the variability of the APOE gene, suitable for use in insurance and other applications — I will use it in the next chapter to investigate the potential costs of adverse selection in the long-term care insurance market. While much uncertainty remains, certain features of the model ought to be robust. Whatever reduction in relative risks was used, the genotype-specific incidence rates of AD were also adjusted so that the aggregated (population) incidence rates were close to those actually observed. The latter is one of the few reasonably reliable benchmarks available. Further, the model produces prevalence rates of AD that fall within the range of those observed in many studies.

As well as the intensities, APOE gene frequencies at ages up to 75 have been estimated, in respect of lives unaffected by AD at these ages. These are needed in (for example) insurance applications.

1.7.1 Discussion

1. The model specification is dictated entirely by the events studied in the medical and epidemiological literature, and not by the events that might be of interest in any particular application. If it is the case that actuarial models might, in future, need to incorporate more medical detail, it would be very useful to try to collaborate with medical and other researchers.

2. The published conclusions of medical papers are usually in the form of summary statistics (means, medians, odds ratios and confidence intervals) and if age-related outcomes are shown they are usually in the form of graphs. These are not ideal for actuarial use. AD is a major condition, much studied, but some crude assumptions had to be made in order to calibrate the model from published data only. There must be many medical data sets that could furnish age-related estimates of incidence rates, if only they could be re-analysed. Again, closer collaboration between actuaries and other researchers would be valuable.

Table 1.9: Frequencies of APOE genotypes among lives free of Alzheimer's disease at ages 65, 70 and 75, estimated by solving the Kolmogorov equations forward from age 60.

	Proportion of relative		Gene Frequencies in AD-free population				
Gender	risk, m	Age	$\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$
M & F	1.00	65	0.0168	0.2103	0.6114	0.0258	0.1357
		70	0.0151	0.2055	0.6168	0.0251	0.1374
		75	0.0133	0.1978	0.6246	0.0241	0.1403
	0.50	65	0.0174	0.2115	0.6100	0.0259	0.1353
		70	0.0164	0.2090	0.6128	0.0255	0.1362
		75	0.0154	0.2050	0.6170	0.0250	0.1377
	0.25	65	0.0177	0.2121	0.6092	0.0259	0.1351
		70	0.0172	0.2109	0.6106	0.0258	0.1355
		75	0.0166	0.2088	0.6128	0.0255	0.1363
F	1.00	65	0.0171	0.2097	0.6116	0.0257	0.1358
		70	0.0159	0.2041	0.6176	0.0247	0.1377
		75	0.0145	0.1951	0.6263	0.0232	0.1409
	0.50	65	0.0175	0.2112	0.6101	0.0258	0.1354
		70	0.0168	0.2081	0.6133	0.0253	0.1364
		75	0.0160	0.2033	0.6181	0.0245	0.1381
	0.25	65	0.0177	0.2119	0.6093	0.0259	0.1351
		70	0.0174	0.2104	0.6110	0.0256	0.1357
		75	0.0169	0.2078	0.6135	0.0252	0.1365
M	1.00	65	0.0169	0.2120	0.6094	0.0260	0.1357
		70	0.0153	0.2110	0.6105	0.0258	0.1373
		75	0.0138	0.2097	0.6105	0.0257	0.1403
	0.50	65	0.0175	0.2125	0.6088	0.0260	0.1352
		70	0.0168	0.2120	0.6094	0.0259	0.1359
		75	0.0160	0.2115	0.6094	0.0258	0.1372
	0.25	65	0.0177	0.2126	0.6086	0.0260	0.1350
		70	0.0174	0.2124	0.6089	0.0259	0.1354
		75	0.0170	0.2122	0.6089	0.0259	0.1360

3. Another common type of medical statistic is prevalence rates. The difference between prevalence rates and incidence rates is exactly the difference between the Manchester Unity approach to modelling Permanent Health Insurance, and the multiple-state model approach. Prevalence rates are often easier to estimate, as they can be based on census-type surveys, but it would be helpful if the greater versatility of incidence rates (transition intensities) was more widely appreciated.

4. As a consequence of fitting the intensities using published summary statistics, it is impossible to estimate even crude confidence intervals for them. In any application, therefore, sensitivity analysis is especially important.

5. Several epidemiological authors have suggested the use of individual patient data rather than summary or published data, partly to avoid publication bias in meta-analyses. Useful references are Friedman, Furberg & DeMets (1998), Green, Benedetti & Crowley (1997) and Piantadosi (1997).

6. It is now about six years since the rôle of the APOE gene in AD was confirmed. Since then, the gene has been intensively studied, to the point that meta-analyses including thousands of lives have been published. Even so, little is known about population risk, and data are very scarce in places, so that:

   (a) the relative risks were reduced in a rather arbitrary way to allow for the selectiveness of case-based studies; and

   (b) the relative risks for males are suspect, with the $\varepsilon 2$ allele apparently conferring such strong protection that carriers of the $\varepsilon 4$ allele are not necessarily at higher risk overall.

   If this is typical, it seems likely that the time between the identification of a gene disorder, and the assessment of its impact on insurance, will be of the order of ten years.

7. Despite the fact that AD is a much-studied condition, many studies reach conflicting conclusions. When setting up an actuarial model, it is necessary to consider the body of medical research in its entirety (hence the large number

of references). Inevitably, some studies must be chosen as the basis of the model, but to confine one's attention only to these risks overlooking important features or sources of variation, and could impair the credibility of the results in the eyes of medical experts.

8. APOE is a relatively simple gene to consider, since it has only three relevant alleles, hence six genotypes. Other genes are more complex; for example, the BRCA1 gene (predisposing to breast and ovarian cancer) has several hundred known mutations, some of which have only been observed in a single family.

9. Human genetics is developing at a great speed. This work started in late 1997 and was finished in late 1999, in which time the volume of papers on AD and the APOE gene increased greatly, as the references show. It is very likely that assessments of the impact of specific genetic tests on insurance will have to be revisited quite frequently, if they are to remain credible.

Points 1 to 3 above suggest ways in which medical data might be made more useful for actuarial models, but there is no *a priori* reason why medical studies should be planned with that in mind. However, actuarial models derived from insurance practice, capable of dealing with fairly general payments while in different states or on transition between different states, could make a useful contribution to health economics, and it would be helpful to pursue collaborations from that point of view.

In the next chapter I apply the model specified and parameterized in this chapter to estimating the costs arising, in respect of AD, in a long-term care insurance contract and then to the question of the potential for adverse selection in the long-term care insurance market.

Chapter 2

The Potential for Adverse Selection in Long-Term Care Insurance from Individuals Knowing their APOE Genotype

2.1 Introduction

The aims of this chapter are:

1. to apply the model of Alzheimer's disease, discussed in the previous chapter, to a single premium LTC insurance contract, and to carry out sensitivity analyses;

2. to model the possible effect of adverse selection on LTC insurance, especially with respect to the size of the market; and

3. to consider the costs of a comprehensive retirement package providing both pension and LTC cover.

In Section 2.2, I describe LTC insurance contracts. Then in Section 2.3, I introduce payment streams to the Alzheimer's disease model (introduced in Section 1.3) and extend the statistical framework discussed in Section 1.3.1 to calculating

moments of the present value of these payment streams. In Section 2.4, the basic costs of single premium LTC contracts covering AD alone are calculated, including:

1. the costs using the aggregate incidence of AD; this represents the pricing assumption that would be appropriate in the absence of any genetic markers for AD;

2. the costs in respect of each APOE genotype (Section 2.4.1) — these allow underwriting ratings to be illustrated; and

3. for comparison with (1) above, the average costs assuming no adverse selection (a weighted average of (2) above).

Then, in Section 2.5 the sensitivity of the results to the main model assumptions is considered. Section 2.6 is the main part of the paper; I estimate the possible costs of adverse selection if the results of genetic tests for the APOE gene are known to applicants but are not used in underwriting.

The model can be used to model benefits other than LTC. In Section 2.7, I consider the combined costs of LTC insurance and a pension. In particular, I look at whether or not there is any offset between the costs of the two benefits, so that the combined costs are more stable than either on its own. If this were so, there would be scope for including both in a comprehensive retirement package. Finally, I provide conclusions in Section 2.8.

It is important to emphasise that little reliance should be placed on the absolute costs shown here, in view of the unreliability or scarcity of much of the data, the use of studies of many different populations, the use of institutionalisation as a proxy for an insurance claim, and the early stage of research into the genetics of AD. These costs should not be used to price any genuine LTC contract. Nevertheless, the results do give useful quantitative information on the relative costs of adverse selection.

2.2 Long-Term Care Insurance

Despite the low volume of business in force, there is great variety among LTC or related products. Dullaway & Elliott (1998) described the current market in the U.K., and the main data sources available for pricing and reserving. Another useful reference is Humble & Ryan (1998), which dealt with the closely related problems of Continuing Care Retirement Communities (CCRCs). Only the broad features of long-term care policies are described here, abstracted as the basis for the model; the model itself is flexible enough to be adapted to many product designs — the above references provide more specific details.

This research deals only with pre-funded LTC products, under which an ordinarily healthy person pays regular or single premiums in return for the entitlement to benefit when the need arises. Immediate care annuities, under which a person in immediate need of care buys an annuity to cover its cost, are not considered.

When a claim arises, the insurer will make regular payments to cover the cost of care, either at home or residential, up to some maximum amount per year (the annual sum assured). There is usually a limit to the annual sum assured that can be chosen by the policyholder, since the cost of reasonable care is bounded; this could affect the costs of adverse selection. The level of benefit payment sometimes depends on the severity of the claim (see Section 3.2) and usually is indexed.

Claims can arise:

1. on failure of a given number of Activities of Daily Living (ADLs) — see Section 3.2 for more detail; or

2. on reaching a certain level of cognitive impairment (often because of AD), resulting in a need for continual care or supervision.

The term 'cognitive impairment' covers AD, which accounts for by far the majority of cases, and other forms of mental deterioration, chiefly vascular in origin (for example, arising from strokes).

LTC insurers will usually measure cognitive impairment using a form of mini-mental-state examination (see Dullaway & Elliott (1998) for examples) plus evidence from relevant health professionals. This is not a precise process; in particular it can

be hard to distinguish AD from other dementias without a post-mortem examination.

2.3 Inclusion of Payment Streams in the Alzheimer's Disease Model

In this section I introduce payment streams to the model and extend the statistical framework introduced in Section 1.3.1 to calculating moments of the present value of these payment streams.

I use the same notation here as was introduced in Section 1.3.1, with the exception that I will ignore genotypes in the superscript of the transition intensities. I first look at how a claim will be represented in the Alzheimer's disease model, shown in Figure 1.1.

A major event in the progression of AD, studied by researchers, is institutionalisation. I use it as the best available proxy for the start of an insurance claim: It might be reasonable, because:

1. if full-time care is needed, a claim will almost certainly succeed; and

2. carers of AD patients with LTC cover may lack incentives to undertake care at home after a claim begins.

It should be noted that point 2 above could cause the insured experience to differ from that of the general population, reducing the amount of time for which an insured AD patient is cared for outside an institution.

Define the rate of benefit payable to a life age $x + t$ in the ith subgroup while in the jth state of the model as b_t^{ij}. Then the LTC benefit in this model, arising from AD only, can be represented by $b_t^{ij} = 1$ if $j = 3$, for all i (level benefit) or by $b_t^{ij} = e^{\delta_b t}$ if $j = 3$, for all i (benefit escalating at rate δ_b per year). The latter is the default, since this is a feature of most LTC policies (Dullaway & Elliot, 1998). I only use level benefits in some sensitivity tests.

Norberg (1995) derived a system of differential equations for moments of prospective reserves in a very general Markov model, with both assurance-type and continuous annuity-type benefits, generalising Thiele's equations for expected values. I now summarise the results from his paper.

Using the notation introduced in Section 1.4.3, the payment function is assumed to be of the form:

$$dB_t = \sum_j I_j(t)\, db_t^j + \sum_{j \neq k} b_t^{jk}\, dN_{jk}(t) \tag{2.19}$$

where b_t^j and b_t^{jk} are payment functions specifying payment due during sojourns in state j at time t and upon transition from state j to state k at time t, respectively. It is assumed that jumps in the payment function, b_t^j, can only occur at a finite set of times, $\mathbf{D} = \{t_0, t_1, \ldots, t_m\}$.

The force of interest at time t is denoted δ_t and the discount function at time t, v_t is defined by:

$$v_t = e^{-\int_0^t \delta_s ds} \tag{2.20}$$

and $dv_t = -v_t\, \delta_t\, dt$. The present value, at any time $t \in [0, n]$, of future benefits less premiums is then:

$$\frac{1}{v_t} \int_t^n v_\tau\, dB_\tau \tag{2.21}$$

where v_τ / v_t is the value at time t of a unit due at time τ. The qth moment of the present value in equation 2.21, conditional on the policy being in state j at time t:

$$V_t^{(q)j} = \mathrm{E}\left[\left(\frac{1}{v_t} \int_t^n v_\tau\, dB_\tau \right)^q \middle| \, I_j(t) = 1 \right] \tag{2.22}$$

can be shown, under the above assumptions, to satisfy:

$$\frac{d}{dt} V_t^{(q)j} = \left(q\delta_t + \mu_t^{j\cdot} \right) V_t^{(q)j} - q b_t^j V_t^{(q-1)j} - \sum_{k \neq j} \mu_t^{jk} \sum_{r=0}^{q} \binom{q}{r} \left(b_t^{jk} \right)^r V_t^{(q-r)k} \tag{2.23}$$

valid on $(0, n) \backslash \mathbf{D}$ and subject to the boundary conditions:

$$V_{n-}^{(q)j} = \sum_{r=0}^{q} \binom{q}{r} \left(\Delta b_n^j \right)^r V_n^{(q-r)j} \tag{2.24}$$

where δ_t^j, b_t^j, b_t^{jk} and μ_t^{jk} are assumed to be piecewise continuous and $\mu_t^{j\cdot} = \sum_{k \neq j} \mu_t^{jk}$. The central moments, $m_t^{(q)j}$, are then defined by:

$$m_t^{(q)j} = \begin{cases} V_t^{(1)j} & \text{if q} = 1 \\ \sum_{p=1}^{q} \begin{pmatrix} q \\ p \end{pmatrix} (-1)^{q-p} V_t^{(p)j} \left(V_t^{(1)j} \right)^{q-p} & \text{if } q > 1 \end{cases} \tag{2.25}$$

Using standard numerical methods, these equations are easily solved recursively, to give the qth moments of the payment streams in the model. I used the same numerical routine as for calculating the occupancy probabilities (see Section 1.3.1) — a 4th order Runge-Kutta algorithm (Press *et al.*, 1993) with step size 0.0005 years. They are solved backwards, the boundary condition being that reserves in all states are 0 at the terminal age (120 in this case).

2.4 The Costs of Alzheimer's Disease in LTC Insurance

I assume that the LTC contract has a single premium paid at outset, so the only policy cash-flows thereafter are the benefits. Benefits are payable continuously, only while institutionalised with AD. The quantum of benefit is £1 per annum at inception of the policy, increasing continuously at rate δ_b per year. I use a force of interest $\delta = 0.05$ throughout. The present value of the benefit is the random variable whose expected value and higher moments are obtained by solving Norberg's equations, the expected value being the relevant quantity for use in the traditional actuarial equation of value.

Table 2.10 gives the first three moments (mean, variance and skewness, denoted $q = 1$, 2 and 3 respectively) of the present value of AD claims payments, for lives entering at ages 60, 65, 70 and 75, under the following assumptions:

1. the rate of onset of AD is the aggregate rate, μ_{x+t}^{AD}, men and women combined (equation (1.3));
2. μ_{x+t}^{23} and μ_{x+t}^{24} are as given in Table 1.3;
3. μ_{x+t}^{34} is the mean value given in Table 1.3;

Table 2.10: Mean, variance and skewness ($q = 1$, 2 and 3) of the present value of AD claims costs, unit benefit increasing continuously ($\delta_b = 0.05$), using the aggregate incidence rate of AD.

State at start of contract	q	Entry Age 60	65	70	75
	1	1.1534	1.1634	1.1739	1.1738
1	2	6.431	6.374	6.238	5.975
	3	52.312	50.591	47.668	43.158
	1	5.2367	4.9939	4.6694	4.2541
2	2	25.898	23.438	20.579	17.427
	3	223.272	185.912	148.209	112.619
	1	5.4984	5.3574	5.1578	4.8850
3	2	28.187	26.115	23.520	20.446
	3	259.646	222.571	182.189	141.315

Table 2.11: Expected present values (EPV) of unit benefit increasing continuously ($\delta_b = 0.05$), depending on the incidence of AD after age 90.

	μ^{12}_{x+t} after age 90			Overstatement of the EPV of benefits under (A) compared with:	
	(A)	(B)	(C)		
Age	Gompertz	Level	Nil	(B)	(C)
60	1.1534	1.0584	0.8479	9.0%	36.0%
65	1.1634	1.0650	0.8468	9.2%	37.4%
70	1.1739	1.0674	0.8358	10.0%	40.5%
75	1.1738	1.0581	0.8016	10.9%	46.4%

4. the force of mortality for 'healthy' lives, μ^{14}_{x+t}, is 65% of ${}^{AF80}\mu_{x+t}$, as the majority of elderly people will be women; and

5. benefits increase continuously, with force $\delta_b = 0.05$, representing indexation to earnings.

Table 2.10 also shows moments for lives starting in state 2 (that is, buying insurance just after the onset of AD) and in state 3 (that is, buying insurance just after institutionalisation). The main feature is that the expected present values (EPVs) are not strongly dependent on age at entry, especially for lives healthy at outset. The increasing incidence of AD with age appears to counter the effect of the shorter future lifetime.

In Section 1.4.2, it was noted that the data for rate of onset of AD did not extend beyond age 90, and that the Gompertz curve was extrapolated. This arguably

Table 2.12: EPV of unit benefit increasing continuously ($\delta_b = 0.05$), with gender-specific mortality and incidence of AD.

		EPV of Benefits for Entry at Age			
μ^{14}_{x+t}	μ^{12}_{x+t}	60	65	70	75
$0.65\,{}^{AF80}\mu_{x+t}$	${}^{Female}\mu^{AD}_{x+t}$	0.8268	0.8425	0.8631	0.8871
$0.65\,{}^{AM80}\mu_{x+t}$	${}^{Male}\mu^{AD}_{x+t}$	0.4370	0.4452	0.4531	0.4584
$0.65\,{}^{AF80}\mu_{x+t}$	${}^{Male}\mu^{AD}_{x+t}$	0.8039	0.8089	0.8123	0.8107
$0.65\,{}^{AM80}\mu_{x+t}$	${}^{Female}\mu^{AD}_{x+t}$	0.4176	0.4345	0.4566	0.4830

overstates the incidence of AD, although the evidence is not clear on this point. Column (B) in Table 2.11 shows by how much this would overstate the expected present value of the increasing benefit, if the true incidence rate levelled off at age 90 or was zero. $\mu^{12}_{90+t} = \mu^{12}_{90}$ almost certainly understates the incidence of AD, so Table 2.11 shows that the Gompertz assumption has no great effect. The last column of the table shows that a significant proportion of the AD costs in the model arise from cases at very old ages; the fact that the rate of increase of the benefit is equal to the force of interest is part of the reason.

Next, I look at the different costs in respect of men and women. Two factors are relevant:

1. the baseline mortality (μ^{14}_{x+t}), which I take to be 65% of AM80 ultimate and 65% of AF80 ultimate; and

2. the gender specific incidence of AD (μ^{12}_{x+t}), which I take from equations 1.4 and 1.5.

The first two lines of Table 2.12 compare the EPVs of £1 p.a. sums assured at outset, increasing at $\delta_b = 0.05$, for females and males entering at different ages. As expected, the EPVs are higher for females. Taking these as a starting point, the third line shows the effect of changing only the rate of onset of AD, from female to male; the effect is not substantial, and this is why the aggregate rate of onset is used for all this work. The fourth line shows the effect of changing only the baseline mortality, from female to male; this reduces the costs very substantially. Females' longer lives mean that they are more likely to become institutionalised, and then

live longer while institutionalised.

The sensitivity of the LTC costs to overall longevity is expected, because mortality while institutionalised is modelled by a constant addition to the assumed overall mortality. The same result will arise, in the model, if mortality at old ages continues to fall in the future as it has in the past. However, the extent to which such falls will be accompanied by a different pattern of infirmity cannot be predicted. Longer lifetimes could result from:

1. people remaining healthy for longer, followed by a swifter decline (so-called 'compression of morbidity'); or

2. people being kept alive for longer while in decline.

These could have dramatically different implications for LTC costs. Here, it is worth noting that no attempt has been made to model changes in the pattern of infirmity.

2.4.1 Long-Term Care Insurance Costs and APOE Genotype

Recall that the incidence rates of AD were estimated to be consistent with 100%, 50% and 25% of the absolute relative risks implied by Farrer *et al.* (1997), to allow for the selective effect of using case-based data rather than population data (see Section 1.5). In the following, these three cases are represented by the parameter $m = 1.0, 0.5$ or 0.25.

Table 2.13 shows the EPV of the £1 p.a. benefit increasing continuously with force $\delta_b = 0.05$ for each genotype, and averages weighted by gene frequencies, for males and females separately and in aggregate. The label i is used to indicate genotype: $i = 1$ represents $\varepsilon 2/\varepsilon 2$ and $\varepsilon 2/\varepsilon 3$, $i = 2$ represents $\varepsilon 3/\varepsilon 3$, $i = 3$ represents $\varepsilon 2/\varepsilon 4$, $i = 4$ represents $\varepsilon 3/\varepsilon 4$ and $i = 5$ represents $\varepsilon 4/\varepsilon 4$. Summation over i denotes summation over all genotypes. Thus, define p_x^i to be the population frequency of genotype i at age x, and $\sum_i p_x^i = 1$. Also, define C_x^i to be the EPV of the unit benefit, for a life with genotype i buying insurance at age x; these are the EPVs

given in Table 2.13. The weighted average EPVs in Table 2.13 are then:

$$\sum_i p_x^i C_x^i. \tag{2.26}$$

I assume that the gene frequencies from Farrer *et al.*, (1997) (Section 1.5) apply at age 60, denoted p_{60}^i, and calculate the frequencies at higher ages by solving the Kolmogorov forward equations for ${}^iP_{60\,60+t}^{11}$, the probability that a healthy life age 60 with genotype i is still healthy at age $60+t$. Then, the p_{60+t}^i are given by Equation 1.17 and the average costs for entrants at age $60+t$ is:

$$\text{Average EPV at age } 60+t = \frac{\sum_i p_{60}^i \, {}^iP_{60\,60+t}^{11} \, C_{60+t}^i}{\sum_i p_{60}^i \, {}^iP_{60\,60+t}^{11}}. \tag{2.27}$$

The EPVs for males and females together are similar to those for females only. Males have significantly lower costs at all ages and levels of relative risk (values of m). This is largely because of the difference in baseline mortality.

The EPVs of the individual genotypes depend strongly on the level of relative risk, but the EPVs averaged over all the genotypes do not. This is because the modelled relative risks were adjusted to reproduce (approximately) the aggregate incidence of AD (see Section 1.5). Note some other consequences of adjusting the relative risks to reproduce the aggregate incidence of AD:

1. Increasing the level of relative risks for high-risk genotypes means decreasing the relative risks for low-risk genotypes. Since the $\varepsilon 3/\varepsilon 4$ genotype is low-risk for males (the relative risks fall below 1.0 eventually) the EPVs for this genotype are lower than those for the $\varepsilon 3/\varepsilon 3$ genotype, especially for $m = 1$ and for older entrants. It is doubtful that this feature is genuine, but it is not very great for $m = 0.25$, so I have not tried to eliminate it.

2. The EPVs of the high-risk genotypes decrease with age, while those of low-risk genotypes increase with age.

Table 2.13 also shows what might be the effect on underwriting, if LTC insurers were allowed to use APOE test results. Increased premiums might be indicated based on the excess of the costs for the $\varepsilon 4/\varepsilon 4$ and $\varepsilon 3/\varepsilon 4$ genotypes over total average costs. However, it is not correct to take the final column of Table 2.13 as the total

Table 2.13: EPVs of unit benefit increasing continuously ($\delta_b = 0.05$) by genotype, and averages over genotypes.

Gender	Proportion of relative risk, m	Age	EPV of benefits for genotype $\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	Average EPV
M & F	1.00	60	2.3619	1.7056	1.1003	1.5955	0.7816	1.223
		65	2.2063	1.6784	1.1101	1.5780	0.7866	1.216
		70	1.9539	1.6180	1.1188	1.5025	0.7900	1.198
		75	1.6483	1.5184	1.1215	1.3662	0.7888	1.166
F	1.00	60	2.0878	1.7565	1.0609	1.7065	0.7106	1.198
		65	1.9660	1.7233	1.0706	1.6799	0.7159	1.190
		70	1.7673	1.6521	1.0793	1.5734	0.7204	1.170
		75	1.5243	1.5370	1.0825	1.3873	0.7211	1.134
M	1.00	60	1.6233	0.6499	0.7728	0.4818	0.3001	0.691
		65	1.4866	0.6363	0.7897	0.4683	0.3042	0.695
		70	1.2788	0.6076	0.8065	0.4214	0.3075	0.693
		75	1.0652	0.5591	0.8183	0.3341	0.3086	0.683
M & F	0.50	60	1.8427	1.4510	1.1234	1.3874	0.9746	1.194
		65	1.7510	1.4402	1.1332	1.3823	0.9823	1.195
		70	1.6034	1.4104	1.1419	1.3449	0.9889	1.190
		75	1.4282	1.3562	1.1442	1.2722	0.9899	1.174
F	0.50	60	1.6933	1.4958	1.1081	1.4649	0.9427	1.189
		65	1.6236	1.4808	1.1179	1.4537	0.9507	1.189
		70	1.5094	1.4433	1.1265	1.3966	0.9577	1.183
		75	1.3702	1.3782	1.1291	1.2932	0.9599	1.164
M	0.50	60	1.1156	0.6506	0.7006	0.5909	0.5068	0.669
		65	1.0530	0.6534	0.7164	0.5949	0.5177	0.679
		70	0.9571	0.6504	0.7326	0.5861	0.5290	0.687
		75	0.8603	0.6378	0.7449	0.5605	0.5379	0.691
M & F	0.25	60	1.5154	1.3012	1.1309	1.2669	1.0590	1.169
		65	1.4689	1.3002	1.1408	1.2690	1.0679	1.174
		70	1.3919	1.2885	1.1495	1.2536	1.0755	1.176
		75	1.2989	1.2611	1.1517	1.2170	1.0773	1.169
F	0.25	60	1.4447	1.3362	1.1309	1.3189	1.0505	1.175
		65	1.4106	1.3327	1.1408	1.3174	1.0595	1.180
		70	1.3519	1.3164	1.1495	1.2908	1.0675	1.181
		75	1.2777	1.2823	1.1517	1.2368	1.0697	1.172
M	0.25	60	0.8778	0.6492	0.6723	0.6228	0.5828	0.658
		65	0.8536	0.6585	0.6877	0.6327	0.5960	0.671
		70	0.8134	0.6654	0.7035	0.6372	0.6096	0.683
		75	0.7720	0.6662	0.7159	0.6324	0.6206	0.691

average LTC costs; it represents AD-related costs alone, and these are only about 25% – 33% of the total claim numbers (Watson, 1998) and perhaps 40% – 50% of the claim costs. For example, the percentage rating indicated for an $\varepsilon4/\varepsilon4$ female aged 60, with $m = 1$, would not be given by:

$$\frac{2.0878 - 1.198}{1.198} = 0.742 \text{ or } 74\%$$

but would be closer to:

$$\frac{2.0878 - 1.198}{2 \times 1.198} = 0.371 \text{ or } 37\%$$

if total LTC costs were about twice AD-related LTC costs. Table 2.14 shows approximate underwriting ratings on that basis. Note that:

1. Ratings decrease with age, because the relative risks all decrease with age.

2. Ratings for $\varepsilon4/\varepsilon4$ males exceed those for $\varepsilon4/\varepsilon4$ females. This is because overall costs for males are significantly lower, but this in turn is because the $\varepsilon2/\varepsilon4$ and $\varepsilon3/\varepsilon4$ genotypes are in fact low-risk for males at most ages. It should not be considered that this is evidence that ratings for males should in fact be higher (see Section 2.6.1) and the following comments are based on females only.

3. If the odds ratios in Farrer *et al.* (1997) are about right ($m = 1$), then ratings could be up to +50% for $\varepsilon4/\varepsilon4$ and +25% for $\varepsilon3/\varepsilon4$ and $\varepsilon2/\varepsilon4$. Most insurers would probably charge extra premiums for these risks.

4. If, however, the relative risks in the population are much lower ($m = 0.25$), then ratings could be up to +15% for $\varepsilon4/\varepsilon4$ and +7% for $\varepsilon3/\varepsilon4$ and $\varepsilon2/\varepsilon4$. Most insurers would probably ignore the latter.

2.5 Sensitivity Analysis

In this section tests of sensitivity to key model assumptions are carried out. I consider:

Table 2.14: Approximate underwriting ratings (equivalent to percentage extra premiums) for the $\varepsilon4/\varepsilon4$ and female $\varepsilon3/\varepsilon4$ & $\varepsilon2/\varepsilon4$ genotypes, assuming total LTC insurance costs are twice AD-related costs. Unit benefit increasing continuously ($\delta_b = 0.05$).

Proportion of relative risk, m	Age	Rating factor for sex and genotype: Male & Female $\varepsilon4/\varepsilon4$	Male & Female $\varepsilon3/\varepsilon4$	Male & Female $\varepsilon2/\varepsilon4$	Female $\varepsilon4/\varepsilon4$	Female $\varepsilon3/\varepsilon4$	Female $\varepsilon2/\varepsilon4$	Male $\varepsilon4/\varepsilon4$
		%	%	%	%	%	%	%
1.00	60	47	20	15	37	23	21	67
	65	41	19	15	33	22	21	57
	70	32	18	13	26	21	17	42
	75	21	15	9	17	18	11	28
0.50	60	27	11	8	21	13	12	33
	65	22	10	8	18	12	11	28
	70	19	9	7	14	11	9	20
	75	11	8	4	9	9	6	12
0.25	60	15	6	4	11	7	6	17
	65	13	5	4	10	6	6	14
	70	9	5	3	7	6	5	10
	75	6	4	2	5	5	3	6

1. the use of institutionalisation as a proxy for the start of a LTC claim;

2. the rate of benefit increases;

3. the level of baseline mortality;

4. mortality in state 2 (after onset of AD but before institutionalisation); and

5. the addition to baseline mortality in state 3 (lives institutionalised from AD).

For brevity, I look only at males and females separately, at entry ages 60 and 70, and at levels of relative risk $m = 1.00$ and $m = 0.25$.

Percentage changes to the EPVs in Table 2.13 are reported. The changes in the EPVs averaged over all genotypes indicate the overall uncertainty of AD costs in LTC insurance, while the relative changes in the EPVs of high-risk and low-risk genotypes indicate uncertainty about the potential for adverse selection.

It has been assumed that lives with AD do not claim LTC benefits before becoming institutionalised because, of all the events studied by epidemiologists, institutionalisation is the best proxy for the start of a claim. In practice, a claim might be

Table 2.15: EPV of benefits commencing 1 year before institutionalisation ($w = 1$ years) as a % of EPV of benefits commencing on institutionalisation ($w = 0$ years), for unit benefit increasing continuously ($\delta_b = 0.05$) by genotype and averages over genotypes.

			EPV of benefits with $w = 1$ years as a % of EPV of benefits with $w = 0$ years for					
Gender	Proportion of relative risk, m	Age	$\varepsilon4/\varepsilon4$ %	$\varepsilon3/\varepsilon4$ %	$\varepsilon3/\varepsilon3$ %	$\varepsilon2/\varepsilon4$ %	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$ %	Average EPV %
F	1.00	60	123.96	125.54	129.17	125.28	130.33	127.82
		70	126.03	126.88	129.91	126.72	131.13	128.94
F	0.25	60	126.66	127.54	128.90	127.46	129.18	128.51
		70	128.18	128.57	129.64	128.55	129.93	129.36
M	1.00	60	125.11	127.91	130.27	125.66	131.65	129.58
		70	128.19	129.84	131.55	127.26	133.18	131.17
M	0.25	60	128.43	130.34	130.73	130.33	131.09	130.63
		70	130.77	131.81	132.00	131.80	132.39	131.98

made sooner (later is less likely). The possibility that claims precede institutionalisation by w years, or start immediately upon entry if a life is institutionalised less than w years after entry, can be considered by paying a lump-sum benefit of:

$$b^{i23}_{x+t} = e^{\delta_b \max(t-w,0)} \int_0^{\min(t,w)} e^{\delta_b s + \delta(\min(t,w)-s)} ds \tag{2.28}$$

upon institutionalisation at age $x + t$. b^{i23}_{x+t} is the accumulated LTC benefit paid over w years preceding institutionalisation, or from outset. This method is only approximate:

1. it ignores the costs in respect of lives who die while in receipt of LTC benefit but before becoming institutionalised; and

2. it overstates the costs in respect of lives who become institutionalised within w years of the onset of AD

but these errors are unlikely to be significant. I consider $w = 1$ year in Table 2.15 and $w = 2$ years in Table 2.16.

The increases in costs range from 23% per additional year of benefit, in the case of high-risk genotypes, to 31% per additional year of benefit, in the case of low-risk

Table 2.16: EPV of benefits commencing 2 years before institutionalisation ($w = 2$ years) as a % of EPV of benefits commencing on institutionalisation ($w = 0$ years), for unit benefit increasing continuously ($\delta_b = 0.05$) by genotype and averages over genotypes.

Gender	Proportion of relative risk, m	Age	EPV of benefits with $w = 2$ years as a % of EPV of benefits with $w = 0$ years for $\varepsilon4/\varepsilon4$ %	$\varepsilon3/\varepsilon4$ %	$\varepsilon3/\varepsilon3$ %	$\varepsilon2/\varepsilon4$ %	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$ %	Average EPV %
F	1.00	60	147.85	151.04	158.34	150.54	160.64	155.62
		70	151.84	153.62	159.77	153.26	162.20	157.80
F	0.25	60	153.29	155.05	157.79	154.91	158.34	157.01
		70	156.25	157.05	159.23	157.00	159.80	158.66
M	1.00	60	150.11	155.76	160.52	151.28	163.27	159.12
		70	156.04	159.50	163.00	154.25	166.26	162.22
M	0.25	60	156.79	160.65	161.44	160.64	162.17	161.23
		70	161.36	163.50	163.91	163.48	164.68	163.86

genotypes, almost independent of age. Therefore, using institutionalisation as a proxy for the start of a claim is likely to understate costs quite significantly, but will not have much effect on underwriting, or the costs of adverse selection, unless the duration of an AD-related claim, prior to institutionalisation, is likely to be greater than that of other LTC insurance claims.

The force of benefit escalation of 0.05 represents indexation to earnings in a relatively low inflation environment, with force of interest $\delta = 0.05$ also. Tables 2.17 and 2.18 give the EPVs of benefits increasing with force $\delta_b = 0.025$, and level benefits ($\delta_b = 0.0$), respectively, as a percentage of the EPV of benefits with $\delta_b = 0.05$.

From Tables 2.18 and 2.17 it can be seen that:

1. claims inflation has a large impact on the EPV of benefits: with $\delta_b = 0.025$ the EPVs of benefits are between 45% and 68% of those with $\delta_b = 0.05$, and with $\delta_b = 0$, between 22% and 49%.

2. claims inflation has a greater impact on low-risk genotypes ($\varepsilon3/\varepsilon3$, $\varepsilon2/\varepsilon2$ and $\varepsilon2/\varepsilon3$) under all scenarios; the lower the claims inflation, the greater the difference between the EPVs of low-risk and high-risk groups, so the greater the scope for adverse selection.

Table 2.17: EPVs of unit benefit increasing continuously ($\delta_b = 0.025$) as a percentage of EPV of unit benefit increasing continuously ($\delta_b = 0.05$) by genotype and averages over genotypes for benefits commencing on institutionalisation.

			EPV of benefits with $\delta_b = 0.025$ as a % of EPV of benefits with $\delta_b = 0.05$ for					
Gender	Proportion of relative risk, m	Age	$\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	Average EPV
F	1.00	60	56.67	52.63	46.71	53.45	45.88	49.05
		70	64.69	62.62	57.74	63.49	56.66	59.41
F	0.25	60	51.22	49.18	46.98	49.47	46.74	47.65
		70	60.65	59.76	58.04	60.02	57.75	58.51
M	1.00	60	60.74	55.07	51.21	57.92	50.38	52.46
		70	67.93	65.02	62.31	68.71	60.95	62.99
M	0.25	60	54.80	51.48	50.75	51.55	50.45	50.98
		70	63.82	62.23	61.83	62.35	61.46	61.92

Table 2.18: EPVs of level unit benefit ($\delta_b = 0$) as a percentage of EPV of unit benefit increasing continuously ($\delta_b = 0.05$) by genotype, and averages over genotypes for benefits commencing on institutionalisation.

			EPV of level benefits as a % of EPV of benefits with $\delta_b = 0.05$ for					
Gender	Proportion of relative risk, m	Age	$\varepsilon4/\varepsilon4$ %	$\varepsilon3/\varepsilon4$ %	$\varepsilon3/\varepsilon3$ %	$\varepsilon2/\varepsilon4$ %	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$ %	Average EPV %
F	1.00	60	33.99	29.28	23.00	30.19	22.28	25.51
		70	43.48	40.69	34.60	41.95	33.42	36.71
F	0.25	60	27.91	25.58	23.25	25.90	23.04	23.97
		70	38.28	37.09	34.94	37.47	34.61	35.54
M	1.00	60	38.69	31.89	27.53	34.90	26.82	28.96
		70	47.63	43.65	40.08	48.43	38.52	40.99
M	0.25	60	31.75	27.89	27.06	27.97	26.77	27.33
		70	42.15	40.04	39.49	40.22	39.05	39.62

Table 2.19: Sensitivity of the EPVs of unit benefit increasing continuously ($\delta_b = 0.05$) to the mortality assumptions, for Females. Level of relative risk, $m = 1.00$.

Baseline Mortality % AF80	State 2 Mortality, % Baseline	Age	EPV of benefits as a % of EPV of benefits before adjusting mortality, for: $\varepsilon4/\varepsilon4$ %	$\varepsilon3/\varepsilon4$ %	$\varepsilon3/\varepsilon3$ %	$\varepsilon2/\varepsilon4$ %	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$ %	Average EPV %
100%	33.5%	60	77.78	72.65	62.32	73.70	59.98	66.26
		70	73.80	71.03	62.53	71.85	59.98	65.34
65%	65%	60	95.06	93.99	91.74	94.20	91.17	92.59
		70	93.61	93.05	91.24	93.20	90.65	91.83
65%	100%	60	90.56	88.60	84.65	89.00	83.73	86.16
		70	87.89	86.88	83.76	87.17	82.80	84.79
100%	100%	60	69.38	63.18	51.65	64.54	49.41	56.11
		70	63.37	60.12	50.97	61.24	48.55	54.04

The second point above arises because, when benefits increase at the force of interest, the effect of discounting is lost. The effect is much reduced if the level of relative risks is low ($m = 0.25$).

The major effect of the difference between male and female mortality has already been considered, in Section 2.4. Turning now to the mortality assumptions made in respect of lives with AD, I consider:

1. μ^{14}_{x+t}, *the mortality of lives without AD.* Table 2.12 showed the effect of mortality using aggregate rates of AD incidence; I now look at individual genotypes and weighted average costs, with μ^{14}_{x+t} given by 65% and 100% of AM80 or AF80 mortality, as appropriate.

2. μ^{24}_{x+t}, *the mortality of lives with AD prior to institutionalisation.* In Section 1.4 this was estimated to be 33.5% of the baseline mortality. This low level could, however, be a feature of the data set used (from a brain-bank study, Jost & Grossberg (1995)) as persons from institutions might have been more likely to have been included. I look at the effects of taking μ^{24}_{x+t} to be 65% or 100% of the baseline mortality.

The results are shown in Tables 2.19–2.22. As before, the EPVs are sensitive to the baseline mortality, increasing this to 100% of AM80 or AF80 decreases the EPVs

Table 2.20: Sensitivity of the EPVs of unit benefit increasing continuously ($\delta_b = 0.05$) to the mortality assumptions, for Females. Level of relative risk $m = 0.25$.

Baseline Mortality % AF80	State 2 Mortality, % Baseline	Age	EPV of benefits as a % of EPV of benefits before adjusting mortality, for: $\varepsilon 4/\varepsilon 4$ %	$\varepsilon 3/\varepsilon 4$ %	$\varepsilon 3/\varepsilon 3$ %	$\varepsilon 2/\varepsilon 4$ %	$\varepsilon 2/\varepsilon 2$ & $\varepsilon 2/\varepsilon 3$ %	Average EPV %
100%	33.5%	60	69.61	66.83	62.96	67.18	62.34	64.09
		70	67.42	66.18	63.16	66.38	62.52	63.97
65%	65%	60	93.31	92.73	91.88	92.80	91.74	92.13
		70	92.28	92.03	91.39	92.06	91.24	91.55
65%	100%	60	87.44	86.38	84.90	86.51	84.66	85.33
		70	85.56	85.10	84.00	85.17	83.76	84.29
100%	100%	60	59.92	56.62	52.30	57.08	51.69	53.58
		70	56.28	54.85	51.60	55.15	50.97	52.48

Table 2.21: Sensitivity of the EPVs of unit benefit increasing continuously ($\delta_b = 0.05$) to the mortality assumptions, for Males. Level of relative risk, $m = 1.00$.

Baseline Mortality % AM80	State 2 Mortality, % Baseline	Age	EPV of benefits as a % of EPV of benefits before adjusting mortality, for: $\varepsilon 4/\varepsilon 4$ %	$\varepsilon 3/\varepsilon 4$ %	$\varepsilon 3/\varepsilon 3$ %	$\varepsilon 2/\varepsilon 4$ %	$\varepsilon 2/\varepsilon 2$ & $\varepsilon 2/\varepsilon 3$ %	Average EPV %
100%	33.5%	60	73.69	64.85	57.81	70.86	55.39	59.98
		70	68.40	63.41	58.27	70.55	55.10	59.50
65%	65%	60	93.95	92.15	90.68	93.45	90.07	91.13
		70	91.82	90.85	89.84	92.29	89.10	90.08
65%	100%	60	88.50	85.34	82.81	87.52	81.85	83.59
		70	84.69	83.03	81.33	85.44	80.17	81.73
100%	100%	60	63.86	54.16	46.77	60.21	44.68	49.09
		70	56.11	50.96	45.82	57.98	43.05	47.09

Table 2.22: Sensitivity of the EPVs of unit benefit increasing continuously ($\delta_b = 0.05$) to the mortality assumptions, for Males. Level of relative risk $m = 0.25$.

Baseline Mortality % AM80	State 2 Mortality, % Baseline	Age	EPV of benefits as a % of EPV of benefits before adjusting mortality, for: $\varepsilon4/\varepsilon4$ %	$\varepsilon3/\varepsilon4$ %	$\varepsilon3/\varepsilon3$ %	$\varepsilon2/\varepsilon4$ %	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$ %	Average EPV %
100%	33.5%	60	63.69	58.03	56.75	58.16	56.00	57.13
		70	60.87	57.93	57.21	58.10	56.38	57.36
65%	65%	60	91.88	90.70	90.44	90.72	90.26	90.52
		70	90.33	89.74	89.61	89.77	89.43	89.64
65%	100%	60	84.91	82.87	82.42	82.91	82.13	82.55
		70	82.18	81.19	80.96	81.24	80.66	81.00
100%	100%	60	53.16	47.13	45.76	47.29	45.07	46.18
		70	48.54	45.60	44.85	45.81	44.11	45.02

by about 35% – 45%. However, the use of 100% of AM80 or AF80 is mainly for illustration, as industry sources suggest that 65% of AM80 or AF80 is a reasonable assumption for LTC pricing. Of more potential concern is the assumption regarding mortality in state 2. Looking at the third scenario in each table, where mortality in state 2 is assumed to be the same as in state 1, the reductions in the EPVs are not large (about 15%) given the very large increase in μ^{i24}_{x+t}.

The EPVs in respect of non-$\varepsilon4$ genotypes are more sensitive to the mortality assumptions than those of the $\varepsilon4$ genotypes, as AD strikes later when mortality is higher. Therefore lighter baseline mortality would be expected to reduce the scope for adverse selection.

Finally, the mortality of institutionalised lives is considered. In Section 1.4, μ^{34}_{x+t} was modelled by adding a constant K to the baseline force of mortality. The estimate of K was 0.173, but estimates from different studies ranged from 0.08 to 0.26. Again for comparison purposes I take values of $K = 0.085$ and 0.34, respectively about half and double the mean estimate.

Table 2.23 shows the results for females with baseline mortality of 65% of AF80; the results for males and for higher baseline mortality are very similar and I omit them. The EPVs of benefits are quite sensitive to the additional term K; doubling K reduces costs by about 35%–40%. As expected, the EPVs in respect of high-risk

Table 2.23: Sensitivity of the EPVs of unit benefit increasing continuously ($\delta_b = 0.05$) to the addition K to the force of mortality in the institutionalised state, for Females. Baseline mortality: 65% of AF80.

Prop'n of relative risk, m	Addition to baseline mortality in state 3	Age	EPV of benefits as a % of EPV of benefits with $K = 0.173$, for: $\varepsilon4/\varepsilon4$ %	$\varepsilon3/\varepsilon4$ %	$\varepsilon3/\varepsilon3$ %	$\varepsilon2/\varepsilon4$ %	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$ %	Average EPV %
1.00	0.085	60	153.82	148.87	140.86	149.95	139.58	144.00
		70	146.52	144.30	138.71	145.18	137.40	140.61
	0.34	60	59.31	60.91	64.03	60.58	64.72	62.83
		70	61.52	62.33	64.74	62.08	65.47	63.94
0.25	0.085	60	146.80	144.24	141.26	144.62	140.90	142.16
		70	142.01	141.04	139.06	141.30	138.71	139.61
	0.34	60	61.81	62.67	63.84	62.56	64.02	63.50
		70	63.35	63.71	64.57	63.65	64.75	64.34

genotypes are more sensitive to the assumed mortality of institutionalised lives, as these lives contract AD at younger ages, where K is relatively larger. A higher value of K, therefore, could increase the scope for adverse selection.

It is also worth noting that:

1. the change in the EPVs are roughly proportional to K; and
2. sensitivity to K is greater for younger ages at entry, again because K is relatively larger at younger ages.

2.6 The Impact of Adverse Selection on LTC Insurance

Adverse selection may arise in the following circumstances:

1. insurers are not aware of, or are not allowed to use, any information regarding an applicant's APOE genotype;
2. a proportion of applicants are aware of their APOE genotype and do use this information in the process of deciding whether or not to purchase LTC insurance; and

3. applicants who know their APOE genotype have an impression of the risks of AD, whether right or wrong, that gives them an incentive to seek LTC insurance.

To model point 3 above, it is assumed that the presence of one $\varepsilon 4$ allele is enough to lead any tested individual to regard themselves as at risk. Someone age 60, male or female, could reasonably infer this from the odds ratios in Farrer *et al.* (1997).

This model of adverse selection differs in four important respects from that used for life insurance by Macdonald (1997, 1999):

1. Only single premium contracts bought at a fixed age are considered, and it is assumed that any genetic testing has been carried out by then.

2. The sum assured applied for is fixed, and does not depend on the genetic test result. Macdonald (1997, 1999) found that large sums assured was the costliest aspect of adverse selection in life insurance, but LTC insurance is limited by the reasonable costs of care, and sums assured much in excess of needs are less likely to arise.

3. The impact of adverse selection in life insurance was found to be modest partly because it was measured against the background of a large, mature market. The LTC market is as yet undeveloped, so the impact of adverse selection will depend on the proportion of the population that, ultimately, is covered.

4. Macdonald (1997, 1999) made extreme assumptions in order to obtain an upper bound on the cost of adverse selection. This research is aiming to make realistic assumptions in order to obtain realistic estimates.

The weighted average EPVs in Table 2.13 (see equation (2.26)) give the net single premiums for a unit benefit, covering AD only, assuming that the proportion of each genotype in the insured population is the same as in the whole population. This is taken to be the premium charged to cover AD in the absence of adverse selection. The cost of adverse selection will be given as the percentage increase in these single premiums resulting from solving Thiele's equations with different proportions of high- and low-risk genotypes; in fact these will also be weighted averages of the

EPVs for each genotype in Table 2.13, with suitably chosen weights. The weights will depend on:

1. the prevalence of genetic testing for APOE (note that APOE is implicated in conditions other than AD, so testing could be carried out for other reasons);

2. the size of the LTC insurance market in the absence of adverse selection, which determines how many low-risk individuals join the risk pool; and

3. the probability that a person with one or two $\varepsilon 4$ alleles will buy LTC insurance, which determines how many high-risk individuals join the risk pool.

Let $\hat{p}_x^i$ be the frequency of genotype i among those buying insurance at age x. If $\hat{p}_x^i \neq p_x^i$, because of adverse selection, then the cost of adverse selection is:

$$\frac{\sum_i \hat{p}_x^i C_x^i - \sum_i p_x^i C_x^i}{\sum_i p_x^i C_x^i} \tag{2.29}$$

which I express as a percentage. As before, assume the gene frequencies from Farrer *et al.* (1997) apply to the whole population at age 60, and that for older ages p_{60+t}^i are found by solving the Kolmogorov forward equations (see equations (1.17) and (2.27)).

More precisely, the following steps are carried out:

1. Without adverse selection, the LTC market is a proportion z of the population. The same proportion z of lives of each genotype buy LTC insurance, and the proportion of purchasers at age x with genotype i is p_x^i.

2. With adverse selection, behaviour depends on genotype, and a proportion z^i of lives with genotype i purchase insurance.

3. Assume that lives with low-risk genotypes ($i = 1$ and $i = 2$) are just as likely as before to buy LTC insurance after a genetic test result: $z^i = z$.

4. A larger proportion of lives with high-risk genotypes ($i = 3$, $i = 4$ and $i = 5$) buy LTC insurance. Suppose that these lives are k times more likely to buy insurance: $z^i = kz$ (up to a maximum of $z^i = 1$).

5. The proportion of purchasers at age x with genotype i is then:

$$\hat{p}_x^i = \frac{z^i p_x^i}{\sum_{j=1}^{j=5} z^j p_x^j} \tag{2.30}$$

and the cost of adverse selection is given by equation (2.29).

At present, the LTC insurance market in the U.K. is tiny; well under 1% of the population is covered. In such a small market, there is scope for severe adverse selection, since the high-risk genotypes could be very much more likely to buy insurance (k could be very large). If the market were to attain large size, say 50% of the population (which would be large under a non-compulsory system) there could not be much adverse selection, since high-risk genotypes could not be more than twice as likely to buy insurance. The potential for adverse selection also depends on the prevalence of genetic testing for the APOE gene. It is not clear how common it might become, but note that APOE is implicated in coronary heart disease, in which context screening might be more likely.

It is also most important to realise that the costs of adverse selection calculated are in respect of AD-related claims only. Since these account for perhaps 25% – 33% of total claims (Watson, 1998) and about 40% – 50% of costs (Watson, personal communication), the costs of adverse selection, as a percentage of the total LTC insurance premiums charged in the absence of adverse selection, should be about 1/4 – 1/3 of those shown here. I will return to this point in Section 2.6.3.

2.6.1 An Anomaly? Adverse Selection for Males

Before proceeding an apparent anomaly needs to be addressed. Using the fitted relative risks for men as estimated in Section 1.5, self-selection by those with one or two $\varepsilon 4$ alleles is not generally adverse. Table 2.24 gives the costs of 'adverse' selection in percentage terms; they are negligible at age 60, and negative at higher ages. The reason lies in a feature of the odds ratios from Farrer *et al.* (1997), discussed in Setion 1.5. For men, the odds ratio for the $\varepsilon 2/\varepsilon 4$ genotype peaks at only just over 1.0, at about age 70, then falls well below 1.0. The odds ratio for the $\varepsilon 3/\varepsilon 4$ genotype peaks at less than 2.0 at a slightly earlier age, then also falls below 1.0. This means that:

1. the $\varepsilon 3/\varepsilon 3$ genotype, which accounts for over 60% of the population at age 60, is high-risk at many ages, relative to the $\varepsilon 2/\varepsilon 4$ and $\varepsilon 3/\varepsilon 4$ genotypes;

2. at age 60, any increased costs caused by having a higher proportion of $\varepsilon 4/\varepsilon 4$ genotypes in the insured population are just about balanced by the decreased costs caused by having the same higher proportion of $\varepsilon 2/\varepsilon 4$ and $\varepsilon 3/\varepsilon 4$ genotypes; and

3. at higher ages, at which the population frequency of $\varepsilon 4/\varepsilon 4$ genotypes is reduced, a higher proportion of $\varepsilon 2/\varepsilon 4$ and $\varepsilon 3/\varepsilon 4$ genotypes in the insured population actually reduces costs.

The decision that must be made is whether to accept this as a genuine feature of the data, or to make adjustments so that the $\varepsilon 4$ allele is never low-risk. The data upon which the odds ratios were based were much scarcer for males than for females, especially at older ages. Therefore, more reliance should be placed on the results for females, and the relative risks for the $\varepsilon 3/\varepsilon 4$ and $\varepsilon 2/\varepsilon 4$ genotypes for males adjusted to remove the apparent anomaly; this can be done by simply imposing a lower limit of 1.0. Define:

$$\bar{f}^i_{x+t} = \max(f^i_{x+t}, 1) \tag{2.31}$$

where f^i_{x+t} is defined in equation (1.14). Accordingly, new adjustment factors, $\bar{r}_m$, can be calculated to maintain a consistent overall incidence of AD (see Section 1.5 and Table 1.8). The adjusted values are: $\bar{r}_1 = 1.12$ and $\bar{r}_{0.25} = 1.02$. These adjusted relative risk functions for males are used for the rest of this chapter.

2.6.2 The Cost of Adverse Selection based on Alzheimer's Disease Alone

All the costs in this section are expressed as percentages of AD care costs in the absence of adverse selection. These correspond directly to the increases in net single premiums, covering AD costs only, needed to absorb adverse selection.

Tables 2.25 to 2.28 give the percentage costs of adverse selection with high-risk lives 2, 4, 10 and 100 times more likely to buy LTC insurance; that is, for $k = 2, 4, 10$

Table 2.24: Costs of adverse selection, for males, as a percentage of AD-related LTC insurance costs in the absence of adverse selection, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes k times as likely to insure as low risk genotypes, for unit benefits increasing continuously ($\delta_b = 0.05$) and commencing w years before institutionalisation.

Increased likelihood of high risk genotypes insuring, k	Period of additional benefits w years	Proportion of relative risk, m	Cost of adverse selection at age: 60	65	70	75
			%	%	%	%
2	0	1.00	0.31	−0.56	−1.86	−3.48
		0.25	0.15	−0.04	−0.31	−0.63
	2	1.00	−0.29	−1.07	−2.25	−3.75
		0.25	0.03	−0.13	−0.37	−0.66
4	0	1.00	0.66	−1.20	−3.97	−7.47
		0.25	0.32	−0.08	−0.66	−1.34
	2	1.00	−0.61	−2.29	−4.82	−8.04
		0.25	0.06	−0.29	−0.80	−1.41
10	0	1.00	1.06	−1.92	−6.40	−12.08
		0.25	0.51	−0.13	−1.05	−2.15
	2	1.00	−0.99	−3.68	−7.76	−12.99
		0.25	0.10	−0.46	−1.28	−2.26
100	0	1.00	1.46	−2.66	−8.87	−16.79
		0.25	0.70	−0.18	−1.46	−2.97
	2	1.00	−1.36	−5.09	−10.75	−18.06
		0.25	0.14	−0.63	−1.76	−3.12

Table 2.25: Costs of adverse selection as a percentage of AD-related LTC insurance costs in the absence of adverse selection, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes twice as likely to insure as low-risk genotypes ($k = 2$), for unit benefits increasing continuously ($\delta_b = 0.05$) and commencing w years before institutionalisation.

Gender	Period of additional benefits w years	Proportion of relative risk, m	Cost of adverse selection at age: 60 %	65 %	70 %	75 %
F	0	1.00	9.86	9.23	8.10	6.46
		0.25	2.92	2.70	2.32	1.80
M	0	1.00	4.08	3.52	2.80	2.22
		0.25	0.95	0.81	0.62	0.46
F	1	1.00	9.28	8.70	7.64	6.10
		0.25	2.73	2.53	2.18	1.69
M	1	1.00	3.89	3.38	2.72	2.19
		0.25	0.90	0.77	0.59	0.45
F	2	1.00	8.91	8.35	7.33	5.87
		0.25	2.61	2.42	2.08	1.62
M	2	1.00	3.78	3.29	2.66	2.17
		0.25	0.87	0.74	0.58	0.44

and 100 respectively. The market size (the proportion z of the population buying insurance) is not mentioned in the tables, because it does not affect the percentage costs for a given value of k; it only limits the value k can take. For example, the costs of adverse selection given in Table 2.25, where high-risk genotypes are twice as likely to insure as low-risk genotypes ($k = 2$), could represent any of the following possibilities:

1. a modest level of adverse selection given market penetration of 1%; or

2. an upper limit for the costs of adverse selection given market penetration of 50%, and given that the prevalence of genetic testing in the population is 100%; or

3. an upper limit for the costs of adverse selection given market penetration of 10%, and given that the prevalence of genetic testing in the population is 20%.

From Tables 2.25 to 2.28 the greatest costs of adverse selection arise for younger ages at policy inception, under the assumption of high relative risk ($m = 1$) and

Table 2.26: Costs of adverse selection as a percentage of AD-related LTC insurance costs in the absence of adverse selection, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes 4 times as likely to insure as low-risk genotypes ($k = 4$), for unit benefits increasing continuously ($\delta_b = 0.05$) and commencing w years before institutionalisation.

Gender	Period of additional benefits w years	Proportion of relative risk, m	Cost of adverse selection at age: 60	65	70	75
			%	%	%	%
F	0	1.00	21.00	19.74	17.43	14.06
		0.25	6.22	5.76	4.96	3.86
M	0	1.00	8.68	7.50	5.98	4.77
		0.25	2.02	1.72	1.32	0.98
F	1	1.00	19.77	18.59	16.44	13.29
		0.25	5.81	5.39	4.65	3.63
M	1	1.00	8.30	7.21	5.81	4.69
		0.25	1.91	1.63	1.27	0.96
F	2	1.00	18.97	17.85	15.79	12.78
		0.25	5.55	5.15	4.45	3.48
M	2	1.00	8.05	7.02	5.70	4.64
		0.25	1.84	1.58	1.23	0.94

Table 2.27: Costs of adverse selection as a percentage of AD-related LTC insurance costs in the absence of adverse selection, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes 10 times as likely to insure as low-risk genotypes ($k = 10$), for unit benefits increasing continuously ($\delta_b = 0.05$) and commencing w years before institutionalisation.

Gender	Period of additional benefits w years	Proportion of relative risk, m	Cost of adverse selection at age: 60	65	70	75
			%	%	%	%
F	0	1.00	33.69	31.80	28.32	23.14
		0.25	9.97	9.26	7.99	6.23
M	0	1.00	13.93	12.06	9.63	7.70
		0.25	3.25	2.76	2.12	1.58
F	1	1.00	31.71	29.96	26.71	21.88
		0.25	9.32	8.66	7.49	5.87
M	1	1.00	13.31	11.59	9.36	7.58
		0.25	3.07	2.62	2.03	1.54
F	2	1.00	30.43	28.75	25.65	21.04
		0.25	8.90	8.28	7.17	5.62
M	2	1.00	12.91	11.29	9.18	7.50
		0.25	2.96	2.53	1.98	1.51

Table 2.28: Costs of adverse selection as a percentage of AD-related LTC insurance costs in the absence of adverse selection, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes 100 times as likely to insure as low-risk genotypes ($k = 100$), for unit benefits increasing continuously ($\delta_b = 0.05$) and commencing w years before institutionalisation.

Gender	Period of additional benefits w years	Proportion of relative risk, m	Cost of adverse selection at age: 60 %	65 %	70 %	75 %
F	0	1.00	46.44	44.03	39.54	32.76
		0.25	13.75	12.77	11.05	8.65
M	0	1.00	19.20	16.65	13.34	10.69
		0.25	4.48	3.80	2.92	2.18
F	1	1.00	43.72	41.48	37.30	30.98
		0.25	12.85	11.95	10.36	8.14
M	1	1.00	18.34	16.01	12.96	10.52
		0.25	4.23	3.61	2.80	2.12
F	2	1.00	41.95	39.81	35.82	29.79
		0.25	12.27	11.42	9.92	7.81
M	2	1.00	17.80	15.59	12.70	10.41
		0.25	4.08	3.49	2.73	2.08

for benefits commencing on institutionalisation ($w = 0$). For $k = 2, 4, 10$ and 100 the greatest costs of adverse selection are 9.86%, 21.00%, 32.69% and 46.44% for females and 4.08%, 8.68%, 13.93% and 19.20% for males respectively. The costs in respect of males are less than half of those in respect of females in all cases. These drop very considerably, by about 2/3, if the population relative risks are much lower those reported in case-based studies ($m = 0.25$). Other features are:

1. the costs of adverse selection are quite stable for ages 60–70 (decreasing slightly with age), but drop substantially at age 75 years;

2. the period of time spent claiming prior to institutionalisation (w) has very little effect and it is omitted from further analysis; and

3. as expected, the degree of adverse selection (equivalently, the size of the market) represented by k has a very large impact on the costs of adverse selection.

In Table 2.29, the costs of adverse selection are given for level benefits ($\delta_b = 0$) and for benefits increasing at a reduced rate ($\delta_b = 0.025$). For brevity, I only look at the

Table 2.29: Costs of adverse selection, for females, as a percentage of AD-related LTC insurance costs in the absence of adverse selection, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes k times as likely to insure as low risk genotypes, for level unit benefits and benefits increasing continuously ($\delta_b = 0$ and 0.025) and commencing on institutionalisation.

Claims inflation, δ_b	Likelihood of high risk genotypes insuring, k	Proportion of relative risk, m	Cost of adverse selection at age: 60	65	70	75
			%	%	%	%
0.025	2	1.00	12.32	11.34	9.70	7.51
		0.25	3.76	3.40	2.84	2.12
0.025	4	1.00	26.24	24.24	20.89	16.35
		0.25	8.00	7.26	6.06	4.55
0.025	10	1.00	42.10	39.05	33.93	26.91
		0.25	12.83	11.66	9.76	7.34
0.025	100	1.00	58.03	54.07	47.38	38.10
		0.25	17.69	16.09	13.50	10.20
0.000	2	1.00	14.90	13.52	11.34	8.57
		0.25	4.69	4.17	3.38	2.45
0.000	4	1.00	31.73	28.90	24.42	18.66
		0.25	9.98	8.88	7.23	5.25
0.000	10	1.00	50.90	46.56	39.67	30.71
		0.25	16.02	14.27	11.63	8.49
0.000	100	1.00	70.16	64.46	55.40	43.48
		0.25	22.08	19.69	16.09	11.78

costs for females. As expected from Section 2.5, a reduced rate of benefit escalation increases the percentage costs of adverse selection. This is because premature cases of AD cost relatively more if $\delta_b < \delta$. This increase in the percentage costs is roughly in proportion to the decrease in the rate of benefit escalation; at worst, it is about 5 percentage points per 1% decrease, but at the lower levels of relative risk ($m = 0.25$) less than 2 percentage points per 1% increase.

It should be remembered that these percentage costs are based on AD-related LTC insurance costs alone, and not on total LTC insurance costs. I defer any conclusions until total LTC costs are considered, in the next section.

2.6.3 The Costs of Adverse Selection Based on Total LTC Insurance Costs

The costs of adverse selection estimated so far are given as a percentage of AD costs only, so they overstate the true percentage costs as the LTC premiums actually charged will be higher to allow for other causes of claiming, such as loss of ADLs not linked to AD and other forms of cognitive impairment.

To estimate the total costs of LTC I take a simplistic approach and assume that the costs of AD make up a constant proportion of all costs. While this is unlikely to be true across ages, it does allow for estimates of the costs of adverse selection to be made in a way that at least allows for other causes of claiming.

Watson (1998) stated that 25% to 33% of all LTC insurance claims are attributable to AD. However, this refers to claim numbers and, based on LTC underwriting sources of four of the major underwriting companies at the time, AD was responsible for between 40% and 50% of all LTC claims expenses (Watson, personal correspondence). The reason AD patients have higher than average claim costs is because they have longer nursing home confinements, on average, than patients from other causes.

Using these figures total LTC insurance costs can be estimated as 2 to 2.5 times the cost of AD alone and, because of the methods used, this translates directly into the total percentage costs of adverse selection being 40% to 50% of the costs of adverse selection as a percentage of AD-related costs alone. Table 2.30 gives the range of costs of adverse selection as a percentage of total LTC costs for females, for the basic scenarios. Other costs can easily be calculated from the tables in the previous section.

From Table 2.30 I conclude the following:

1. In a large LTC insurance market, the costs of adverse selection will not exceed 5%, and are probably negligible.

2. If the risks conferred by the $\varepsilon 4$ allele in the whole population are much smaller than those observed to date ($m = 0.25$), then the same conclusion holds.

3. If the risks conferred by the $\varepsilon 4$ allele in the whole population are comparable

Table 2.30: Costs of adverse selection as a percentage of total LTC insurance costs, with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes k times as likely to insure as low-risk genotypes, for benefits increasing continuously ($\delta_b = 0.05$) and commencing on institutionalisation.

Gender	Likelihood of high risk genotypes insuring, k	Prop'n of relative risk, m	Cost of adverse selection at age: 60	65	70	75
			%	%	%	%
F	2	1.00	3.9 – 4.9	3.7 – 4.6	3.2 – 4.1	2.6 – 3.2
		0.25	1.2 – 1.5	1.1 – 1.4	0.9 – 1.2	0.7 – 0.9
M	2	1.00	1.6 – 2.0	1.4 – 1.8	1.1 – 1.4	0.9 – 1.1
		0.25	0.4 – 0.5	0.3 – 0.4	0.2 – 0.3	0.2 – 0.2
F	4	1.00	8.4 – 10.5	7.9 – 9.9	7.0 – 8.7	5.6 – 7.0
		0.25	2.5 – 3.1	2.3 – 2.9	2.0 – 2.5	1.5 – 1.9
M	4	1.00	3.5 – 4.3	3.0 – 3.8	2.4 – 3.0	1.9 – 2.4
		0.25	0.8 – 1.0	0.7 – 0.9	0.5 – 0.7	0.4 – 0.5
F	10	1.00	13.5 – 16.8	12.7 – 15.9	11.3 – 14.2	9.3 – 11.6
		0.25	4.0 – 5.0	3.7 – 4.6	3.2 – 4.0	2.5 – 3.1
M	10	1.00	5.6 – 7.0	4.8 – 6.0	3.9 – 4.8	3.1 – 3.9
		0.25	1.3 – 1.6	1.1 – 1.4	0.8 – 1.1	0.6 – 0.8
F	100	1.00	18.6 – 23.2	17.6 – 22.0	15.8 – 19.8	13.1 – 16.4
		0.25	5.5 – 6.9	5.1 – 6.4	4.4 – 5.5	3.5 – 4.3
M	100	1.00	7.7 – 9.6	6.7 – 8.3	5.3 – 6.7	4.3 – 5.3
		0.25	1.8 – 2.2	1.5 – 1.9	1.2 – 1.5	0.9 – 1.1

with those observed to date ($m = 1$) then costs of adverse selection begin to exceed 10% when high-risk genotypes are about 4 times more likely to buy insurance ($k = 4$). This would be feasible, though extreme, if about 25% or fewer of the population had LTC insurance; in a larger market, k would necessarily be less than 4.

4. With the same level of $\varepsilon 4$ risks in a smaller LTC insurance market, say less than 10% of the population, the cost of adverse selection could reach or exceed 20%. This would probably be regarded as a significant problem by most observers. From Table 2.14, it can be seen that extra single premiums of between +20% and +40% would then be needed (for females).

5. These levels of adverse selection assume that the whole population is tested for the APOE gene. This is unlikely, although as previously noted the gene is also a risk factor for coronary heart disease. Lower rates of genetic testing would cut the costs of adverse selection. If a proportion y of all genotypes were tested, the effect would be as if high-risk lives were ky times more likely to buy insurance. However, it is possible that higher proportions of lives with an $\varepsilon 4$ allele would be tested, for example the relatives of someone known to carry the allele might be more likely to be tested. This assumption is conservative.

6. It has been assumed that benefits increase at rate $\delta_b = 0.05$, which is also the force of interest δ. This eliminates the effect of the $\varepsilon 4$ allele in advancing the age of onset of AD; the benefit escalation cancels out the discounting. Lower rates of benefit escalation would increase the cost of adverse selection.

7. No conclusion is drawn in respect of males, because of the apparently anomalous relative risks discussed in Section 2.6.1.

2.7 The Cost of a Combined Pension and Long-Term Care Package

LTC insurance avoids the need to use savings to pay for care, for example by selling a house. It is needed most by those whose pensions are inadequate to pay for care.

Warren *et al.* (1999) compared the single premium costs of a pension and a LTC policy. Here I consider something different; under the assumption that any pension will be used to pay for care as needed, then the ideal, and cheapest, form of LTC insurance would be a top-up benefit, to pay for the excess of care costs over a pension.

In 1996–97, the average retirement pension in the U.K. was £3,254, which is certainly not sufficient to meet care costs. From discussions with LTC pricing actuaries, I took a typical sum assured under a LTC policy to be £9,600. Tables 2.31 to 2.34 show, for females and males, with high ($m = 1$) and low ($m = 0.25$) relative risks:

1. The cost at ages 60–75 of a pension starting at £3,254 per year, payable continuously and increasing continuously at rate $\delta_p = 0.03$ per year. These costs are found by using the model with the pension benefit payable while in any of the three live states.

2. the costs at ages 60–75 of a combined retirement package, providing the pension above throughout life, increased to a care benefit payable while institutionalised with AD of £9,600 per year at outset, increasing at rate $\delta_b = 0.05$ per year. I assume that care costs are linked to earnings inflation while pensions are linked to price inflation.

3. The additional cost of the LTC insurance 'top-up' as a percentage of the cost of the pension alone.

The cost of a comprehensive LTC 'top-up' would be rather higher than shown in these tables, for two reasons:

1. the LTC benefits in these tables are payable only in the event of AD; as before, this might account for 40% – 50% of the total;

2. the use of institutionalisation as a proxy for the start of a claim further underestimates the cost; see Tables 2.15 and 2.16.

Allowing for these together, the percentage extra costs might be three to four times those shown.

Table 2.31: Comparison of the EPV of a pension of £3,254 p.a. ($\delta_b = 0.03$) with the EPV of a package of a pension of £3,254 p.a. ($\delta_b = 0.03$) and a care benefit of £9,600 p.a. ($\delta_b = 0.05$) while institutionalised from AD, for females. High level of relative risk m= 1.

		Expected Present Value					
Scheme	Age	$\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	Average EPV
	60	60,812	63,087	65,774	62,956	66,314	65,178
Pension	65	53,571	55,213	57,665	54,967	58,219	57,086
only	70	46,759	47,583	49,547	47,508	50,086	49,126
	75	40,191	40,349	41,663	40,597	42,139	41,427
Pension	60	76,562	76,545	74,088	75,989	71,901	74,484
with LTC	65	68,150	68,155	65,881	67,527	63,734	66,147
benefit	70	59,680	59,758	57,651	59,065	55,518	57,855
while inst'd	75	51,197	51,478	49,614	50,647	47,460	49,721
	60	25.90 %	21.33 %	12.64 %	20.70 %	8.43 %	14.28 %
Percentage	65	27.21 %	23.44 %	14.25 %	22.85 %	9.47 %	15.87 %
increase	70	27.63 %	25.59 %	16.36 %	24.33 %	10.85 %	17.77 %
	75	27.38 %	27.58 %	19.09 %	24.76 %	12.63 %	20.02 %

Table 2.32: Comparison of the EPV of a pension of £3,254 p.a. ($\delta_b = 0.03$) with the EPV of a package of a pension of £3,254 p.a. ($\delta_b = 0.03$) and a care benefit of £9,600 p.a. ($\delta_b = 0.05$) while institutionalised from AD, for females. Low level of relative risk, $m = 0.25$.

		Expected Present Value					
Scheme	Age	$\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	Average EPV
	60	64,146	64,854	65,633	64,816	65,781	65,505
Pension	65	56,305	56,813	57,523	56,742	57,675	57,351
only	70	48,595	48,844	49,412	48,826	49,559	49,283
	75	41,123	41,168	41,545	41,244	41,677	41,470
Pension	60	75,276	75,229	74,488	75,045	74,013	74,685
with LTC	65	66,970	66,949	66,269	66,747	65,806	66,375
benefit	70	58,634	58,653	58,033	58,435	57,575	58,122
while inst'd	75	50,444	50,531	49,995	50,280	49,535	50,058
	60	17.35 %	16.00 %	13.49 %	15.78 %	12.51 %	14.01 %
Percentage	65	18.94 %	17.84 %	15.20 %	17.63 %	14.10 %	15.73 %
increase	70	20.66 %	20.08 %	17.45 %	19.68 %	16.17 %	17.94 %
	75	22.67 %	22.74 %	20.34 %	21.91 %	18.85 %	20.71 %

Table 2.33: Comparison of the EPV of a pension of £3,254 p.a. ($\delta_b = 0.03$) with the EPV of package of a pension of £3,254 p.a. ($\delta_b = 0.03$) and a care benefit of £9,600 p.a. ($\delta_b = 0.05$) while institutionalised from AD, for males. High level of relative risk $m = 1$.

		Expected Present Value					
Scheme	Age	$\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	Average EPV
	60	52,884	55,675	55,897	55,738	56,361	55,910
Pension	65	45,897	47,989	48,134	47,996	48,578	48,122
only	70	39,376	40,576	40,635	40,573	41,010	40,653
	75	33,138	33,622	33,625	33,624	33,861	33,650
Pension	60	63,943	61,476	61,335	61,463	58,434	61,085
with LTC	65	55,861	53,683	53,578	53,681	50,638	53,242
benefit	70	47,841	46,135	46,082	46,136	43,051	45,705
while inst'd	75	40,118	39,048	39,045	39,045	35,869	38,615
	60	20.91 %	10.42 %	9.73 %	10.27 %	3.68 %	9.26 %
Percentage	65	21.71 %	11.87 %	11.31 %	11.85 %	4.24 %	10.64 %
increase	70	21.50 %	13.70 %	13.40 %	13.71 %	4.98 %	12.43 %
	75	21.06 %	16.14 %	16.12 %	16.12 %	5.93 %	14.75 %

Table 2.34: Comparison of the EPV of a pension of £3,254 p.a. ($\delta_b = 0.03$) with the EPV of package of a pension of £3,254 p.a. ($\delta_b = 0.03$) and a care benefit of £9,600 p.a. ($\delta_b = 0.05$) while institutionalised from AD, for males. Low level of relative risk $m = 0.25$.

		Expected Present Value					
Scheme	Age	$\varepsilon4/\varepsilon4$	$\varepsilon3/\varepsilon4$	$\varepsilon3/\varepsilon3$	$\varepsilon2/\varepsilon4$	$\varepsilon2/\varepsilon2$ & $\varepsilon2/\varepsilon3$	Average EPV
	60	55,223	55,915	55,966	55,930	56,077	56,012
Pension	65	47,653	48,167	48,201	48,169	48,309	48,198
only	70	40,389	40,679	40,693	40,678	40,787	40,697
	75	33,551	33,665	33,665	33,665	33,731	33,672
Pension	60	61,749	61,077	61,044	61,074	60,480	61,049
with LTC	65	53,886	53,310	53,286	53,310	52,717	53,225
benefit	70	46,231	45,797	45,784	45,797	45,201	45,716
while inst'd	75	39,003	38,740	38,739	38,739	38,132	38,661
	60	11.82 %	9.23 %	9.07 %	9.20 %	7.85 %	8.99 %
Percentage	65	13.08 %	10.68 %	10.55 %	10.67 %	9.13 %	10.43 %
increase	70	14.46 %	12.58 %	12.51 %	12.58 %	10.82 %	12.33 %
	75	16.25 %	15.08 %	15.07 %	15.07 %	13.05 %	14.82 %

The level of relative risks affects the cost for each genotype, but not the average cost. More important, the cost is much less dependent on the APOE genotype than LTC insurance alone; pension costs are relatively insensitive to genotype, but what effect there is offsets the genetic risk under the LTC part of the contract, and reduces the differences in cost. Allowing for non-AD LTC claims as well will reduce the dependence even further. Even with a high level of relative risks ($m = 1$), the difference between the cost in respect of $\varepsilon4/\varepsilon4$ lives and the average cost is negligible, and so is the scope for adverse selection.

I have picked a particular combination of pension and LTC benefits, and the effect of combining them will clearly vary with the circumstances. However, it is clear that adverse selection can be much reduced if the same insurer underwrites both pension and LTC top-up.

2.8 Conclusions

1. The cost of AD-related LTC insurance depends strongly on the relative risks of the APOE genotypes. If these are consistent with observations to date, the costs for the $\varepsilon4/\varepsilon4$ genotype could be about 3 times those for the $\varepsilon2/\varepsilon2$ and $\varepsilon2/\varepsilon3$ genotypes. These are based on studies that selected at-risk subjects; the relative risks in the population could be lower. If they were 25% of those above, the above difference in cost would fall to about 1.5 times.

2. At low levels of relative risk, underwriting ratings, for females, could be up to +10% ($\varepsilon4/\varepsilon4$ genotype) or +7% ($\varepsilon3/\varepsilon4$ and $\varepsilon2/\varepsilon4$ genotypes). At high levels of relative risk these could increase to about +40% and +25% respectively. The model suggests higher ratings for male $\varepsilon4/\varepsilon4$ lives, and none at all for male $\varepsilon3/\varepsilon4$ lives, but these may be unreliable.

3. The costs are very sensitive to the background level of mortality, and to mortality while institutionalised. The latter is quite uncertain, and the different studies have given very different estimates; using these could change costs by almost ±40%. However, costs are not very sensitive to mortality after the onset of AD but before institutionalisation, which is also subject to great

uncertainty.

4. The model almost certainly underestimates AD-related costs, because institutionalisation is used as a proxy for the start of a claim. This affects the level of costs but not the cost of adverse selection. On the other hand, the use of a Gompertz formula for the rate of onset of AD at ages over 90 probably overstates costs.

5. The costs of adverse selection for males appear to be negative, because the $\varepsilon 2$ allele confers such protection that the $\varepsilon 3/\varepsilon 3$ genotype is high-risk at many ages. For the study of adverse selection only, I adjusted the relative risks so that the $\varepsilon 4$ allele never conferred lower risk, but no reliance is placed on these results.

6. The cost of adverse selection is only likely to be significant if:

   (a) the level of relative risks in the population is as high as observed to date;

   (b) the LTC insurance market is small;

   (c) carriers of the $\varepsilon 4$ allele are very much more likely to buy insurance (more than 4 times more likely); and

   (d) a high proportion of the population is tested for the APOE gene.

   In the worst-case scenario, adverse selection would increase costs, and single premiums, for females by 18%–23% at age 60, and 13%–16% at age 75. This is likely to be a substantial overestimate.

7. Adverse selection might sometimes be reduced by combining pension and 'top-up' LTC benefits into a comprehensive retirement package.

Chapter 3

Modelling Disability in Long-Term Care Insurance

3.1 Introduction

In Chapters 1 and 2 the main emphasis was on estimating the costs that arise under a long-term care insurance contract in respect of Alzheimer's disease. In these chapters, in order to look at the potential costs of adverse selection arising from variants of the APOE gene, the cost of other events in the ageing process (mainly disability) that trigger benefits were very simply assumed to be a multiple of those costs arising from Alzheimer's disease (see Chapter 2 for more detail).

The research in the next five chapters is motivated by wanting independently (from Alzheimer's disease) to estimate the costs of disability arising under a long-term care contract, providing a better basis for estimating the potential for adverse selection.

The aim of this chapter is to describe a continuous-time Markov model of the disability process and to describe the data I will use to estimate the model parameters, which are from the 1982, 1984, 1989 and 1994 National Long-Term Care Surveys (NLTCS) in the U.S.A.. Maximum likelihood estimates of the model parameters are discussed and obtained in Chapter 4. Then in Chapter 5, the methodology for calculating variance estimates of the transition intensities is discussed, and these variance estimates are used as weights in graduating the transition intensities. In Chapter 6,

aggregate mortality in the disability model is investigated and adjustments are made to some of the disability models. Finally, in Chapter 7, the parameterized models are used to calculate the expected present value (EPV) of model long-term care benefits in respect of disability, which together with the estimated costs of Alzheimer's disease (in Chapter 2) are used to revisit the potential costs of adverse selection in the long-term care insurance market arising from people's knowledge that they are a carrier of a high risk allele of the APOE gene (and therefore at higher risk of Alzheimer's disease).

In this chapter I concentrate on the disability process of ageing only. For a detailed discussion of long-term care insurance products in general see Section 2.2. The model presented here does not aim to be product specific, rather it is based on the underlying process, so it can be modified to many long-term care product designs. In Section 3.2 disability is defined in terms of Activities of Daily Living (ADLs), and a model of disability is proposed in Section 3.3. I then provide an overview of the four National Long-Term Care Surveys (NLTCS) in Section 3.4. Then, before looking at the data in more detail, previous research, relevant to this work, that has used these data sets is discussed in Section 3.5. I deal with some anomalies found in the data in Section 3.6, before providing a detailed discussion of each pair of consecutive surveys (1982–84, 1984–89 and 1989–94), as they are used, in Sections 3.7 – 3.9.

I have presented some of the research in this chapter to the RSC2001 conference in March 2001.

3.2 Activities of Daily Living (ADLs)

A typical set of ADLs is that used as a benchmark by the Association of British Insurers: Washing; Dressing; Mobility; Toiletting; Feeding; and Transferring. A typical LTC policy would pay benefits upon failure of a given number of ADLs, often 3 or 4. Sometimes a reduced benefit might be paid on failure of a smaller number of ADLs.

Failure of ADLs is not irreversible; the studies by Manton (1988) and Manton,

Corder & Stallard (1993) show quite high rates of recovery. Evidently, LTC insurance shares many of the features of Permanent Health Insurance (PHI) both in the difficulty of claims underwriting, and in statistical analysis.

An insurance policy that pays claims depending on a complex series of events is conveniently represented by a multiple state model, which is the approach adopted in Section 3.3. The order in which ADLs fail is then of potential importance, leading to the unpleasant possibility that, with 6 ADLs, $6! = 720$ states might be needed to represent all possibilities, even ignoring recoveries. Some studies have suggested that ADLs typically fail in a given order. Dullaway & Elliott (1998) stated, without mentioning a source, that ADLs usually fail in the order given above, and that recoveries are typically in reverse order. Dunlop, Hughes & Manheim (1997) suggest the following orders, based on a 1984–90 study with 5,151 subjects:

1. Mobility; Washing; Transferring; Dressing; Toiletting; Feeding; or
2. Mobility; Washing; Transferring; Toiletting; Dressing; Feeding.

These two lists agree on the first three ADLs to fail, and therefore about the events that would trigger a claim under a typical LTC policy. It is therefore a reasonable simplification to consider only the number of failing ADLs, and to ignore the order of failure.

3.3 A Model of Disability in Long-Term Care Insurance

Figure 3.19 shows a simple continuous time, discrete state model of disability, where disability is defined in terms of loss of ADLs (see Section 3.2). The reasons for grouping disability states in this way are discussed in Section 3.7. The states can easily be expanded using the same methodology if the data are available. Disability is a reversible process (see Section 4.2) and is modelled as such, though this does require the estimation of a large number of transition intensities. However, given sufficient data this is just a matter of 'number crunching'. Cognitive impairment is also covered by LTC insurance (discussed in Chapter 1), however only claims arising

from disability will be considered in this chapter. This will include a proportion of claims whose underlying cause is cognitive impairment — those lives impaired in a sufficient number of ADLs caused by cognitive impairment for them to be eligible for benefits, before being eligible from cognitive impairment itself.

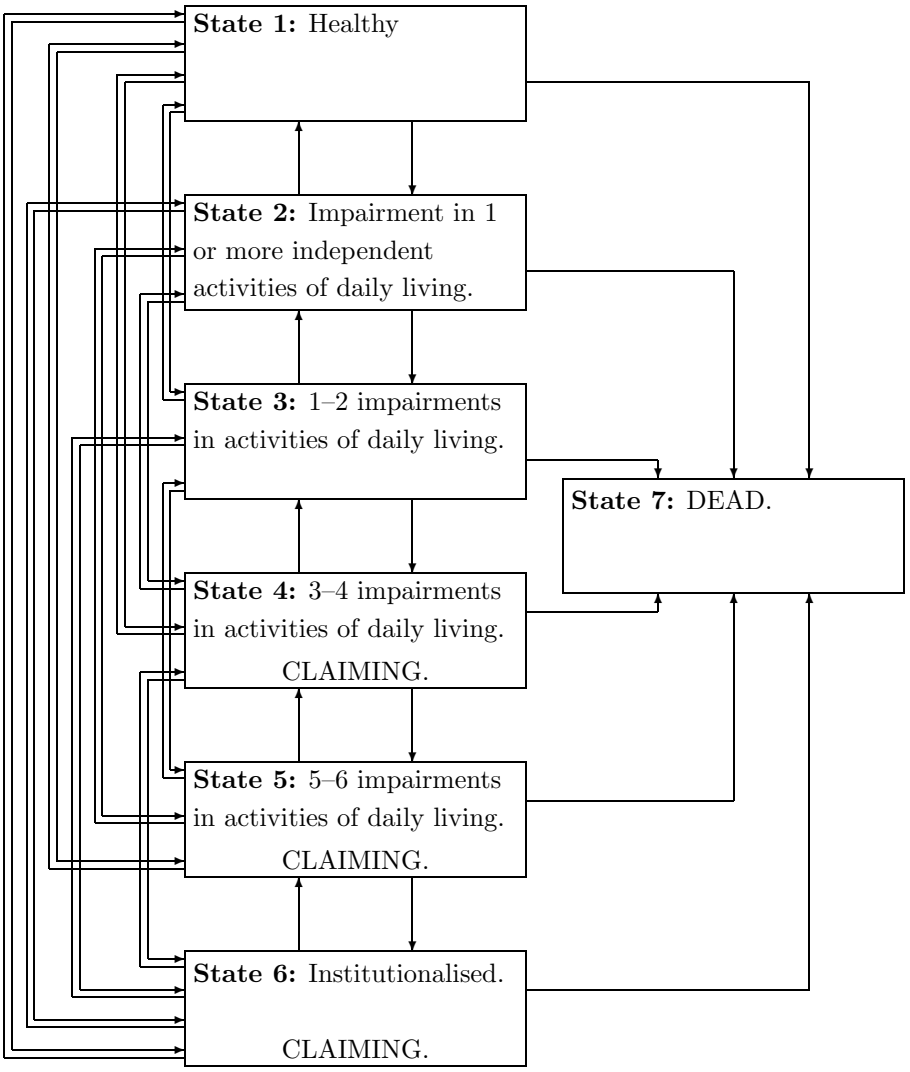

Figure 3.19: A model of disability for the lifetime of an individual and long term care insurance.

A comprehensive model of LTC costs would need to include all forms of cognitive impairment as well as disability. However, this would require data at the individual level, of progression through ADLs and all forms of cognitive impairment. Until such data becomes available (from demographic studies or as the experience of insured lives increases), such detailed models cannot be utilized. A major cause of claiming

in LTC insurance is onset of Alzheimer's disease — this was discussed in detail in Chapters 1 and 2.

3.4 Overview of the U.S. National Long-Term Care Surveys

The primary source of useful data on Activities of Daily Living (ADLs) is the series of 1982, 1984, 1989 and 1994 U.S. National Long-Term Care Surveys (NLTCS) based at Duke University. The 'raw' data sets were kindly made available to me by Professor Kenneth Manton at Duke University. The data, with separate keys, are in four main files:

1. an analysis file which provides summary data from all four surveys, though not in enough detail for the purposes of this analysis;

2. a file of raw data for the 1982 and 1984 surveys combined (the merging of these two years seems to have resulted in a loss of detailed information about the status of lives in 1984); and

3. two separate 'raw data' files for the 1989 and 1994 surveys.

The first five digits of each of these files is a unique identifier number that allows individual lives to be traced though all four data sets. All lives are accounted for in the analysis file in all survey years with lives not in a particular survey year classified as 'Not in Survey Year'. However, in the 'raw' data files, only those lives taking part in that particular survey are included.

These surveys are based on samples of Medicare enrollees aged over 65 years, representing more than 97% of the U.S. population (Manton, 1988). Medicare is a health insurance program for:

1. people aged 65 years or older;

2. some people with disability under age 65 years; and

3. people with end stage renal disease (permanent kidney failure requiring dialysis or a transplant).

Medicare has two parts: Part A: Hospital Insurance — most people pay for this through medicare taxes while they are working, though people not otherwise eligible can pay a monthly premium for cover; and Part B: Medical Insurance — for which most people pay a monthly premium. The sample for the NLTCS were taken from people covered under Part A.

In 1982, in order to identify approximately 6,000 chronically disabled persons, over 35,000 records were screened, also identifying over 1,900 institutionalised lives. Of these sampled lives, 25,541 showed no sign of disability. In 1984 and subsequent years, only a subset of 12,100 of these 25,541 lives were used. The other 13,441 lives were screened out of the surveys and there is no information about them. The cut-off date used for this screen was 1 April 1982, so that people were categorised as of this date. The 1984 NLTCS was designed to track changes in functional level and returns to the community, as were the 1989 and 1994 surveys. They are longitudinal in the sense that they follow the same cohort of lives, but they only report 'snapshot' views of the cohort at fixed times — they do not continuously track the cohort, but individual lives are traceable between each pair of surveys. Therefore, complete life histories are not available.

All four surveys had a two stage design — first a community screener questionnaire was adminstered to the selected lives to determine if they had any physical disability. Then, in general, if they showed any signs of disability a community detail questionnaire was undertaken at a later date to clarify the type and level of disability as well as many other personal factors. In 1982, if no disability was identified during the screener questionnaire the person was not interviewed further. Once a life has been given a detailed interview (including an institutional questionnaire), they were given detailed interviews in all later surveys until death. The specifics of each individual survey will be discussed further in Sections 3.7 – 3.9.

The 1984, 1989 and 1994 surveys incorporated aged-in samples to maintain adequate coverage of the U.S. population aged over 65 years. For example, the sample used in 1982 who were over 65 years old would be over 67 years old in 1984, so in order to maintain a sample of over–65 year old lives in 1984 a new sample, the aged-in population, of people aged 65–67 year old in 1984 was combined with the

original sample.

The same measures of disability were used in all years and were defined by the inability to perform one or more of 8 Independent Activities of Daily Living (IADLs — light housework; laundry; meal preparation; grocery shopping; getting around outside; getting to places outside within walking distance; money management; using the telephone) or one or more of 6 ADLs (eating; getting in and out of bed; getting around inside; dressing; bathing; getting to the bathroom or using the toilet) without using personal assistance or special equipment. To be classified as chronically disabled in respect of any of the ADLs or IADLs the person would have to be unable to (or be expected to be unable to) perform the activity for more than 90 days because of disability or by reason of health. This may be taken to be a proxy for a deferred period under a LTC policy (the period during which the claimant must continuously meet the claims criteria before payments commence). In the U.K. the most common deferred period is 3 months, or approximately 90 days (Dullaway & Elliot, 1998).

Figure 3.20 is an overview of the four surveys, with the total number of people in each classification given and, where the figures are in brackets, the number of these lives from the aged-in population that are joining the survey for the first time. Transitions between all the classifications in Figure 3.20 are given in Appendix A. The surveys include a total of 35,848 lives, though not all are used in any one survey. The following classifications have been included to account for all lives in all survey years:

1. 'Aged-in population' — these are lives that are not over age 65 years in the current survey year, but will be included in future survey years; and

2. 'Not in Survey Year' — these are lives that have previously died, as well as other lives that have, somewhat ambiguously, been left out of the survey year. (I assume that there is no systematic reason why these lives have been left out.)

The large number of lives in the classification 'Not in Survey Year' in the 1989 (9,524 lives) and 1994 (12,730 lives) NLTCS that are not accounted for (i.e. not

classified as dead in a previous survey) raises the question: Is there any evidence in the data to suggest what may have happened to these lives? It will become apparent in Section 4.2, when looking at overall trends in the data, that the number of deaths recorded between the 1984 and 1989 NLTCS and between the 1989 and 1994 NLTCS is substantially understated when compared with the number between the 1982 and 1984 NLTCS, which suggests that a substantial proportion of the lives unaccounted for, were dead. There is further support for this in Chapter 6, when comparing overall mortality in the parameterized disability models with a benchmark force of mortality: for the models parameterized using the 1982 and 1984 NLTCS, overall model mortality is comparable to the benchmark; whereas the overall forces of mortality for the models parameterized using the 1984–89 NLTCs and the 1989–94 NLTCS are substantially below the benchmark (see Chapter 6 for more details).

The 'Non-response dead' classification has been used in Figure 3.20 in two slightly different ways. In 1982, the first survey, people who were alive as of the cut-off date (1 April 1982), but died before the screener questionnaire took place were classified as 'Community screener only — Non-response dead' (classification **B.**). It is not known if these lives had any disability or not. Lives that were given the screener in 1982, but died before answering the detailed questionnaire are classified as 'Community detail questionnaire - Non-response dead' (classification **E.**). It is known that these lives may have had some form of disability (as they had been chosen for the detailed interview) though they need not necessarily have been disabled in accordance with the definitions used. (Some lives that completed a detailed questionnaire were found to be non-disabled.)

In 1984, a 'deceased' questionnaire was administered to the next of kin, so lives that died between 1982 and 1984 were classified as 'dead', and there is no 'Non-response dead' classification. However, in 1989 and 1994, there was no such survey and lives that died between surveys were classified as 'Non-response dead' in the questionnaire in which they were found to have died. So, for example, between 1984 and 1989, 728 (529 + 131 + 68) lives died, of which 49 (38 + 11 + 0) lives were from the aged-in population.

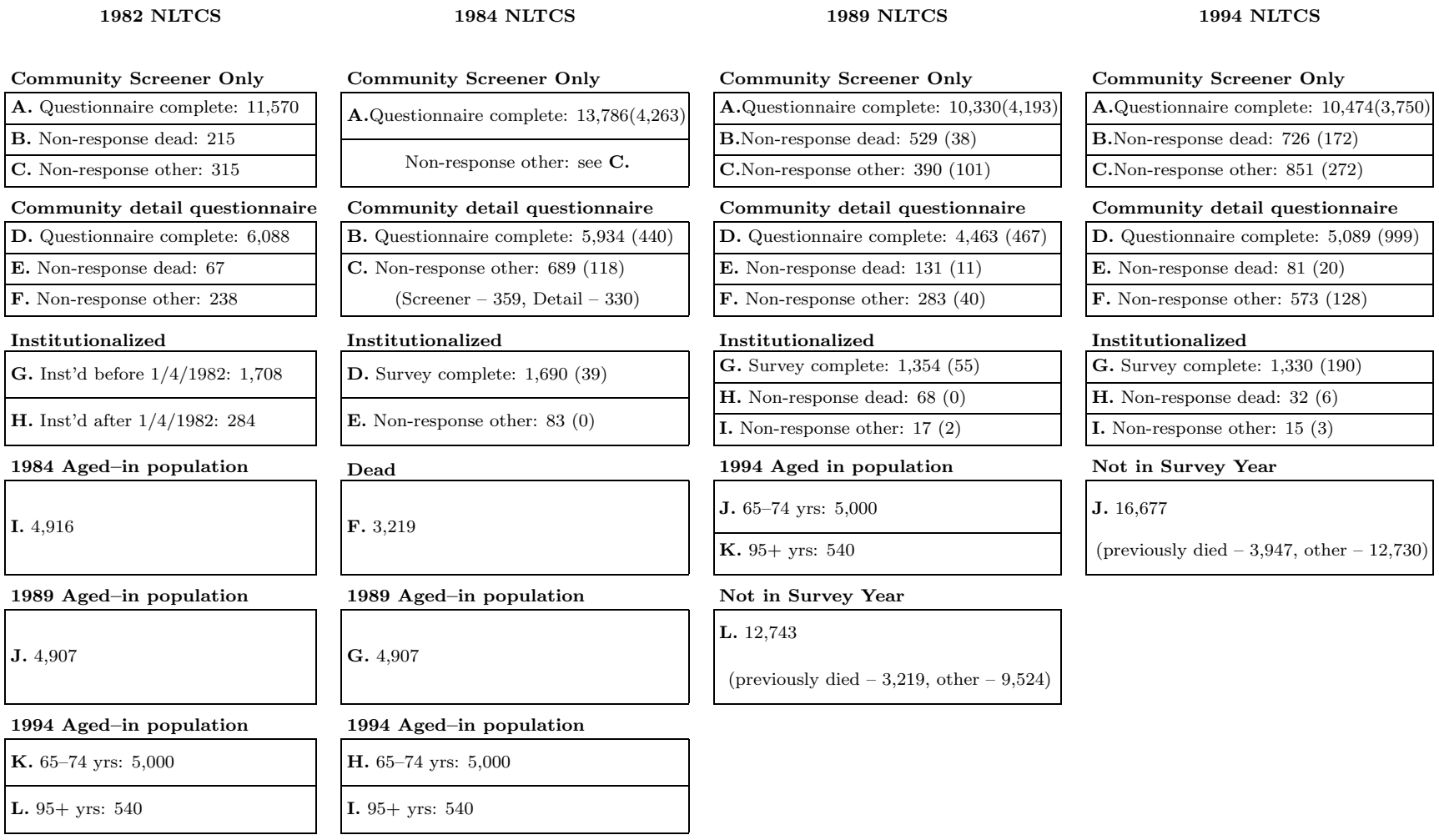

Figure 3.20: An overview of the 1982, 1984, 1989 and 1994 U.S. National Long-Term Care Surveys.

The '1984 aged-in' group (4,916 lives) are lives that were aged 63–65 years in 1982, and were added to the survey in 1984, when they were aged 65–67 years to maintain coverage of the over 65 years old population. Similarly, the '1989 aged-in' group (4,907 lives) were added to the survey in 1989 when the lives were aged 65-70 years old. In 1994, two groups of lives were added to the survey — the usual aged in sample of lives aged 65–70 years in 1994 (5,000 lives), as well as a group of lives then aged over 95 years (540 lives) to supplement the sample of old age lives, which diminished over time because of mortality. (Strictly speaking these are not aged-in lives, but a supplementary sample to ensure adequate coverage of the entire over 65 years old population.) However, neither of these groups are used in this work as either their status is unknown in 1989 (for the 95+ years old group) or they are too young to be included in the 1989 survey (the group aged 65-74 year olds in 1994). I include them here for completeness only.

In 1982, lives that were found to be in institutions were split into those that had been institutionalised before or after the cut-off date of 1 April 1982. This is because these two groups represent different types of institutionalised lives. Those institutionalised after 1 April 1982 include lives that had been admitted for acute medical reasons who may quickly recover (for example, lives admitted from physical injury) or die relatively soon (for example, lives admitted towards the end of a fatal disease). Those admitted before 1 April 1982, represent the 'stayers', the group left after lives with acute medical conditions have either stabilised in their condition or left the institutions. This is discussed in more detail in Section 3.7.

These data have been used before by other researchers to investigate trends of disability. Previous research, relevant to this work, is summarized in the next section and the differences between their work and that presented here are highlighted before the data are discussed in more detail in Section 3.6.

3.5 Previous Research Using the NLTCS

These data have already been used for research into trends of disability. Papers by Manton (1988) and Manton, Corder & Stallard (1993) are of particular interest

as they look at transition probabilities between disability levels. In both of these studies disability was grouped into 6 categories: Healthy, loss of IADLs only, loss of 1–2 ADLs, loss of 3–4 ADLs, loss of 5–6 ADLs and Institutionalised (Inst'd).

Manton (1988) studied changes in functional level between 1982 and 1984 producing 2-year adjusted (for mortality and institutionalisation) and unadjusted transition probabilities between disability levels, stratified by gender and age (grouped into: 65–74 years, 75–84 years and 85+ years as of 1 April 1982). His aim was to look at trends of disability in the general U.S. population using longitudinal data.

Manton, Corder & Stallard (1993) studied the secular trends in changes in functional level between 1982, 1984 and 1989, providing 2-year and 5-year transition probabilities for changes in functional level, stratified by age (as above), but not by gender. This paper looked at changes in trends of disability over time by comparing transition probabilities between 1982–1984 and 1984–1989. I look at it in more detail as it raises some interesting methodological questions relevant to this research.

After adjusting for censored data, Manton (1988) and Manton, Corder & Stallard (1993) calculated maximum likelihood estimates of the t-year probabilities of a person aged x moving from state i to state j ($P^{i\ j}_{x\ x+t}$) using equation (3.32)

$$P^{i\ j}_{x\ x+t} = \frac{n^{i\,j}_{x\ x+t}}{\sum_j n^{i\,j}_{x\ x+t}} \tag{3.32}$$

where, for example in the case of the 1982 and 1984 NLTCS, $n^{i\,j}_{x\ x+t}$ are the number of people in state i, aged x in 1982 and in state j, aged $x+2$ in 1984. More specifically, lives are grouped by their age in 1982 and this cohort of lives is then followed to 1984, when they are 2 years older.

To compare transition probabilities between the two time periods 1982–1984 and 1984–1989, it is necessary to transform the 5-year transition probabilities (from 1984–1989) into 2-year transition probabilities, or vice versa. Assuming constant transition intensities, 2-year transition probabilities can be calculated by raising the matrix of 5-year transition probabilities to the power 2/5 (or 5-year transition probabilities, by raising the matrix of 2-year transition probabilities to the power 5/2). However, some of the 2-year transition probabilities calculated in this way are

negative. A simple solution to this is to set any negative probabilities to zero, as in Manton, Corder & Stallard (1993). However, this presents more difficulty if it is the transition intensities that are of interest because:

1. in a multiple state model in which all non-dead states commute, if one transition probability between two non-dead states is zero, then all transition probabilities are necessarily zero; and

2. it implies that at least some of the transition intensities calculated from these data will be negative or complex, as a set of positive transition intensities cannot produce negative probabilities.

This problem arises because not all discrete-time homogeneous Markov chains have transition probabilities consistent with continuous-time Markov processes. In fact, it would be slightly surprising if real data led to a set of transition probabilities that were consistent in this way.

The difference between this work and the research described above is that I aim to:

1. model the underlying disability process by estimating the transition intensities of disability, which are needed for actuarial applications (discussed in more detail in Chapter 1); and

2. set out a consistent methodology for dealing with such data, that would be of use for modelling purposes and for the calculation of transition probabilities.

Before looking at the data in more detail, some anomalies in the data are highlighted and discussed in the next section.

3.6 Anomalies in the Data

As is clear from the tables of transitions between the classifications of Figure 3.20 (given in Appendix A), some of the data are misclassified in some years. Not all of the lives that are classified as 'Community screener only — Non-response dead' in 1982 and 1989 are classified as dead in the 1984 and 1994 surveys, respectively. I look at these two years next.

3.6.1 Anomalies in the 1982 NLTCS

There are 12 lives classified as 'Community screener only — non-response dead' in 1982 that are not classified as 'Dead' in 1984: 6 lives as 'Community screener only — complete'; 4 lives as 'Community detail questionnaire — non-response other'; and 2 lives as 'Institutionalised — questionnaire complete'.

To aid in reclassifying these lives I assume that if a given life has completed a questionnaire (or at least, that there are entries in the data file for the individual in question) then the person is alive in that survey year. So, of the 8 lives that have been classified as having completed a questionnaire in 1984 all 8 have data entries in the detailed surveys in 1984 (6 have answers to the community detail questionnaire and 2 have answers to the institutionalised questionnaire). As there is no information about these lives in 1982 (as they did not even answer the screener interview) I reclassify these 8 lives as 'Community screener only — non-response other' in 1982. This adjustment only affects the 1982 survey.

The four lives classified as 'Community detail questionnaire — non-response other' in 1984 are more difficult to reclassify as there is no information about them in 1984. Futhermore, they are classified as 'Not in survey year' in 1989 and 1994, providing no more information about them. As it is not clear whether these lives were misclassified in 1982 or 1984 and as they are effectively censored observations in all four survey years, providing little extra information, it seems reasonable to exclude them from all surveys.

This adjustment affects all survey years in the following way:

1. in 1982, 4 lives are removed from the 'Community screener only — non-response dead' classification;

2. in 1984, 4 lives are removed from the 'Community detail — non-response other' classification;

3. in 1989 and 1994, 4 lives are removed from the 'Not in survey year' classification; and

4. the total number of lives in each survey year is reduced to 35,844.

I now look at the anomalies in the 1989 NLTCS, which are dealt with in a similar fashion.

3.6.2 Anomalies in the 1989 NLTCS

In the 1989 survey there are 44 lives classified as 'Community screener only — non-response dead' that are not classified as dead (i.e. in the classification 'Non-response dead' in the community screener, the community detail or the institutionalised questionnaire) or as 'Not in survey year' in 1994.

Of these 44 lives, 25 answered questionnaires (19 lives have data entries for the screener interview and 6 have data entries for the institutionalised questionnaire). Using the same reasoning as in Section 3.6.1, I reclassify these lives as 'Community screener only — non-response other' in 1989. Seven of these lives are part of the '1989 aged-in population', which has to be allowed for when looking at the surveys in more detail, as in Section 3.8. This adjustment only affects the 1989 survey.

There are 16 non-responders in 1994 (15 from the screener only group and 1 from the community detail group) from the 19 lives not yet accounted for. The other 3 lives, although classified as having completed the screener questionnaire, have no such data entries and so I treat them as if they were non-responders. It seems reasonable to assume (or without loss of information) that these 19 lives did actually die prior to the 1989 survey, are correctly classified in 1989 and are, accordingly, taken out of the 1994 survey (reclassified as 'Not in survey year' in 1994). This adjustment only affects the 1994 survey.

These adjustments were made before looking in more detail at the individual surveys as some of these adjustments affect several survey years and once the adjustments are made to the raw data, they can easily be incorporated in any futher analysis.

3.7 Details of the 1982 and 1984 NLTCS

Table 3.35 shows the 1982 and 1984 NLTCS in more detail, after all the adjustments in Section 3.6 have been carried out and some the classifications merged. I first

Table 3.35: Transitions between disability states in the 1982 and 1984 National Long-Term Care Surveys, unadjusted for censored data.

1982 Status	1984 Status							
	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd[1]	Dead	N-R[2]
Healthy	9866	470	391	116	111	166	773	213
IADL only	274	541	335	80	80	100	261	58
1–2 ADLs	115	220	660	232	135	151	389	76
3–4 ADLs	16	32	145	194	161	81	197	25
5–6 ADLs	16	35	65	89	298	96	363	32
Inst'd before 1/4/82	12	10	9	15	14	953	688	7
1982 detail non-responders	20	12	23	16	21	44	123	46
Inst'd after 1/4/82	8	9	10	6	7	118	122	4
N–R[3]	121	9	7	4	7	25	247	106
Total	10448	1338	1645	752	834	1734	3163	567

(1) Including institutionalized non-responders.
(2) These are non-responders of the screener and the detail questionnaire.
(3) These are non-responders of the screener only, including lives that died after 1 April 1982 but before the screener took place.

clarify what the states/classifications (I refer to lives as being in states from now on as the level of disability of lives is introduced) used in Table 3.35 are in more detail, then discuss how the censored data are dealt with, and finally provide a table of data after adjusting for censored lives.

All lives that completed a screener only are healthy lives (as they were not chosen for a detailed questionnaire), lives that completed a detailed questionnaire are classified in the analysis data file as: Healthy (not disabled), IADL only, 1–2 ADLs, 3–4 ADLs or 5–6 ADLs. So the 'healthy' state consists of some lives that completed the detail questionnaire and all of those that completed the screener only. I grouped disability into the same 6 categories as did Manton (1998) and Manton, Corder & Stallard (1993) (see Section 3.5). I chose the same set of groupings because:

1. grouping disability in this way had desirable features — this grouping was predictive of mortality and disability according to Manton (1988);

2. the groups were not so small as to reduce their credibility, and gave a reasonable number of groups to model; and

3. the raw data were already classified into these groupings, making data extraction easier.

There are many other ways and instruments to classify disability; however the use of ADLs is consistent with methods used by insurance companies (see Section 3.2). For more information on the classification of disability see Manton (1988).

For more specific modelling it may be necessary to split some of these states further, providing the data are available. For example, to model a product that pays out half the sum assured on the loss of 2 ADLs and the full sum assured after the loss of three or more ADLs, the 1–2 ADL group would need to be split into two separate groups. However, this grouping is sufficient for this research — to model contracts that pay out the full sum assured on the loss of 3 or more ADLs.

Looking at the 1982 survey first, for the same reasons as in Manton (1988), two groups of lives were classified separately, as their experiences were unlike those of any other groups:

1. non-responders in 1982 had a poor disability experience and higher mortality which can be explained by disability or poor health being a reason for non-response; and

2. people institutionalised after 1 April 1982, but before the actual survey took place, had very high mortality rates and high recovery rates, which may be expected from admissions to institutions for acute medical reasons (as discussed in Section 3.4).

For these reasons, these two groups of lives are left out of any further analysis in the 1982 and 1984 surveys (though they are included in the later surveys). Lives in the 1984 aged-in group are also excluded here as they are too young (<65 years old) and their status is unknown in 1982.

In 1982, the categories 'Screener only — non-response other' and 'Screener only — non-response dead' have been merged, as all lives in this second classification died

after 1 April 1982 but before the survey took place, so to leave them out would be to understate mortality. As these lives did not answer a questionnaire it is not known whether they were disabled or not, thus it seems reasonable to distribute them *pro-rata* to states covered by the questionnaires (excluding the lives institutionalised after 1 April 1982), using as weights both lives that were only given a screener and those selected for a detail questionnaire (as they could have been either).

In 1984 it is not possible to distinguish between non-responders to the screener only and non-responders to the detail questionnaire (as the data are not available). So these two groups of censored lives are dealt with together from necessity. They are distributed to states in 1984 in the same way as non-responders to the screener only were dealt with in 1982. This is because this joint group contains lives that did not answer the screener and thus could be in either group (screener only or community detail). A better way to distribute lives that were censored on the detail questionnaire (and thus had completed the screener questionnaire), were it possible to distinguish between these two groups, would be to distribute them pro-rata over disability states using as weights only those lives that answered the detail questionnaire (i.e. excluding the non-disabled lives that completed the screener only). Then censored lives that only completed the screener could be distributed as the 1982 censored lives were.

In redistributing the non-responders in 1982 and 1984 all the data were used (i.e. over both genders and all ages) to calculate the proportions used to allocate the lives. This assumes that non-responders do not differ in their level of disability with respect to age or gender. An alternative method would be only to use lives within the age group of the censored lives in the re-allocation. However, as it is not clear whether non-responders would differ in their level of disability by age or gender, using all the data provides a more robust method as more lives are used.

The data after the above adjustments for censored lives are summarized, for all age groups and both genders, in Table 3.36. The same data stratified by gender and age group (see below) are given in Appendix B. The age groupings I chose to look at are:

1. 10-year age bands (65–74, 75–84 and 85+ years) — to allow comparison with

Table 3.36: Transitions between disability states in the 1982 and 1984 National Long-Term Care Surveys, adjusted for censored data.

	1984 Status						
1982 Status	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	10129.12	499.07	423.91	131.46	129.78	210.18	941.67
IADL only	326.96	547.18	342.23	83.34	83.90	108.55	282.39
1–2 ADLs	180.54	227.73	669.11	236.20	139.86	161.48	411.95
3–4 ADLs	39.07	34.68	148.14	195.45	162.70	84.73	206.54
5–6 ADLs	43.16	38.21	68.80	90.75	300.02	100.33	372.09
Inst'd[(1)]	28.12	11.65	10.78	15.85	15.10	955.85	703.37
Total	10746.97	1358.52	1662.97	753.05	831.36	1621.12	2918.00

(1) These are lives that were institutionalized before 1 April 1982.

the work done by the previous researchers; and

2. 5-year age bands (65–69, 70–74, 75–79, 80–84 and 85+ years) — to get more point estimates, which may help highlight any patterns in the data with age.

3.8 Details of the 1984 and 1989 NLTCS

Table 3.37 shows the 1984 and 1989 NLTCS in more detail, after the adjustments for anomolous data have been made and some of the classifications merged/renamed. This pair of surveys includes all the lives in the 1982 survey that survived to 1984, including those lives that were excluded from the 1982 and 1984 analysis (the 1982 detail non-responders and those persons institutionalised after 1 April 1982), as well as the 1984 aged-in population (4,916 lives) that survived to 1984 (4,860 lives).

In both survey years the institutionalised state includes lives that did not respond to the institutional questionnaire, as they were identified as living in an institution. There is only one institutionalised state in 1984 as the survey does not provide enough information to classify them as institutionalised before or after the cut-off date of 1 April 1984. It may then be that lives institutionalised in 1984 have higher mortality and recovery rates than those in 1982, as these were exactly the features of the lives left out of the 1982 institutionalised population — lives institutionalised after the cut-off date, but before the survey took place (see Section 3.7 for more detail).

Table 3.37: Transitions between disability states in the 1984 and 1989 National Long-Term Care Surveys, unadjusted for censored data.

	1989 Status								
1984 Status	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd[1]	Dead	N-R[2] A	N-R[2] B
Healthy	6581	416	465	194	159	336	271	171	109
IADL only	157	219	228	91	78	120	79	21	37
1–2 ADLs	85	91	350	182	102	175	78	28	40
3–4 ADLs	10	17	77	122	70	86	54	17	13
5–6 ADLs	10	12	43	46	117	73	38	6	4
Inst'd[1]	4	4	7	3	4	492	112	10	6
N-R other[3]	106	18	24	26	15	32	29	54	34
Total	6953	777	1194	664	545	1314	661	307	243

(1) Including institutionalized non-responders.
(2) A: Non-response other to the screener questionnaire, B: Non-response other to the detail questionnaire.
(3) These are non-responders of the screener and the detail questionnaire.

The dead state in 1989 includes the total number of lives classified as non-response dead in any questionnaire in 1989. I aggregated these lives as the survey in which a life is found to be deceased does not provide any further useful information.

In the 1982 survey, lives that did not respond to the detail questionnaire were left in a separate classification and excluded from the analysis due to their very high mortality (see Section 3.7 for more detail). In 1984, it is not possible to distinguish (the data are not available) between non-responders of the screener only and those of the detail questionnaire. So it is not possible to see if this is also the case with the 1984 detail non-responders — both of these groups of lives have to be dealt with together. However, in 1989 it is possible to assign these non-responders to either the screener only or the detail questionnaire, and thus when redistributing them, to use slightly different weights to allow for the survey to which they did not respond. Even so, it is not possible to look at the mortality experience of the 1989 detail non-responders as they are all left out of the 1994 survey (classified as 'Not in survey year'). They may have been left out of the 1994 survey exactly because they were detail non-responders in 1989, although there is no further evidence to support this.

Non-responders to the screener and detail questionnaire in 1984 are dealt with

Table 3.38: Transitions between disability states in the 1984 and 1989 National Long-Term Care Surveys, adjusted for censored data.

	1989 Status						
1984 Status	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	6805.15	461.29	532.09	239.96	192.39	413.65	292.06
IADL only	187.48	228.42	242.22	99.78	84.77	136.04	81.31
1–2 ADLs	121.44	101.6	366.01	191.86	109.62	193.05	80.54
3–4 ADLs	27.49	21.16	83.28	125.87	72.99	93.08	54.99
5–6 ADLs	18.31	13.86	45.77	47.86	118.37	76.19	38.78
Inst'd	17.91	7.04	11.52	6.05	6.23	497.21	113.33
Total	7177.78	833.37	1280.89	711.38	584.36	1409.21	661.00

in the same way as those in 1982 — they are allocated pro-rata to the other states in 1984 using as weights all the other lives (i.e. including non-disabled lives that only took the screener). This may understate levels of disability among these lives, as some of them were scheduled for the detailed questionnaire, an indication that they may have been disabled, but they are allocated using all lives — including the non-disabled lives that only took the screener.

In 1989, the censored lives are classified by the survey to which they did not respond. The non-responders to the screener only were distributed in the same way as the 1984 non-responders, as they may or may not have been asked to take a detail questionnaire. For the detail non-responders, it is known that they were asked to take a detailed questionnaire, so when allocating these lives pro-rata to states in 1989, the non-disabled screener only group is excluded before allocating these lives.

For the same reasons as in Section 3.7, when calculating the proportions for allocating non-responders to disability states all the data were used (over all ages and both genders).

The data after the above adjustments for censored lives are summarized, for all age groups and both genders, in Table 3.38. The same data stratified by gender and age group (10-year and 5-year age bands, with an upper age group of lives aged over 85 years) are given in Appendix C.

Table 3.39: Transitions between disability states in the 1989 and 1994 National Long-Term Care Surveys, unadjusted for censored data.

	1994 Status								
1989 Status	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd[1]	Dead	N-R[2] A	N-R[2] B
Healthy	5189	349	462	180	163	370	272	246	236
IADL only	96	110	113	61	41	64	46	38	24
1–2 ADLs	66	59	181	138	78	136	69	42	31
3–4 ADLs	14	13	41	70	58	88	48	22	19
5–6 ADLs	4	3	11	19	70	47	31	15	8
Inst'd[1]	1	2	5	3	2	291	55	21	15
N-R other[3]	109	11	9	12	13	32	11	46	6
Total	5479	547	822	483	425	1028	532	430	339

(1) Including institutionalized non-responders.
(2) A: Non-response other to the screener questionnaire, B: Non-response other to the detail questionnaire.
(3) These are non-responders of the screener only.

3.9 Details of the 1989 and 1994 NLTCS

Table 3.39 shows the 1989 and 1994 NLTCS in more detail after the adjustments for anomolous data have been made and some of the classifications merged/renamed. This pair of surveys includes all lives that are in the 1984 survey, that survived to 1989 and are not classified as 'Not in survey year' in either 1989 or 1994. Also included in this pair of surveys is the 1989 aged-in population (4,907 lives) that survived to 1989 (4,858 lives). The details of the institutionalised state (in 1989 and 1994) and the dead state (in 1994) are the same as in Section 3.8.

An additional feature of this pair of surveys is that a supplementary sample of 922 lives that would otherwise have been scheduled for the screener interview only in 1994 were also given a detailed questionnaire to improve the description of non-disabled persons (because of other factors of interest in the survey, such as use of Medicare-funded health services). For the purpose of redistributing censored cases these 922 lives are classified as screener only, as they would not have been asked to take a detailed questionnaire as part of the normal screening procedure.

As already observed in Section 3.8 all non-responders of the detailed questionnaire are excluded from the 1994 survey (classified as 'not in survey year') and so are left out of this analysis. The reason why they have been excluded from the 1994 survey

Table 3.40: Transitions between disability states in the 1989 and 1994 National Long-Term Care Surveys, adjusted for censored data.

	1994 Status						
1989 Status	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	5491.93	407.93	545.39	233.61	211.85	488.53	280.59
IADL only	131.82	116.54	122.49	66.88	46.29	76.81	46.62
1–2 ADLs	108.36	67.07	192.65	145.27	84.55	151.87	69.84
3–4 ADLs	35.68	17.49	47.56	74.03	61.60	96.73	48.36
5–6 ADLs	17.27	5.31	14.37	21.07	71.86	51.50	31.20
Inst'd	21.69	5.92	10.67	6.53	5.17	298.69	55.39
Total	5806.75	620.26	933.13	547.39	481.32	1164.13	532.00

is not clear, whether by sampling or selection. This may have been a fairly disabled group of lives, as disability/illness is often a reason for non-response (see Section 3.7) and it is unfortunate that their experience is lost.

This leaves only the non-responders of the screen in 1989, which I redistribute in the same way as the non-responders in 1984 — they are allocated pro-rata over all disability states using all lives as weights. Including in the weights lives that only answered the screener is justified here as these lives may have been either screener only or have been disabled enough to be asked to complete a detailed questionnaire.

In 1994, the non-responders are classified by the survey to which they did not respond — exactly the same as the 1989 non-responders in the 1984–89 analysis. So, these are dealt with in exactly the same way (see Section 3.8 for details).

Again, when calculating the proportions used to allocate the non-responders to disability states all the data were used (all ages and both genders) for the same reasons as in Section 3.7.

The data after the above adjustments for censored lives are summarized, for all age groups and both genders, in Table 3.40. The same data stratified by gender and age group (10-year and 5-year age bands, with an upper age group of lives aged over 85 years) are given in Appendix D.

With the data classified in to disability states and censored cases dealt with, in the next chapter they are used to estimate the transition intensities defining the disability model (illustrated in Figure 3.19).

Chapter 4

Estimating the Transition Intensities in the Disability Model

4.1 Introduction

The aim of this chapter is to estimate the transition intensities in the disability model using the data described in the previous chapter. However, the data do not provide sufficient information to estimate the transition intensities directly. This is because the usual maximum likelihood estimators (occurrence–exposure rates) require information at the individual level of the numbers and times of transitions between states (Waters 1984). The NLTCS do not provide this much detail (see Section 3.4), they only provide information at discrete points in time and no information about what happened in between.

As in Section 3.5, I calculate transition probabilities between disability states (in Section 4.2) and show in Section 4.3, that these are not consistent with a set of positive transition intensities (as some of the implied intensities are negative). To avoid negative transition intensities, the maximum likelihood estimates have to be constrained to lie in the feasible region of the parameter space, but the unconstrained estimates are still a useful starting point for the fitting process. In Section 4.5 I describe a method for estimating constrained (non-negative) maximum likelihood estimates and in Section 4.6 I describe the numerical routines used to estimate them. In Chapter 5, I find approximate confidence intervals and graduate the estimated

intensities.

In the previous section, the data was split into 10-year age bands (to allow comparison with previous research) and 5-year age bands (to give more data points). Looking forward to section 5.5 (graduation of the transition intensities) it becomes clear that having more data points is very beneficial (it allows a more robust graduation procedure to be used) and so I concentrate the results on the data grouped into 5-year age bands. I initially used both sets of age groupings for all of the following work, but I report only the main results for the data grouped into 10-year age bands (final estimates of the transition intensities). Also, for brevity, in this chapter, I only use data from the 1982–84 NLTCS to illustrate the methods. The corresponding results for the 1984–89, 1989–94 surveys are given in the specified Appendices. Comments on these results are given in the text, where they differ from those illustrated.

I now introduce some notation. The theoretical (or population) transition intensity between states i and j (the states are numbered as in Figure 3.19) for a person aged $x+t$ I denote as μ_{x+t}^{ij}, the maximum likelihood estimate as $\hat{\mu}_{x+t}^{ij}$, the constrained maximum likelihood estimate as $\overline{\mu}_{x+t}^{ij}$ and the graduated transition intensity as $\overset{\circ}{\mu}_{x+t}^{ij}$.

4.2 Calculation of Transition Probabilities

The MLEs of the transition probabilities for men, women and in aggregate in 5-year age bands, for the 1982–84 NLTCS, calculated using equation (3.32), are given in Tables 4.41, 4.42 and 4.43 respectively. Corresponding transition probabilities for the 1984–89 and 1989–94 NLTCS are given in Appendix E. It is noticeable that the probability of recovery from disability can be fairly high, especially for younger age groups, justifying the use of a disability-recovery model.

It is worth noting a few features of mortality, from Tables 4.41, 4.42 and 4.43, which are also generally true for the 1984–89 and 1989–94 NLTCS in Appendix E (however I focus on mortality in the 1982–84 NLTCS, since it becomes clear in Sections 6.4 and 6.5 that mortality in the 1984–89 NLTCS and the 1989–94 NLTCS is anomalous, see below):

Table 4.41: The 2-year transition probabilities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys as a percentage, for males using 5 year age groupings.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	89.04	2.17	1.47	0.65	0.61	0.63	5.43
	70–74	81.22	2.69	2.27	0.78	0.94	0.98	11.11
	75–79	75.99	4.37	3.17	0.85	1.34	1.47	12.81
	80–84	66.02	6.54	5.98	1.04	1.74	2.94	15.73
	85+	46.16	9.48	10.46	3.42	1.84	4.39	24.25
IADL only	65–69	30.39	30.89	13.35	1.39	6.70	1.62	15.68
	70–74	30.11	31.85	11.30	1.50	2.10	3.02	20.11
	75–79	16.46	30.67	11.74	4.96	8.60	7.03	20.55
	80–84	9.94	28.30	17.24	6.17	9.84	7.57	20.95
	85+	2.15	33.64	13.35	7.34	4.42	6.18	32.93
1–2 ADLs	65–69	14.52	8.63	31.01	11.69	10.17	3.44	20.54
	70–74	9.13	10.73	29.86	16.20	5.09	6.58	22.41
	75–79	7.54	13.50	26.86	7.15	10.37	0.28	34.30
	80–84	4.33	5.65	30.79	9.99	5.50	5.68	38.07
	85+	5.22	4.06	18.34	9.98	13.52	15.76	33.13
3–4 ADLs	65–69	10.74	1.96	20.52	15.63	15.66	5.22	30.27
	70–74	11.71	1.90	16.23	27.34	15.94	0.68	26.21
	75–79	4.89	2.05	7.30	20.89	14.07	10.81	39.98
	80–84	0.65	8.63	11.36	14.17	19.84	8.64	36.72
	85+	0.39	3.81	3.89	11.32	20.73	13.37	46.49
5–6 ADLs	65–69	6.13	5.21	7.72	10.90	27.39	6.57	36.08
	70–74	7.84	4.13	11.08	9.03	29.75	5.22	32.96
	75–79	3.60	4.09	2.86	6.56	31.03	9.32	42.53
	80–84	2.85	5.77	5.80	9.61	23.10	7.18	45.68
	85+	0.41	0.04	4.20	6.14	24.48	6.31	58.41
Inst'd	65–69	6.94	1.79	0.37	1.79	0.23	68.12	20.74
	70–74	4.02	0.15	4.55	0.18	3.00	41.10	47.00
	75–79	2.65	2.21	1.20	1.11	0.14	49.12	43.56
	80–84	0.43	0.09	0.02	1.18	1.19	44.74	52.35
	85+	0.25	0.03	0.08	0.01	0.02	37.96	61.66

Table 4.42: The 2-year transition probabilities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys as a percentage, for females using 5 year age groupings.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	90.78	2.59	2.27	0.54	0.52	0.35	2.94
	70–74	85.58	4.27	2.51	1.08	0.93	1.20	4.43
	75–79	77.25	6.10	4.70	1.60	1.17	2.26	6.92
	80–84	64.77	6.34	7.71	2.49	1.66	5.35	11.68
	85+	47.82	8.29	11.35	2.20	4.11	9.96	16.27
IADL only	65–69	30.93	32.02	17.51	6.97	2.43	2.63	7.52
	70–74	22.61	32.73	21.54	4.26	3.23	6.01	9.62
	75–79	14.69	30.55	23.49	5.04	3.09	7.88	15.25
	80–84	9.19	30.15	28.39	3.84	4.35	8.19	15.90
	85+	2.24	26.68	24.29	7.31	7.37	11.42	20.70
1–2 ADLs	65–69	17.23	16.17	38.76	9.82	3.71	2.66	11.65
	70–74	11.35	14.79	37.25	9.93	6.12	7.13	13.41
	75–79	9.89	11.89	38.15	11.66	5.19	8.50	14.73
	80–84	6.20	12.20	32.31	12.30	5.80	11.21	19.97
	85+	3.10	8.00	31.96	14.62	8.94	13.01	20.38
3–4 ADLs	65–69	4.79	6.54	33.43	28.16	11.53	4.42	11.14
	70–74	6.19	4.25	21.35	29.69	20.25	6.21	12.06
	75–79	3.27	4.14	23.12	20.54	17.57	11.10	20.26
	80–84	2.85	3.69	15.65	23.94	22.29	13.06	18.52
	85+	1.27	3.79	6.73	21.17	24.12	17.13	25.79
5–6 ADLs	65–69	7.90	6.32	9.76	7.22	38.49	5.15	25.15
	70–74	4.51	5.96	6.89	11.39	27.30	10.67	33.28
	75–79	4.19	3.62	6.25	8.50	36.98	12.99	27.47
	80–84	4.27	3.73	7.09	10.84	28.86	13.58	31.63
	85+	1.57	0.72	5.49	7.99	26.92	13.51	43.80
Inst'd	65–69	8.04	0.29	2.67	2.71	0.03	75.61	10.65
	70–74	2.32	1.11	0.13	1.90	1.97	59.34	33.24
	75–79	1.29	1.14	0.71	0.56	2.13	60.67	33.49
	80–84	1.07	0.71	0.72	0.66	0.36	62.70	33.78
	85+	0.51	0.38	0.05	0.86	0.71	51.81	45.68

Table 4.43: The 2-year transition probabilities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys as a percentage, for males and females using 5 year age groupings.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	90.01	2.41	1.92	0.59	0.56	0.47	4.04
	70–74	83.72	3.60	2.41	0.95	0.93	1.11	7.28
	75–79	76.74	5.40	4.08	1.30	1.24	1.94	9.30
	80–84	65.24	6.42	7.07	1.95	1.69	4.46	13.18
	85+	47.30	8.67	11.07	2.58	3.40	8.20	18.78
IADL only	65–69	30.69	31.53	15.70	4.55	4.28	2.19	11.06
	70–74	25.31	32.42	17.86	3.27	2.82	4.94	13.39
	75–79	15.40	30.60	18.75	5.01	5.31	7.54	17.39
	80–84	9.40	29.64	25.34	4.47	5.85	8.02	17.28
	85+	2.21	28.72	21.09	7.32	6.50	9.88	24.28
1–2 ADLs	65–69	16.25	13.44	35.96	10.50	6.05	2.94	14.87
	70–74	10.49	13.22	34.40	12.35	5.72	6.92	16.89
	75–79	9.20	12.36	34.83	10.34	6.71	6.08	20.48
	80–84	5.68	10.39	31.89	11.66	5.72	9.68	24.99
	85+	3.69	6.91	28.19	13.33	10.21	13.77	23.91
3–4 ADLs	65–69	7.19	4.69	28.22	23.10	13.19	4.74	18.85
	70–74	8.40	3.31	19.30	28.75	18.52	4.00	17.72
	75–79	3.76	3.51	18.31	20.65	16.51	11.01	26.25
	80–84	2.34	4.83	14.66	21.69	21.72	12.03	22.73
	85+	1.03	3.79	5.94	18.43	23.17	16.08	31.56
5–6 ADLs	65–69	7.02	5.77	8.75	9.05	32.98	5.86	30.57
	70–74	6.13	5.07	8.92	10.24	28.49	8.02	33.12
	75–79	3.96	3.81	4.91	7.73	34.63	11.54	33.41
	80–84	3.73	4.50	6.60	10.38	26.68	11.16	36.93
	85+	1.33	0.58	5.22	7.61	26.42	12.04	46.80
Inst'd	65–69	7.54	0.97	1.63	2.30	0.12	72.22	15.22
	70–74	2.97	0.75	1.81	1.25	2.36	52.39	38.48
	75–79	1.74	1.50	0.87	0.74	1.48	56.86	36.81
	80–84	0.93	0.58	0.57	0.77	0.54	58.88	37.74
	85+	0.46	0.31	0.05	0.71	0.58	49.25	48.64

1. mortality generally increases with disability level (that is, people with greater disability have a higher probability of death), the most notable exceptions are from relatively low mortality in the institutionalised state;

2. mortality increases with age, with only a few exceptions — for which mortality is usually very similar between age groups; and

3. also as expected, mortality is greater for men than for women for almost all levels of disability and age groups, the only exception is for the 5–6 ADLs state, age group 70–74 years.

The first point above supports the inclusion of 'IADL only' as a separate state in the process of disability as it is predictive of mortality, as noted by Manton (1988). Also, it may be expected that the institutionalised state would have relatively low mortality, since lives admitted to an institution after a given cut-off date were excluded from the analysis, removing lives that may recover of die relatively quickly (see Section 3.7 for more detail).

From the tables in Appendix E, mortality in the 1984–89 and 1989–94 surveys is roughly comparable. However, 2–year probabilities of death in the 1982–84 surveys is roughly double that of the 5-year probabilities of death in the 1984–89 and 1989–94 surveys, even though it is based on a shorter time period (2 years compared with 5 years). This does raise some concerns about the data. All lives in the 1982 survey are accounted for in the 1984 survey as a deceased questionnaire was administered in 1984 (see Section 3.7). However, in the 1989 and 1994 surveys this was not the case, and a new classification was introduced — 'Not in survey year'. It may be because of this change in accounting for all lives between surveys that deceased lives in 1989 and 1994 were more likely to be left out of the survey, thus reducing the overall mortality in the 1984–89 and 1989–94 surveys. However, as there is no information as to why certain lives were left out of some of the surveys it is difficult to be certain. I look at the overall mortality in more detail in Sections 6.3 to 6.5.

Table 4.44 summarises the trends of disability and recovery — it gives the overall percentage change in disability status between surveys. In this table 'less disabled'

Table 4.44: Summary of percentage change in disability for males and females over the 1982, 1984, 1989 and 1994 NLTCS.

Change in disability	% change between surveys					
	1982–84		1984–89		1989–94	
	F	M	F	M	F	M
Less disabled	6.9	5.6	5.9	5.3	5.8	3.9
Same disability	63.9	65.1	62.0	68.5	58.9	67.4
More disabled	16.1	12.0	27.3	20.4	30.4	22.8
Died	13.1	17.3	4.9	5.8	4.9	5.9

covers those lives that moved to a less disabled state between surveys; 'same disability' covers those lives that were in the same state of disability in consecutive surveys; 'more disabled' covers those lives that moved to a more disabled state (other than dead) between surveys; and 'died' covers all lives that died between the surveys. This table needs to be interpreted carefully as the surveys are over different time periods, the 1982–84 survey is over 2 years while the 1984–89 and 1989–1994 surveys are over 5-year periods and the age structure of the population could be quite different over different time periods. Even so, some observations are worth noting:

1. perhaps surprisingly, females consistently have a slightly higher probability of recovery from disability than males;

2. males have a higher tendency to stay in the same disability state;

3. females have a considerably higher probability of becoming more disabled than males; and

4. as expected, males have a higher probability of death than females.

As already mentioned above, an apparent anomaly in these surveys is that the 5-year probabilities of dying over the periods 1984–89 and 1989–94 are considerably less (1/2 – 1/3) than the 2-year probability dying in the period 1982–84. The other main difference between the surveys is that the probabilities of becoming more disabled in the 5-year periods 1984–89 and 1989–94 are almost double those of becoming more disabled in the 2-year period 1982–84. This may be expected though, as although the lives cover the same age range at the start of any period (>65 years), at the end of the 2-year period (1982–84) they are then aged over 67 years, whereas they are

aged over 70 years at the end of the 5-year periods (1984–89 and 1989–94). Thus, at the end of a 5-year period the population will be older and they would be expected to be more disabled than after a 2-year period.

4.3 Transformation of Transition Probabilities to Transition Intensities

By assuming the transition intensities are constant for each age group in the NLTCS data, the matrix of intensities can easily be computed from the matrix of two-year transition probabilities, calculated in Section 4.2. If $P(t)$ is the matrix of transition probabilities over t years, and Q is the matrix of constant transition intensities (in rates per annum), then:

$$P(t) = \exp(Qt). \tag{4.33}$$

The usual problem is to calculate $P(t)$ from Q; here it is the other way round. The method given in Section 6.4.2 of Kulkarni (1995) is easily inverted once it is noticed that $P(t)$ and Qt have the same eigenvectors, and the eigenvalues of $P(t)$ are the exponentiated eigenvalues of Qt.

Tables 4.45, 4.46 and 4.47 give the annual transition intensities, calculated from the 1982–84 NLTCS grouped in 5-year age bands, for men, women and in aggregate, respectively. Corresponding transition intensities are given for the 1984–89 and 1989–94 surveys (5-year age bands) in Appendix F. As predicted in Section 3.5, a number of the transition intensities are negative, which have no meaning in models of physical processes. I include the estimates here, which I will refer to as 'unconstrained MLEs', where they exist, because:

1. they are a good starting point for a search for the constrained MLEs (see Section 4.6); and

2. they are used to compare methods of calculating confidence intervals for the transition intensities in Chapter 5.

For some categories (in 1984–89: males aged 65–69 years, 80–84 years and 85+ years; in 1989–94: males and females aged 85+ years; females aged 85+ years; and

Table 4.45: The MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0192	0.0080	0.0064	0.0015	0.0034	0.0247
	70–74		0.0232	0.0174	0.0028	0.0071	0.0061	0.0540
	75–79		0.0391	0.0270	0.0034	0.0053	0.0089	0.0602
	80–84		0.0721	0.0493	−0.0065	0.0106	0.0189	0.0713
	85+		0.1133	0.1504	0.0091	−0.0146	0.0150	0.1253
IADL only	65–69	0.2665		0.2656	−0.0984	0.1132	0.0072	0.0893
	70–74	0.2873		0.1964	−0.0419	0.0326	0.0207	0.1233
	75–79	0.1561		0.2030	0.0589	0.1012	0.0768	0.0613
	80–84	0.1007		0.2873	0.0279	0.1756	0.0710	0.0252
	85+	0.0087		0.2754	0.1117	−0.0413	0.0089	0.2066
1–2 ADLs	65–69	0.0849	0.1588		0.3335	0.0848	0.0190	0.0904
	70–74	0.0150	0.1997		0.3432	−0.0132	0.1019	0.1004
	75–79	0.0424	0.2469		0.1248	0.1307	−0.0475	0.2381
	80–84	0.0466	0.0572		0.2919	−0.0449	0.0479	0.2640
	85+	0.0895	0.0477		0.3514	0.1591	0.2143	0.0715
3–4 ADLs	65–69	0.0982	−0.1033	0.6488		0.4301	0.0481	0.2269
	70–74	0.1082	−0.0407	0.2888		0.3098	−0.0372	0.1612
	75–79	0.0433	−0.0223	0.1556		0.2716	0.1455	0.2552
	80–84	−0.0417	0.1885	0.2328		0.7223	0.1048	0.1530
	85+	0.0001	0.1134	0.0255		0.7467	0.2733	0.1615
5–6 ADLs	65–69	0.0116	0.1030	−0.0195	0.3437		0.0640	0.2655
	70–74	0.0472	0.0407	0.1539	0.1245		0.0626	0.2259
	75–79	0.0208	0.0593	0.0197	0.1262		0.1020	0.2985
	80–84	0.0328	0.0713	0.0336	0.3402		0.0807	0.3702
	85+	−0.0016	−0.0199	0.1075	0.1872		0.0505	0.5036
Inst'd	65–69	0.0393	0.0209	−0.0132	0.0347	−0.0076		0.1190
	70–74	0.0334	−0.0130	0.0685	−0.0248	0.0470		0.3423
	75–79	0.0174	0.0262	0.0093	0.0145	−0.0060		0.2973
	80–84	0.0043	−0.0031	−0.0043	0.0244	0.0084		0.3752
	85+	0.0030	−0.0001	0.0009	−0.0002	0.0001		0.4809

Table 4.46: The MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for females using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0204	0.0146	0.0012	0.0033	0.0014	0.0132
	70–74		0.0377	0.0102	0.0068	0.0052	0.0054	0.0192
	75–79		0.0584	0.0233	0.0108	0.0054	0.0100	0.0292
	80–84		0.0619	0.0563	0.0174	0.0060	0.0312	0.0525
	85+		0.1046	0.1072	−0.0137	0.0423	0.0738	0.0614
IADL	65–69	0.2622		0.2233	0.0884	0.0117	0.0189	0.0368
only	70–74	0.1939		0.3393	0.0124	0.0221	0.0454	0.0377
	75–79	0.1261		0.3783	0.0075	0.0238	0.0617	0.0825
	80–84	0.0801		0.5520	−0.0880	0.0850	0.0406	0.0570
	85+	0.0143		0.4446	0.0057	0.0924	0.0842	0.0870
1–2	65–69	0.0955	0.2555		0.1439	0.0246	0.0157	0.0734
ADLs	70–74	0.0600	0.2345		0.1535	0.0415	0.0587	0.0714
	75–79	0.0741	0.1921		0.2419	0.0048	0.0602	0.0657
	80–84	0.0527	0.2293		0.2822	−0.0256	0.0955	0.1228
	85+	0.0355	0.1253		0.3034	0.0037	0.0967	0.0913
3–4	65–69	−0.0187	−0.0462	0.5734		0.1706	0.0323	0.0348
ADLs	70–74	0.0331	−0.0493	0.3634		0.3879	0.0158	−0.0056
	75–79	−0.0061	−0.0292	0.4904		0.3481	0.0901	0.1033
	80–84	0.0021	−0.0215	0.3142		0.5072	0.0882	0.0303
	85+	0.0055	0.0784	0.0574		0.5702	0.1708	0.0312
5–6	65–69	0.0390	0.0705	0.0692	0.0954		0.0417	0.1812
ADLs	70–74	0.0180	0.0976	0.0156	0.2165		0.1221	0.2715
	75–79	0.0298	0.0450	0.0133	0.1636		0.1211	0.1743
	80–84	0.0400	0.0494	0.0405	0.2291		0.1338	0.2339
	85+	0.0174	−0.0140	0.0921	0.1598		0.1497	0.3416
Inst'd	65–69	0.0475	−0.0020	0.0129	0.0280	−0.0042		0.0587
	70–74	0.0138	0.0122	−0.0097	0.0208	0.0178		0.2089
	75–79	0.0075	0.0116	0.0013	0.0035	0.0214		0.2084
	80–84	0.0076	0.0070	0.0027	0.0076	0.0007		0.2096
	85+	0.0050	0.0040	−0.0030	0.0132	0.0034		0.3094

Table 4.47: The MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1982		1984 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0198	0.0119	0.0027	0.0029	0.0022	0.0184
	70–74		0.0315	0.0132	0.0056	0.0059	0.0057	0.0338
	75–79		0.0507	0.0254	0.0072	0.0055	0.0092	0.0420
	80–84		0.0647	0.0523	0.0106	0.0077	0.0263	0.0599
	85+		0.1072	0.1157	−0.0071	0.0277	0.0582	0.0802
IADL	65–69	0.2607		0.2321	0.0339	0.0440	0.0138	0.0562
only	70–74	0.2274		0.2931	−0.0139	0.0297	0.0376	0.0640
	75–79	0.1391		0.3007	0.0314	0.0518	0.0670	0.0795
	80–84	0.0857		0.4693	−0.0542	0.1085	0.0493	0.0461
	85+	0.0122		0.3906	0.0350	0.0503	0.0635	0.1223
1–2	65–69	0.0943	0.2214		0.1966	0.0466	0.0179	0.0822
ADLs	70–74	0.0457	0.2217		0.2213	0.0192	0.0710	0.0880
	75–79	0.0635	0.2058		0.2034	0.0398	0.0320	0.1090
	80–84	0.0494	0.1804		0.2711	−0.0286	0.0816	0.1657
	85+	0.0471	0.1040		0.3084	0.0410	0.1180	0.0951
3–4	65–69	0.0161	−0.0583	0.5779		0.2342	0.0373	0.0967
ADLs	70–74	0.0600	−0.0416	0.3288		0.3539	0.0013	0.0577
	75–79	0.0119	−0.0313	0.3876		0.3170	0.1132	0.1394
	80–84	−0.0035	0.0162	0.2897		0.5482	0.0962	0.0387
	85+	0.0032	0.0851	0.0521		0.5996	0.1823	0.0774
5–6	65–69	0.0325	0.0788	0.0418	0.1637		0.0519	0.2283
ADLs	70–74	0.0312	0.0672	0.0797	0.1783		0.0936	0.2504
	75–79	0.0264	0.0510	0.0139	0.1490		0.1131	0.2227
	80–84	0.0354	0.0654	0.0328	0.2464		0.1137	0.2897
	85+	0.0138	−0.0149	0.0957	0.1670		0.1330	0.3682
Inst'd	65–69	0.0441	0.0081	0.0030	0.0277	−0.0047		0.0856
	70–74	0.0192	0.0034	0.0154	0.0088	0.0270		0.2533
	75–79	0.0106	0.0161	0.0028	0.0071	0.0138		0.2355
	80–84	0.0069	0.0055	0.0017	0.0099	0.0022		0.2407
	85+	0.0047	0.0031	−0.0023	0.0117	0.0028		0.3368

males aged 85+ years), these estimates are not even real (the matrix of transition intensities is complex). In Section 4.4, I calculate approximate MLEs of the transition intensities in these categories by maximizing the log-likelihood over the real plane. It is interesting to note that this problem mostly arises at ages over 85 years, where the assumption of constant transition intensities may be least realistic.

4.4 Approximate (or Constrained to Real) Maximum Likelihood Estimates of the Transition Intensities

In the previous section, for some of the categories, transformation of the transition probabilities resulted in complex transition intensities (with the complex component being significantly non-zero). The aim of this section is, for these categories, to calculate sets of transition intensities that lie in the real plane, consistent with the maximum likelihood approach. The approach I adopt is to express transition probabilities $P^{ij}_{r\,r+t}$ as functions of the transition intensities, $P^{ij}_{r\,r+t} = P^{ij}_{r\,r+t}(\mu^{12}_r, \ldots, \mu^{n\,n-1}_r)$, and then maximise the likelihood function, L_r, for each age group r, where:

$$L_r \propto \prod_{all\,i,j} \left[P^{ij}_{r\,r+t}\left(\mu^{12}_r, \ldots, \mu^{n\,n-1}_r\right)\right]^{n^{ij}_{r\,r+t}} \tag{4.34}$$

which is equivalent to maximising the log-likelihood function, l_r:

$$l_r \propto \sum_{all\,i,j} \left(n^{ij}_{r\,r+t}\ \log\left[P^{ij}_{r\,r+t}\left(\mu^{12}_r, \ldots, \mu^{n\,n-1}_r\right)\right]\right) \tag{4.35}$$

The transition intensities, μ^{ij}_r, I assume are constant for each age group. Given a set of transition intensities, transition probabilities can be calculated from them by solving Kolmogorov's forward equations. These can then be used to evaluate the log-likelihood function. The set of transition intensities that maximises the log-likelihood over the real region (positive and negative) of the parameter space will then be the closest set consistent with maximum likelihood approach. Although these are not the actual maximum likelihood estimates, they are the closest real equivalent (as some of the first derivatives of the likelihood function will be non-zero). These estimates, for all those groups whose transition intensities could not be

calculated using the method described in Section 4.3, are given in Tables 4.48 and 4.49 for the 1984–89 and 1989–94 surveys, respectively. I refer to these estimates as 'constrained (real) MLEs'. For those groups whose transition intensities *could* be calculated using the method described in Section 4.3, this method would give identical results. The numerical method I used for this maximization process is discussed in Section 4.6.

4.5 Constrained (Positive) Maximum Likelihood Estimates of the Transition Intensities

The problem is now to estimate a set of transition intensities consistent with a maximum likelihood approach while ensuring that all intensities are non-negative. This can be done by constraining the transition intensities to lie in the non-negative region of the parameter space while maximising the log-likelihood by introducing a penalty function:

$$\max_{\mu_r^{ij}\, i\neq j} \left\{ \sum_{all\, i,j} \left\{ n_{r\, r+t}^{ij} \, \log \left[P_{r\, r+t}^{ij} \left(\mu_r^{1\,2}, \ldots, \mu_r^{n\,n-1} \right) \right] \right\} + F\left(\mu_r^{1\,2}, \ldots, \mu_r^{n\,n-1} \right) \right\} \quad (4.36)$$

where:

$$F\left(\mu_r^{1\,2}, \ldots, \mu_r^{n\,n-1} \right) = \begin{cases} 0 & \text{if } \mu_r^{ij} \geq 0 \;\; i \neq j \\ -K + \sum_{i\neq j} \min\left(0, \mu_r^{ij}\right) & \text{otherwise} \end{cases}$$

for each age group r. The penalty function $F\left(\mu_r^{1\,2}, \ldots, \mu_r^{n\,n-1}\right)$ ensures that all transition intensities are kept positive during a computational maximization scheme — if all intensities are non-negative, F is 0; if one or more of the intensities are negative, then F has a large negative value that decreases as the intensities move towards the positive region (which ensures the gradient is in the correct direction). The constant K is chosen with respect to the size of the log-likelihood to ensure that the 'cost' of a negative intensity is sufficiently large for the intensity to be kept non-negative — it does not punish a zero transition intensity, as this is a reasonable estimate (unlike a zero transition probability). The transition intensities calculated in this way I refer to as 'constrained (positive) MLEs'. It is now only a matter of choosing a suitable maximization algorithm.

Table 4.48: Approximate (constrained to real values, but not positive) MLEs of the annual transition intensities between disability states in the 1984 and 1989 National Long-Term Care Surveys.

Group	1984 Status	1989 Status						
		Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Males aged 65–69 years	Healthy		0.0127	0.0122	0.0009	0.0015	0.0017	0.0043
	IADL only	0.1501		0.1860	−0.0036	0.0146	0.0222	0.0207
	1–2 ADLs	0.0056	0.1731		0.1602	0.0387	0.0107	0.0008
	3–4 ADLs	0.0394	−0.0015	0.2190		0.1297	0.0160	0.0168
	5–6 ADLs	0.0167	0.0180	0.0534	0.0663		0.0500	0.0186
	inst'd	0.0024	0.0155	−0.0004	0.0011	0.0004		0.0197
Males aged 80–84 years	Healthy		0.0385	0.0312	0.0172	0.0037	0.0124	0.0172
	IADL only	0.0476		0.0484	0.0624	0.1670	0.0168	0.0410
	1–2 ADLs	0.0519	0.0929		−0.0130	0.0097	0.0915	0.0078
	3–4 ADLs	−0.0078	−0.0088	0.0873		0.1883	0.2523	−0.0009
	5–6 ADLs	−0.0104	−0.0087	0.0697	0.4707		−0.0168	−0.0175
	inst'd	0.0171	−0.0002	−0.0022	0.0231	−0.0038		0.1031
Males aged over 85 years	Healthy		0.1052	0.0118	−0.0048	0.0020	0.0310	0.0184
	IADL only	0.0587		0.2818	0.1817	0.0513	0.0387	0.0520
	1–2 ADLs	−0.0087	0.2350		0.1037	0.1358	0.0168	0.0299
	3–4 ADLs	−0.0003	0.0006	0.0000		0.0021	0.1947	0.0997
	5–6 ADLs	−0.0007	0.0004	0.0000	0.0638		0.1140	0.0331
	inst'd	0.0177	−0.0017	0.0040	0.0014	0.0010		0.0420

Table 4.49: Approximate (constrained to real but not positive values) MLEs of the annual transition intensities between disability states in the 1989 and 1994 National Long-Term Care Surveys.

Group	1989 Status	1994 Status						
		Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Males aged over 85 years	Healthy		0.0092	0.0525	0.0100	0.0296	0.0595	0.0187
	IADL only	0.2609		−3.2488	1.9198	−0.0350	1.2899	−0.3004
	1–2 ADLs	0.0316	0.0166		−0.1336	0.0648	0.0171	0.0551
	3–4 ADLs	−0.1499	−0.3665	2.5812		0.2076	−0.9818	0.2731
	5–6 ADLs	−0.0123	−0.0091	0.0095	−0.0111		0.2329	0.1139
	Inst'd	0.0233	0.0107	0.0102	0.0099	0.0023		0.0749
Females aged over 85 years	Healthy		0.0367	0.0969	−0.0388	0.0581	0.0625	0.0102
	IADL only	−0.0260		0.0528	0.5240	−0.0600	−0.0332	−0.0420
	1–2 ADLs	−0.0173	0.1430		0.2788	0.0199	0.1178	0.0053
	3–4 ADLs	0.1259	−0.0902	0.3304		0.2056	0.1750	0.1131
	5–6 ADLs	0.0392	0.0125	0.0119	0.0869		0.0875	0.0336
	Inst'd	0.0124	0.0020	0.0021	0.0013	0.0001		0.0298
Males and Females aged over 85 years	Healthy		0.0279	0.0967	−0.0474	0.0513	0.0594	0.0183
	IADL only	−0.0434		0.0398	0.6372	−0.0585	−0.0674	−0.0488
	1–2 ADLs	−0.0080	0.1263		0.3126	0.0082	0.1364	0.0010
	3–4 ADLs	0.1286	−0.0653	0.4683		0.2380	0.1422	0.1287
	5–6 ADLs	0.0312	0.0114	−0.0009	0.0912		0.1042	0.0408
	Inst'd	0.0135	0.0036	0.0033	0.0013	0.0007		0.0352

4.6 Numerical Calculation of the Constrained (Real and Positive) Maximum Likelihood Estimates

I now discuss the numerical methods used to calculate the constrained (positive) MLEs and the constrained (real) MLEs, all of which can be found in Press *et al.* (1993).

The first step is: given a set of transition intensities, to calculate the corresponding matrix of 2 or 5-year transition probabilities. This is done by solving Kolmogorov's forward equations — a set of n^2 simultaneous differential equations, where n is the number of states, in this case 7 (for more detail see Section 2.3). The method I chose was a 4th order Runge-Kutta algorithm with adaptable step size. Although initially more effort to set up, it is much more efficient (in terms of computer run-time) than an equivalent algorithm with constant step size. I found the extra effort worthwhile, as the number of calculations required in the maximization routine is considerable ($> 1,000$).

The log-likelihood function (equation (4.35)), can now be evaluated using these transition probabilities. This process can be thought of as one function evaluation during the maximization process.

The maximization method I chose was a form of conjugate gradient method in multidimensions, called the Polak-Ribiere method. I chose this method because:

1. other methods, not involving derivative information, took a very long time to converge to a maximum; and

2. although it requires derivative information, it makes very efficient use of this information, converging rapidly.

The basic routine is to start at an initial point P_0 (in our case a matrix of transition intensities) and move from point P_i to point P_{i+1} by maximizing along the uphill gradient from point P_i, with the restriction that the new gradient is conjugate to the old gradient. The method for choosing a new direction was modified by Polak & Ribiere, making the process more efficient near the maximum.

To calculate the constrained (positive) MLEs I used as a starting point the MLE transition intensities calculated in Section 4.3, where they exist (for some groups, the transition intensity matrix was complex). These contain negative transition intensities, but the penalty function F, (in equation (4.36)) ensures that these quickly become positive.

To calculate the constrained (real) MLEs, (for the groups where the MLEs of the transition intensities are complex) I use a simple starting matrix, with every annual transition intensity set to an arbitrary value of 0.01 — from which the maximization routine still converged but took considerably longer to do so than from the MLEs of the transition intensities. It is then possible to use these constrained (real) MLEs, as a starting point to calculate the corresponding constrained (positive) MLEs.

4.7 Constrained Maximum Likelihood Estimates Compared with Unconstrained Maximum Likelihood Estimates

I now compare the constrained MLEs, calculated using the above routines with the transition intensities calculated in Section 4.3 (transformed from the maximum likelihood estimates of the transition probabilities), which I will simply refer to as the MLEs for convenience (I include under this heading those approximate MLEs calculated in Section 4.5). I also compare 2 and 5-year transition probabilities calculated from these constrained MLEs with the original 2 and 5-year transition probabilities calculated directly from the data (given in Section 4.2). For males, females and in aggregate the constrained MLEs of the annual transition intensities calculated from the 1982–84 NLTCS in 5-year age bands are given in Tables 4.50, 4.51 and 4.52, respectively (these are directly comparable to Tables 4.45, 4.46 and 4.47). The constrained MLEs in 10-year age bands for the 1982–84 NLTCS and in 5 and 10-year age bands for the 1984–89 and 1989–94 NLTCS are given in Appendix G.

It is noticeable that for the majority of transition intensities there is very little

Table 4.50: The constrained MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, males using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0188	0.0090	0.0051	0.0021	0.0034	0.0247
	70–74		0.0231	0.0176	0.0026	0.0072	0.0061	0.0540
	75–79		0.0393	0.0268	0.0036	0.0054	0.0085	0.0602
	80–84		0.0713	0.0467	0.0000	0.0065	0.0187	0.0718
	85+		0.1145	0.1448	0.0000	0.0000	0.0168	0.1218
IADL	65–69	0.2608		0.1946	0.0000	0.0733	0.0065	0.0855
only	70–74	0.2820		0.1567	0.0000	0.0197	0.0263	0.1206
	75–79	0.1552		0.1974	0.0644	0.1016	0.0604	0.0647
	80–84	0.0986		0.2917	0.0225	0.1758	0.0713	0.0255
	85+	0.0087		0.2708	0.0724	0.0000	0.0145	0.1971
1–2	65–69	0.0933	0.1197		0.2108	0.1310	0.0200	0.0959
ADLs	70–74	0.0255	0.1698		0.2763	0.0123	0.0824	0.1078
	75–79	0.0445	0.2204		0.1022	0.1177	0.0000	0.2310
	80–84	0.0384	0.0667		0.2356	0.0000	0.0485	0.2579
	85+	0.0882	0.0429		0.4066	0.0925	0.2068	0.0866
3–4	65–69	0.0759	0.0000	0.4438		0.3163	0.0467	0.2183
ADLs	70–74	0.0949	0.0000	0.2107		0.2723	0.0000	0.1508
	75–79	0.0407	0.0000	0.1376		0.2611	0.1145	0.2629
	80–84	0.0000	0.1393	0.1863		0.5751	0.1038	0.1734
	85+	0.0000	0.0859	0.0431		0.7191	0.2653	0.1722
5–6	65–69	0.0193	0.0703	0.0447	0.2497		0.0644	0.2680
ADLs	70–74	0.0501	0.0307	0.1692	0.1114		0.0511	0.2286
	75–79	0.0214	0.0552	0.0221	0.1251		0.0897	0.2994
	80–84	0.0201	0.0849	0.0429	0.2904		0.0808	0.3640
	85+	0.0000	0.0000	0.0918	0.1486		0.0576	0.4924
Inst'd	65–69	0.0400	0.0154	0.0000	0.0180	0.0000		0.1192
	70–74	0.0292	0.0000	0.0419	0.0000	0.0368		0.3390
	75–79	0.0175	0.0236	0.0089	0.0111	0.0000		0.2944
	80–84	0.0032	0.0000	0.0000	0.0151	0.0112		0.3746
	85+	0.0029	0.0000	0.0008	0.0000	0.0001		0.4809

Table 4.51: The constrained MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, females using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0205	0.0144	0.0013	0.0033	0.0014	0.0132
	70–74		0.0376	0.0104	0.0068	0.0053	0.0054	0.0192
	75–79		0.0583	0.0235	0.0107	0.0055	0.0101	0.0292
	80–84		0.0613	0.0581	0.0155	0.0071	0.0312	0.0523
	85+		0.1052	0.0985	0.0000	0.0352	0.0731	0.0626
IADL	65–69	0.2624		0.2231	0.0874	0.0123	0.0189	0.0366
only	70–74	0.1929		0.3320	0.0142	0.0214	0.0454	0.0380
	75–79	0.1265		0.3727	0.0096	0.0231	0.0616	0.0824
	80–84	0.0818		0.4742	0.0000	0.0398	0.0420	0.0644
	85+	0.0143		0.4450	0.0000	0.0950	0.0844	0.0866
1–2	65–69	0.0888	0.2385		0.1386	0.0262	0.0159	0.0726
ADLs	70–74	0.0620	0.2185		0.1491	0.0443	0.0584	0.0704
	75–79	0.0720	0.1810		0.2350	0.0076	0.0604	0.0658
	80–84	0.0514	0.2105		0.2147	0.0106	0.0947	0.1173
	85+	0.0352	0.1232		0.2886	0.0112	0.0976	0.0902
3–4	65–69	0.0000	0.0000	0.4933		0.1579	0.0316	0.0386
ADLs	70–74	0.0273	0.0000	0.3115		0.3688	0.0172	0.0000
	75–79	0.0000	0.0000	0.4447		0.3375	0.0897	0.1033
	80–84	0.0028	0.0000	0.2701		0.4703	0.0889	0.0354
	85+	0.0065	0.0662	0.0570		0.5585	0.1695	0.0334
5–6	65–69	0.0363	0.0647	0.0771	0.0926		0.0418	0.1805
ADLs	70–74	0.0207	0.0789	0.0286	0.2117		0.1216	0.2689
	75–79	0.0283	0.0388	0.0217	0.1589		0.1212	0.1742
	80–84	0.0399	0.0417	0.0540	0.2112		0.1334	0.2320
	85+	0.0166	0.0000	0.0817	0.1506		0.1500	0.3401
Inst'd	65–69	0.0460	0.0000	0.0124	0.0239	0.0000		0.0582
	70–74	0.0140	0.0075	0.0000	0.0149	0.0192		0.2082
	75–79	0.0075	0.0115	0.0014	0.0035	0.0213		0.2084
	80–84	0.0076	0.0068	0.0032	0.0071	0.0011		0.2096
	85+	0.0047	0.0026	0.0000	0.0118	0.0034		0.3093

Table 4.52: The constrained MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0198	0.0119	0.0028	0.0029	0.0023	0.0184
	70–74		0.0314	0.0134	0.0054	0.0060	0.0057	0.0338
	75–79		0.0507	0.0254	0.0072	0.0056	0.0092	0.0420
	80–84		0.0644	0.0533	0.0094	0.0084	0.0263	0.0598
	85+		0.1069	0.1115	0.0000	0.0243	0.0579	0.0807
IADL	65–69	0.2598		0.2285	0.0347	0.0435	0.0138	0.0562
only	70–74	0.2257		0.2769	0.0000	0.0241	0.0384	0.0645
	75–79	0.1388		0.2978	0.0321	0.0515	0.0668	0.0795
	80–84	0.0871		0.4307	0.0000	0.0757	0.0493	0.0525
	85+	0.0122		0.3886	0.0323	0.0517	0.0637	0.1220
1–2	65–69	0.0977	0.1991		0.1893	0.0500	0.0179	0.0819
ADLs	70–74	0.0490	0.2032		0.2038	0.0269	0.0699	0.0873
	75–79	0.0645	0.1940		0.2001	0.0413	0.0327	0.1089
	80–84	0.0477	0.1749		0.2275	0.0000	0.0818	0.1605
	85+	0.0469	0.1026		0.2980	0.0463	0.1186	0.0945
3–4	65–69	0.0083	0.0000	0.5143		0.2174	0.0374	0.0989
ADLs	70–74	0.0529	0.0000	0.2829		0.3386	0.0032	0.0595
	75–79	0.0095	0.0000	0.3554		0.3102	0.1114	0.1401
	80–84	0.0000	0.0197	0.2696		0.5076	0.0963	0.0454
	85+	0.0042	0.0692	0.0563		0.5877	0.1808	0.0798
5–6	65–69	0.0342	0.0676	0.0523	0.1586		0.0519	0.2276
ADLs	70–74	0.0337	0.0535	0.0923	0.1715		0.0930	0.2497
	75–79	0.0271	0.0442	0.0197	0.1467		0.1135	0.2225
	80–84	0.0339	0.0638	0.0408	0.2263		0.1135	0.2867
	85+	0.0130	0.0000	0.0847	0.1570		0.1337	0.3663
Inst'd	65–69	0.0442	0.0065	0.0052	0.0228	0.0000		0.0851
	70–74	0.0191	0.0036	0.0153	0.0088	0.0269		0.2533
	75–79	0.0107	0.0158	0.0030	0.0070	0.0138		0.2355
	80–84	0.0069	0.0055	0.0019	0.0094	0.0025		0.2406
	85+	0.0044	0.0022	0.0000	0.0107	0.0027		0.3368

difference between the two methods of estimation. It is known that there will be no negative transition intensities using the contrained MLE method. However, in adjusting these to non-negative values, it may be expected that other transition intensities will change to compensate, especially transition intensities complementary to those that are negative (acting between the same states but in the opposite direction, for example $\mu_{x+t}^{j\,i}$ is complementary to $\mu_{x+t}^{i\,j}$). I make the following observations:

1. almost all negative transition intensities in the MLE routine, were estimated as zero using the constrained MLE routine, however there are exceptions (for example, the transition intensity from 1–2 ADLs to 5–6 ADLs for females aged 80–84 years);

2. the differences between transition intensities complementary to negative MLE transition intensities, are not much larger than the differences in general, which may be surprising; and

3. there are significant differences between constrained (positive) and unconstrained MLEs of some intensities that are not complementary to negative MLE transition intensities (for example, for males aged 65–75, the transition between 1–2 ADLs and Healthy).

The first point above suggests that it may be more efficient to calculate the maximum likelihood estimates of the transition probabilities (from data), transform them to transition intensities and then set any negative transition intensities to zero, rather than using a constrained maximum likelihood approach. I would argue against this method as:

1. not all of the constrained MLEs of negative transition intensities were zero — all intensities should have the possibility to take a positive value;

2. there is no intuitive reason why they should be set to zero, unless the physical process being modelled dictates this — which is not the case here;

3. quite a few transition intensities changed significantly, compensating in some way for the transition intensities that were forced to be non-negative; and

Table 4.53: Comparison of log-likelihood values for the unconstrained MLEs (and constrained (real) MLEs), adjusted MLEs and the constrained (positive) MLEs of the transition intensities calculated from the 1982-1984 NLTCS.

Gender	Age group	Log-likelihood value for: Original MLE	Adjusted MLE	Constrained MLE
M & F	65–69	−4186.71	−4188.62	−4187.52
	70–74	−4780.57	−4782.02	−4781.23
	65–74	−9036.11	−9038.72	−9037.15
	75–79	−4580.38	−4580.93	−4580.62
	80–84	−3576.82	−3580.53	−3577.39
	75–84	−8202.97	−8203.06	−8203.00
	85+	−3283.32	−3284.97	−3284.39
F	65–69	−2279.62	−2281.16	−2280.45
	70–74	−2718.12	−2719.78	−2718.94
	65–74	−5044.61	−5045.82	−5045.07
	75–79	−2863.02	−2863.43	−2863.17
	80–84	−2478.92	−2484.38	−2480.03
	75–84	−5376.17	−5378.18	−5376.72
	85+	−2435.97	−2438.00	−2437.24
M	65–69	−1863.88	−1869.68	−1865.89
	70–74	−1999.99	−2004.51	−2002.36
	65–74	−3907.77	−3914.12	−3910.72
	75–79	−1660.43	−1664.38	−1663.37
	80–84	−1066.74	−1068.54	−1067.42
	75–84	−2752.60	−2752.62	−2752.61
	85+	−813.83	−815.24	−814.45

4. this transformation of transition probabilities to transition intensities does not alway produce a set of real transition intensities from which to start.

Table 4.53 provides further support for not simply setting negative intensities to zero. The table shows for the transition intensities calculated from the 1982–84 NLTCS the log-likelihood (equation (4.35)) for: the unconstrained MLEs (and the constrained (real) MLEs) — highest value by definition; the unconstrained MLEs (and the constrained (real) MLEs) with all negative transition intensities set to zero (I refer to these as adjusted MLEs); and the constrained (positive) MLEs. The same likelihood values for the 1984–89 and 1989–94 NLTCS are given in Appendix H. The constrained (positive) MLEs, as well as always existing, are always better and in some cases substantially better than the adjusted MLEs, as well as making sense intuitively.

Further comparisons can be made by solving the Kolmogorov equations to calculate 2 or 5-year transition probabilities from the constrained (positive) MLEs — which are then directly comparable to the 2 or 5-year transition probabilities calculated from the data (given in Tables 4.41 – 4.43). These transition probabilities calculated from the constrained (positive) MLEs of the transition intensities for the 1982–84 NLTCS grouped in 5-year age bands for men, women and in aggregate are given in Tables 4.54, 4.55 and 4.56, respectively. The same transition probabilities in 5-year age bands for the 1984–89 and 1989–94 NLTCS are given in Appendix I. These can be thought of as the matrices of transition probabilities closest (in the sense of likelihood) to the original transition probabilities, that are consistent with a continuous-time Markov chain with constant (positive and real) transition intensities. This means that they can also be transformed into non-negative transition probabilities over any time span, using the methods in Section 3.5.

As expected there are no '0 transition probabilities' in these estimates (even though some of the transition intensities are zero) and overall there is not much difference between the probabilities using these two methods of estimation. It is not estimation of the transition probabilities that is of primary interest though — the main aim is the estimation of the underlying transition intensities, which can then be applied to the problem of estimating the cost of disability in LTC insurance.

Looking back at Table 3.36 it is noticeable that the group in 1982 with the least exposure is the 3–4 ADL group. It is then not surprising that differences in the transition probabilities out of the 3–4 ADL state are generally the greatest. This is a function of the maximum likelihood technique as the log-likelihood function is a weighted sum of log-probabilities, the weight being the number of transitions (see equation (4.35)) — the fewer the number of transitions, the less effect the probability has on the overall log-likelihood value, which gives that probability greater freedom to adjust to other effects. Another interesting observation is that the force of mortality from all states is almost unaffected, even from the 3–4 ADL state.

In the next chapter, I estimate variances for the constrained (positive) MLEs, and use them to graduate these point estimates using parametric methods, to get smoothed, time-continuous transition intensities, better for use in applications.

Table 4.54: The 2-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys as a percentage, for males using 5 year age groupings.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	89.05	2.17	1.47	0.64	0.61	0.63	5.43
	70–74	81.22	2.69	2.28	0.77	0.94	0.98	11.11
	75–79	76.00	4.37	3.17	0.85	1.34	1.47	12.81
	80–84	66.04	6.54	5.88	1.18	1.70	2.94	15.72
	85+	46.20	9.49	10.26	3.20	2.24	4.39	24.23
IADL only	65–69	30.27	31.16	12.30	2.97	6.06	1.60	15.64
	70–74	30.03	32.08	10.09	2.70	2.02	3.03	20.05
	75–79	16.51	31.15	11.72	5.12	8.66	6.25	20.59
	80–84	9.71	28.44	17.56	5.73	10.01	7.59	20.96
	85+	2.13	33.75	13.38	6.67	5.04	6.16	32.87
1–2 ADLs	65–69	14.61	8.02	32.54	10.02	10.71	3.47	20.63
	70–74	9.20	10.33	31.49	14.95	5.19	6.29	22.55
	75–79	7.48	12.55	27.33	6.32	9.42	2.70	34.20
	80–84	4.23	5.64	31.09	9.47	5.81	5.68	38.08
	85+	5.19	3.92	18.51	10.98	12.36	15.80	33.23
3–4 ADLs	65–69	10.61	3.71	18.73	17.09	14.57	5.16	30.13
	70–74	11.49	2.94	14.01	28.29	15.07	2.21	25.99
	75–79	4.89	2.67	6.97	21.94	14.08	9.27	40.19
	80–84	2.17	7.69	10.38	15.81	18.55	8.62	36.78
	85+	0.45	3.40	3.87	11.30	21.00	13.40	46.58
5–6 ADLs	65–69	6.15	4.83	7.97	10.04	28.24	6.60	36.17
	70–74	7.88	3.82	11.67	8.66	30.24	4.69	33.05
	75–79	3.62	4.01	2.89	6.72	31.82	8.28	42.66
	80–84	2.54	5.87	5.80	9.53	23.37	7.19	45.70
	85+	0.47	0.79	3.96	5.41	24.91	6.28	58.19
Inst'd	65–69	6.91	1.58	0.80	1.30	0.51	68.18	20.72
	70–74	3.94	0.67	3.43	1.05	2.75	41.37	46.79
	75–79	2.63	2.05	1.12	0.94	0.51	49.42	43.33
	80–84	0.39	0.23	0.26	0.92	1.10	44.79	52.31
	85+	0.25	0.03	0.07	0.02	0.02	37.96	61.66

Table 4.55: The 2-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys as a percentage, for females using 5 year age groupings.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	90.78	2.59	2.27	0.55	0.52	0.35	2.94
	70–74	85.59	4.27	2.50	1.08	0.93	1.20	4.43
	75–79	77.25	6.10	4.70	1.60	1.17	2.26	6.92
	80–84	64.78	6.34	7.72	2.48	1.66	5.35	11.68
	85+	47.90	8.34	10.84	2.69	4.03	9.94	16.26
IADL only	65–69	30.96	32.06	17.45	6.97	2.43	2.62	7.51
	70–74	22.60	32.86	21.44	4.26	3.22	6.01	9.61
	75–79	14.69	30.60	23.45	5.04	3.09	7.88	15.25
	80–84	9.15	30.43	26.73	5.57	4.11	8.16	15.86
	85+	2.24	26.78	24.53	6.89	7.44	11.42	20.71
1–2 ADLs	65–69	16.61	15.77	39.72	9.80	3.73	2.68	11.69
	70–74	11.37	14.36	37.65	9.91	6.15	7.14	13.41
	75–79	9.77	11.65	38.49	11.64	5.21	8.51	14.74
	80–84	6.22	12.06	33.41	11.04	5.98	11.25	20.03
	85+	3.11	7.83	32.37	14.28	8.97	13.03	20.40
3–4 ADLs	65–69	6.48	7.64	30.67	28.83	10.95	4.34	11.10
	70–74	6.13	5.77	19.86	30.17	19.74	6.18	12.15
	75–79	3.65	4.81	22.13	20.75	17.36	11.06	20.23
	80–84	2.83	4.04	15.16	24.60	21.94	12.99	18.45
	85+	1.29	3.61	6.42	21.61	24.17	17.12	25.78
5–6 ADLs	65–69	7.64	6.20	9.92	7.18	38.74	5.17	25.16
	70–74	4.53	5.52	6.93	11.42	27.64	10.70	33.25
	75–79	4.09	3.48	6.35	8.46	37.13	13.01	27.48
	80–84	4.28	3.60	7.21	10.58	29.09	13.59	31.65
	85+	1.51	1.21	5.26	7.64	27.08	13.50	43.80
Inst'd	65–69	7.92	0.54	2.42	2.41	0.41	75.67	10.64
	70–74	2.31	0.90	0.70	1.55	1.95	59.37	33.21
	75–79	1.29	1.14	0.72	0.56	2.13	60.67	33.49
	80–84	1.07	0.71	0.72	0.65	0.36	62.70	33.78
	85+	0.49	0.28	0.22	0.83	0.67	51.83	45.67

Table 4.56: The 2-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys as a percentage, for males and females using 5 year age groupings.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	90.01	2.41	1.92	0.59	0.56	0.47	4.04
	70–74	83.73	3.60	2.41	0.95	0.93	1.11	7.28
	75–79	76.74	5.40	4.08	1.30	1.24	1.94	9.30
	80–84	65.24	6.41	7.08	1.95	1.69	4.46	13.18
	85+	47.35	8.67	10.83	2.81	3.37	8.20	18.78
IADL only	65–69	30.67	31.61	15.67	4.55	4.27	2.19	11.05
	70–74	25.27	32.53	17.48	3.64	2.77	4.93	13.38
	75–79	15.39	30.65	18.73	5.01	5.31	7.53	17.38
	80–84	9.38	29.79	24.44	5.51	5.62	8.00	17.25
	85+	2.21	28.85	21.15	7.10	6.53	9.88	24.27
1–2 ADLs	65–69	16.29	12.84	36.49	10.47	6.08	2.95	14.89
	70–74	10.52	12.85	35.00	11.99	5.78	6.94	16.92
	75–79	9.22	12.06	35.08	10.33	6.73	6.09	20.50
	80–84	5.67	10.36	32.47	10.87	5.92	9.69	25.03
	85+	3.70	6.76	28.44	13.17	10.23	13.78	23.93
3–4 ADLs	65–69	7.15	6.27	26.77	23.56	12.68	4.73	18.83
	70–74	8.31	4.64	18.09	29.12	18.18	3.98	17.67
	75–79	3.73	4.37	17.57	20.78	16.35	10.97	26.23
	80–84	2.46	4.81	14.48	22.31	21.26	12.01	22.68
	85+	1.04	3.53	5.76	18.78	23.22	16.09	31.58
5–6 ADLs	65–69	7.03	5.54	8.85	9.01	33.15	5.86	30.57
	70–74	6.15	4.71	9.13	10.16	28.68	8.03	33.15
	75–79	3.97	3.64	4.98	7.71	34.73	11.56	33.42
	80–84	3.68	4.50	6.66	10.10	26.94	11.17	36.94
	85+	1.27	1.13	4.99	7.28	26.54	12.02	46.77
Inst'd	65–69	7.53	0.94	1.59	2.01	0.48	72.24	15.21
	70–74	2.97	0.75	1.81	1.24	2.36	52.40	38.48
	75–79	1.74	1.48	0.88	0.74	1.48	56.87	36.81
	80–84	0.93	0.58	0.57	0.77	0.53	58.88	37.74
	85+	0.44	0.25	0.18	0.69	0.55	49.26	48.63

Chapter 5

Confidence Intervals and Graduation of Transition Intensities in the Disability Model

5.1 Introduction

The main aim of this chapter is to graduate the transition intensities that were calculated in the previous chapter. I first, however, calculate variance estimates of the transition intensities. These are useful for two reasons:

1. to use as weights in the graduation process; and
2. to estimate confidence intervals for the transition intensities, against which the graduated transition intensities can be compared.

In Section 5.2, I look at two different methods for calculating variance estimates of the transition intensities, both of which use asymptotic maximum likelihood theory:

1. a standard method, which I will refer to as the asymptotic method, for which the likelihood function factorises to provide concise variance estimates — which is only valid given complete life-history data (i.e. the timing of each transition between states); and
2. a method which requires explicit calculation of the information matrix, which I will refer to as the information matrix method – which is valid given only

the partial data available from the NLTCS (i.e. the number of lives in each state at the start and end of the survey period).

The reason for looking at the first method is that this theory provides a very concise variance estimate, one which can even be calculated using the partial data available — this is, however, only an estimate of an estimate. The interest lies in comparing estimates from these two methods, the difference between them is an indication of the amount of 'information' lost by having access only to partial data, as the NLTCS provides, rather than complete life-history data.

I investigate these differences further, in Section 5.3, by comparing these variance estimates in three simple models. Then in Section 5.4, I describe a method for estimating the variance of the constrained (positive) MLEs of the transition intensities, and provide tables of these estimates. I use these as weights in Sections 5.5 and 5.6 where I graduate the constrained MLEs using parametric methods, to get smoothed, time-continuous transition intensities, more reasonable for use in applications. At the end of each of these graduation sections I provide graphs of the graduated transition intensities and confidence intervals, show that the functions do provide a good fit and give a table of parameter values for the functional forms of the transition intensities.

For brevity, in this chapter as in the previous one, I only use data from the 1982–84 NLTCS to illustrate the methods and concentrate on the data grouped by 5-year age bands (65–69 years, 70–74 years, 75–80 years and 80–84 years and 85+ years). The corresponding results for the 1984–89, 1989–94 surveys and for data grouped into 10-year age bands are given in the specified Appendices. Comments on these results are given in the text, where they differ from those illustrated.

5.2 Comparison of Two Methods for Estimating the Variance of the Transition Intensities

In this section I illustrate the results using one specific data set, that for males and females aged 65–74 years in the 1982–84 NLTCS, though the results presented here hold for all the other data sets. I first look at a standard method for estimating the

variance of the transition intensities, which can be used when complete life histories are known.

Given the life histories of each individual life in the NLTCS (i.e. the timing of each transition between states), the calculation of variance estimates of the transition intensities would be straightforward. As summarised by Macdonald (1996), for age group r over period t, let:

$$
\begin{aligned}
{}^{k}W^{i}_{r\,r+t} &= \text{waiting time in state } i \text{ for } k\text{th life} && (5.37)\\
{}^{k}N^{i\,j}_{r\,r+t} &= \text{number of transitions from state } i \text{ to state } j \text{ by } k\text{th life} && (5.38)
\end{aligned}
$$

Let $W^{i}_{r\,r+t} = \sum_k {}^{k}W^{i}_{r\,r+t}$ and $N^{i\,j}_{r\,r+t} = \sum_k {}^{k}N^{i\,j}_{r\,r+t}$, and using lower case symbols for observed samples, then the likelihood function for age group r, L_r can be shown to be:

$$
L_r\left(\mu_r^{1\,2}\ldots\mu_r^{6\,7}\right) = \prod_{i=1}^{7}\left[\mathrm{e}^{-\mu_r^{i\cdot}w^{i}_{r\,r+t}}\prod_{\substack{j=1\\ j\neq i}}^{7}\left(\mu_r^{i\,j}\right)^{n^{i\,j}_{r\,r+t}}\right] \tag{5.39}
$$

where $\mu_r^{i\cdot} = \sum_{j\neq i}\mu_r^{i\,j}$. From which it can be shown that the $\hat{\mu}_r^{i\,j}$ are asymptotically independent with distribution:

$$
\hat{\mu}_r^{i\,j} - \mu_r^{i\,j} \overset{asy.}{\sim} \mathrm{N}\left(0, \frac{\mu_r^{i\,j}}{\mathrm{E}\left[W^{i}_{r\,r+t}\right]}\right) \tag{5.40}
$$

So, given the life history of each individual, $\mu_r^{i\,j}/\mathrm{E}\left[W^{i}_{r\,r+t}\right]$ is an asymptotically unbiased estimate of the variance of $\hat{\mu}_r^{i\,j}$, which can be estimated by using the MLEs as approximations of the actual population transition intensities, and the actual waiting times as approximations for the expected waiting times. Even with the incomplete data provided by the the NLTCS, this variance estimate can be approximated — by using the unconstrained MLEs of the transition intensities (where they are positive) as approximations to the actual population transition intensities, and the expected waiting times can be approximated using the census method:

$$
\mathrm{E}\left[W^{i}_{r\,r+t}\right] \approx t \times \left(\frac{\mathrm{n}^{i\cdot}_{r\,r+t} + \mathrm{n}^{\cdot j}_{r\,r+t}}{2}\right) \tag{5.41}
$$

where $\mathrm{n}^{i\cdot}_{r\,r+t}$ and $\mathrm{n}^{\cdot j}_{r\,r+t}$ are the number of lives in state i for age group r and in state j in age group $r+t$, respectively. These variance estimates for the data set of males and females aged 65–74 years in the 1982–84 NLTCS are given in Table 5.57

— it should be noted that these are approximate estimates as if the life history of each individual were given (which is not the case in the NLTCS). I now look at a method for estimating the variance estimates which does not assume that complete life histories are known.

This standard method from MLE theory uses the information matrix (for those groups for which there exist unconstrained MLEs of the transition intensities), see Morgan (2000) for example. Although the method is straightforward, it is notationally difficult to represent, partly due to the parameters (transition intensities) themselves being in matrix notation. I will write, for convenience, the 36 non-zero transition intensities in the model as $\mu_r = (\mu_r^1 \dots \mu_r^{36})$, where r denotes age group. The elements of the information matrix $I(\mu_r)$ can now easily be defined:

$$[I(\mu_r)]_{ij} = -\mathrm{E}\left[\frac{\partial^2 l_r}{\partial \mu_r^i \, \partial \mu_r^j}\right] \text{ for } i, j = 1, 2, \dots, 36.$$

and where l_r is the log-likelihood function for age group r, defined in equation (4.35). We then have, from Morgan (2000), that if $\hat{\mu}_r^i$ are any unbiased estimates of the μ_r^i:

$$\mathrm{Var}(\hat{\mu}_r^i) \geq \left[I(\mu_r)^{-1}\right]_{ii}$$

and that for large samples that this is a good approximation to the variance or more specifically that:

$$\hat{\mu}_r^i - \mu_r^i \sim \mathrm{N}\left(0, \left[I(\mu_r)^{-1}\right]_{ii}\right) \text{ for } i = 1, 2, \dots, 36.$$

$I(\mu_r)$ can be approximated by substituting the maximum likelihood estimates of the transition intensities ($\hat{\mu}_{x+t}^i$) in place of the population (theoretical) transition intensities (μ_r^i). The elements of $I(\mu_r)$ can then be calculated by numerically differentiating the log-likelihood function (see Press *et al.* (1988) for more detail).

When estimating the variances of the transition intensities it is necessary to use the unconstrained MLEs, rather than the constrained (positive or real) MLEs (the set of first derivatives of the log-likelihood will be non-zero for the constrained MLEs). This is not a problem for the data set that I have chosen here, as the unconstrained MLEs are real. The variance estimates for the data set of males and females aged 65–74 years in the 1982–84 NLTCS are given in Table 5.58 — these are directly comparable to the variance estimates calculated using the asymptotic method.

Table 5.57: Variance estimates of the unconstrained MLEs of the transition intensities using the asymptotic method, for males and females aged 65–74 years in the 1982–84 NLTCS.

1984 Status	1989 Status						
	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy		1.55×10^{-6}	7.81×10^{-7}	2.46×10^{-7}	2.63×10^{-7}	2.27×10^{-7}	1.57×10^{-6}
IADL only	1.65×10^{-4}		1.82×10^{-4}	4.78×10^{-6}	2.44×10^{-5}	1.78×10^{-5}	4.20×10^{-5}
1–2 ADLs	4.58×10^{-5}	1.49×10^{-4}		1.41×10^{-4}	2.27×10^{-5}	2.94×10^{-5}	5.85×10^{-5}
3–4 ADLs	6.49×10^{-5}	- (1)	6.80×10^{-4}		4.57×10^{-4}	2.78×10^{-5}	1.14×10^{-4}
5–6 ADLS	4.52×10^{-5}	1.06×10^{-4}	8.75×10^{-5}	2.50×10^{-4}		1.05×10^{-4}	3.51×10^{-4}
Inst'd	4.48×10^{-5}	8.00×10^{-6}	1.48×10^{-5}	2.57×10^{-5}	1.53×10^{-5}		2.52×10^{-4}

(1) Undefined, since the unconstrained MLE is negative.

Table 5.58: Variance estimates of the unconstrained MLEs of the transition intensities using the information matrix, for males and females aged 65–74 years in the 1982–84 NLTCS.

1984 Status	1989 Status						
	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy		3.95×10^{-6}	3.41×10^{-6}	1.47×10^{-6}	1.02×10^{-6}	4.67×10^{-7}	2.00×10^{-6}
IADL only	3.62×10^{-4}		1.06×10^{-3}	3.69×10^{-4}	2.00×10^{-4}	7.10×10^{-5}	1.38×10^{-4}
1–2 ADLs	2.21×10^{-4}	8.57×10^{-4}		1.23×10^{-3}	4.81×10^{-4}	1.15×10^{-4}	2.12×10^{-4}
3–4 ADLs	3.80×10^{-4}	1.15×10^{-3}	5.78×10^{-3}		3.27×10^{-3}	3.12×10^{-4}	7.60×10^{-4}
5–6 ADLS	1.95×10^{-4}	6.00×10^{-4}	1.31×10^{-3}	1.64×10^{-3}		2.88×10^{-4}	7.42×10^{-4}
Inst'd	7.51×10^{-5}	4.19×10^{-5}	9.96×10^{-5}	1.13×10^{-4}	6.92×10^{-5}		3.62×10^{-4}

Table 5.59: Variance estimates of the unconstrained MLEs of the transition intensities using the asymptotic method as a percentage of those using the information matrix, for males and females aged 65–74 years in the 1982–84 NLTCS.

1984 Status	1989 Status						
	Healthy %	IADL only %	1–2 ADLs %	3–4 ADLs %	5–6 ADLs %	Inst'd %	Dead %
Healthy		39.25	22.90	16.71	25.81	48.72	78.29
IADL only	45.45		17.20	1.29	12.24	25.01	30.47
1–2 ADLs	20.73	17.37		11.49	4.72	25.50	27.62
3–4 ADLs	17.10	- (1)	11.75		13.99	8.91	14.95
5–6 ADLs	23.18	17.62	6.68	15.24		36.60	47.24
Inst'd	59.64	19.09	14.90	22.73	22.11		69.61

(1) Undefined, since the unconstrained MLE is negative.

Table 5.59 illustrates the differences between these two variance estimates — it gives the variance estimates calculated using the asymptotic method (which can be thought of as an estimate of a variance estimate) as a percentage of those calculated using the information matrix. The variance estimates calculated using the asymptotic method are between 79.3% and 1.3% of those calculated using the information matrix. The fact that the variance estimates using the asymptotic method are smaller than those calculated using the information matrix is not surprising, since the asymptotic method assumes that more information is available — complete life histories of all the individuals (aggregated), whereas the information matrix method only uses the information of which state an individual is in at the start and end of the survey period. Given that the variance is a measure of the uncertainty of a point estimate of a transition intensity, it would be expected that using more (relevant) information would provide a more certain estimate and, correspondingly, a smaller variance. This heuristic argument may not be the only reason why the variance estimates differ, and I first investigate the asymptotic estimate further, before looking at simulations of both methods.

Another possible reason for these differences may lie in using the census method to approximate the expected waiting time in the asymptotic method — the census method will only give a reasonable approximation if the change in exposure in a given state is roughly linear over time.

Table 5.60: Comparison of methods for calculating the expected waiting time for males and females aged 65–74 years in the 1982–84 NLTCS.

	Expected waiting time for state:					
Method used to Calculate $\mathrm{E}\left[W_r^i\right]$	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd
Census Method	15931.50	1468.81	1489.13	650.96	688.71	689.89
Expectations	15931.60	1444.07	1495.43	654.58	679.68	699.05

As a check of how good the census method approximation is, it can be compared to the actual expected waiting time (using the unconstrained MLEs as approximations for the population transition intensities), which can easily be calculated using Norberg's equations (Norberg, (1995)). For example, in the notation of Section 2.3, by setting the force of interest, δ, equal to 0%, and the payment function $b^i = 1$ (with $b^j = 0$ for $i \neq j$), the solutions to Norberg's equations, using the unconstrained MLEs of the transition intensities, will give (an approximation to), $\mathrm{E}\left[W_r^i(j)\right]$, the expected waiting time in state i, given that a life started in state j for age group r. The expected waiting time in state i is then simply the weighted sum of these conditional waiting times — the weights being the probability of starting in a given state:

$$\mathrm{E}\left[W_r^i\right] = \frac{\sum_{j=1}^{7} \mathrm{n}^{j\cdot}_{r\,r+t} \times \mathrm{E}\left[W_r^i(j)\right]}{\sum_{j=1}^{7} \mathrm{n}^{j\cdot}_{r\,r+t}} \tag{5.42}$$

Table 5.60 compares the census method approximations to the expected waiting times and the expected waiting times, calculated as described above. The census method does give very good approximations to the expected waiting times, with a maximum difference of 1.71% — clearly not enough to account for the differences in the variance calculations. Furthermore, the census method does not always overestimate the expected waiting times (for example in the 1–2 ADLs state), which would need to be the case to account for the underestimated variances.

To investigate further this discrepancy between variance estimates, it is possible to run simulations from a given set of transition intensities, to see how these two variance estimates behave — and the simulations themselves will provide a third method to estimate the variances of the transition intensities, as further comparison. For a given matrix of transition intensities, the corresponding matrix of transition probabilities, $\hat{\mathrm{P}}^{i\,j}_{r\,r+t}$, can be calculated by solving Kolmogorov's equations. Then for

each starting state i ($i = 1, 2, \ldots, 7$), the number of lives in state i in age group r who are in state j after time t, $\mathrm{N}^{ij}_{r\,r+t}$, are multinomially distributed with:

$$\mathrm{P}\left[\mathrm{N}^{ij}_{r\,r+t} = \mathrm{n}^{ij}_{r\,r+t} \text{ for } 1 \leq j \leq 7\right] = \frac{\mathrm{n}^{i\cdot}_{r\,r+t}}{\prod_{j=1}^{7} \mathrm{n}^{ij}_{r\,r+t}!} \prod_{j=1}^{7} \left(\hat{\mathrm{P}}^{ij}_{r\,r+t}\right)^{\mathrm{n}^{ij}_{r\,r+t}} \tag{5.43}$$

where $\mathrm{n}^{i\cdot}_{r\,r+t}$ are the number of lives starting in state i in age group r and for any collection $\mathrm{n}^{i1}_{r\,r+t}, \mathrm{n}^{i2}_{r\,r+t}, \ldots, \mathrm{n}^{i7}_{r\,r+t}$ of non-negative integers with sum $\mathrm{n}^{i\cdot}_{r\,r+t}$. Then given a simulated matrix of numbers of transitions between states, the corresponding matrix of transition probabilities can be calculated (as in Section 4.2), which can then be transformed, using the method described in Section 4.3, to a matrix of transition intensities. These can then be used to calculate variance estimates of the transition intensities using both of the methods described earlier in this Section. Also, given sufficient simulations (investigated further in Section 5.4), these simulated transition intensities themselves can be used to calculate a 'boot-strap' estimate of the variance of the transition intensities.

Figures 5.21 to 5.26 show the results for 500 simulations based on the probabilities calculated from the annual constrained (positive) MLEs of transition intensities for males and females aged 65–74 years in the 1982–84 NLTCS, for starting states 'Healthy' through to 'Institutionalized', respectively. It should be noted that the 'bootstrap' variance estimates are not accurate in these figures as they are only based on 500 simulations, and it takes more that 5,000 simulations before the 'bootstrap' variance estimates converge (see Section 5.4). I include them here only as an indication of the variance of the simulated samples. I used the constrained (positive) MLEs rather than the unconstrained MLEs to avoid any problems associated with negative transition intensities and because none of the 'true' population transition intensities would be negative.

A few points are worth noting about these simulations:

1. the asymptotic variance estimates change almost linearly with varying transition intensity, which is not surprising, as the variance is estimated as $\frac{\mu^{ij}_r}{\mathrm{E}[W^i_{r\,r+t}]}$ — also suggesting that the expected waiting time for any given state remains relatively constant with varying intensities;

2. the variance estimates calculated using the information matrix are much more scattered for any given value of μ_r^{ij}, showing dependency on other transition intensities, which is expected as they are not independent, though overall they do exhibit a similar trend — the variance estimates increase as the transition intensities increase in magnitude;

3. even though the 'population' transition intensities are all non-negative (as the constrained (positive) MLEs were used), from random fluctuation quite a number of the unconstrained MLEs based on the simulated samples contained negative transition intensities;

4. the MLEs of the transition intensities for 2 of the 500 simulated samples were complex, showing that, although not common, random deviation in data sets generated by non-negative 'population' transition intensities, can lead to MLEs that are complex; and

5. even though the 'bootstrap' variance estimates have not yet converged, they do give very good agreement with the variance estimates calculated using the information matrix, which is expected since they are based on the same information.

Given the data that are available, the variance estimates calculated using the information matrix are approximate lower bounds (they would be lower bounds if the population transition intensities, μ_{x+t}^{jk}, were used in the calculations). The asymptotic method, even though it can be approximated using the same data as the information matrix method, assumes that more information is known than the NLTCS provides and so the method is not valid. The differences in the variance estimates between the two methods gives an indication of how much information is lost by only knowing which states individual lives start and end in and not their life history. (By life history I mean the knowledge of the states each life moved to and the timing of these transitions.) In the disability model (7 states and 36 non-zero transition intensities), having only the limited information means that the variance estimates of the transition intensities are up to 77 times greater than if the life history of each individual was known (or equivalently, the confidence intervals of

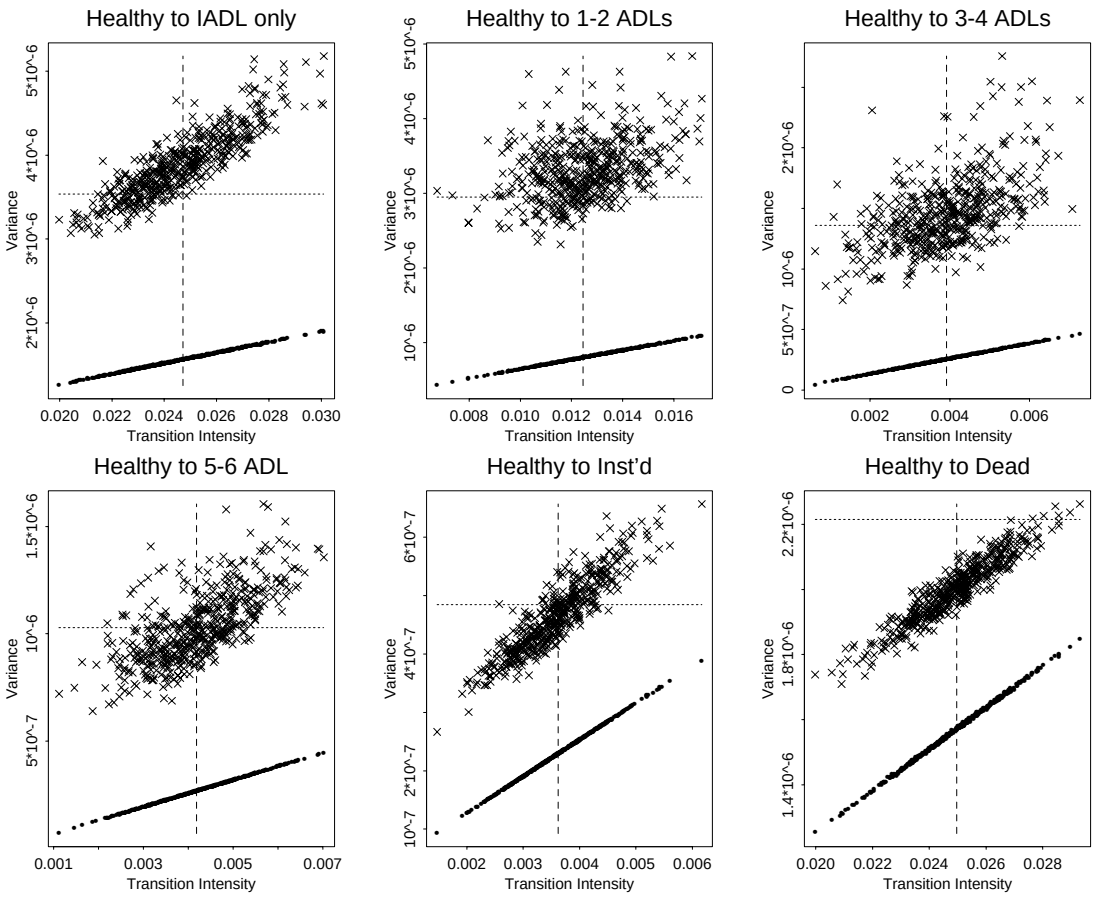

Figure 5.21: Variance estimates of the transition intensities out of the 'Healthy' state using the asymptotic method (•), the information matrix (×) and the 'bootstrap' variance estimate (horizontal line) for 500 simulations, based on the annual constrained (positive) MLEs of transition intensities (vertical line) for males and females aged 65–74 years in the 1982–84 NLTCS.

the point estimates are up to 8.8 times larger) — a substantial loss of information. In the next section I will look at some simpler models to compare these variance estimates further.

5.3 Comparison of Variance Estimates in Three Simple Models

In this section I look at three very simple Markov models, two 2-state models and a 3-state model each with known constant transition intensities and compare variance

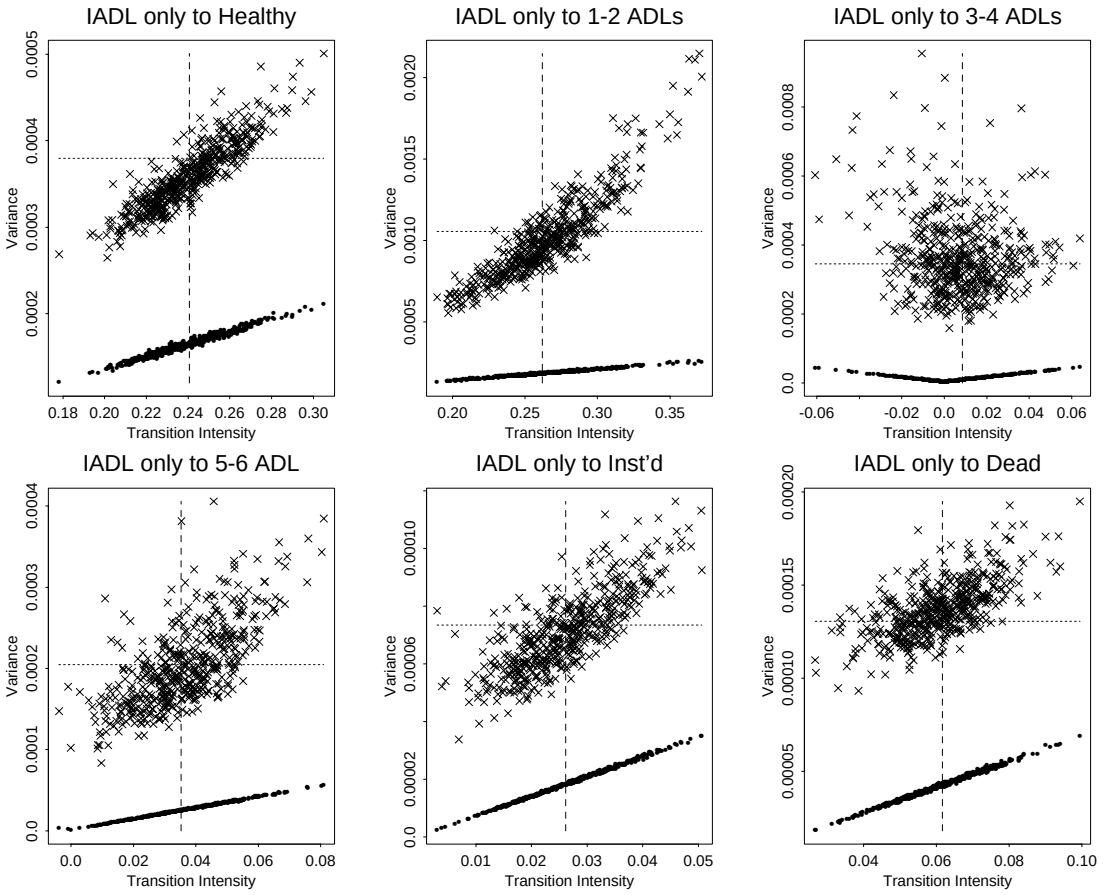

Figure 5.22: Variance estimates of the transition intensities out of the 'IADL only' state using the asymptotic method (•), the information matrix (×) and the 'bootstrap' variance estimate (horizontal line) for 500 simulations, based on the annual constrained (positive) MLEs of transition intensities (vertical line) for males and females aged 65–74 years in the 1982–84 NLTCS.

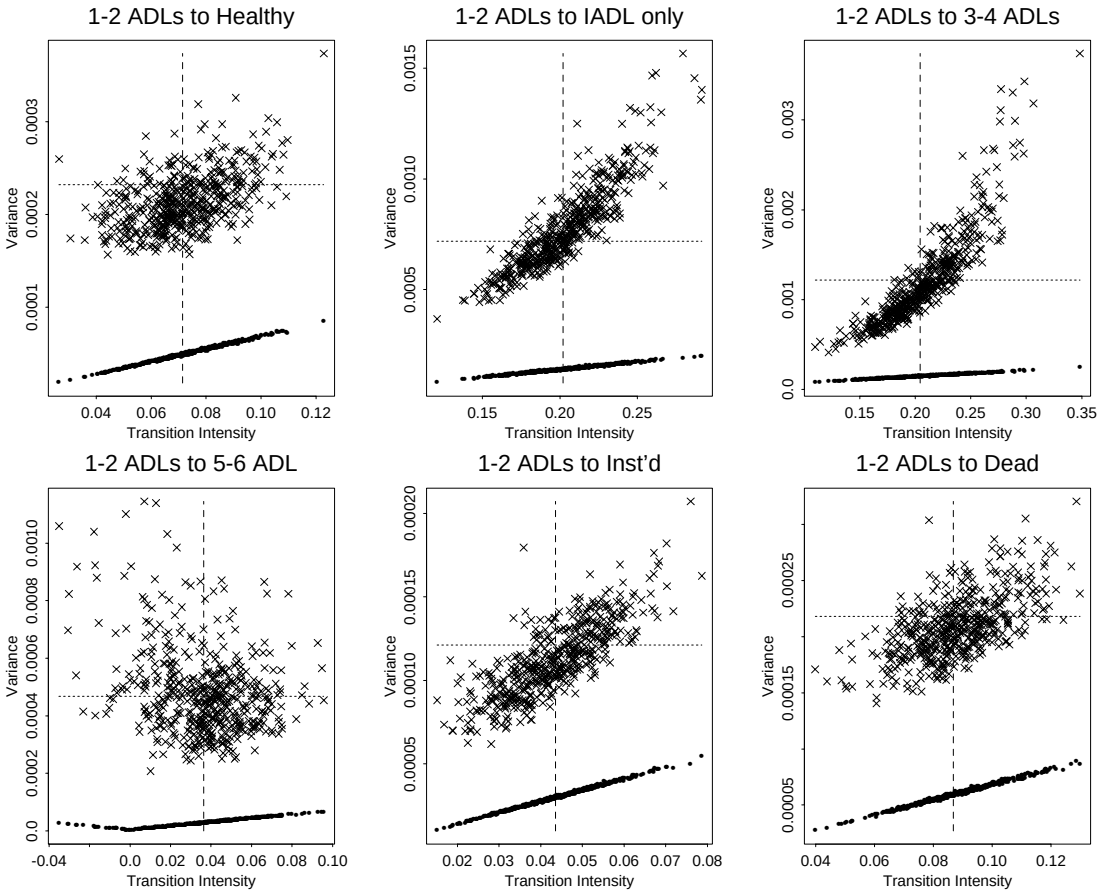

Figure 5.23: Variance estimates of the transition intensities out of the '1–2 ADLs' state using the asymptotic method (•), the information matrix (×) and the 'bootstrap' variance estimate (horizontal line) for 500 simulations, based on the annual constrained (positive) MLEs of transition intensities (vertical line) for males and females aged 65–74 years in the 1982–84 NLTCS.

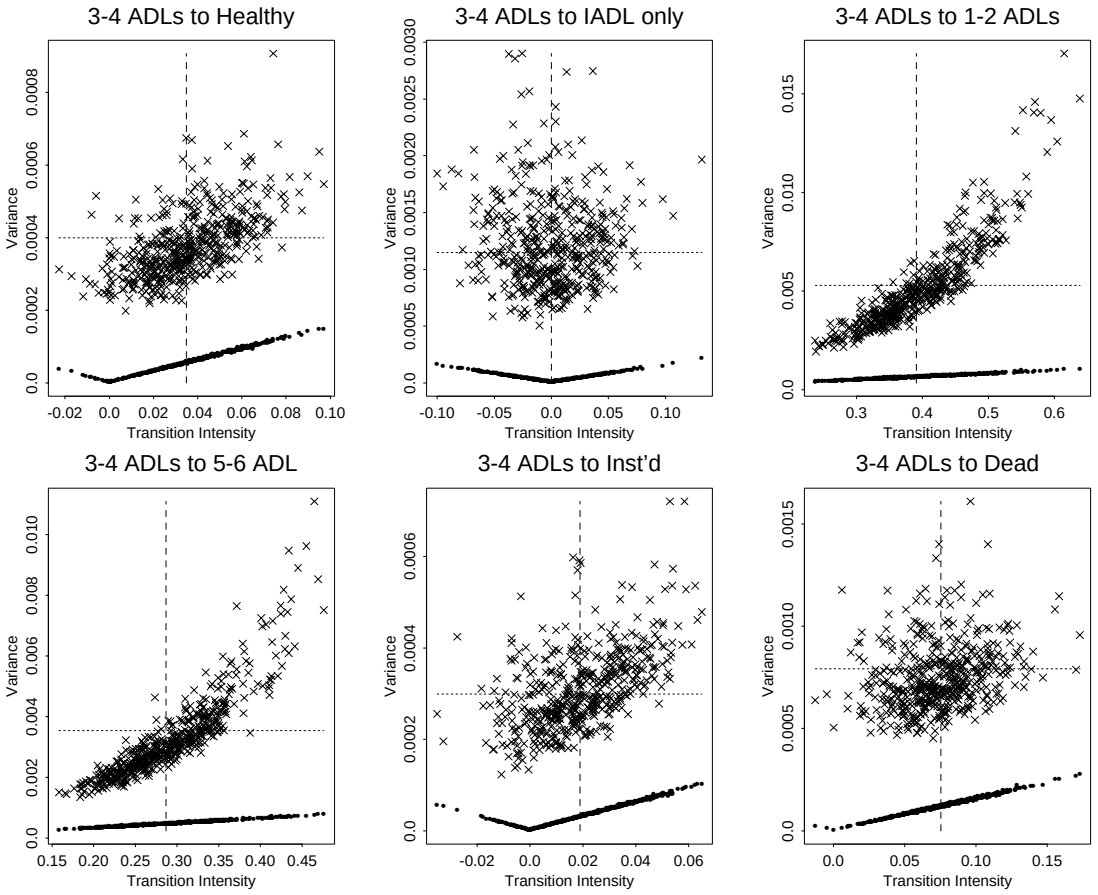

Figure 5.24: Variance estimates of the transition intensities out of the '3–4 ADLs' state using the asymptotic method (•), the information matrix (×) and the 'bootstrap' variance estimate (horizontal line) for 500 simulations, based on the annual constrained (positive) MLEs of transition intensities (vertical line) for males and females aged 65–74 years in the 1982–84 NLTCS.

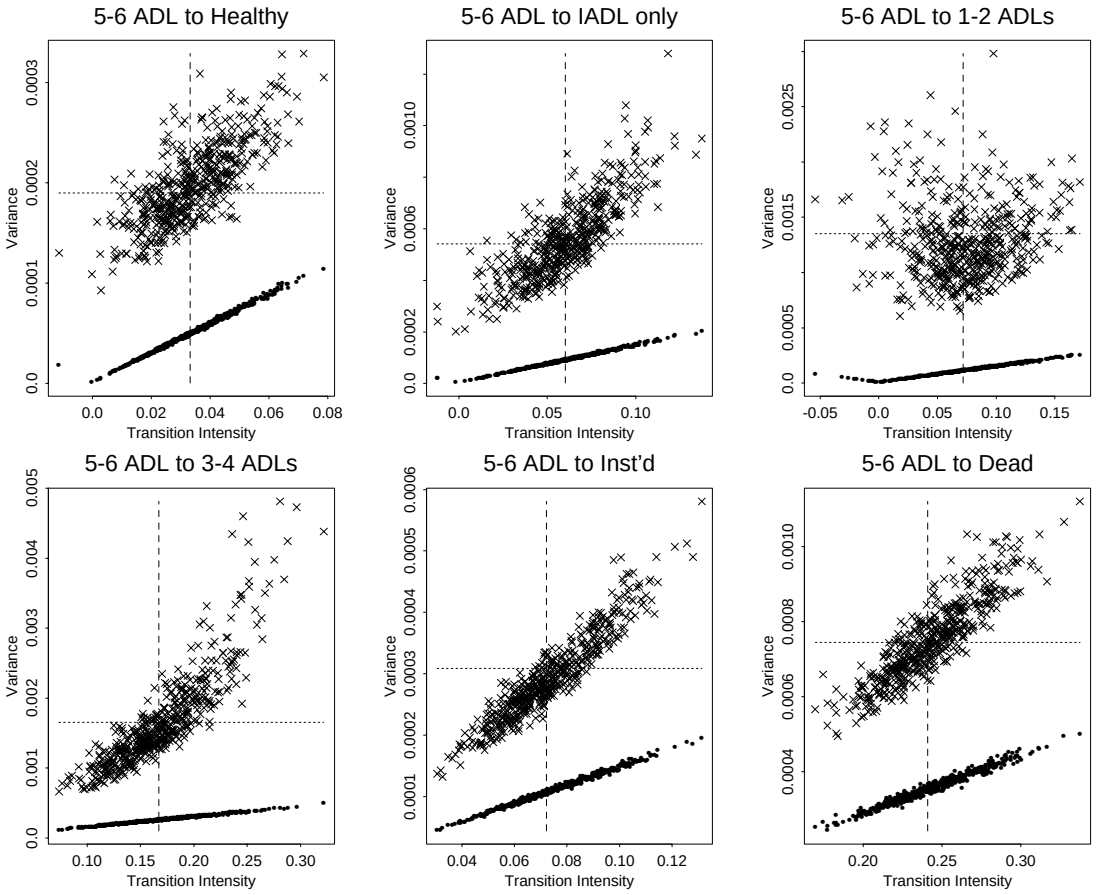

Figure 5.25: Variance estimates of the transition intensities out of the '5–6 ADLs' state using the asymptotic method (•), the information matrix (×) and the 'bootstrap' variance estimate (horizontal line) for 500 simulations, based on the constrained (positive) MLEs of transition intensities (vertical line) for males and females aged 65–74 years in the 1982–84 NLTCS.

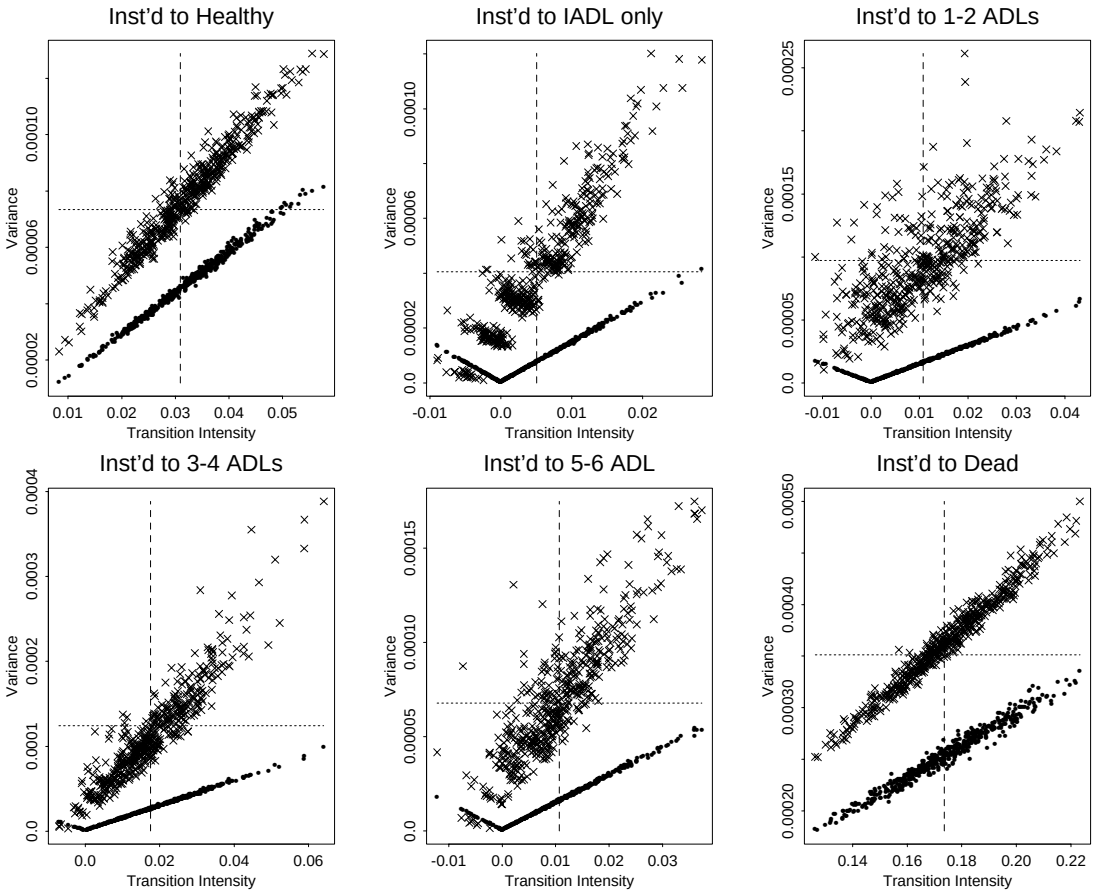

Figure 5.26: Variance estimates of the transition intensities out of the 'Institutionalized' state using the asymptotic method (•), the information matrix (×) and the 'boot-strap' variance estimate (horizontal line) for 500 simulations, based on the constrained (positive) MLEs of transition intensities (vertical line) for males and females aged 65–74 years in the 1982–84 NLTCS.

estimates using the asymptotic method and information matrix method, described in the previous section. For the first and simplest model (illustrated in Figure 5.27) I calculate the variance of the transition intensity algebraically using both of the methods described in the previous section and compare them by looking at the ratio of the variance estimate using the asymptotic method to that using the information matrix. For the next two models (illustrated in Figures 5.30 and 5.33), the algebra becomes unwieldly, but, for constant transition intensities it is possible, with the help of Maple software, to look at the ratios of the variance estimates with variable numbers of people in each starting state. For these two models I also look at 1,000 simulated samples to see how the variance estimates behave, and compare the average ratio of the variance estimates from these simulated samples with the theoretical ratios.

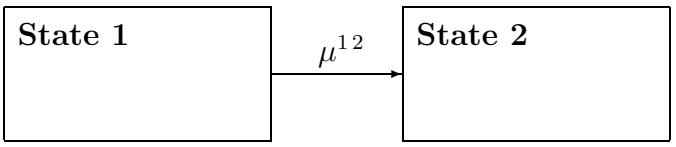

Figure 5.27: A simple 2-state model with one transition intensity.

The first model is illustrated in Figure 5.27. It is a simple model of mortality, with no competing risks. For brevity I will write the one year probability of staying in state 1 (or one year probability of survival) as P_{01}^{11}, the one year probability of moving to state 2 (or annual probability of dying) as P_{01}^{12} and the constant transition intensity from state 1 to state 2 (or force of mortality) as μ^{12}. It is easily shown that:

$$\begin{aligned} \mathrm{P}_{01}^{11} &= \mathrm{e}^{-\mu^{12}} \\ \mathrm{P}_{01}^{12} &= 1 - \mathrm{e}^{-\mu^{12}} \end{aligned}$$

The log-likelihood, l, can then be written as:

$$\begin{aligned} l &\propto n_{01}^{11} \log\left(\mathrm{P}_{01}^{11}\right) + n_{01}^{12} \log\left(\mathrm{P}_{01}^{12}\right) \\ &= -n_{01}^{11}\mu^{12} + n_{01}^{12} \log\left(1 - \mathrm{e}^{-\mu^{12}}\right) \end{aligned}$$

where $n_{r\,r+t}^{i\,j}$ is defined in the previous section. By setting the first partial derivative:

$$\frac{\partial l}{\partial \mu^{1\,2}} = -n_{0\,1}^{1\,1} + n_{0\,1}^{1\,2}\,\frac{\mathrm{e}^{-\mu^{1\,2}}}{1-\mathrm{e}^{-\mu^{1\,2}}} \tag{5.44}$$

to zero, we obtain:

$$\hat{\mu}^{1\,2} = \log\left(\frac{n_{0\,1}^{1\,1} + n_{0\,1}^{1\,2}}{n_{0\,1}^{1\,1}}\right) \tag{5.45}$$

and from the second partial derivative:

$$\frac{\partial^2 \mu^{1\,2}}{\partial (\mu^{1\,2})^2} = -n_{0\,1}^{1\,2}\,\frac{\mathrm{e}^{-\mu^{1\,2}}}{\left(1-\mathrm{e}^{-\mu^{1\,2}}\right)^2} \tag{5.46}$$

we get that:

$$\mathrm{Var}_I\left(\hat{\mu}^{1\,2}\right) \geq \frac{\left(1-\mathrm{e}^{-\mu^{1\,2}}\right)^2}{n_{0\,1}^{1\,2}\,\mathrm{e}^{-\mu^{1\,2}}} \tag{5.47}$$

which can be approximated by substituting $\hat{\mu}_{0\,1}^{1\,2}$ (from equation (5.45)) for $\mu^{1\,2}$, which gives:

$$\mathrm{Var}_I\left(\hat{\mu}^{1\,2}\right) \geq \frac{n_{0\,1}^{1\,2}}{n_{0\,1}^{1\,1}\left(\mu^{1\,1} + \mu^{1\,2}\right)} \tag{5.48}$$

If we now look at the asymptotic method, we are given that $\frac{\hat{\mu}^{1\,2}}{\mathrm{E}\left[W_{0\,1}^{1}\right]}$, is an asymptotically unbiased estimate of the variance of $\hat{\mu}_r^{1\,2}$. Using the census method to calculate the central exposed to risk, we have:

$$\mathrm{E}\left[W_{0\,1}^{1}\right] \approx n_{0\,1}^{1\,1} + \frac{n_{0\,1}^{1\,2}}{2} \tag{5.49}$$

and combining this with equation (5.45) we have:

$$\mathrm{Var}_A\left(\hat{\mu}^{1\,2}\right) \approx \frac{\log\left(\frac{n_{0\,1}^{1\,1}+n_{0\,1}^{1\,2}}{n_{0\,1}^{1\,1}}\right)}{n_{0\,1}^{1\,1} + \frac{n_{0\,1}^{1\,2}}{2}} \tag{5.50}$$

The variance ratio, $R_{\hat{\mu}^{i\,j}}$, which I define as $\mathrm{Var}_A\left(\hat{\mu}^{i\,j}\right)/\mathrm{Var}_I\left(\hat{\mu}^{1\,2}\right)$ can then be written as:

$$R_{\hat{\mu}^{1\,2}}(k) = \frac{1+k}{k(1+\frac{k}{2})}\log\left(1+k\right) \tag{5.51}$$

where $k = n_{0\,1}^{1\,2}/n_{0\,1}^{1\,1}$. It is worth noting here that this variance ratio only depends on the ratio $n_{0\,1}^{1\,2}/n_{0\,1}^{1\,1}$ and not on the number of lives under observation, $n_{0\,1}^{1\,2} + n_{0\,1}^{1\,1}$ and is thus independent of sample size. Figure 5.28 shows $R_{\hat{\mu}^{1\,2}}(k)$ for $0 \leq k \leq 1$ — it is always less than unity and decreases with increasing k (which is equivalent to increasing $n_{0\,1}^{1\,2}$). For small k it is very close to unity, suggesting that for small

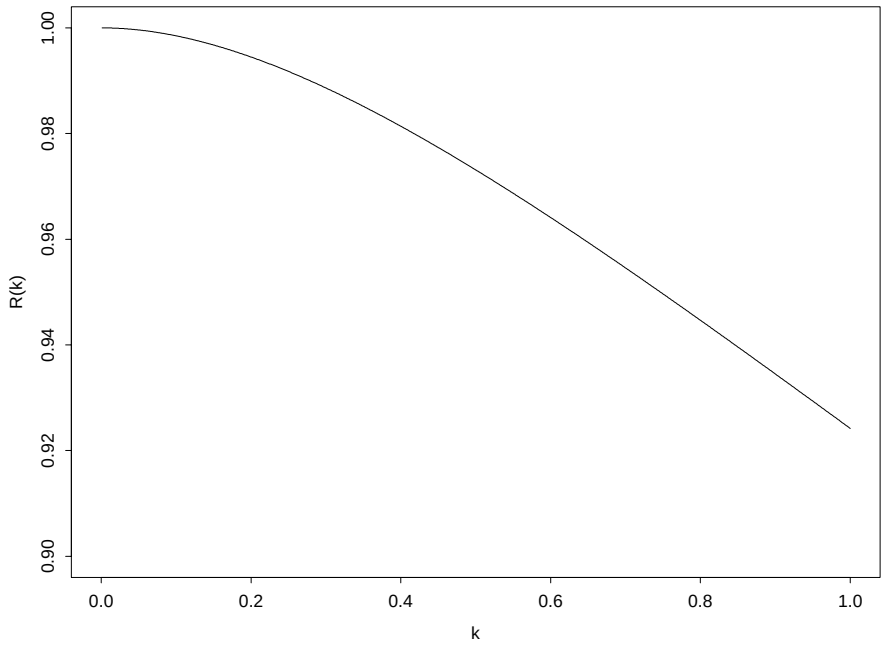

Figure 5.28: Variance ratio, $R_{\hat{\mu}^{12}}(k)$, for $0 \leq k \leq 1$ for the 2-state model with one transition intensity.

μ^{12} (and thus small k), the asymptotic method and the information matrix method provide very similar variance estimates in this 2-state model. This result is not surprising considering the following points:

1. the only extra information that is assumed in using the asymptotic method is the precise timing of any transitions, since there is no uncertainty as to how many transitions took place (once a life moves to state 2 they stay there); and

2. the fewer transitions that occur (the smaller μ^{12} is), the less impact the precise timings of the transitions have on the estimation of the expected waiting times, and thus on the variance estimates — as observed by Sverdrup (1965).

It is also possible to look at the effect of using the census approximation to estimate the expected waiting time — by using expectations we have that:

$$\begin{aligned} \mathrm{E}\left[W_{01}^{1}\right] &= \left(n_{01}^{11} + n_{01}^{12}\right) \int_0^1 \mathrm{e}^{-\mu^{12}t} \, \mathrm{dt} \\ &= \left(n_{01}^{11} + n_{01}^{12}\right) \frac{1 - \mathrm{e}^{-\mu^{12}}}{\mu^{12}} \end{aligned}$$

which gives a variance estimate of:

$$\mathrm{Var}_{A'}\left(\hat{\mu}^{12}\right) \approx \frac{\left(\mu^{12}\right)^2}{\left(n_{01}^{11} + n_{01}^{12}\right)\left(1 - \mathrm{e}^{-\mu^{12}}\right)} \tag{5.52}$$

and a variance ratio, $R'_{\hat{\mu}^{12}}$, of:

$$R'_{\hat{\mu}^{12}}\left(\mu^{12}\right) = n_{01}^{12}\left[\frac{\left(\mu^{12}\right)^2 \mathrm{e}^{-\mu^{12}}}{\left(n_{01}^{11} + n_{01}^{12}\right)\left(1 - \mathrm{e}^{-\mu^{12}}\right)^3}\right] \tag{5.53}$$

As all the terms in the square brackets in equation (5.53) are constant and by noting that:

$$\begin{aligned} \mathrm{E}\left[n_{01}^{12}\right] &= \left(n_{01}^{11} + n_{01}^{12}\right)\mathrm{P}_{01}^{12} \\ &= \left(n_{01}^{11} + n_{01}^{12}\right)\left(1 - \mathrm{e}^{-\mu^{12}}\right) \end{aligned}$$

we have that:

$$\mathrm{E}\left[R'_{\hat{\mu}^{12}}\left(\mu^{12}\right)\right] = \left(\frac{\mu^{12}}{\mathrm{e}^{\frac{\mu^{12}}{2}} - \mathrm{e}^{-\frac{\mu^{12}}{2}}}\right)^2 \tag{5.54}$$

This expected variance ratio is also independent of sample size. It only depends on the transition intensity μ^{12} and not on the number of lives under observation. Figure 5.29 shows $\mathrm{E}[R'_{\hat{\mu}^{12}}(\mu^{12})]$ for $0 \leq \mu^{12} \leq 1$ — it is always less than unity and decreases with increasing μ^{12} (which is equivalent to increasing n_{01}^{12}). Figures 5.28 and 5.29 are comparable if we note that:

$$\begin{aligned} k &= \frac{n_{01}^{12}}{n_{01}^{11}} \\ &\approx \frac{\left(n_{01}^{11} + n_{01}^{12}\right)\mathrm{P}_{01}^{12}}{\left(n_{01}^{11} + n_{01}^{12}\right)\mathrm{P}_{01}^{11}} \\ &= \mathrm{e}^{\mu^{12}} - 1 \\ &= \mu^{12} + O\left((\mu^{12})^2\right) \end{aligned}$$

which shows that the asymptotic method using the census approximation to the expected waiting time slightly underestimates the variance. This is because the census approximation slightly overestimates the expected waiting time, and so underestimates the variance, in the case of this simple two state model.

I now look at the effects on these variance estimates when there are two transition intensities in the two state model. The second model is illustrated in Figure 5.30. I

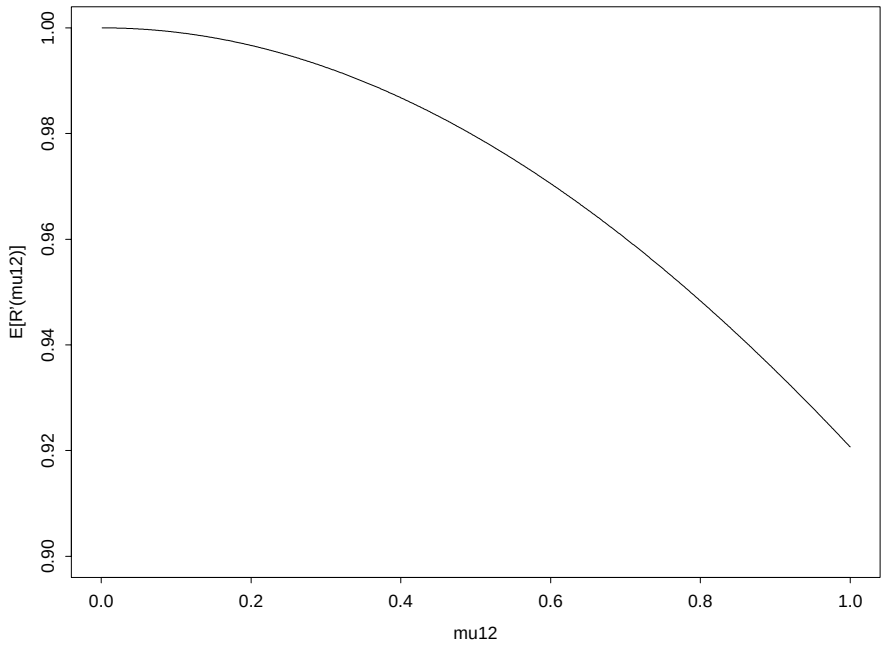

Figure 5.29: Expected variance ratio, $\mathrm{E}[R'_{\hat{\mu}^{12}}(\mu^{12})]$, for $0 \leq \mu^{12} \leq 1$ for the 2-state model with one transition intensity.

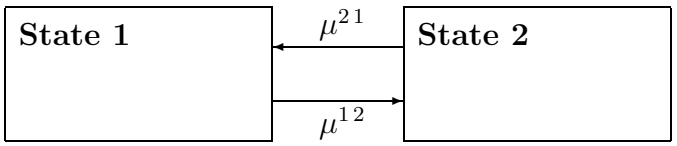

Figure 5.30: A simple 2-state model with two transition intensities.

will use the same notation as for the previous model. It is easily shown that:

$$\begin{aligned}
\mathrm{P}_{01}^{11} &= \frac{1}{\mu^{12}+\mu^{21}}\left\{\mu^{21}+\mu^{12}\mathrm{e}^{-(\mu^{12}+\mu^{21})}\right\} \\
\mathrm{P}_{01}^{12} &= \frac{\mu^{12}}{\mu^{12}+\mu^{21}}\left\{1-\mathrm{e}^{-(\mu^{12}+\mu^{21})}\right\} \\
\mathrm{P}_{01}^{22} &= \frac{1}{\mu^{12}+\mu^{21}}\left\{\mu^{12}+\mu^{21}\mathrm{e}^{-(\mu^{12}+\mu^{21})}\right\} \\
\mathrm{P}_{01}^{21} &= \frac{\mu^{21}}{\mu^{12}+\mu^{21}}\left\{1-\mathrm{e}^{-(\mu^{12}+\mu^{21})}\right\}
\end{aligned}$$

and the asymptotic variance estimates, $V_A(\mu^{12})$ and $V_A(\mu^{21})$, using the census

method to approximate the expected waiting times, are:

$$\begin{aligned} V_A\left(\mu^{12}\right) &= \frac{\mu^{12}}{n_{01}^{11} + \frac{n_{01}^{12}+n_{01}^{21}}{2}} \\ V_A\left(\mu^{21}\right) &= \frac{\mu^{21}}{n_{01}^{22} + \frac{n_{01}^{12}+n_{01}^{21}}{2}} \end{aligned}$$

To simplify the following equations, I now fix the transition intensities as $\mu^{12} = 0.1$ and $\mu^{21} = 0.05$. Then using Maple software, the variance ratios $R_{\hat{\mu}^{12}}(\cdot)$ and $R_{\hat{\mu}^{21}}(\cdot)$ are:

$$\begin{aligned} R_{\hat{\mu}^{12}}(\cdot) &= 1.028\left[n_{01}^{11}\left(-2.137n_{01}^{11} - 3.982n_{01}^{22} + 27.73n_{01}^{12} - 55.47n_{01}^{22}\right)\right. \\ &+ n_{01}^{22}\left(-1.871n_{01}^{22} - 25.73n_{01}^{12} - 51.47n_{01}^{21}\right) \\ &+ \left.n_{01}^{12}\left(-66.98n_{01}^{12} + 3.215\times10^5 n_{01}^{21}\right) - 267.91\left(n_{01}^{21}\right)^2\right] \\ &\div \left[\left(2n_{01}^{11} + n_{01}^{12} + n_{01}^{21}\right)\right. \\ &\times \left.\left(1.461n_{01}^{11} - 1.290n_{01}^{22} - 3.441n_{01}^{12} + 1.652\times10^4 n_{01}^{21}\right)\right] \\ R_{\hat{\mu}^{21}}(\cdot) &= 2.055\left[n_{01}^{11}\left(2.137n_{01}^{11} + 3.982n_{01}^{22} - 27.73n_{01}^{12} + 55.47n_{01}^{22}\right)\right. \\ &+ n_{01}^{22}\left(1.871n_{01}^{22} + 25.73n_{01}^{12} - 51.47n_{01}^{21}\right) \\ &+ \left.n_{01}^{12}\left(66.98n_{01}^{12} - 3.215\times10^5 n_{01}^{21}\right) + 267.91\left(n_{01}^{21}\right)^2\right] \\ &\div \left[\left(2n_{01}^{11} + n_{01}^{12} + n_{01}^{21}\right)\right. \\ &\times \left.\left(2.814n_{01}^{11} - 2.677n_{01}^{22} - 1.652\times10^4 n_{01}^{12} + 13.77n_{01}^{21}\right)\right] \end{aligned}$$

By substituting the mean values for n_{01}^{ij} into the above equations they can be simplified further. Let $n_{01}^{1\cdot} = n_{01}^{11} + n_{01}^{12}$ and $n_{01}^{2\cdot} = n_{01}^{22} + n_{01}^{21}$, then $\mathrm{E}[n_{01}^{ij}] = n_{01}^{i\cdot} \times \mathrm{P}_{01}^{ij}$ and:

$$\begin{aligned} \overline{R}_{\hat{\mu}^{12}}(K) &= \frac{7.081\times10^5 K}{\left(6.705 + 5.110\times10^4 K\right)\left(143.0 + 3.482K\right)} \\ \overline{R}_{\hat{\mu}^{21}}(K) &= \frac{5.311\times10^5 K}{\left(7.659\times10^3 + 9.568K\right)\left(3.482 + 73.26K\right)} \end{aligned}$$

where $K = n_{01}^{2\cdot}/n_{01}^{1\cdot}$. These are again independent of sample size, and only depend on the ratio of the numbers of lives initially under observation in each starting state. These variance ratios are illustrated in Figure 5.31. It is noticeable that they are always less than unity and that with increasing K (increasing proportion of lives starting in state 2) the variance ratio, $\overline{R}_{\hat{\mu}^{12}}(K)$ tends downwards away from unity,

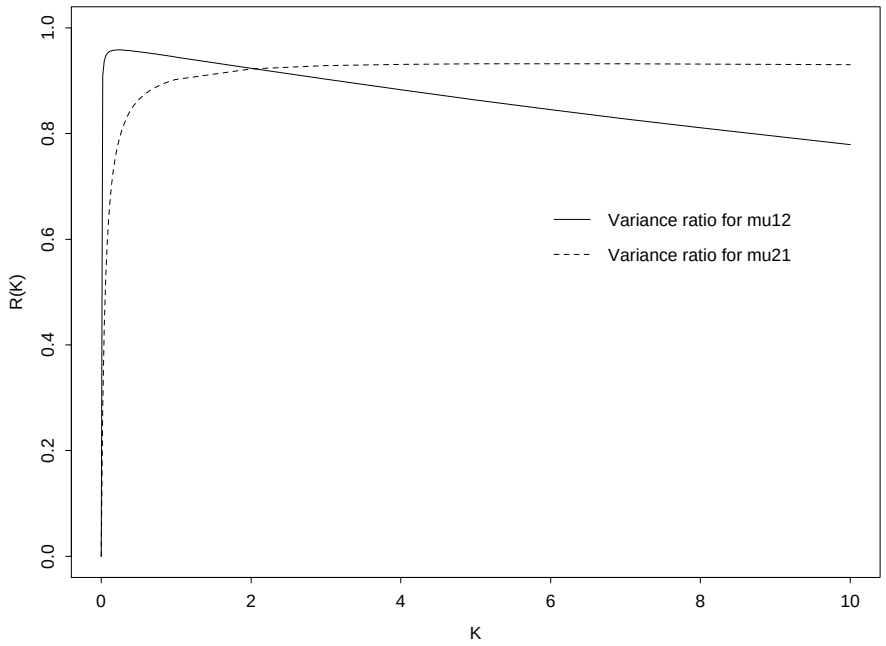

Figure 5.31: Variance ratios, $R_{\hat{\mu}^{ij}}(K)$ $(0 \leq K \leq 10)$ for μ^{12} and μ^{21} when $n_{01}^{ij} = \mathrm{E}[n_{01}^{ij}]$ in the 2-state model with two transition intensities.

while $\overline{R}_{\hat{\mu}^{21}}(K)$ tends towards unity. For the specific case that $K = \frac{1}{2}$ (as I chose for the following simulations) the values of the variance ratios are:

$$\overline{R}_{\hat{\mu}^{12}}(0.5) = 0.9548 \tag{5.55}$$

$$\overline{R}_{\hat{\mu}^{21}}(0.5) = 0.8638 \tag{5.56}$$

The results from 1,000 simulations (using the same method as described in the previous section), with $\mu^{12} = 0.1$, $\mu^{21} = 0.05$, $n_{01}^{1\cdot} = 2,000$ and $n_{01}^{2\cdot} = 1,000$ $(K = 0.5)$ are illustrated in Figure 5.32. The 'bootstrap' variance estimates in these graphs may not be accurate as they are only based on 1,000 simulations — I have included them for illustration only. The mean value of the estimated transition intensities for these 1,000 simulations are $\overline{\mu}^{12} = 0.1006$ and $\overline{\mu}^{21} = 0.0496$. From the graphs in Figure 5.32 the difference between the two variance estimates seems fairly constant, so it is then not surprising that the mean of the variance ratios in the simulations for μ^{12} and μ^{21}, which are 0.9550 and 0.8632, respectively — are in very close agreement with the variance ratios calcuated in equations 5.55 and 5.56,

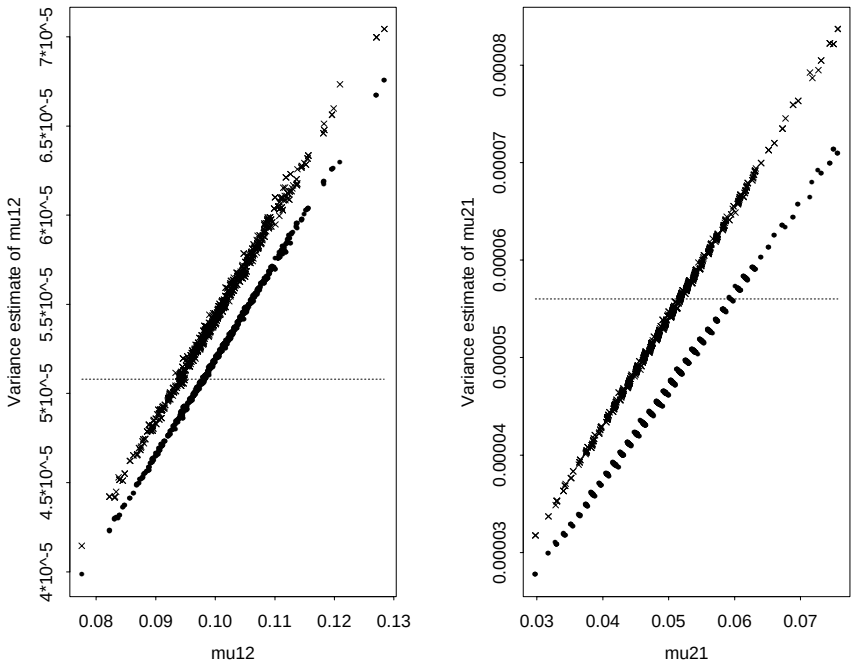

Figure 5.32: Variance estimates of the transition intensities for the 2-state model with two transition intensities using the asymptotic method (•), the information matrix (×) and the 'boot-strap' variance estimate (horizontal line) for 1,000 simulations, with $\mu^{12} = 0.1$, $\mu^{21} = 0.05$, $N^1 = 2,000$ and $N^2 = 1,000$ ($K = 0.5$).

even though they are slightly different measures.

It is very noticeable that the variance ratio has become considerably lower with the addition of just one more transition intensity in the two state model. In the first model with $\mu^{12} = 0.1$ the variance ratio is 0.9983, whereas for the second model, depending on the transition intensity and ratio of lives in each starting state, it is between 0.86 and 0.95 — quite a large reduction.

This is not surprising, considering the information that is lost with only the knowledge of the states that each life starts and ends in (as assumed in the information matrix method) rather then the life histories (as assumed in the asymptotic method) — not only is the timing of each transition not known (as in the 2-state model with one transition intensity), but in this model the number of transitions between states is also unknown (since knowing the state a life starts and ends in does not tell you how many times they have travelled between states). So the difference in information used by the asymptotic method and the information matrix method

is greater than in the previous model, which is clearly shown by lower values of the variance ratio.

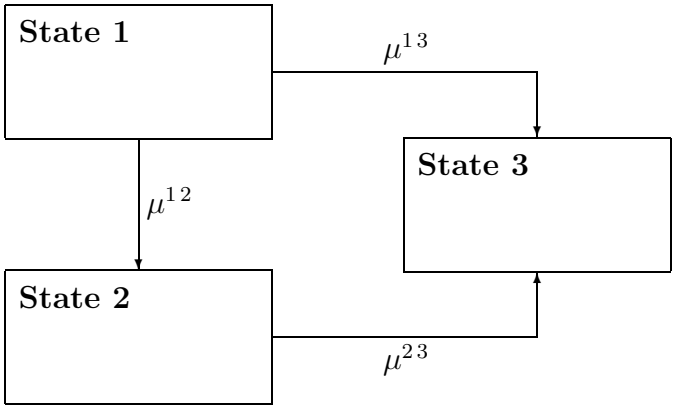

Figure 5.33: A simple 3-state model with three transition intensities.

The final model that I investigate, a 3-state model with three transition intensities, is illustrated in Figure 5.33. I use the same methodology and notation in this model as the previous one. The transition probabilities can be written as:

$$\begin{aligned}
\mathrm{P}_{01}^{11} &= \mathrm{e}^{-\left(\mu^{12}+\mu^{13}\right)} \\
\mathrm{P}_{01}^{12} &= \frac{\mu^{12}}{\mu^{12}+\mu^{13}-\mu^{23}}\left[\mathrm{e}^{-\mu^{23}}-\mathrm{e}^{-\left(\mu^{12}+\mu^{13}\right)}\right] \\
\mathrm{P}_{01}^{13} &= 1-\frac{1}{\mu^{12}+\mu^{13}-\mu^{23}}\left[\mu^{12}\mathrm{e}^{-\mu^{23}}+\left(\mu^{13}-\mu^{23}\right)\mathrm{e}^{-\left(\mu^{12}+\mu^{13}\right)}\right] \\
\mathrm{P}_{01}^{22} &= \mathrm{e}^{-\mu^{23}} \\
\mathrm{P}_{01}^{23} &= 1-\mathrm{e}^{-\mu^{23}}
\end{aligned}$$

and the asymptotic variance estimates, $V_A(\mu^{12})$, $V_A(\mu^{13})$ and $V_A(\mu^{23})$, using the census method to approximate the expected waiting times are:

$$\begin{aligned}
V_A\left(\mu^{12}\right) &= \frac{\mu^{12}}{n_{01}^{11}+\frac{n_{01}^{12}+n_{01}^{13}}{2}} \\
V_A\left(\mu^{13}\right) &= \frac{\mu^{13}}{n_{01}^{11}+\frac{n_{01}^{12}+n_{01}^{13}}{2}} \\
V_A\left(\mu^{23}\right) &= \frac{\mu^{23}}{n_{01}^{22}+\frac{n_{01}^{12}+n_{01}^{23}}{2}}
\end{aligned}$$

To simplify the following equations, I now fix the transition intensities as $\mu^{12} = 0.1$, $\mu^{13} = 0.025$ and $\mu^{23} = 0.05$. Then using Maple software, the variance ratios

$R_{\hat{\mu}^{12}}(\cdot)$, $R_{\hat{\mu}^{13}}(\cdot)$ and $R_{\hat{\mu}^{23}}(\cdot)$ are:

$$\begin{aligned}
R_{\hat{\mu}^{12}}(\cdot) &= 2.000\left[\left(n_{01}^{12}\right)^2\left(20.89n_{01}^{12}-1.214\times10^{10}n_{01}^{12}-3.332\times10^{9}n_{01}^{23}\right)\right.\\
&+ \left(n_{01}^{13}\right)^2\left(1.669\times10^{11}n_{01}^{12}-4.391\times10^{12}n_{01}^{13}-1.939\times10^{12}n_{01}^{23}\right)\\
&+ \left.5.284\times10^{13}n_{01}^{12}n_{01}^{23}n_{01}^{23}\right]\div\left[\left(2n_{01}^{11}+n_{01}^{12}+n_{01}^{13}\right)\right.\\
&\times \left(2.089\left(n_{01}^{12}\right)^2-1.214\times10^{9}n_{01}^{12}n_{01}^{13}-3.332\times10^{8}n_{01}^{12}n_{01}^{23}\right.\\
&+ \left.\left.1.629\times10^{10}\left(n_{01}^{13}\right)^2+5.288\times10^{12}n_{01}^{13}n_{01}^{23}\right)\right]\\
R_{\hat{\mu}^{13}}(\cdot) &= 5\times10^{-7}\left[\left(n_{01}^{12}\right)^2\left(20.89n_{01}^{12}-1.214\times10^{10}n_{01}^{12}-3.332\times10^{9}n_{01}^{23}\right)\right.\\
&+ \left(n_{01}^{13}\right)^2\left(1.669\times10^{11}n_{01}^{12}-4.391\times10^{12}n_{01}^{13}-1.939\times10^{12}n_{01}^{23}\right)\\
&+ \left.5.284\times10^{13}n_{01}^{12}n_{01}^{23}n_{01}^{23}\right]\div\left[\left(2n_{01}^{11}+n_{01}^{12}+n_{01}^{13}\right)\right.\\
&\times \left(-8.331\left(n_{01}^{12}\right)^2+4.446\times10^{2}n_{01}^{12}n_{01}^{13}+3.996\times10^{4}n_{01}^{12}n_{01}^{23}\right.\\
&- \left.\left.2.723\times10^{2}\left(n_{01}^{13}\right)^2+1.924\times10^{2}n_{01}^{13}n_{01}^{23}\right)\right]\\
R_{\hat{\mu}^{23}}(\cdot) &= 5\times10^{-7}\left[\left(n_{01}^{12}\right)^2\left(20.89n_{01}^{12}-1.214\times10^{10}n_{01}^{12}-3.332\times10^{9}n_{01}^{23}\right)\right.\\
&+ \left(n_{01}^{13}\right)^2\left(1.669\times10^{11}n_{01}^{12}-4.391\times10^{12}n_{01}^{13}-1.939\times10^{12}n_{01}^{23}\right)\\
&+ \left.5.284\times10^{13}n_{01}^{12}n_{01}^{23}n_{01}^{23}\right]\div\left[\left(2n_{01}^{11}+n_{01}^{12}+n_{01}^{13}\right)\right.\\
&\times \left.\left(-4.165\left(n_{01}^{12}\right)^2+6.607\times10^{4}n_{01}^{12}n_{01}^{13}+2.424\times10^{2}\left(n_{01}^{13}\right)^2\right)\right]
\end{aligned}$$

By substituting the mean values for n_{01}^{ij} into the above equations they can be simplified further. Let $n_{01}^{1\cdot}=\sum_j n_{01}^{1j}$ and $n_{01}^{2\cdot}=\sum_j n_{01}^{2j}$, then $\mathrm{E}[n_{01}^{ij}]=n_{01}^{i\cdot}\times\mathrm{P}_{01}^{ij}$ and:

$$\begin{aligned}
\overline{R}_{\hat{\mu}^{12}}(K) &= \frac{6480.57K-1.912\times10^{-7}}{6668.23K+8.015}\\
\overline{R}_{\hat{\mu}^{13}}(K) &= \frac{810.07K-2.390\times10^{-8}}{894.18K+4.008}\\
\overline{R}_{\hat{\mu}^{43}}(K) &= \frac{195.04K-5.755\times10^{-9}}{195.12K+9.164}
\end{aligned}$$

where $K=n_{01}^{2\cdot}/n_{01}^{1\cdot}$. These are again independent of sample size, and only depend on the ratio of the numbers of lives initially under observation in each starting state. These variance ratios are illustrated in Figure 5.34. It is noticeable that they are always less than unity and that with increasing K (increasing proportion of lives starting in state 2) all the variance ratios increase, but with an upper limit of less than unity. For the specific case that $K=\frac{1}{2}$ (as I chose for the following simulations)

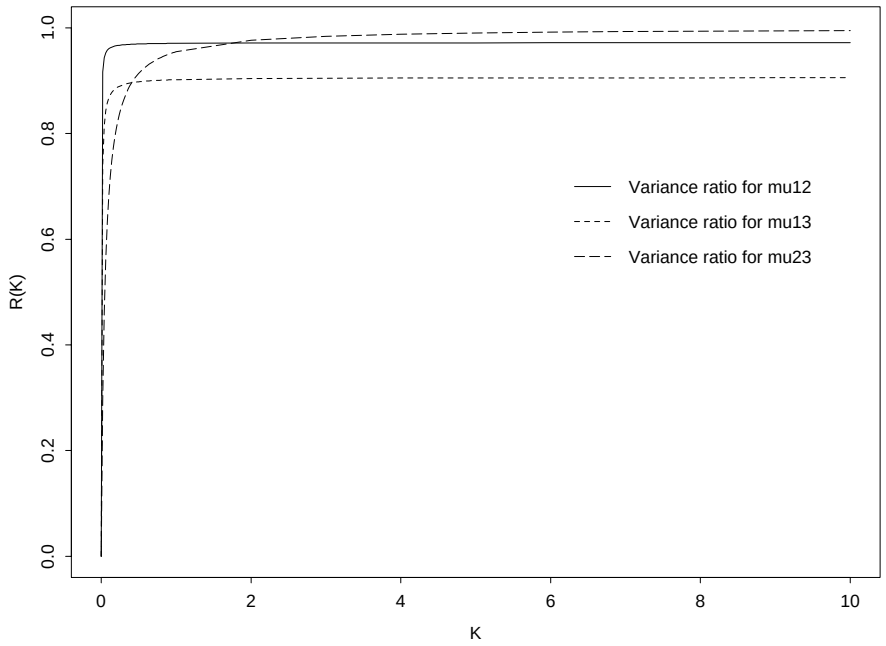

Figure 5.34: Variance ratios, $R_{\hat{\mu}^{ij}}(K)$ $(0 \leq K \leq 10)$ for μ^{12}, μ^{13} and μ^{23} when $n_{01}^{ij} = \mathrm{E}[n_{01}^{ij}]$ in the 3-state model.

the values of the variance ratios are:

$$\overline{R}_{\hat{\mu}^{12}}(0.5) = 0.9695 \tag{5.57}$$

$$\overline{R}_{\hat{\mu}^{13}}(0.5) = 0.8979 \tag{5.58}$$

$$\overline{R}_{\hat{\mu}^{23}}(0.5) = 0.9138 \tag{5.59}$$

The results from 1,000 simulations (using the same method as described in the previous section), with $\mu^{12} = 0.1$, $\mu^{13} = 0.025$, $\mu^{23} = 0.05$, $N^1 = 5,000$ and $N^2 = 2,500$ $(K = 0.5)$ are illustrated in Figure 5.35. The 'bootstrap' variance estimates in these graphs may not be accurate as they are only based on 1,000 simulations — I have included them for illustration only. The mean value of the transition intensities for these 1,000 simulations are $\overline{\mu}^{12} = 0.1003$, $\overline{\mu}^{13} = 0.0249$ and $\overline{\mu}^{23} = 0.0499$. From the graphs in Figure 5.32 the difference between the two variance estimates seems fairly constant, so it is then not surprising that the mean of the variance ratios in the simulations for μ^{12}, μ^{13} and μ^{23}, which are 0.9696, 0.8968 and 0.9136, respectively — are in very close agreement with the variance ratios given

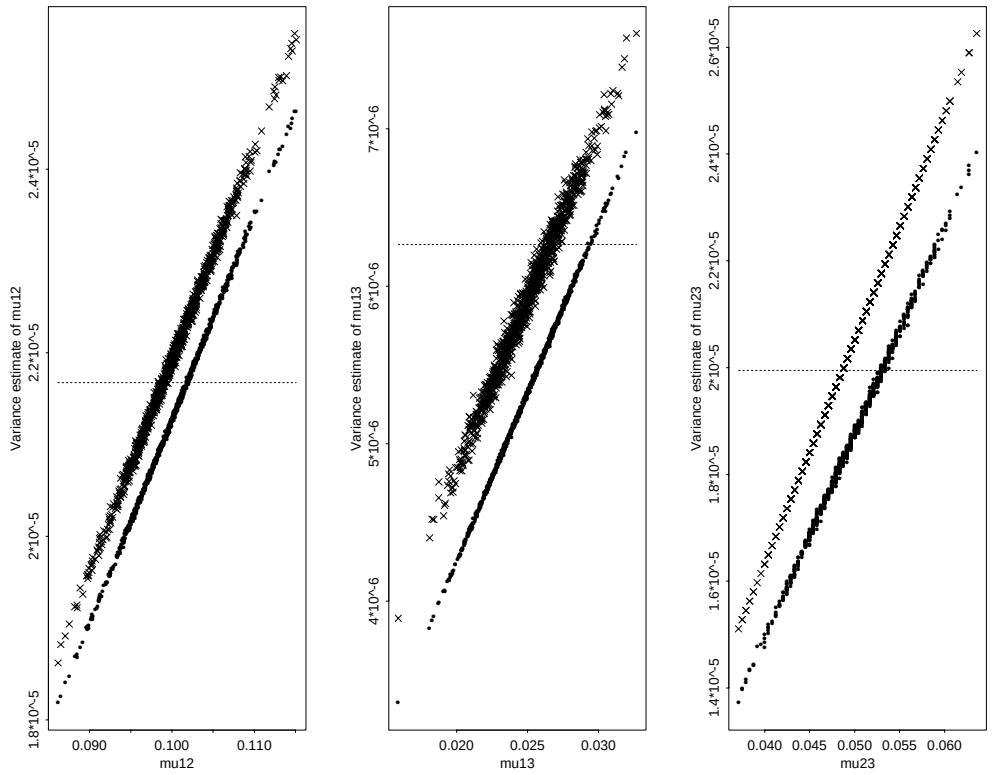

Figure 5.35: Variance estimates of the transition intensities for the 3-state model using the asymptotic method (•), the information matrix (×) and the 'boot-strap' variance estimate (horizontal line) for 1,000 simulations, with $\mu^{12} = 0.1$, $\mu^{13} = 0.025$, $\mu^{23} = 0.05$, $N^1 = 5,000$ and $N^2 = 2,500$ ($K = 0.5$).

in equations 5.57 to 5.59, even though they are slightly different measures.

The variance ratios in this 3-state model (0.90 to 0.97) are comparable to the variance ratios in the 2-state model with two transition intensities (0.86 to 0.95). While they are considerably less than unity, they have not become yet smaller with the introduction of an extra state and transition intensity. This may be because, while the model has become more complex, the number of transitions a life can make is limited to a maximum of two and in some cases, from the knowledge of the state in which a life started and finished, the exact number of transitions that a life has made is known (i.e. if a life starts in state 1 and ends in state 2, then that life has only made one transition in that time).

Care is required in interpreting the results presented in this section, as the variance ratios depend not only on the model, the magnitude of the transition intensities

and the numbers of lives starting in each state, but also on the numbers of transitions between states. However, these results have shown that variance estimates of transition intensities can be substantially increased, even in simple models, if the only data available are the states in which a life is at the start and end of a survey period, rather than their complete life histories.

In summary, in this section I have compared, in three simple models, two methods of calculating variance estimates of transition intensities, one which uses complete life history data (i.e. the knowledge of the states each life moved to and the timing of these transitions), the asymptotic method, and one which uses only partial data (i.e. the knowledge of which state a life starts and ends in), the information matrix method. For the simplest model, a 2-state model with one transition intensity, the difference between the variance estimates of the two methods is negligible, especially when the underlying transition intensity is small in magnitude. The reason for this may be because the only difference in the information used by the two methods is small — the timing of any transition. However, for the slightly more complex models, the difference between variance estimates increases rapidly, with variance estimates from the asymptotic method up to 14% smaller than those using the information matrix method. This is representative of the information lost in only knowing the starting and ending states that a life is in, rather than their complete life history (i.e. the timing, and number of transitions between states). Given the complexity of the disability model (7 states with 36 transition intensities), it seems unreasonable to use the asymptotic variance estimator, given that the NLTCS only provides partial information about the lives in the survey. The figures in Table 5.59 indicate that, in the disability model, if complete life histories had been available, and thus the asymptotic method would have been valid then the variance estimates would be reduced by up to 98%, compared with those using the information matrix with partial data.

Applied to data collection, this is strong motivation, for any model more complex than the basic mortality model, to collect complete life history data and not just data on the states in which lives are at fixed points in time. The cost of using partial data is an increase in the variance estimates of the transition intensities —

an increase which, even for simple models, can be substantial.

In the next section I look at methods for estimating the variance of the constrained (positive) MLEs of the transition intensities in the disability model.

5.4 Calculation of Variance Estimates for the Transition Intensities in the Disability Model

The aim of this section is to estimate the variance of the constrained (positive) MLEs of the transition intensities in the disability model. The maximum likelihood method used in the previous section cannot be used to calculate variance estimates of the constrained (positive) MLEs of the transition intensities, since, for these estimates, the set of first derivatives of the log-likelihood function will be non-zero. (The unconstrained MLEs maximise the log-likelihood, whereas the constrained (positive) MLEs maximise the log-likelihood in the non-negative region of the parameter space). I first look at calculating 'boot-strap' variance estimates (explained below), however, the amount of computer time required to calculate these estimates is prohibitive. Using the same methodology as the 'boot-strap' method, it is possible to adjust the maximum likelihood technique to calculate equivalent variance estimates in a much more efficient manner. The variance estimates from the 'boot-strap' method are then used to check the reasonableness of those calculated using the adjusted maximum likelihood method — and it is illustrated that the former do converge (given sufficient simulations) to the latter.

The first method is to calculate 'boot-strap' variance estimates. That is, run a large number of simulations using the method described in Section 5.2, and from these, estimate the corresponding transition intensities. The variance of these simulated transition intensities are estimates of the variance of the original transition intensities. However, it is necessary to find how many simulations are required for these 'boot-strap' variance estimates to converge.

I initially ran 10,000 simulations based on the constrained (positive) MLEs of the transition intensities for the data set of males and females aged 65–74 in the 1982–84 NLTCS. I then found the maximum and minimum variance estimate of 100

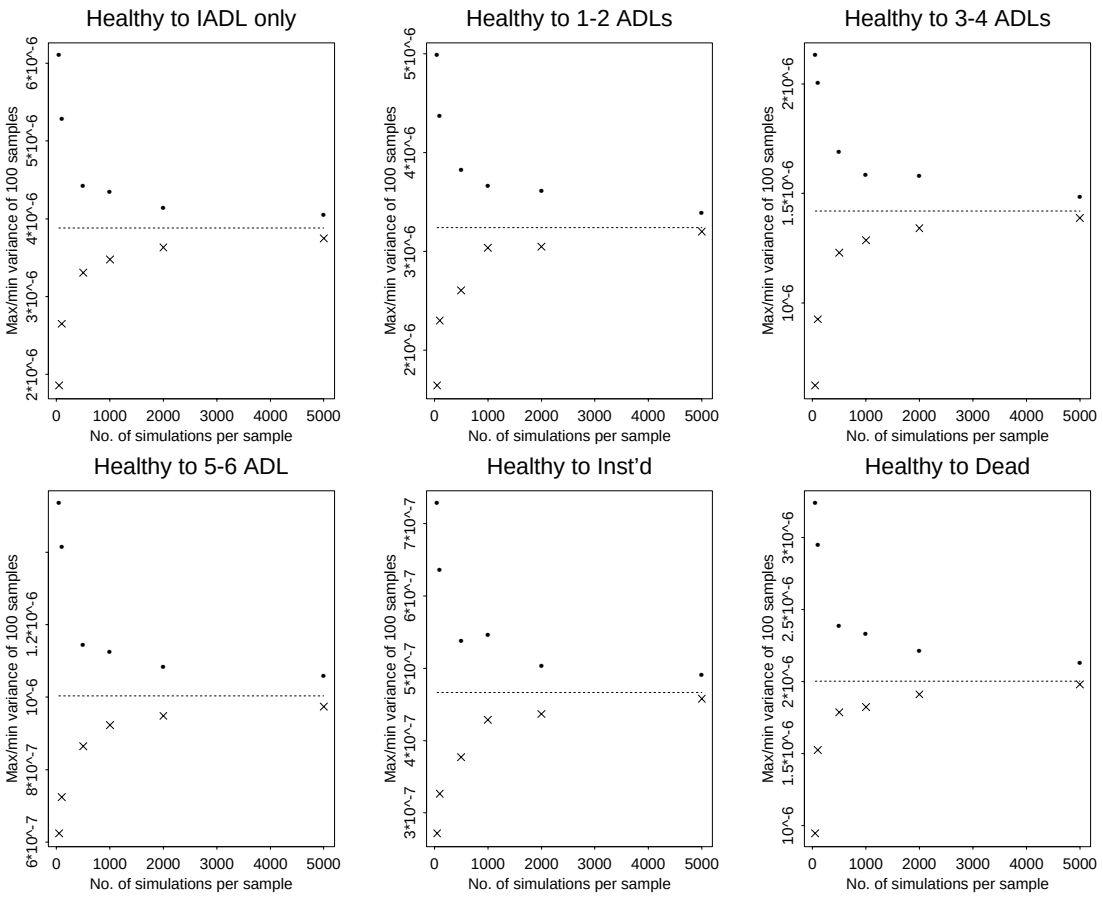

Figure 5.36: Convergence of the 'boot-strap' variance estimates for the constrained (positive) MLEs of the transition intensities out of the 'Healthy' state, showing the maximum (•) and minimum (×) 'boot-strap' variance estimate of 100 samples, for samples of size n ($n = 50$, 100, 500, 1,000, 2,000 and 5,000) and where the horizontal line is the variance estimate using an adjusted maximum likelihood approach, for males and females aged 65–74 years in the 1982–84 NLTCS.

samples, each based on n ($n = 50$, 100, 500, 1,000, 2,000 and 5,000) simulations from the initial 10,000 simulations (the n simulations from 10,000 were chosen at random without replacement). The reason for selecting from only 10,000 simulations is lack of computer time — each simulation took about 30 seconds to run, with an additional 10 seconds to calculate the covariance matrix using the information matrix. The convergence of the variance estimates is illustrated in Figures 5.36 to 5.41 — even using 5,000 simulations there is still some uncertainty. I will now discuss how the information matrix can be used to calculate the variance estimates much more efficiently (in terms of computer time).

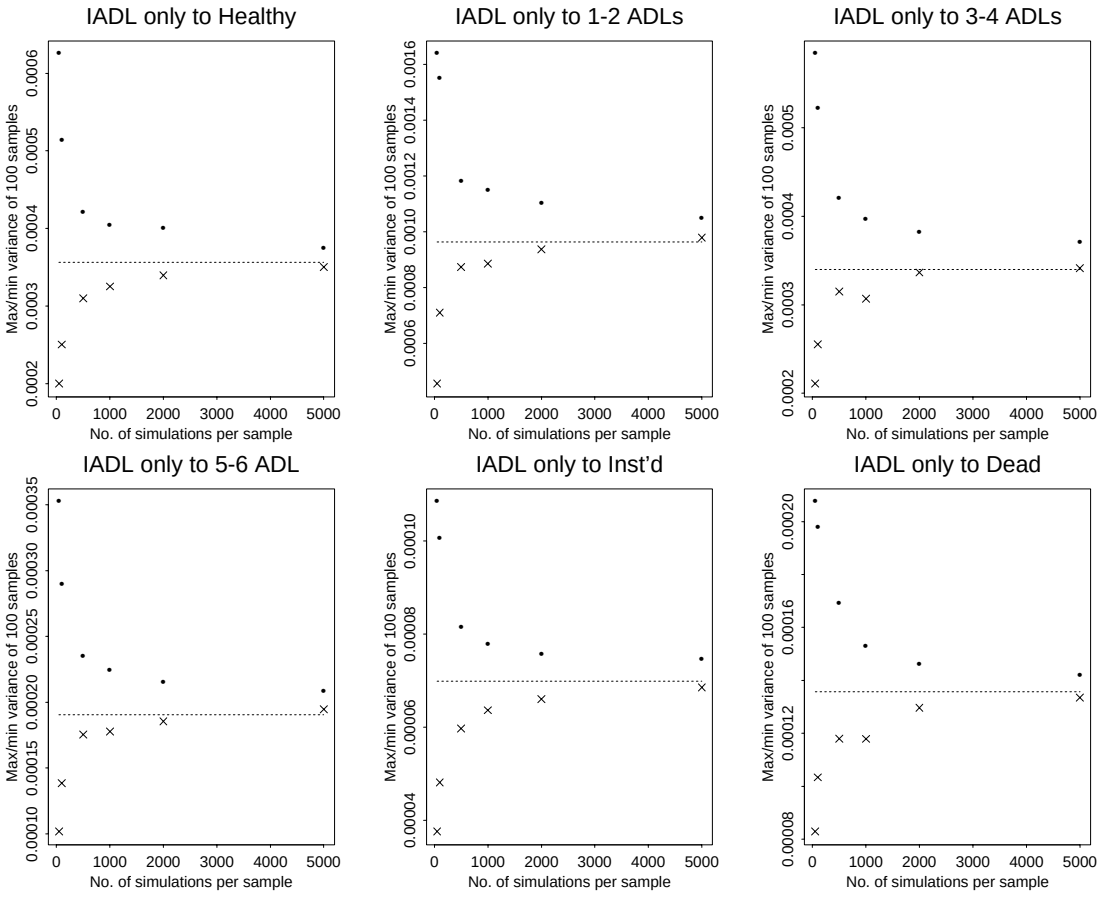

Figure 5.37: Convergence of the 'boot-strap' variance estimates for the constrained (positive) MLEs of the transition intensities out of the 'IADL only' state, showing the maximum (•) and minimum (×) 'boot-strap' variance estimate of 100 samples, for samples of size n ($n = 50$, 100, 500, 1,000, 2,000 and 5,000) and where the horizontal line is the variance estimate using an adjusted maximum likelihood approach, for males and females aged 65–74 years in the 1982–84 NLTCS.

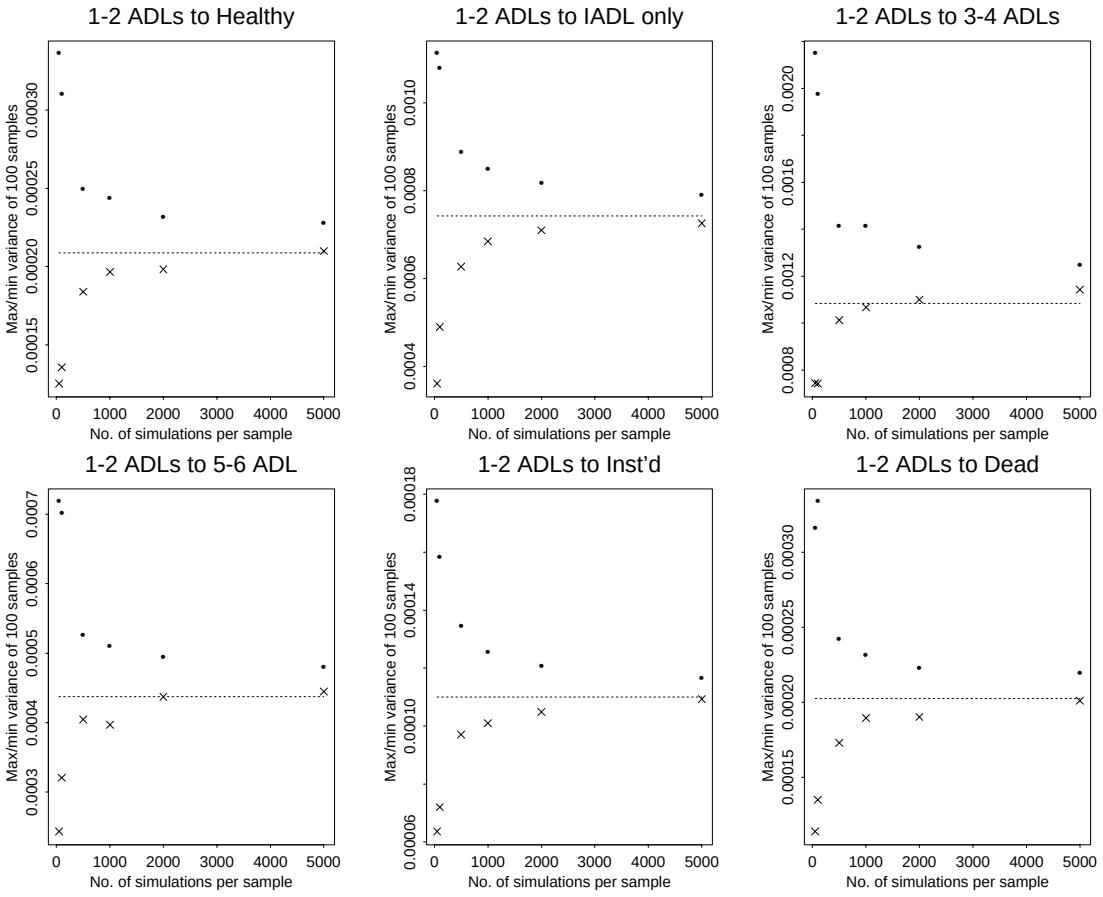

Figure 5.38: Convergence of the 'boot-strap' variance estimates for the constrained (positive) MLEs of the transition intensities out of the '1–2 ADLs' state, showing the maximum (•) and minimum (×) 'boot-strap' variance estimate of 100 samples, for samples of size n ($n = 50$, 100, 500, 1,000, 2,000 and 5,000) and where the horizontal line is the variance estimate using an adjusted maximum likelihood approach, for males and females aged 65–74 years in the 1982–84 NLTCS.

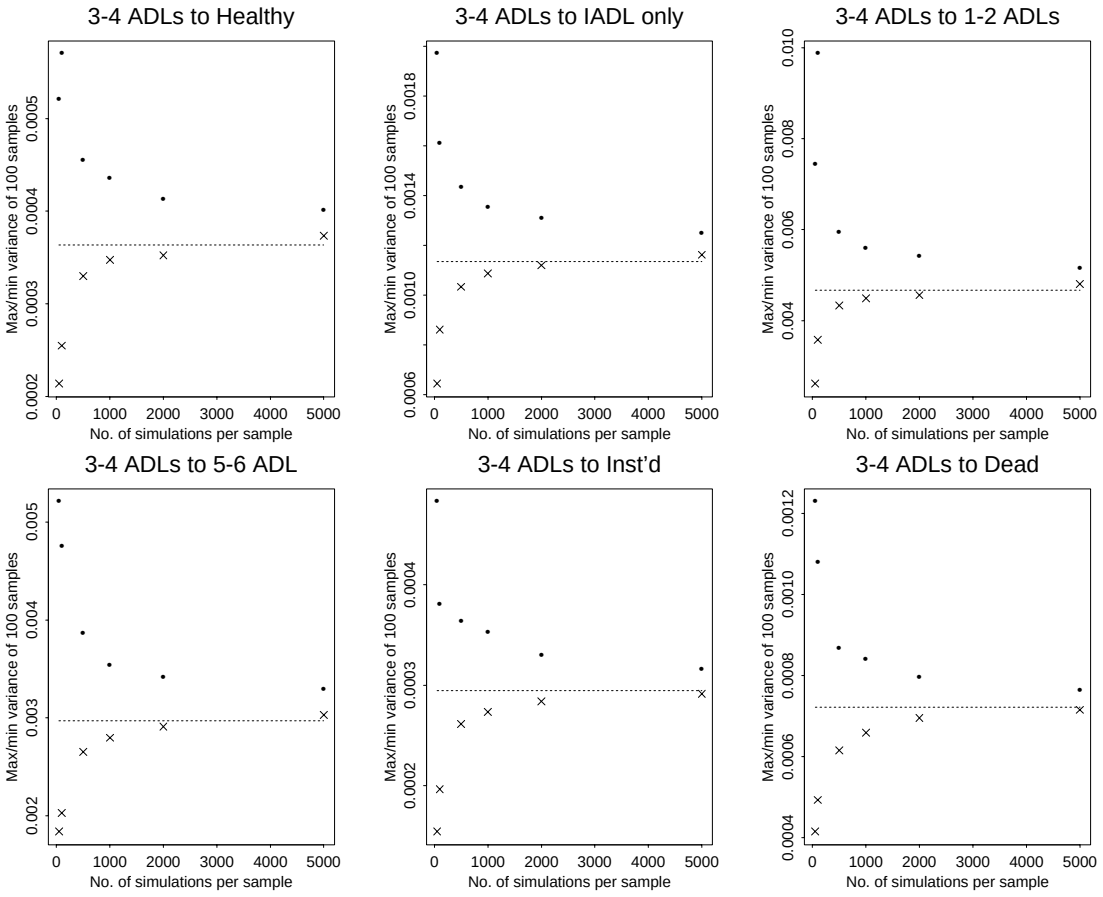

Figure 5.39: Convergence of the 'boot-strap' variance estimates for the constrained (positive) MLEs of the transition intensities out of the '3–4 ADLs' state, showing the maximum (•) and minimum (×) 'boot-strap' variance estimate of 100 samples, for samples of size n ($n = 50$, 100, 500, 1,000, 2,000 and 5,000) and where the horizontal line is the variance estimate using an adjusted maximum likelihood approach, for males and females aged 65–74 years in the 1982–84 NLTCS.

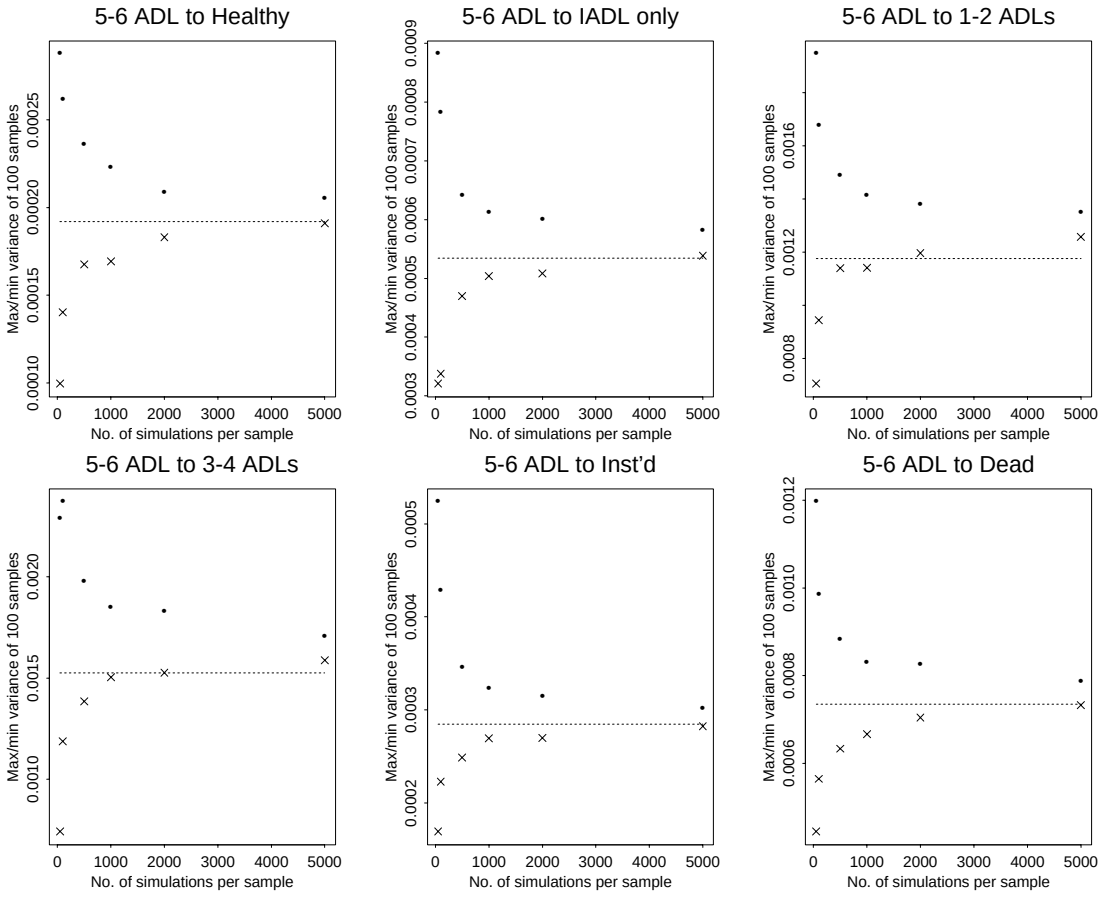

Figure 5.40: Convergence of the 'boot-strap' variance estimates for the constrained (positive) MLEs of the transition intensities out of the '5–6 ADLs' state, showing the maximum (•) and minimum (×) 'boot-strap' variance estimate of 100 samples, for samples of size n ($n = 50$, 100, 500, 1,000, 2,000 and 5,000) and where the horizontal line is the variance estimate using an adjusted maximum likelihood approach, for males and females aged 65–74 years in the 1982–84 NLTCS.

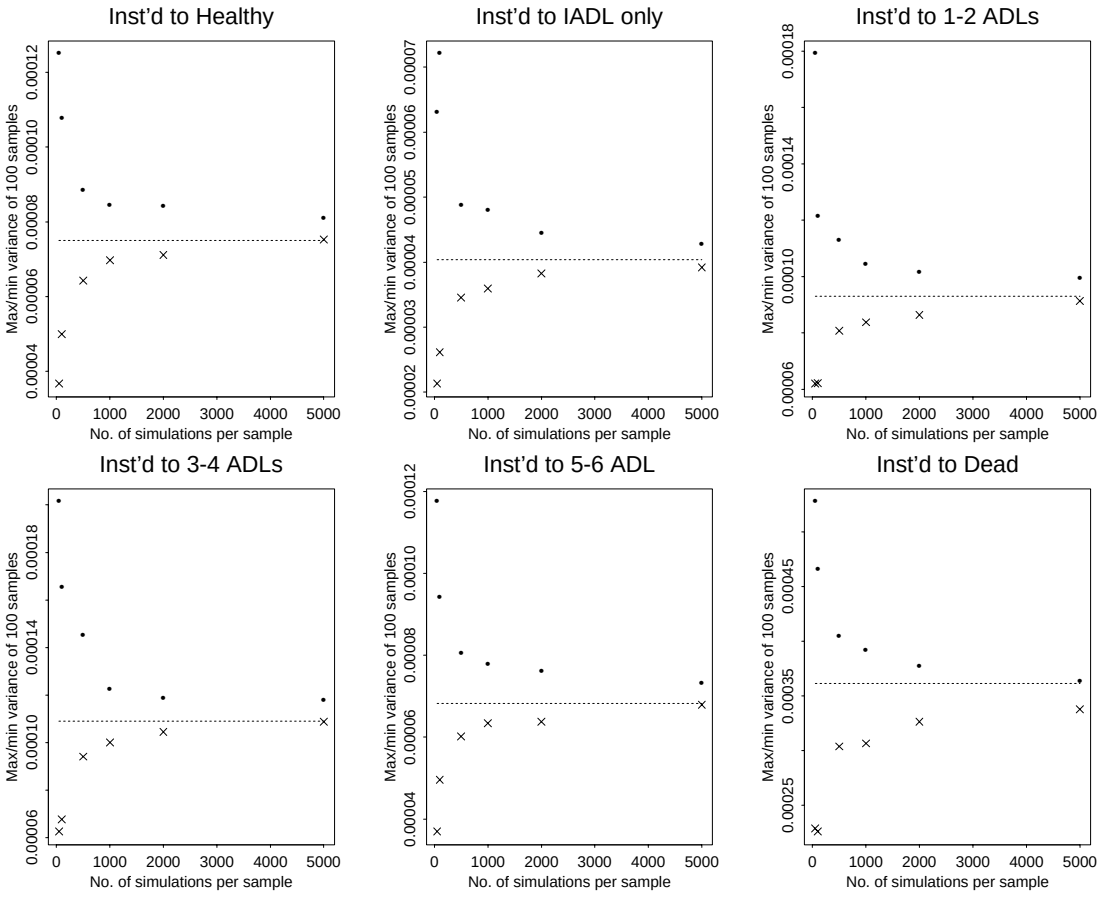

Figure 5.41: Convergence of the 'boot-strap' variance estimates for the constrained (positive) MLEs of the transition intensities out of the 'Institutionalized' state, showing the maximum (•) and minimum (×) 'boot-strap' variance estimate of 100 samples, for samples of size n (n = 50, 100, 500, 1,000, 2,000 and 5,000) and where the horizontal line is the variance estimate using an adjusted maximum likelihood approach, for males and females aged 65–74 years in the 1982–84 NLTCS.

It is not possible to use the information matrix method as it stands as the constrained (positive) MLEs will result in non-zero first derivatives of the log-likelihood function. However, this method can be adapted, to calculate variance estimates consistent with these 'boot-strap' estimates. The 'boot-strap' method assumes that the estimated transition intensities (constrained (positive) MLEs in this case) are the true population transition intensities. From these the transition probabilities are calculated, which are then used as the probabilities in the multinomial distribution for the purposes of sampling (see Section 5.2). The resulting variance estimates are those of the constrained (positive) MLEs of the transition intensities, under the assumption that the constrained (positive) MLEs of the transition intensities are the true population transition intensities. Under the same assumption, variance estimates using the information matrix can be calculated as follows:

1. as before, from the constrained (positive) MLEs of the transition intensities, calculate the corresponding transition probabilities;

2. use these transition probabilities to calculate the expected number of lives that move between states, by multiplying the number of lives starting in a given state from the data, by the corresponding transition probability; then

3. as the constrained (positive) MLEs of the transition intensities maximise the likelihood function using these numbers of transitions, the information matrix method can be used to estimate the variance (as the first derivatives of the log-likelihood will now be zero).

Or more specifically, let $n^{ij}_{r\,r+t}$ be the actual number of lives in state i and age group r at the start of the survey period who are in state j after t years and let $n^{i\cdot}_{r} = \sum_{j=1}^{7} n^{ij}_{r\,r+t}$ be the number of lives in age group r starting in state i. Then the expected number of lives, consistent with the transition probabilities calculated from the constrained (positive) MLEs of the transition intensities ($\mathrm{P}^{ij}_{r\,r+t}(\overline{\mu}^{1\,2}_{r}, \ldots, \overline{\mu}^{n\,n-1}_{r})$) that moved from state i to state j in age group r over t years, $\overline{n}^{ij}_{r\,r+t}$, can be calculated as:

$$\overline{n}^{ij}_{r\,r+t} = n^{i\cdot}_{r}\,\mathrm{P}^{ij}_{r\,r+t}\left(\overline{\mu}^{1\,2}_{r}, \ldots, \overline{\mu}^{n\,n-1}_{r}\right) \tag{5.60}$$

Table 5.61: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		1.06×10^{-5}	9.01×10^{-6}	7.32×10^{-6}	4.09×10^{-6}	1.34×10^{-6}	7.83×10^{-6}
	70–74		1.96×10^{-5}	1.94×10^{-5}	8.29×10^{-6}	7.37×10^{-6}	5.24×10^{-6}	2.38×10^{-5}
	75–79		6.00×10^{-5}	5.02×10^{-5}	1.65×10^{-5}	1.90×10^{-5}	1.14×10^{-5}	4.82×10^{-5}
	80–84		2.62×10^{-4}	2.40×10^{-4}	1.53×10^{-4}	1.46×10^{-4}	6.72×10^{-5}	1.63×10^{-4}
IADL only	65–69	1.75×10^{-3}		3.37×10^{-3}	2.12×10^{-3}	1.74×10^{-3}	1.32×10^{-4}	8.23×10^{-4}
	70–74	2.09×10^{-3}		2.58×10^{-3}	9.74×10^{-4}	4.99×10^{-4}	4.42×10^{-4}	1.06×10^{-3}
	75–79	1.21×10^{-3}		3.41×10^{-3}	1.77×10^{-3}	2.04×10^{-3}	7.62×10^{-4}	1.32×10^{-3}
	80–84	1.82×10^{-3}		1.01×10^{-2}	1.11×10^{-2}	1.14×10^{-2}	2.27×10^{-3}	3.48×10^{-3}
1–2 ADLs	65–69	1.33×10^{-3}	2.58×10^{-3}		1.06×10^{-2}	5.18×10^{-3}	4.26×10^{-4}	1.74×10^{-3}
	70–74	9.50×10^{-4}	2.68×10^{-3}		6.23×10^{-3}	2.28×10^{-3}	1.09×10^{-3}	1.57×10^{-3}
	75–79	8.76×10^{-4}	4.70×10^{-3}		3.30×10^{-3}	3.34×10^{-3}	5.70×10^{-4}	2.83×10^{-3}
	80–84	5.92×10^{-4}	2.83×10^{-3}		1.62×10^{-2}	1.02×10^{-2}	1.44×10^{-3}	3.80×10^{-3}
3–4 ADLs	65–69	3.73×10^{-3}	6.05×10^{-3}	4.07×10^{-2}		3.09×10^{-2}	2.34×10^{-3}	9.05×10^{-3}
	70–74	2.24×10^{-3}	2.09×10^{-3}	1.08×10^{-2}		1.08×10^{-2}	1.02×10^{-3}	3.97×10^{-3}
	75–79	1.37×10^{-3}	2.89×10^{-3}	7.69×10^{-3}		1.31×10^{-2}	4.35×10^{-3}	8.57×10^{-3}
	80–84	2.41×10^{-3}	2.76×10^{-2}	3.76×10^{-2}		1.61×10^{-1}	1.44×10^{-2}	2.84×10^{-2}
5–6 ADLs	65–69	1.00×10^{-3}	2.54×10^{-3}	7.41×10^{-3}	1.70×10^{-2}		1.22×10^{-3}	4.42×10^{-3}
	70–74	9.30×10^{-4}	1.38×10^{-3}	4.94×10^{-3}	4.25×10^{-3}		1.11×10^{-3}	2.85×10^{-3}
	75–79	5.48×10^{-4}	1.84×10^{-3}	1.64×10^{-3}	4.43×10^{-3}		1.98×10^{-3}	4.65×10^{-3}
	80–84	8.44×10^{-4}	6.77×10^{-3}	7.47×10^{-3}	4.06×10^{-2}		3.89×10^{-3}	9.89×10^{-3}
Inst'd	65–69	4.66×10^{-4}	3.07×10^{-4}	2.30×10^{-4}	5.67×10^{-4}	1.44×10^{-4}		1.20×10^{-3}
	70–74	4.41×10^{-4}	2.35×10^{-4}	1.28×10^{-3}	5.08×10^{-4}	9.36×10^{-4}		4.11×10^{-3}
	75–79	1.91×10^{-4}	4.08×10^{-4}	2.63×10^{-4}	2.50×10^{-4}	1.09×10^{-4}		2.29×10^{-3}
	80–84	4.05×10^{-5}	6.68×10^{-5}	7.70×10^{-5}	6.10×10^{-4}	5.24×10^{-4}		3.45×10^{-3}

Table 5.62: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for females using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		1.07×10^{-5}	9.20×10^{-6}	2.87×10^{-6}	1.60×10^{-6}	5.33×10^{-7}	3.28×10^{-6}
	70–74		2.46×10^{-5}	1.65×10^{-5}	7.40×10^{-6}	6.59×10^{-6}	3.32×10^{-6}	6.94×10^{-6}
	75–79		6.20×10^{-5}	5.51×10^{-5}	2.90×10^{-5}	1.04×10^{-5}	1.00×10^{-5}	1.76×10^{-5}
	80–84		1.54×10^{-4}	2.11×10^{-4}	8.47×10^{-5}	4.93×10^{-5}	4.87×10^{-5}	6.59×10^{-5}
IADL only	65–69	1.42×10^{-3}		3.28×10^{-3}	1.54×10^{-3}	3.36×10^{-4}	1.41×10^{-4}	3.02×10^{-4}
	70–74	9.12×10^{-4}		3.59×10^{-3}	9.84×10^{-4}	7.50×10^{-4}	3.82×10^{-4}	3.60×10^{-4}
	75–79	7.78×10^{-4}		5.17×10^{-3}	2.38×10^{-3}	6.13×10^{-4}	5.62×10^{-4}	5.98×10^{-4}
	80–84	7.30×10^{-4}		8.72×10^{-3}	2.91×10^{-3}	1.68×10^{-3}	7.37×10^{-4}	8.35×10^{-4}
1–2 ADLs	65–69	7.79×10^{-4}	2.78×10^{-3}		2.03×10^{-3}	4.77×10^{-4}	1.35×10^{-4}	4.17×10^{-4}
	70–74	5.01×10^{-4}	2.35×10^{-3}		2.04×10^{-3}	1.45×10^{-3}	4.50×10^{-4}	4.96×10^{-4}
	75–79	4.15×10^{-4}	1.91×10^{-3}		4.22×10^{-3}	1.06×10^{-3}	5.09×10^{-4}	4.90×10^{-4}
	80–84	3.89×10^{-4}	2.66×10^{-3}		3.96×10^{-3}	2.30×10^{-3}	7.49×10^{-4}	7.85×10^{-4}
3–4 ADLs	65–69	1.05×10^{-3}	4.90×10^{-3}	1.64×10^{-2}		3.83×10^{-3}	6.24×10^{-4}	1.27×10^{-3}
	70–74	8.22×10^{-4}	3.47×10^{-3}	1.02×10^{-2}		1.34×10^{-2}	1.37×10^{-3}	1.88×10^{-3}
	75–79	7.02×10^{-4}	3.90×10^{-3}	1.77×10^{-2}		1.00×10^{-2}	2.54×10^{-3}	2.52×10^{-3}
	80–84	6.45×10^{-4}	3.41×10^{-3}	1.23×10^{-2}		1.95×10^{-2}	2.87×10^{-3}	2.80×10^{-3}
5–6 ADLs	65–69	7.15×10^{-4}	2.01×10^{-3}	3.32×10^{-3}	2.65×10^{-3}		5.25×10^{-4}	1.88×10^{-3}
	70–74	5.69×10^{-4}	2.60×10^{-3}	3.83×10^{-3}	6.80×10^{-3}		1.92×10^{-3}	3.28×10^{-3}
	75–79	3.47×10^{-4}	1.03×10^{-3}	2.43×10^{-3}	4.66×10^{-3}		1.43×10^{-3}	1.70×10^{-3}
	80–84	5.83×10^{-4}	1.69×10^{-3}	4.03×10^{-3}	7.40×10^{-3}		1.95×10^{-3}	2.64×10^{-3}
Inst'd	65–69	3.86×10^{-4}	9.39×10^{-5}	3.94×10^{-4}	4.17×10^{-4}	5.28×10^{-5}		4.58×10^{-4}
	70–74	1.06×10^{-4}	1.28×10^{-4}	1.17×10^{-4}	2.61×10^{-4}	3.35×10^{-4}		1.26×10^{-3}
	75–79	3.72×10^{-5}	9.38×10^{-5}	7.16×10^{-5}	8.22×10^{-5}	1.41×10^{-4}		7.11×10^{-4}
	80–84	2.17×10^{-5}	3.72×10^{-5}	4.45×10^{-5}	4.60×10^{-5}	2.42×10^{-5}		4.30×10^{-4}

Table 5.63: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		5.42×10^{-6}	4.87×10^{-6}	2.13×10^{-6}	1.20×10^{-6}	4.18×10^{-7}	2.53×10^{-6}
	70–74		1.16×10^{-5}	8.90×10^{-6}	3.92×10^{-6}	3.46×10^{-6}	1.98×10^{-6}	6.50×10^{-6}
	75–79		3.16×10^{-5}	2.61×10^{-5}	1.25×10^{-5}	6.65×10^{-6}	5.44×10^{-6}	1.39×10^{-5}
	80–84		9.57×10^{-5}	1.20×10^{-4}	4.97×10^{-5}	3.50×10^{-5}	2.73×10^{-5}	4.74×10^{-5}
IADL only	65–69	7.95×10^{-4}		1.87×10^{-3}	8.38×10^{-4}	3.98×10^{-4}	7.10×10^{-5}	2.53×10^{-4}
	70–74	6.47×10^{-4}		1.84×10^{-3}	5.59×10^{-4}	3.49×10^{-4}	2.21×10^{-4}	2.87×10^{-4}
	75–79	4.74×10^{-4}		2.27×10^{-3}	1.07×10^{-3}	5.24×10^{-4}	3.32×10^{-4}	4.12×10^{-4}
	80–84	5.19×10^{-4}		5.26×10^{-3}	2.28×10^{-3}	1.66×10^{-3}	5.31×10^{-4}	7.01×10^{-4}
1–2 ADLs	65–69	5.24×10^{-4}	1.59×10^{-3}		2.36×10^{-3}	7.44×10^{-4}	1.14×10^{-4}	3.96×10^{-4}
	70–74	3.32×10^{-4}	1.34×10^{-3}		1.78×10^{-3}	9.48×10^{-4}	3.33×10^{-4}	4.07×10^{-4}
	75–79	2.82×10^{-4}	1.33×10^{-3}		2.15×10^{-3}	7.64×10^{-4}	2.87×10^{-4}	4.68×10^{-4}
	80–84	2.48×10^{-4}	1.53×10^{-3}		3.34×10^{-3}	2.06×10^{-3}	5.03×10^{-4}	7.29×10^{-4}
3–4 ADLs	65–69	8.57×10^{-4}	3.20×10^{-3}	1.43×10^{-2}		4.81×10^{-3}	5.65×10^{-4}	1.63×10^{-3}
	70–74	6.65×10^{-4}	1.57×10^{-3}	5.84×10^{-3}		6.60×10^{-3}	6.16×10^{-4}	1.29×10^{-3}
	75–79	4.29×10^{-4}	2.09×10^{-3}	8.04×10^{-3}		5.87×10^{-3}	1.69×10^{-3}	2.09×10^{-3}
	80–84	4.73×10^{-4}	3.17×10^{-3}	1.00×10^{-2}		1.94×10^{-2}	2.38×10^{-3}	3.10×10^{-3}
5–6 ADLs	65–69	4.08×10^{-4}	1.14×10^{-3}	2.48×10^{-3}	3.12×10^{-3}		3.85×10^{-4}	1.40×10^{-3}
	70–74	3.74×10^{-4}	9.98×10^{-4}	2.19×10^{-3}	2.84×10^{-3}		7.89×10^{-4}	1.54×10^{-3}
	75–79	2.16×10^{-4}	6.62×10^{-4}	1.14×10^{-3}	2.42×10^{-3}		8.76×10^{-4}	1.32×10^{-3}
	80–84	3.45×10^{-4}	1.42×10^{-3}	2.72×10^{-3}	6.10×10^{-3}		1.20×10^{-3}	2.19×10^{-3}
Inst'd	65–69	2.13×10^{-4}	8.37×10^{-5}	1.78×10^{-4}	2.62×10^{-4}	4.42×10^{-5}		3.76×10^{-4}
	70–74	9.75×10^{-5}	7.73×10^{-5}	1.93×10^{-4}	1.57×10^{-4}	2.65×10^{-4}		1.02×10^{-3}
	75–79	3.61×10^{-5}	8.65×10^{-5}	5.93×10^{-5}	6.94×10^{-5}	7.47×10^{-5}		5.58×10^{-4}
	80–84	1.58×10^{-5}	2.51×10^{-5}	2.97×10^{-5}	5.17×10^{-5}	3.31×10^{-5}		4.03×10^{-4}

The adjusted log-likelihood, $\bar{l}_r$, where:

$$\bar{l}_r \propto \sum_{all\, i,j} \left(\overline{n}^{i\,j}_{r\,r+t} \ \log \left[P^{i\,j}_{r\,r+t} \left(\overline{\mu}^{1\,2}_r, \ldots, \overline{\mu}^{n\,n-1}_r \right) \right] \right) \tag{5.61}$$

is then (by definition) maximised by the constrained (positive) MLEs of the transition intensities, and so can be used to calculate variance estimates of these transition intensities using the information matrix, as in Section 5.2. These estimates are illustrated in Figures 5.36 to 5.41 as horizontal lines, to which the 'boot-strap' estimates converge. Thus, by adjusting the numbers of lives moving between states to be consistent with the transition probabilities calculated from the constrained (positive) MLEs of the transition intensities, the information matrix can be used to calculate, in a much more efficient way than the 'boot-strap' method, variance estimates of the constrained (positive) MLEs of the transition intensities. These estimates for male, females and in aggregate for the 1982–1984 NLTCS are given in 5-year age bands in Tables 5.61 to 5.63, respectively (except for the 85+ age group which is included in the tables for data grouped in 10-year age bands). The same variance estimates in 10-year age bands for the 1982–84 NLTCS and in 5 and 10-year age bands for the 1984–89 and 1989–94 NLTCS are given in Appendix J.

These variance estimates are useful for the two reasons:

1. they provide a set of weights to use in the graduation process (see Sections 5.5 and 5.6); and

2. the 95% confidence intervals for the MLEs (for $\hat{\mu}^{i\,j}_{x+t}$) can be calculated and used to check if the graduated transition intensities lie within these limits.

There are a few abnormally large (> 1) variance estimates for males aged over 85 years in the 1989–94 NLTCS (for example, the variance of the transition intensity from '3–4 ADLs' to '1–2 ADLs' is 24.8). These large variance estimates only occur for transition intensities out of two states — 'IADL only' and '3–4 ADLs', suggesting that it may be caused by a small exposed to risk for these states. From Table D.100, the exposed to risk for the 'IADL only' state is 69.55 life years (over 5 years) and for the '3–4 ADLs' state it is only 45.7 life years (over 5 years), compared to, for example, 110.55 life years (over 5 years) for the '1–2 ADLs state'. A second factor

is that the number of observed transitions for those transition intensities with large variance, is relatively high (for example, there were 3.15 observed transitions from '3–4 ADLs' to '1–2 ADLs', when the 5-year exposed to risk is only 45.7). These two factors in combination, coupled with uncertainty in the other parameters, give rise to very high variance estimates. With such uncertainty, caution is advised when estimating or extrapolating from this dataset.

In the next two sections, I graduate the constrained (positive) MLEs of the transition intensities, in 10 and 5-year age bands, respectively.

5.5 Graduating the Transition Intensities Grouped in 10-year Age Bands

I chose a very simple graduation procedure. With the data grouped in 10-year age bands (65–74 years, 75–84 years and 85+ years) there are only 3 data points and so the maximum number of parameters that can be used for the fit is also 3. In the fitting procedure, for simplicity, I assume that the transition intensity for an age group is actually a point estimate of the transition intensity for the mid-point of that age group plus half of the duration between surveys or age 90 years plus half of the duration between surveys for the 85+ years group (i.e. for the 1982–84 NLTCS, $\overline{\mu}^{ij}_{71} = \overline{\mu}^{ij}_{65–74}$, $\overline{\mu}^{ij}_{81} = \overline{\mu}^{ij}_{75–84}$ and $\overline{\mu}^{ij}_{91} = \overline{\mu}^{ij}_{85+}$). From Table 5.64, which gives the mean ages for each 10-year age band at the start of each survey period, this assumption seems reasonable, even though, as expected, the mean ages are slightly less than the mid-points for most age-groups.

The parametric form I chose was a Makeham curve, which has 3 parameters and, when the form of the data allow, can be fitted exactly to the data points (to fit a Makeham curve it is necessary that $\overline{\mu}^{ij}_{65–74} - \overline{\mu}^{ij}_{75–84} < \overline{\mu}^{ij}_{75–84} - \overline{\mu}^{ij}_{85+}$ and that $\overline{\mu}^{ij}_{65–74} \neq \overline{\mu}^{ij}_{75–84} \neq \overline{\mu}^{ij}_{85+}$). Where it was not possible to fit a Makeham curve, a straight line was fitted using weighted least squares, the weights (w^{ij}_{x+t}) being the inverse of the variance (i.e. $w^{ij}_{x+t} = 1/\mathrm{Var}[\overline{\mu}^{ij}_{x+t}]$). The only exception to this procedure was if the Makeham fit had a very high exponential parameter (i.e. very steep gradient), in which case a linear fit was chosen — however, this was only the

Table 5.64: Mean ages of lives within each 10-year age band by gender and in aggregate, in the 1982, 1984 and 1989 NLTCS.

Survey year	Gender	Mean age for age group: 65–74 yrs years	75–84 yrs years	85+ yrs years
1982	M	69.56	79.02	88.51
	F	69.77	79.45	88.87
	M & F	69.68	79.31	88.79
1984	M	70.09	78.69	87.64
	F	70.38	79.15	88.08
	M & F	70.27	79.01	88.00
1989	M	69.88	78.31	87.14
	F	70.26	78.52	87.54
	M & F	70.13	78.46	87.49

case for very few transition intensities (in the 1982–84 NLTCS from '3–4 ADLs' to 'IADL only', for females and in aggregate; in the 1984–89 NLTCS from '3–4 ADLs' to 'Healthy' and from 'Institutionalized' to 'IADL only' for males; and in the 1989–94 NLTCS from '3–4 ADLs' to 'IADL only' for females and in aggregate). The form of the graduated transition intensities, for $65 \leq x + t \leq 120$, is then:

$$\mathring{\mu}^{ij}_{x+t} = \begin{cases} A_{ij} + B_{ij}\, e^{C_{ij}((x-(70+M))+t)} & \text{if } (\overline{\mu}^{ij}_{70+M} - \overline{\mu}^{ij}_{80+M}) < (\overline{\mu}^{ij}_{80+M} - \overline{\mu}^{ij}_{90+M})\ , \\ & \overline{\mu}^{ij}_{70+M} \neq \overline{\mu}^{ij}_{80+M} \neq \overline{\mu}^{ij}_{90+M} \text{ and } |C_{ij}| < 0.5 \\ A_{ij} + D_{ij}\,(x+t) & \text{otherwise} \end{cases}$$

and with a lower bound of zero on all intensities at all ages and where $M = 1$ for the 1982–84 NLTCS and $M = 2.5$ for the 1984–89 and 1989–94 NLTCS. The values for the parameters A, B, C and D are given in Table 5.65 for males and females together. The same tables for males and females separately in the 1982–84 NLTCS and for males and females separately and aggregated for the 1984–89 and 1989–94 NLTCS are given in Appendix K.

The reasons for choosing a Makeham curve where possible and a linear form otherwise are:

1. for modelling mortality a Makeham curve is a reasonable assumption, given the number of data points;

2. increasing disability is a predictor of mortality and as such can be seen to be

Table 5.65: Parameter values for the parametric transition intensities for males and females grouped in 10-year age bands, calculated from the 1982 and 1984 NLTCS.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-1.84×10^{-2}	4.31×10^{-2}	5.34×10^{-2}	-
	1–2 ADLs	4.67×10^{-3}	7.78×10^{-3}	1.31×10^{-1}	-
	3–4 ADLs	-2.04×10^{-2}	-	-	3.44×10^{-4}
	5–6 ADLs	3.94×10^{-3}	2.48×10^{-4}	2.20×10^{-1}	-
	Inst'd	-1.10×10^{-4}	3.73×10^{-3}	1.37×10^{-1}	-
	Dead	-2.57×10^{-2}	5.07×10^{-2}	3.71×10^{-2}	-
IADL only	Healthy	-6.12×10^{-1}	8.53×10^{-1}	-1.56×10^{-2}	-
	1–2 ADLs	-2.72×10^{-1}	-	-	7.61×10^{-3}
	3–4 ADLs	-1.78×10^{-2}	-	-	3.41×10^{-4}
	5–6 ADLs	-1.05×10^{-1}	-	-	2.01×10^{-3}
	Inst'd	-1.59×10^{-1}	-	-	2.62×10^{-3}
	Dead	6.07×10^{-2}	9.21×10^{-4}	2.10×10^{-1}	-
1–2 ADLs	Healthy	3.40×10^{-2}	3.74×10^{-2}	-5.32×10^{-2}	-
	IADL only	5.61×10^{-1}	-	-	-4.88×10^{-3}
	3–4 ADLs	1.88×10^{-1}	1.61×10^{-2}	9.59×10^{-2}	-
	5–6 ADLs	7.32×10^{-2}	-	-	-5.68×10^{-4}
	Inst'd	4.09×10^{-2}	2.64×10^{-3}	1.69×10^{-1}	-
	Dead	-1.90×10^{-2}	-	-	1.60×10^{-3}
3–4 ADLs	Healthy	4.14×10^{-3}	3.07×10^{-2}	-3.09×10^{-1}	-
	IADL only	-2.44×10^{-1}	-	-	3.29×10^{-3}
	1–2 ADLs	1.66×10^{0}	-	-	-1.73×10^{-2}
	5–6 ADLs	5.30×10^{-2}	2.34×10^{-1}	4.13×10^{-2}	-
	Inst'd	-5.66×10^{-1}	-	-	8.25×10^{-3}
	Dead	1.69×10^{-2}	-	-	8.65×10^{-4}
5–6 ADLs	Healthy	1.11×10^{-1}	-	-	-1.05×10^{-3}
	IADL only	3.01×10^{-1}	-	-	-3.25×10^{-3}
	1–2 ADLs	7.68×10^{-3}	-	-	6.11×10^{-4}
	3–4 ADLs	1.83×10^{-1}	-	-	-1.35×10^{-4}
	Inst'd	-1.65×10^{-1}	-	-	3.36×10^{-3}
	Dead	2.40×10^{-1}	9.76×10^{-4}	2.43×10^{-1}	-
Inst'd	Healthy	3.57×10^{-3}	2.74×10^{-2}	-1.75×10^{-1}	-
	IADL only	2.96×10^{-2}	-	-	-2.95×10^{-4}
	1–2 ADLs	-5.61×10^{-4}	1.13×10^{-2}	-1.50×10^{-1}	-
	3–4 ADLs	2.59×10^{-2}	-	-	-1.83×10^{-4}
	5–6 ADLs	4.12×10^{-2}	-	-	-4.21×10^{-4}
	Dead	4.90×10^{-2}	1.25×10^{-1}	4.19×10^{-2}	-

part of the ageing process, so movement through stages of disability may then behave in a similar fashion; and

3. recovery from disability decreases with age and a negative exponential may be appropriate.

In the next section, when there are more data points (because of smaller age groups) I look at the trends more closely, however in this case I felt the data did not justify a more complex procedure.

Figures 5.42 to 5.47 give graphs of the transition intensities, for males and females, out of states 1–6, respectively, for the 1982–84 NLTCS. They show the point estimates, in 10-year age bands (constrained (positive) MLEs), the confidence intervals using the variance estimates from the previous section and the parametric form of the transition intensities. While all graphs in a given figure use the same scale, it is worth noting that different figures use different scales and that the scales were chosen such that the confidence intervals could be shown on the graphs, with the exception of when the confidence intervals were very large requiring very large scales which would obscure the patterns of the point estimates — in these cases the scale is limited to -0.2 to 0.5 (or the point estimate of the largest transition intensity plus 0.05 of its standard deviation, if the largest point estimate is greater than 0.5). All the Makeham curves pass though the confidence intervals (as all the point estimates lie inside the confidence intervals). It is also noticeable that almost all of the linear fits also pass through the confidence intervals (in fact only 13 of the linear fits do not pass through the confidence intervals of the transition intensities, which is less than 5% of all the parametric fits).

For brevity, I exclude similar graphs for all of the other datasets, since there is no new information in them — they just provide a visual summary of information given elsewhere.

In general the transition intensities are as expected: the force of disability and mortality increases with increasing age; and the force of recovery decreases with increasing age. There are, however, some notable exceptions (e.g. from the healthy state to 3–4 ADLs), all of these involve small transition intensities though, calculated from relatively few data (they are all linear fits, as well, suggesting no clear pattern

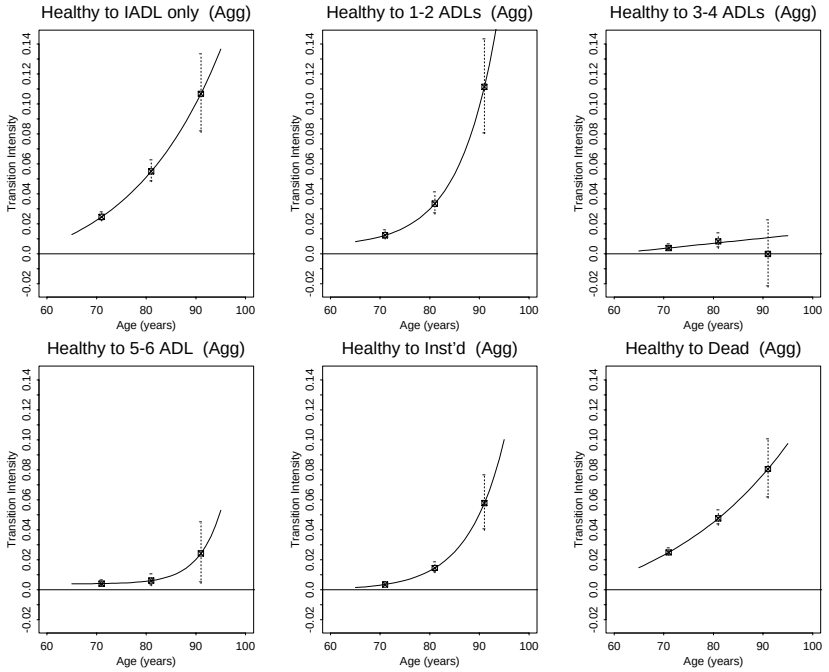

Figure 5.42: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males and females grouped in 10-year age bands in the 1982–84 NLTCS.

between the transition intensities).

Figure 5.48, which compares a Makeham fit to a weighted least squares fit for the transition intensity 'Healthy' to '1–2 ADLs' for males in the 1982–84 NLTCS, illustrates a potential problem with fitting a Makeham curve to three data points — it gives equal weight (necessarily, as it must pass through all three points) to all three points, irrespective of the variance of each point estimate. This means that, in some cases, the size of the exponential parameter in a Makeham fit is most sensitive to the estimate that has the largest variance — that for the oldest age group (which generally has the largest variance as it generally has the scarcest data). While this instability in the fitting procedure is worth noting, with so few data points, it is difficult to overcome. However, in the next section, with the data grouped into 5-year age bands (providing 5 data points), the fitting procedure can be made more robust.

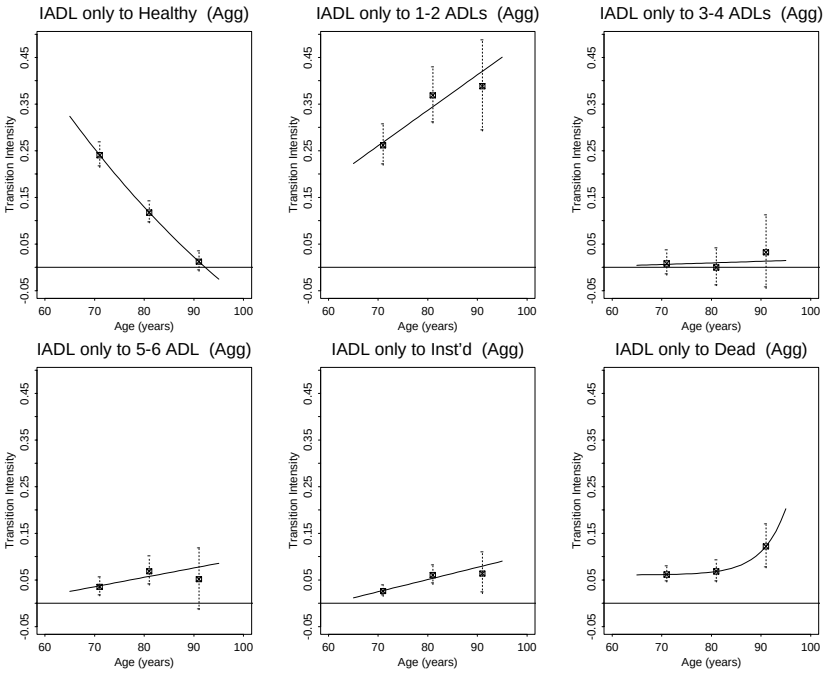

Figure 5.43: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males and females grouped in 10-year age bands in the 1982–84 NLTCS.

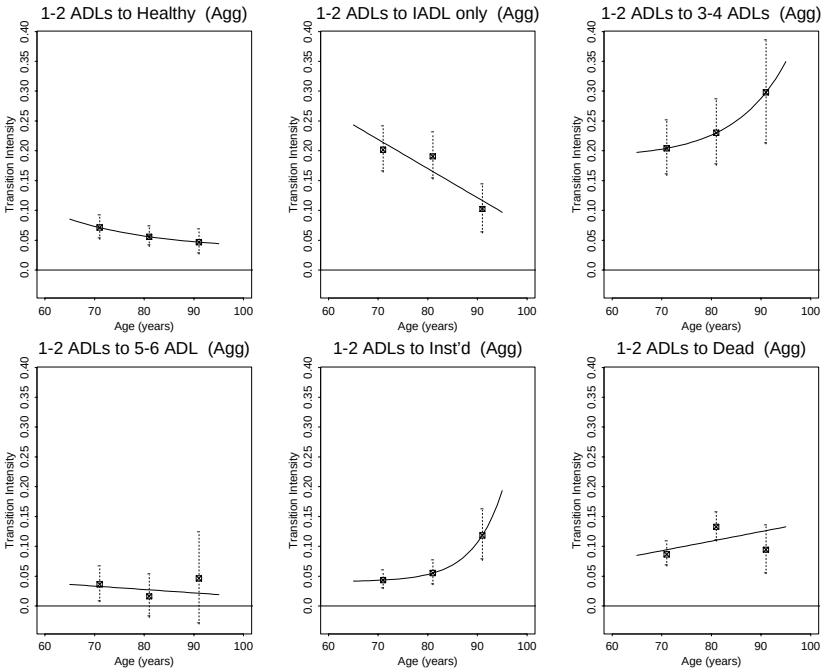

Figure 5.44: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males and females grouped in 10-year age bands in the 1982–84 NLTCS.

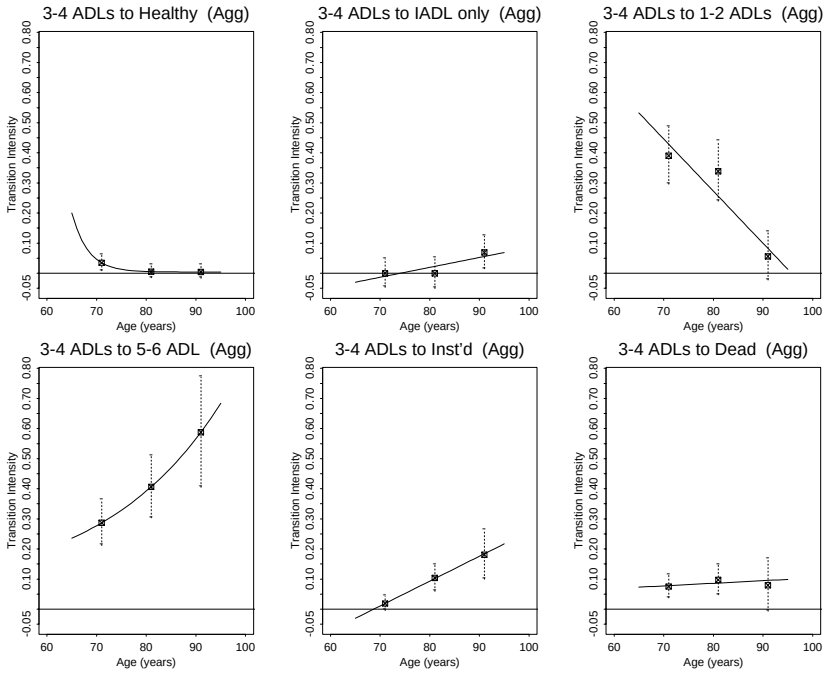

Figure 5.45: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males and females grouped in 10-year age bands in the 1982–84 NLTCS.

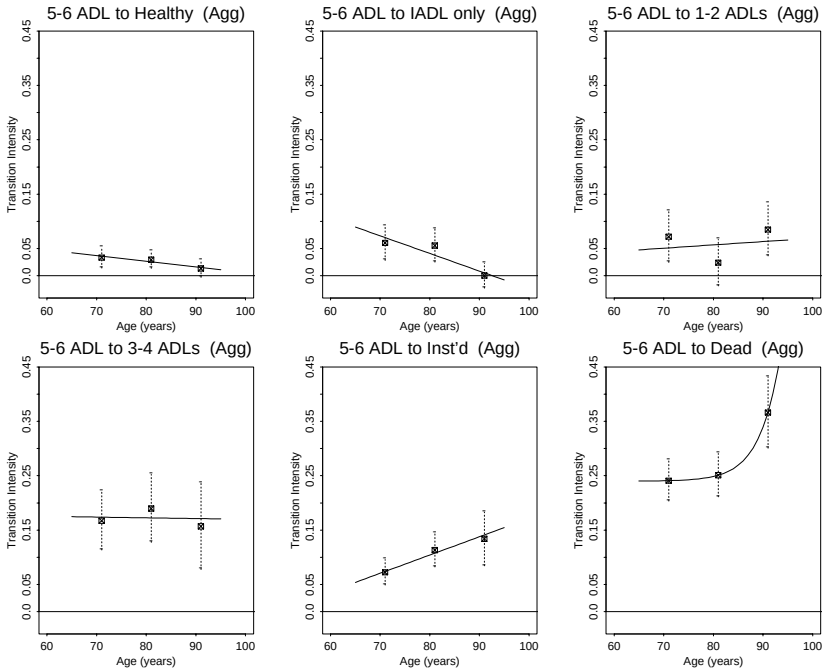

Figure 5.46: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males and females grouped in 10-year age bands in the 1982–84 NLTCS.

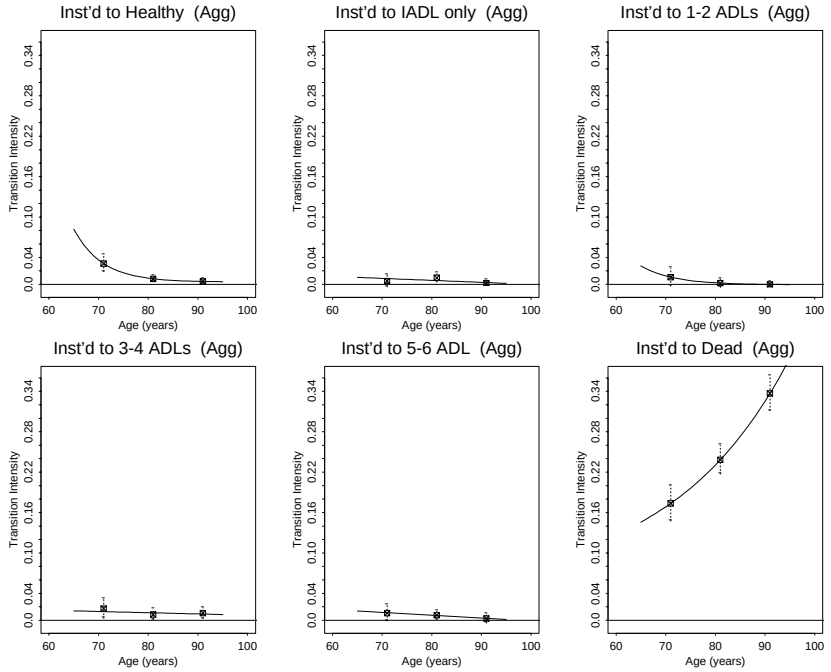

Figure 5.47: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males and females grouped in 10-year age bands in the 1982–84 NLTCS.

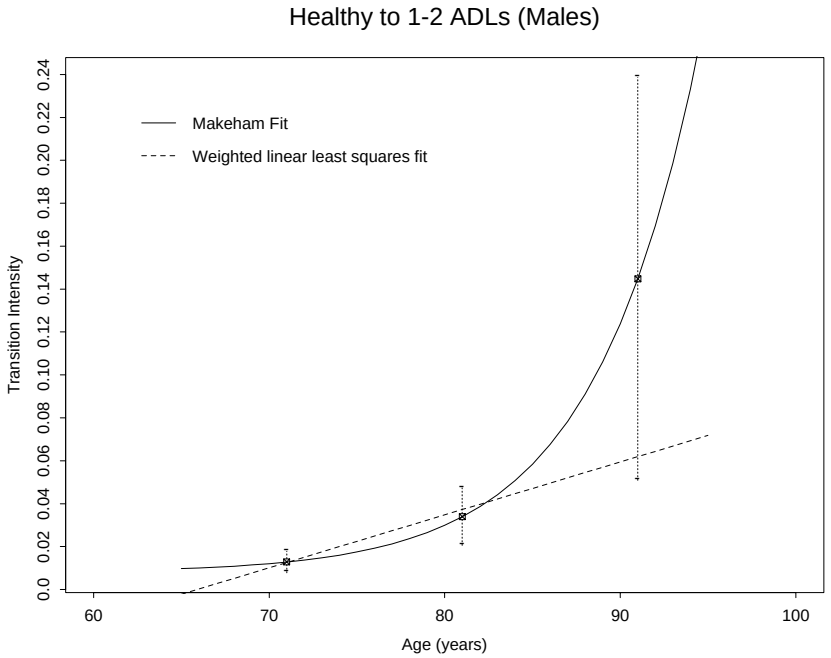

Figure 5.48: Comparison of a Makeham fit with a weighted linear least squares fit for the transition intensity 'Healthy' to '1–2 ADLs' for males grouped in 10-year age bands in the 1982–84 NLTCS.

Table 5.66: Mean ages of lives within each 5-year age band by gender and in aggregate, in the 1982, 1984 and 1989 NLTCS.

Survey year	Gender	Mean age for age group				
		65–69 yrs	70–74 yrs	75–79 yrs	80–84 yrs	85+ yrs
		years	years	years	years	years
1982	M	67.46	72.37	77.33	82.29	88.51
	F	67.49	72.45	77.43	82.34	88.87
	M & F	67.48	72.42	77.39	82.32	88.79
1984	M	67.72	72.28	77.20	82.31	87.64
	F	67.76	72.43	77.37	82.28	88.08
	M & F	67.74	72.37	77.32	82.29	88.00
1989	M	67.68	72.19	77.10	82.25	87.14
	F	67.73	72.38	77.22	82.07	87.54
	M & F	67.72	72.32	77.19	82.11	87.49

5.6 Graduating the Transition Intensities Grouped in 5-year Age Bands

I chose a similar graduation procedure to the previous section. With the data grouped in 5-year age bands (65–69 years, 70–74 years, 75–79 year, 80–84 and 85+ years) there are 5 data points. I assume, again, that the transition intensity for an age group is actually a point estimate of the transition intensity for the mid-point of that age group plus half of the duration between surveys or age 90 years plus half of the duration between surveys for the 85+ years group (i.e. for the 1982–84 NLTCS, $\overline{\mu}^{ij}_{68.5} = \overline{\mu}^{ij}_{65\text{–}69}$, $\overline{\mu}^{ij}_{73.5} = \overline{\mu}^{ij}_{70\text{–}74}$, $\overline{\mu}^{ij}_{78.5} = \overline{\mu}^{ij}_{75\text{–}79}$, $\overline{\mu}^{ij}_{83.5} = \overline{\mu}^{ij}_{80\text{–}84}$ and $\overline{\mu}^{ij}_{91} = \overline{\mu}^{ij}_{85+}$). From Table 5.66, which gives the mean ages for each 5-year age band at the start of each survey period, this assumption seems reasonable, even though, as expected, the mean ages are slightly less than the mid-points for most age-groups.

For the graduation, I chose the same two parametric forms as in the previous section — either a Makeham curve or a straight line, which from the shape of the data, (illustrated in Figures 5.49 to 5.54 and in Appendices M to O) seem to provide adequate flexibility. It would be possible to fit a more complex parametric form (more parameters), but this would be at the cost of the smoothness of the fit, and with only five data points, I feel this is not justified.

The method I use is, for each transition intensity, to fit both a staight line and a

Makeham curve using weighted least squares, the weights (w_{x+t}^{ij}) being the inverse of the variance (i.e. $w_{x+t}^{ij} = 1/\mathrm{Var}[\overline{\mu}_{x+t}^{ij}]$). Then I chose the parametric form which provides the best fit (in terms of the smallest sum of weighted squared residuals). The form of the graduated transition intensities, for $65 \leq x + t \leq 120$, is then, depending on which provides a better fit, either:

$$\mathring{\mu}_{x+t}^{ij} = A_{ij} + B_{ij}\, e^{C_{ij}((x-(67.5+M))+t)}$$

or:

$$\mathring{\mu}_{x+t}^{ij} = A_{ij} + D_{ij}\,(x+t)$$

and with a lower bound of zero on all intensities at all ages and where $M = 1$ for the 1982–84 NLTCS and $M = 2.5$ for the 1984–89 and 1989–94 NLTCS. There was no explicit limit placed on the C_{ij} here, since this fitting procedure did not produce any unreasonably large estimates of C_{ij} (discussed in more detail later in this section). The values for the parameters A, B, C and D are given in Table 5.67 for males and females together. The same tables for males and females separately in the 1982–84 NLTCS and for males and females separately and aggregated for the 1984–89 and 1989–94 NLTCS are given in Appendix L.

Figures 5.49 to 5.54 give graphs of the transition intensities, for males and females, out of states 1–6, respectively, for the 1982–84 NLTCS. They show the point estimates, in 5-year age bands (constrained (positive) MLEs), the confidence intervals using the variance estimates from the previous section and the parametric form of the transition intensities. The same graphs for males and females separately for the 1982–84 NLTCS are given in Appendix M and those for males and females separately and aggregated for the 1984–89 and 1989–94 NLTCS are given in Appendices N and O, respectively. While all graphs in a given figure use the same scale, it is worth noting that different figures use different scales and that the scales were choosen such that the confidence intervals could be shown on the graphs, with the exception of when the confidence intervals were very large requiring very large scales which would obscure the patterns of the point estimates — in these cases the scale is limited to -0.2 to 0.5 (or the point estimate of the largest transition intensity plus 0.05 of its standard deviation, if the largest point estimate is greater than 0.5). In

general, the parametric fits are similar to those in the previous section when using only 3 data points. The main differences are:

1. in the choice a linear fit over a Makeham fit in some cases — in the previous section if a Makeham curve could be fitted it was, whereas with 5 data points a linear fit was chosen over a Makeham fit if it provided a better fit (for example for males and females in the 1982–84 NLTCS: a linear fit was chosen over a Makeham fit (which was used with 3 data points) for the transition intensities 'Healthy' to 'Dead', 'IADL only' to 'Dead', '1–2 ADLs' to 'Inst'd', '3–4 ADLs' to 'Healthy', etc.);

2. where a Makeham fit was chosen using 3 and 5 data points, the exponential parameter of the fit using 5 data points is generally less (sometimes substantially less) than that using 3 data points (for example for males and females in the 1982–84 NLTCS: 'Healthy' to '5–6 ADLs', 'IADL only' to 'Dead', '1–2 ADLs' to 'Inst'd', '5–6 ADLs' to 'Dead' and 'Inst'd' to '1–2 ADLs').

In the previous section, with three data points, there were problems of sensitivity with the Makeham fits — as all data points had equal weighting (as the Makeham fit had to pass through all three points, irrespective of the size of the variance estimates). In some cases this led to high exponential paramaters (C_{ij}), for the Makeham fit — which I then, subjectively, rejected in favor of a linear fit. This problem does not arise in this fitting procedure as the Makeham fit is now a weighted fit, and so parameters with high variance are given less weight accordingly. In fact, the largest positive exponential parameter is 0.174 (for males and females from 'Healthy' to '1–2 ADLs' in the 1982–84 NLTCS) and the largest negative exponential parameter is 0.277 (for males from '3–4 ADLs' to '1–2 ADLs' in the 1989–94 NLTCS). This provides support for limiting the exponential parameter in the Makeham fits in the previous section.

When analysing age-dependent data like that from the NLTCS, there is always the trade-off between:

1. having age bands large enough to ensure that there are enough observations in each age band for the estimates to be useful (too few observations lead to

Table 5.67: Parameter values for the parametric transition intensities for males and females grouped in 5-year age bands, calculated from the 1982 and 1984 NLTCS.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-3.22×10^{-2}	5.19×10^{-2}	4.35×10^{-2}	-
	1–2 ADLs	9.58×10^{-3}	2.11×10^{-3}	1.74×10^{-1}	-
	3–4 ADLs	-2.34×10^{-2}	-	-	3.85×10^{-4}
	5–6 ADLs	-1.37×10^{-4}	3.16×10^{-3}	8.01×10^{-2}	-
	Inst'd	-9.05×10^{-4}	3.15×10^{-3}	1.32×10^{-1}	-
	Dead	-1.62×10^{-1}	-	-	2.64×10^{-3}
IADL only	Healthy	1.04×10^{0}	-	-	-1.13×10^{-2}
	1–2 ADLs	-3.38×10^{-1}	-	-	8.32×10^{-3}
	3–4 ADLs	2.94×10^{-2}	-	-	-1.59×10^{-4}
	5–6 ADLs	-9.89×10^{-2}	1.33×10^{-1}	8.16×10^{-3}	-
	Inst'd	-1.81×10^{-1}	-	-	2.90×10^{-3}
	Dead	-3.19×10^{-2}	8.80×10^{-2}	1.60×10^{-2}	-
1–2 ADLs	Healthy	1.74×10^{-1}	-	-	-1.45×10^{-3}
	IADL only	5.45×10^{-1}	-	-	-4.71×10^{-3}
	3–4 ADLs	1.85×10^{-1}	5.62×10^{-3}	1.33×10^{-1}	-
	5–6 ADLs	-6.10×10^{-2}	1.04×10^{-1}	-1.11×10^{-2}	-
	Inst'd	-5.61×10^{-2}	7.72×10^{-2}	3.48×10^{-2}	-
	Dead	-4.68×10^{-2}	-	-	1.93×10^{-3}
3–4 ADLs	Healthy	1.03×10^{-1}	-	-	-1.11×10^{-3}
	IADL only	-4.26×10^{-3}	2.14×10^{-3}	1.48×10^{-1}	-
	1–2 ADLs	1.61×10^{0}	-	-	-1.69×10^{-2}
	5–6 ADLs	1.64×10^{-2}	2.13×10^{-1}	4.51×10^{-2}	-
	Inst'd	-9.20×10^{-2}	1.09×10^{-1}	3.52×10^{-2}	-
	Dead	1.27×10^{-1}	-	-	-5.50×10^{-4}
5–6 ADLs	Healthy	1.06×10^{-1}	-	-	-9.93×10^{-4}
	IADL only	2.85×10^{-1}	-	-	-3.08×10^{-3}
	1–2 ADLs	-1.81×10^{-1}	2.23×10^{-1}	4.62×10^{-3}	-
	3–4 ADLs	1.40×10^{-1}	-	-	3.16×10^{-4}
	Inst'd	-2.00×10^{-1}	-	-	3.80×10^{-3}
	Dead	1.76×10^{-1}	4.53×10^{-2}	5.28×10^{-2}	-
Inst'd	Healthy	2.39×10^{-3}	2.84×10^{-2}	-1.19×10^{-1}	-
	IADL only	2.89×10^{-2}	-	-	-2.90×10^{-4}
	1–2 ADLs	-3.10×10^{-2}	3.89×10^{-2}	-1.02×10^{-2}	-
	3–4 ADLs	-1.94×10^{-1}	2.05×10^{-1}	-3.68×10^{-4}	-
	5–6 ADLs	9.87×10^{-3}	-	-	-6.85×10^{-5}
	Dead	-5.71×10^{-1}	-	-	9.98×10^{-3}

estimates with very large confidence intervals); and

2. having enough separate groups (i.e. smaller age bands) to ensure that the trends of the transition intensities with age are clear.

The main reason for grouping the data into 3 age bands (65–74, 75–84 and 85+ years), as in the previous section, is to allow comparison with previous research (see Section 3.5), since these are the groupings that were used there. However, while using 3 age bands ensures greater numbers of observations in each age band, this is at the cost of having only three data points with which to graduate the estimates — which caused the fitting procedure to be unstable in some cases as the trend of the transition intensities with age was not clear. With 5 data points, this instability in the fitting procedure does not occur, as there are enough estimates to use a more robust method (weighted least squares) when fitting a Makeham curve. Also, using 5 age bands does not, in general, worsen the reliability of the estimates of the transition intensities by much (as indicated by slightly larger confidence intervals). In fact, this problem of scarce data is only apparent for the oldest age band, 85+ years, in some cases (for example, in the 1989–94 NLTCS, for males, the variance estimates of the transition intensities: IADL only to 3–4 ADLs and 3–4 ADLs to 1–2 ADLs) and this age group is the same, whether the data is grouped into 3 or 5 age bands. Given that using 5 age bands has the advantage of then being able to use a more stable fitting procedure for graduating the transition intensities, and that this does not worsen the estimates by much, I will place more reliance on the graduated transition intensities that used data grouped into 5 age bands. The graduated transition intensities using data grouped into 3 age bands, will still be useful for two reasons:

1. for comparison with previous research; and

2. for comparing with those fitted to the data in 5 age bands.

In the next chapter, I look at the overall forces of mortality in the models and compare them to mortality from other investigations.

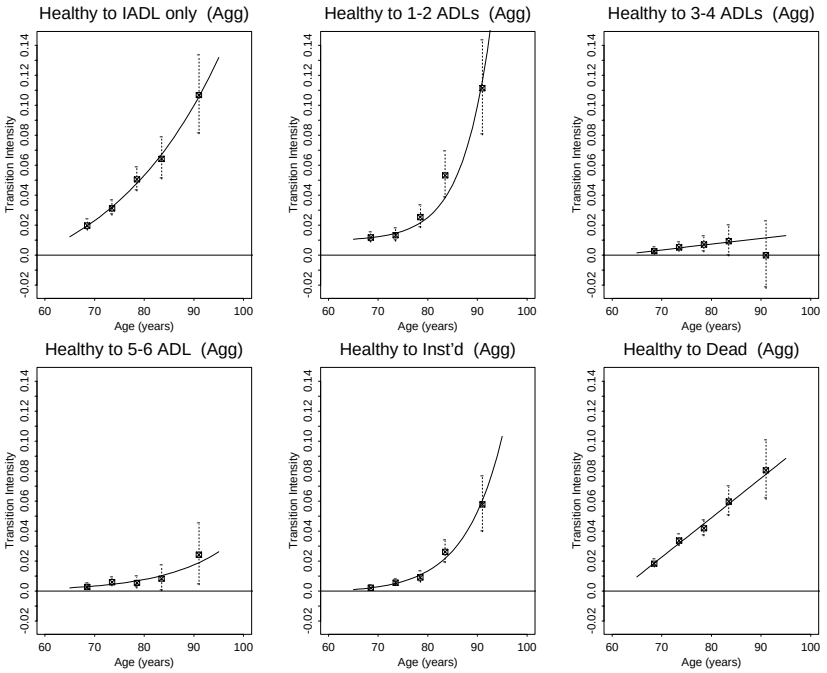

Figure 5.49: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males and females grouped in 5-year age bands in the 1982–84 NLTCS.

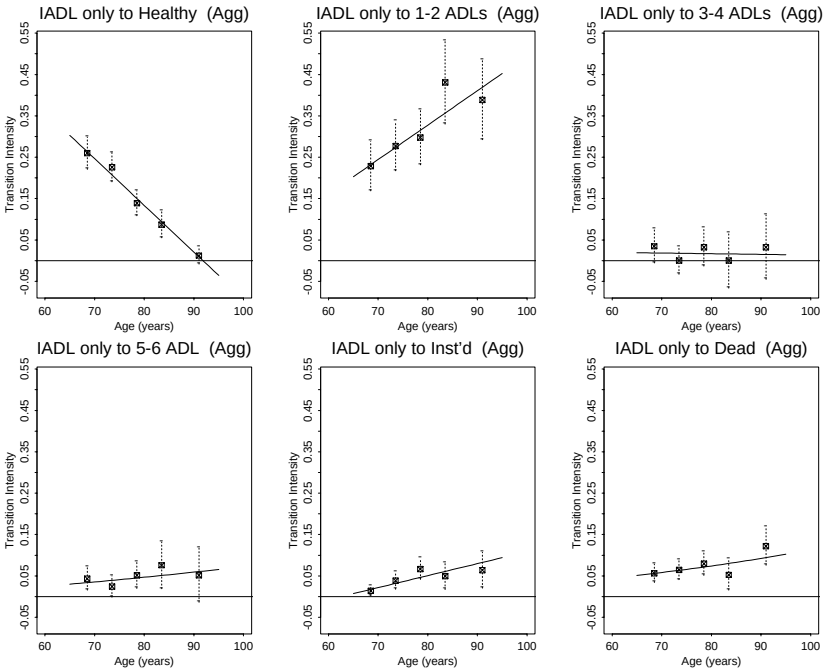

Figure 5.50: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males and females grouped in 5-year age bands in the 1982–84 NLTCS.

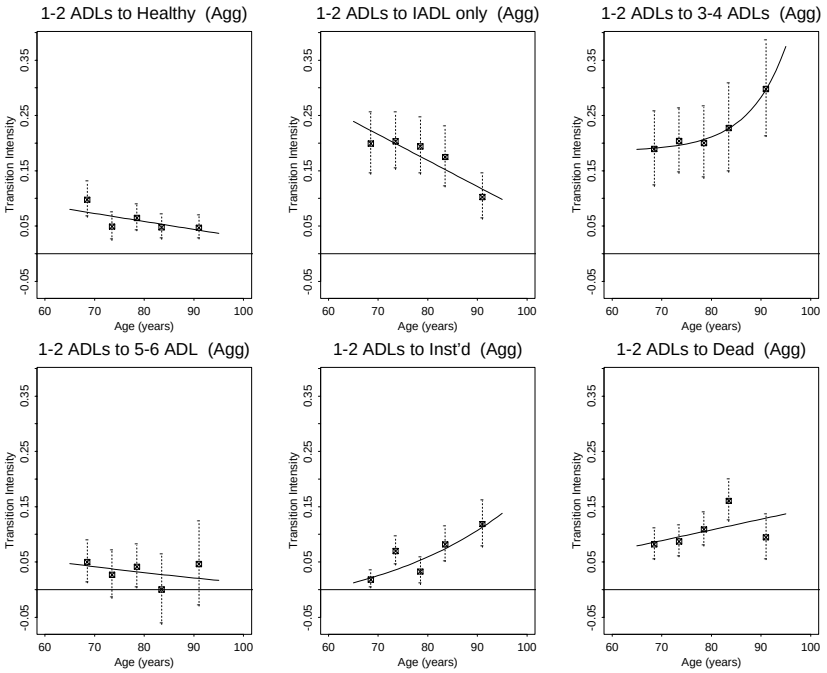

Figure 5.51: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males and females grouped in 5-year age bands in the 1982–84 NLTCS.

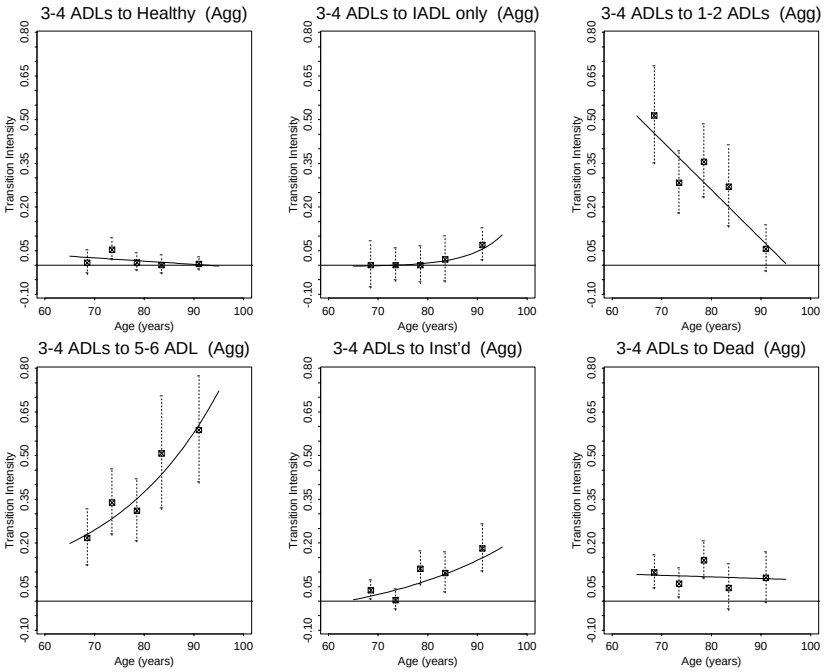

Figure 5.52: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males and females grouped in 5-year age bands in the 1982–84 NLTCS.

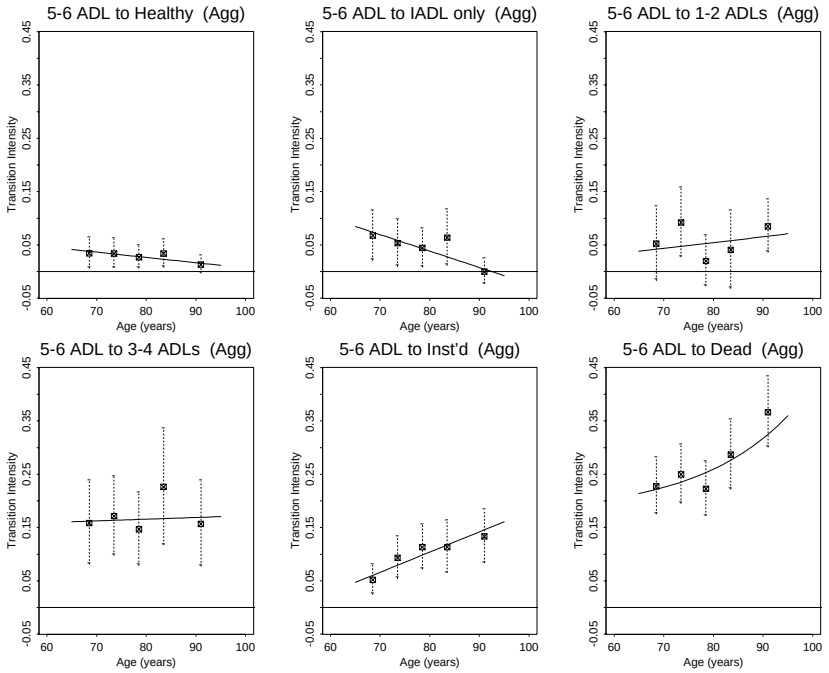

Figure 5.53: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males and females grouped in 5-year age bands in the 1982–84 NLTCS.

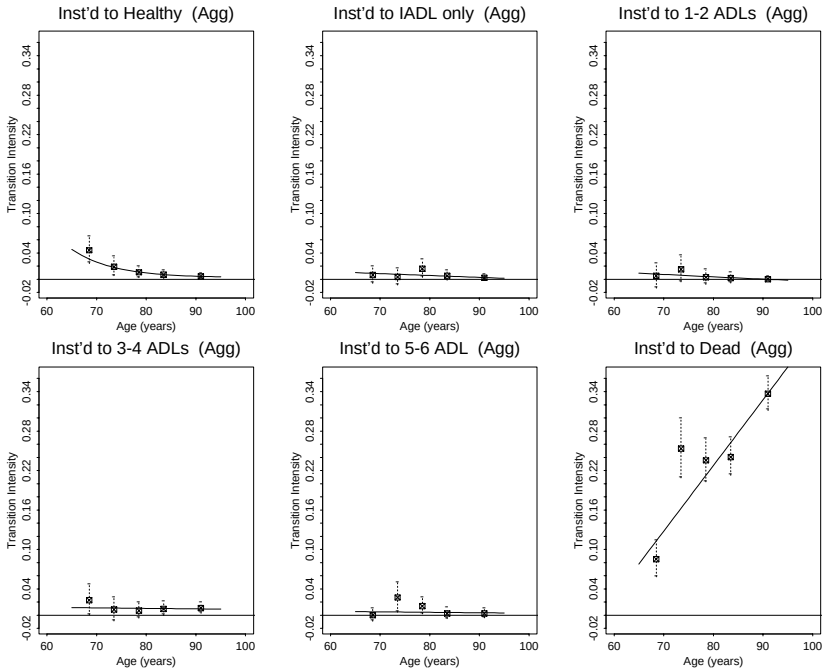

Figure 5.54: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males and females grouped in 5-year age bands in the 1982–84 NLTCS.

Chapter 6

Overall Mortality in the Disability Model

6.1 Introduction

The aim of this chapter is to compare implied aggregate (across disability states), or overall, mortality from the parameterized disability models to that of standard mortality tables, or other mortality investigations.

To avoid confusion, I will refer to the mortality of males and females combined as *aggregate* mortality and mortality aggregated across all disability states for a given model as *overall* mortality.

Overall mortality at any given age in the disability model is the weighted sum of the force of mortality in each non-dead state, where the weights are the probability of a life being in any given state. Assuming that a life is healthy at age 65 (a reasonable assumption for an insured life), the probability of the life being in state j at age $65+t$ is then simply ${}_{65}\mathrm{P}^{1\,j}_{65+t}$, the probability that the life moves from state 1 (Healthy) to state j in time t. The overall force of mortality, μ_{65+t}, at any age $65+t$ can then be calculated as:

$$\mu_{65+t} = \frac{\sum_{i=1}^{6} \mu^{i\,7}_{65+t} {}_{65}\mathrm{P}^{1\,i}_{65+t}}{\sum_{i=1}^{6} {}_{65}\mathrm{P}^{1\,i}_{65+t}} \tag{6.62}$$

where ${}_{65}\mathrm{P}^{1\,i}_{65+t}$ can be calculated by solving Kolmogorov's equations, for any given set of transition intensities and $\mu^{i\,7}_{65+t}$ is the force of mortality in state i at age $65+t$.

The motivation for comparing overall model mortality with a benchmark force of mortality is two-fold:

1. the force of mortality has been subject to large scale investigations, which provide benchmarks to compare the overall force of mortality from the disability model to see if it produces reasonable estimates; and

2. if necessary, the transition intensities in the model can be adjusted to be consistent with the findings from these large scale investigations, to make the model more reasonable for use in application (for example, in estimating the cost of disability in a long-term care insurance contract).

First, in Section 6.2, I summarise an investigation into mortality at old ages and extract a benchmark model of mortality from it. Then in Section 6.3, I compare this benchmark mortality with overall model mortality for the disability models parameterized from the 1982–84 NLTCS (which I refer to as the 1982–84 models). In Sections 6.4 and 6.5, I compare this benchmark force of mortality with overall model mortality for the disability models parameterized using the 1984–89 NLTCS and 1989–94 NLTCS, respectively — which leads to adjustments being made to mortality in these models (the models after these adjustments have been made, I refer to as the adjusted 1984–89 and the adjusted 1989–94 models). Finally, in Section 6.6, I compare overall mortality from the (adjusted) disability models across all three survey periods and comment on their application to estimating costs of disability in a long-term care insurance contract.

6.2 A Benchmark Force of Mortality at Older Ages

The monograph by Thatcher, Kannisto, & Vaupel (1998) provides a comprehensive discussion and analysis of the force of mortality at ages 80–120. In summary:

1. they use data from 13 countries — Austria, Denmark, England and Wales, Finland, France, Germany (West), Iceland, Italy, Japan, the Netherlands, Sweden and Switzerland;

2. these include almost 40 million lives that survived to age 80, over 120,000 that reached age 100 and covers over 32 million deaths at age 80 and over;

3. the period covered is 1960–1990, and the analysis is done in 10-year age periods and cohorts (1960–70, 1970–80 and 1980–90); and

4. the analysis was done by gender for each country separately and all countries together;

The approach taken was, after a comprehensive literature review, to choose 6 models for comparison — 4 explanatory models and 2 descriptive, as follows:

1. Gompertz's law of mortality:

$$\mu_{x+t} = a\,\mathrm{e}^{b(x+t)} \tag{6.63}$$

   This was chosen over a Makeham fit as the difference between them was negligible — the constant term in the Makeham fit was very small, especially at older ages where the exponential term dominates.

2. the logistic model:

$$\mu_{x+t} = c + \frac{a\,\mathrm{e}^{b(x+t)}}{1 + \sigma^2\,\frac{a}{b}\,(\mathrm{e}^{b(x+t)} - 1)} \tag{6.64}$$

   Of which the Makeham fit is a special case — when $\sigma = 0$.

3. the Kannisto model (another special case of the logistic model):

$$\mu_{x+t} = \frac{a\,\mathrm{e}^{b(x+t)}}{1 + a\,\mathrm{e}^{b(x+t)}} \tag{6.65}$$

4. the Weibull model:

$$\mu_{x+t} = a\,(x+t)^b \tag{6.66}$$

5. the Heligman & Pollard model, which describes the form for the one year probability of dying between ages $x+t$ and $x+t+1$, q_{x+t}:

$$q_{x+t} = \frac{a\,\mathrm{e}^{b(x+t)}}{1 + a\,\mathrm{e}^{b(x+t)}} \tag{6.67}$$

6. and the quadratic model

$$\mu_{x+t} = \mathrm{e}^{a+b(x+t)+c(x+t)^2} \quad \text{with } c < 0 \tag{6.68}$$

These models were fitted to the data using the method of maximum likelihood at ages 80–98 years (85–98 years for the quadratic model) and then extrapolated up to age 120 years. The extrapolated values for each model were then compared to the actual values estimated from the data (with the data from all countries combined), by looking at the value of the likelihood function for each fit and by comparing values of the chi-squared statistic. For three of the models the conclusions are clear, the Gompertz, Weibull and Heligman & Pollard models always provided a worse (by overstating mortality at the oldest ages), and generally far worse, fit than the logistic, Kannisto and quadratic models.

The usefulness of this monograph is that it gives a benchmark force of mortality with which to compare overall mortality from the disability model. In this respect, all three of the models that provide superior fits (the logistic, Kannisto and quadratic models) would be suitable, as they are the same general shape (in the age range 85–120) and are almost at the same level. Another consideration is extrapolation of the fitted mortality down to age 65 (since the disability model is parameterized over the age range 65–120 years) — for which the quadratic model is not suitable. A disadvantage of the quadratic model is that it is only a descriptive model and while it provides a good fit in the age range 85–120 years, it cannot be sensibly extrapolated to younger or older ages.

Of the remaining two models, in some cases the Kannisto model provided a better fit than the logistic model, which, as the authors pointed out, is anomalous, because the Kannisto model is a special case of the logistic model and so cannot provide a better fit with both models fitted to an entire data set. The problem here lies in fitting the models to one data set (80–98 years) and then comparing them on another set (98–120 years).

For the purposes of comparison with overall mortality in the disability model, the fits from either the Kannisto model or the logistic model would be adequate, as the differences between them are small. I therefore chose the logistic model, since it is more general. Table 6.68 gives the parameter values for the logistic model fitted to

Table 6.68: Parameter values for the logistic model fitted to data from 13 countries at ages 80–98 years for males and females separately over the period 1980–90, from Thatcher, Kannisto & Vaupel (1998).

Parameter	Gender	
	males	females
a	2.99258×10^{-5}	1.46725×10^{-5}
b	1.02220×10^{-1}	1.08791×10^{-1}
c	9.93451×10^{-6}	9.99586×10^{-6}
σ	2.89834×10^{-1}	3.47119×10^{-3}

the data for all countries combined at ages 80–98 for males and females separately, in the period 1980–90 (as most of the NLTCS took place during this period), taken from Thatcher, Kannisto & Vaupel (1998). These fits are given in Figure 6.55 for comparison with the overall mortality from the disability model — unfortunately, for males and females combined no logistic fit was given and I use the logistic fit for females (as the majority of the population is female at older ages due to the usual mortality differential) for comparison with overall mortality for males and females combined in the disability model.

6.3 Overall Model Mortality, Using the Fitted Parametric Transition Intensities from the 1982–84 NLTCS

In this section, I compare the overall force of mortality in the disability models using the parametric transition intensities fitted to the 1982–84 NLTCS (which I will refer to as the 1982–84 models) with the benchmark force of mortality described in the previous section. I then investigate the shape of overall model mortality by looking at its separate components and finally discuss whether adjustments should be made to the 1982–84 models. For each pair of NLTCS and for each gender classification, two sets of parametric transition intensities were fitted: those with the data grouped into 10-year age bands; and those with the data grouped into 5-year age bands. For brevity and to avoid confusion I will refer to the former as the '1982–84 (10) models'

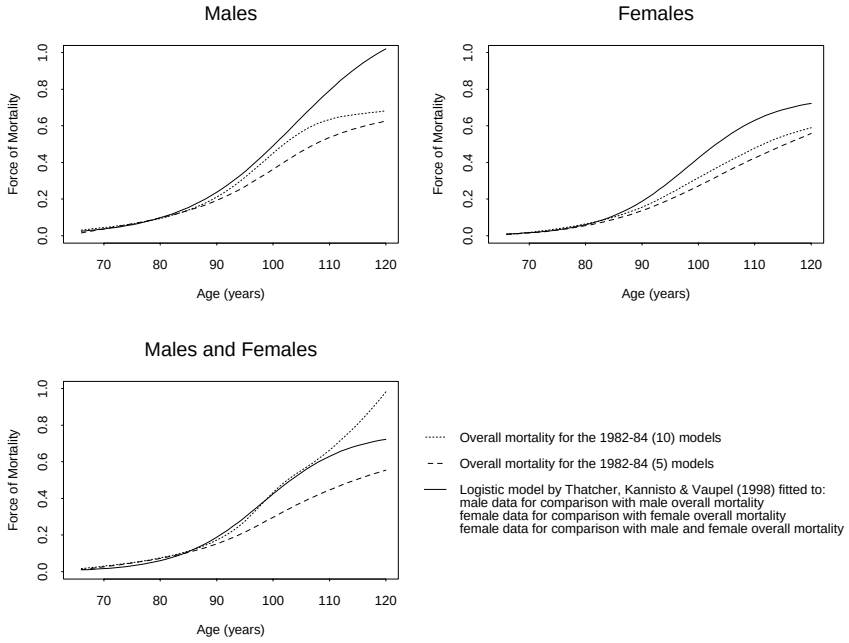

Figure 6.55: Overall model mortality for the 1982–84 (5) models and the 1982–84 (10) models compared with a logistic model of mortality by Thatcher, Kannisto & Vaupel (1998).

and the latter as the '1982–84 (5)' models, for the 1982–84 NLTCS.

In Figure 6.55, overall mortality in the 1982–84 (10) models and the 1982–84 (5) models, for males, females and in aggregate, are compared to those of the logistic model (see previous section for more detail). In general, overall mortality in the disability model is in reasonable agreement with that of the logistic model, though a few noticeable features are:

1. overall mortality in the disability models is in very good agreement with the logistic model upto ages 90–95 years (which is the oldest age group for which there was a point estimate when estimating the parametric transition intensities);

2. after age 95 years overall mortality in the disability models falls below that of the logistic model;

3. the 1982–84 (10) models always produce greater overall mortality than the 1982–84 (5) models;

4. overall mortality in the 1982–84 (10) model, for males and females combined, is the closest to that of the logistic model — they are in very close agreement up to just below age 110 years (however, it should be noted that the logistic fit is that for females, so it would be expected that it should be below that for males and females combined); and

5. as expected, overall model mortality for males is always above that for females, though overall mortality in the 1982–84 (5) model for males and females combined seems anomalous — it is below that for females (this is possible because the parametric transition intensities were fitted separately for each gender classification, so that overall model mortality for males and females combined is not a weighted sum of that for males and females taken separately).

The form of overall model mortality can be investigated by looking at its separate components — the occupancy probabilities of each state (conditional on being alive) and the force of mortality for each non-dead state. For males, these are illustrated in Figures 6.56 and 6.57, for females in Figures 6.58 and 6.59 and for males and females combined in Figures 6.60 and 6.61.

Looking at the overall model mortality for males first, it is clear (from Figures 6.56 and 6.57) why overall mortality is higher for the 1982–84 (10) model, since the parametric force of mortality is higher for each individual state (except the IADL only state, which has little influence, as it has low occupancy probabilities). It can also be seen that overall model mortality levels off at older ages because of the increasing occupancy probability of the 1–2 ADLs state, which has a fairly low and linearly increasing force of mortality.

For females, the reason why overall mortality is higher for the 1982–84 (10) model is not so clear, though after age 90 for the three states with the highest occupancy probabilities (1–2 ADLs state, 5–6 ADLs state and the Institutionalised state), the parametric force of mortality in the 1982–84 (10) model is greater than, or equal to, that of the 1982–84 (5) model. The reason why it levels off at older ages differs from that in the case of males:

1. for the 1982–84 (10) model, it is because of the increasing proportion of lives

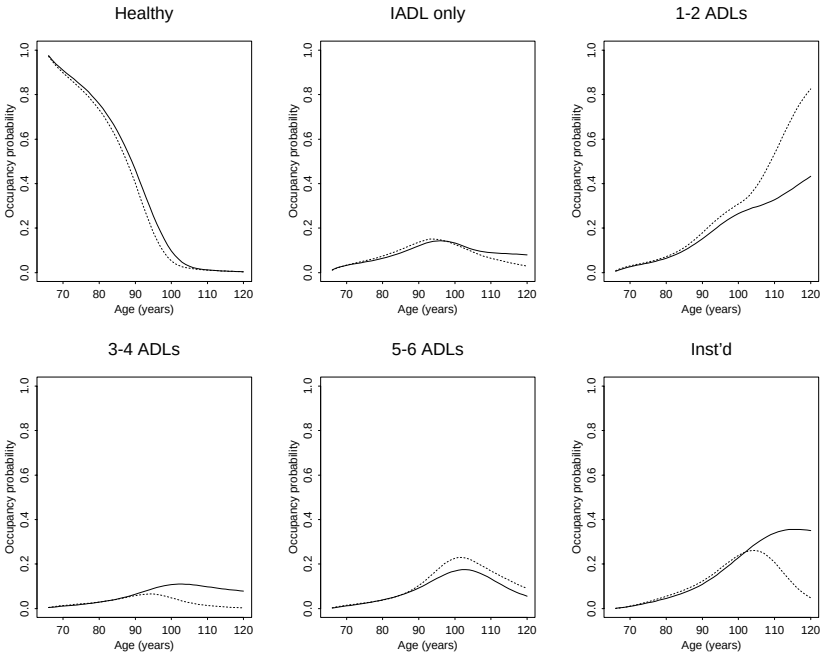

Figure 6.56: Model occupancy probabilities, conditional on being alive, for the 1982–84 (5) models (solid line) and the 1982–84 (10) models (dotted line), for males.

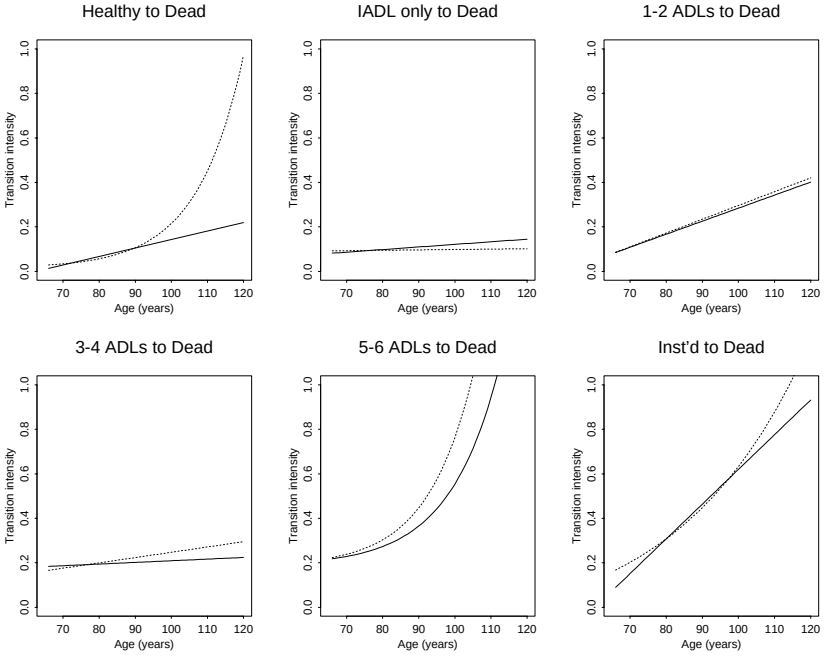

Figure 6.57: The parametric force of mortality for the 1982–84 (5) NLTCS models (solid line) and 1982–84 (10) NLTCS models (dotted line), for males.

in the 1–2 ADLs state, the 3–4 ADLs state and the 5–6 ADLs state (all of which have low and linearly increasing fitted forces of mortality); but

2. for the 1982–84 (5) model, the main influence is from the high proportion of lives in the institutionalised state (which has a linearly increasing force of mortality).

For males and females combined, the reason why overall mortality is higher for the 1982–84 (10) model is similar to that in the case of males. Overall mortality for the 1982–84 (5) model is the same shape as that for males and females separately — it levels off at older ages (mainly due to the high occupancy of the institutionalised state at older ages, which has a linearly increasing force of mortality). However, overall mortality for the 1982–84 (10) model continues to increase exponentially with age, even at the oldest ages. This is because, even though the force of mortality in the institutionalised state increases exponentially with age, the occupancy probability (conditional on being alive) of this state is almost 1 by age 100, and remains at this level. This must be because of very high rates of institutionalizion (directly or indirectly) from all the other non-dead states at older ages and low rates of recovery.

Having compared the overall force of mortality in the 1982–84 models with a benchmark force of mortality, the question is whether or not mortality in the model should be adjusted in some manner, to be more consistent with the benchmark force of mortality. In adjusting model mortality a number of factors need to be considered:

1. whether mortality should be uniformly adjusted across all disability groups and if not, in what manner should they be adjusted;

2. whether the choice between a linear fit and an exponential fit for the parametric transition intensities should be considered;

3. the nature of the adjustment, for example whether the force of mortality in each disability state should be adjusted by a constant amount, a muliplicative factor or an increase/decrease in the exponential factor;

4. how appropriate is the benchmark force of mortality (for example, the USA is not one of the 13 countries upon which it is based); and

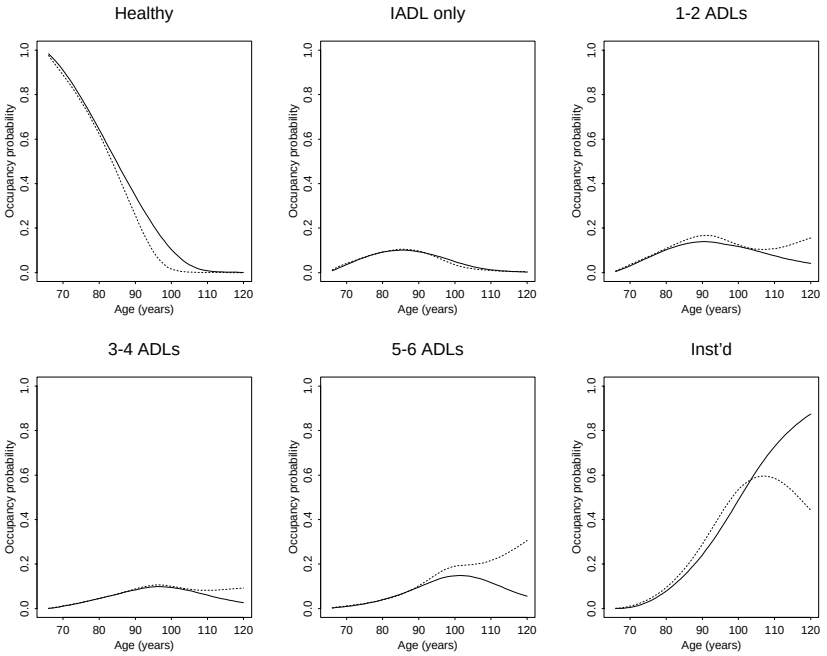

Figure 6.58: Model occupancy probabilities, conditional on being alive, for the 1982–84 (5) models (solid line) and the 1982–84 (10) models (dotted line), for females.

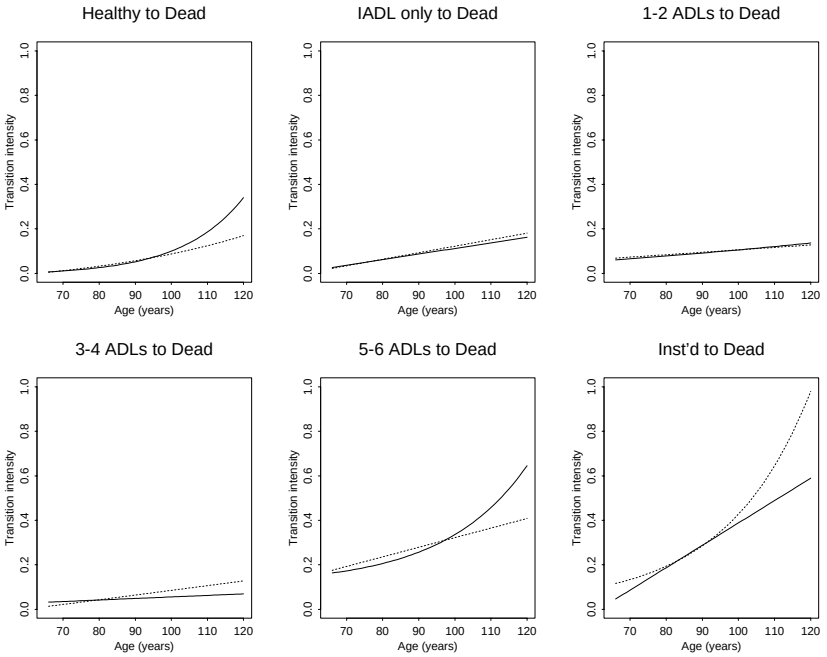

Figure 6.59: The parametric force of mortality for the 1982–84 (5) models (solid line) and the 1982–84 (10) models (dotted line), for females.

Table 6.69: Model survival probabilities for lives in the healthy state at age 65 calculated from the 1982–84 (10) models and the 1982–84 (5) models, for males, females and in aggregate (agg).

	Survival probability from age 65 for the adjusted:					
	1982–84 (10) model			1982–84 (5) model		
Age	males	females	agg	males	females	agg
	%	%	%	%	%	%
70	83.010	94.007	89.114	87.277	94.578	90.915
75	62.866	81.566	73.128	66.596	84.233	75.137
80	42.112	63.636	54.029	43.983	68.115	55.754
85	23.518	42.752	34.397	24.447	47.662	36.092
90	9.946	23.144	17.411	10.869	27.256	19.411
95	2.739	9.161	5.921	3.545	11.925	8.038
100	0.409	2.391	0.992	0.760	3.728	2.332
105	0.031	0.396	0.074	0.099	0.794	0.444
110	0.001	0.043	0.003	0.008	0.114	0.056

5. whether a factor of mortality improvement with time should be incorporated into the adjustments (the NLTCS is over the period 1982–84, while the data used for the logistic fit is over the period 1980–1990).

and most of these choices would necessarily be arbitrary (for example, due to lack of information about the force of mortality by disability status).

Another important consideration is the application of this model. I will use this parameterized model to look at the costs of disability in a long-term care contract, in which the most important age range is at younger ages (65–90 years), especially when the costs are discounted over time. Given that overall model mortality at younger ages is reasonably consistent with the benchmark force of mortality (so any adjustments made would need to be aimed primarily at older ages), together with the consideration of having to make some arbitrary decisions when adjusting mortality, I would argue against making any mortality adjustments here. However, for use in an insurance application, careful consideration would need to be made as to how mortality could be adjusted appropriately.

With no adustments made, a common feature of overall model mortality, irrespective of the method and set of data used to estimate the transition intensities, is that overall model mortality is much closer to the logistic fit at younger ages (65–90

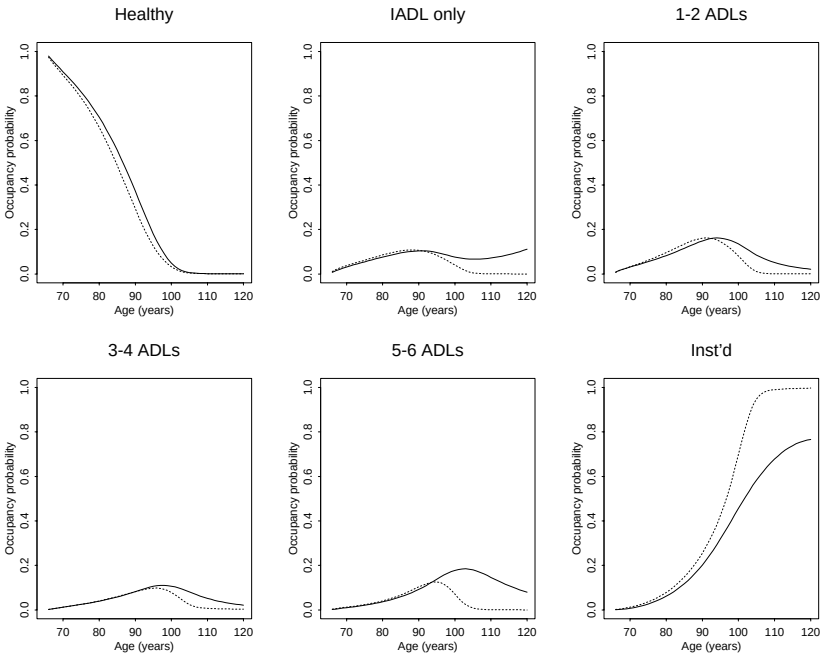

Figure 6.60: Model occupancy probabilities, conditional on being alive, for the 1982–84 (5) models (solid line) and the 1982–84 (10) models (dotted line), for males and females combined.

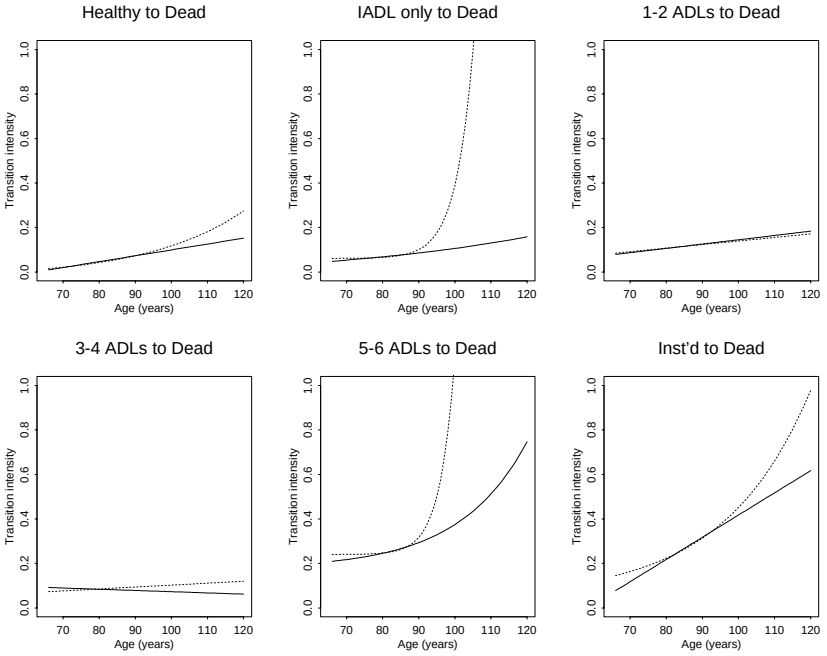

Figure 6.61: The parametric force of mortality for the 1982–84 (5) models and the 1982–84 (10) models (dotted line), for males and females combined.

years) than at older ages (90+ years). This is not surprising though, considering that the oldest data point used in any of the fits is age 91 years and that after this age, all of the parametric transition intensities are extrapolations of the fits before this age — which may not be a reasonable assumption. Furthermore, the graphs of occupancy probabilities (Figures 6.56, 6.58 and 6.60) may be misleading since these occupancy probabilities have been adjusted for deaths (to aid in illustrating the components of overall model mortality). Table 6.69 gives the survival probabilities (as percentages) for each set of parametric transition intensities, assuming that a person is alive and in the healthy state at age 65. For the 1982–84 models, the probability that a person who was alive and in the healthy state at age 65 is still alive at age 100 is between 0.41% and 3.73% — a small percentage, which decreases to less than 0.12% by age 110. So, overall mortality in the model is determined by small occupancy probabilities at ages over 100 and must then be sensitive to small changes in the occupancy probabilities and thus the transition intensities — which are, at these ages, extrapolations. Given these uncertainties in the parameter estimates at older ages, when I estimate the costs of disability in long-term care insurance in Section 7, I look at the sensitivity of these costs to arbitrary adjustments in the force of mortality at older ages (95+ years), as well as isolating the model costs attributable to disabled lives over age 95.

6.4 Overall Model Mortality, Using the Fitted Parametric Transition Intensities from the 1984–89 NLTCS

In this section I compare the overall force of mortality in the disability models using the parametric transition intensities fitted to the 1984–89 NLTCS (which I will refer to, using the notation of the previous section, as the 1984–89 (10) models and the 1984–89 (5) models, depending on the age bands used to group the original data) with the same benchmark force of mortality as the previous section. Figure 6.62 shows the overall model force of mortality in the disability model (as defined in

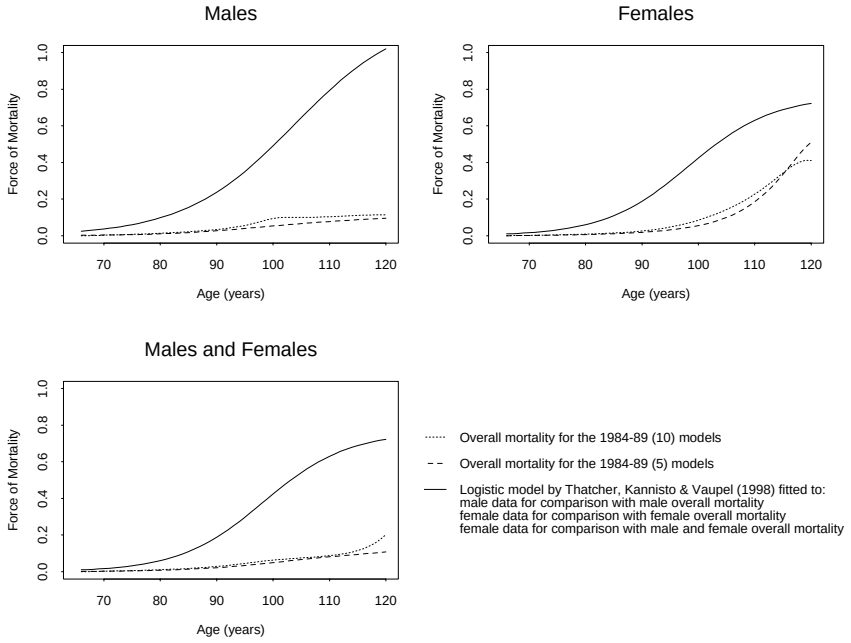

Figure 6.62: Overall model mortality for the 1984–89 (5) models and the 1984–89 (10) models compared with a logistic model of mortality by Thatcher, Kannisto & Vaupel (1998).

Equation 6.62) for males, females and in aggregate compared with a logistic model of mortality (described in Section 6.3). It is clear that overall mortality in the 1984–89 models is substantially lighter than those given by the logistic model in all cases — though overall model mortality for females is considerably higher than that for males and females together or males separately. This is not altogether surprising, since it was noted in Section 4.2 that an apparent anomaly in the NLTCS surveys was that the 5-year probabilities of dying over the periods 1984–89 and 1989–94 were considerably less (1/2 – 1/3) than the 2-year probability of dying in the period 1982–84 — which in turn, may be a consequence of the classification 'Not in survey year' being used in the 1989 and 1994 NLTCS (and not in the 1984 NLTCS).

The reason that overall model mortality is so low becomes clear when it is decomposed into its separate components — the occupancy probabilities of each state (conditional on being alive) and the force of mortality for each non-dead state. For males, these are illustrated in Figures 6.63 and 6.64, for females in Figures 6.65 and 6.66 and for males and females combined in Figures 6.67 and 6.68. At younger

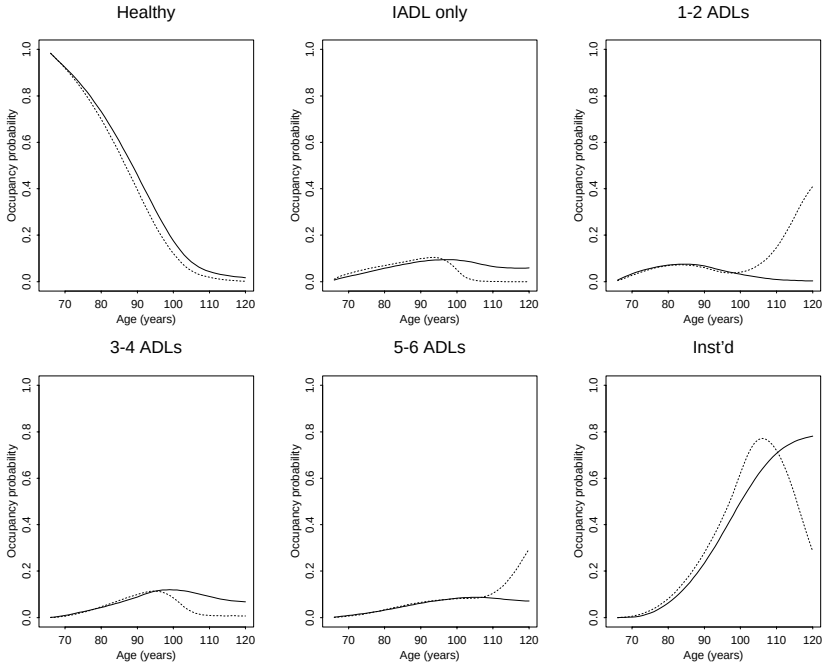

Figure 6.63: Model occupancy probabilities, conditional on being alive, for the 1984–89 (5) models (solid line) and the 1984–89 (10) models (dotted line), for males.

ages (< 90 years), overall model mortality is heavily influenced by the mortality from the healthy state (as the occupancy probability, conditional on being alive, of the healthy state starts at 1 at age 65 and drops to about 1/2 by age 85–90 years), which, for all three gender classifications, are very low and remain very low, compared with, for example, mortality from the healthy state in the previous section. At older ages (> 90 years), the dominent influence on overall mortality is (in general) the mortality of the institutionalised state, which for males and females combined and males alone, is very low and linearly increasing. For females, mortality in the institutionalised state is exponentially increasing, and the effect of this is clear in the exponentially increasing overall mortality for females — although it is still substantially below that of the logistic model.

With such low overall model mortality, some adjustments need to be made. The options for adjusting mortality in the models are either:

1. to increase mortality in each state in some way, such that overall mortality in the model is consistent with mortality from the logistic model (in the sense of least squares, or some similar criterion); or

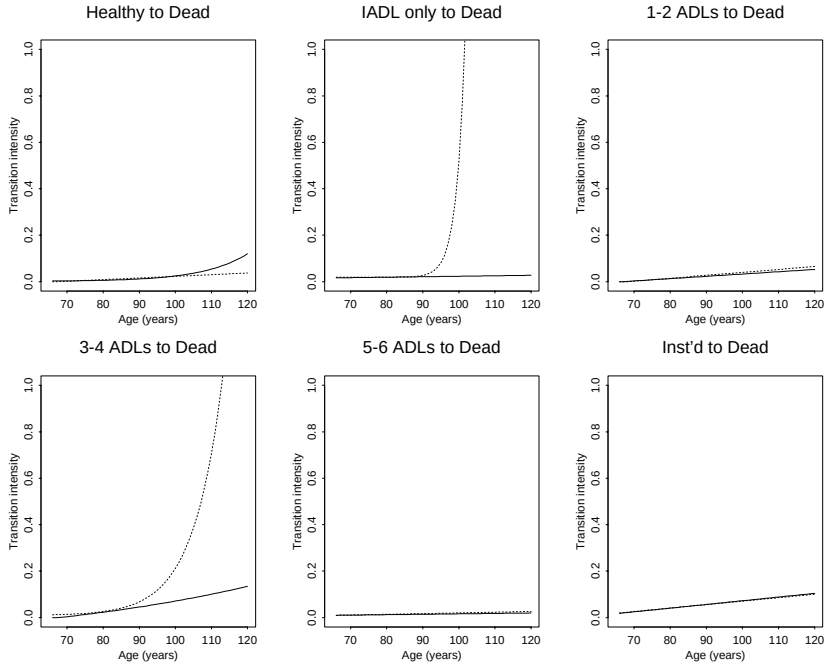

Figure 6.64: The parametric force of mortality for the 1984–89 (5) models (solid line) and the 1984–89 (10) models (dotted line), for males.

2. to use the mortality transition intensities from the 1982–84 NLTCS.

The advantage of the first method above is that overall mortality in the model could be adjusted to be arbitarily close to that of the logistic fit. The disadvantage is that a number of choices need to be made about how mortality in each state is adjusted (as discussed in Section 6.3), some of which will necessarily be arbitrary (for example, whether mortality should be uniformly adjusted by disability level). The main advantage of using the mortality estimates from the 1982–84 NLTCS is that these are based on actual experience, and it may be expected that mortality experience by disability level would not change much between the periods 1982–84 and 1984–89. Further support for using the mortality estimates from the 1982–84 NLTCS is gained by comparing the occupancy probabilities, conditional on being alive, of the disability models from the 1982–84 and 1984–89 NLTCS, as they show the same patterns across survey periods:

1. the occupancy probabilities for the healthy state start at 1 (by assumption) and decrease to about 0.5 by age 85–90 years, then rapidly drop to zero;

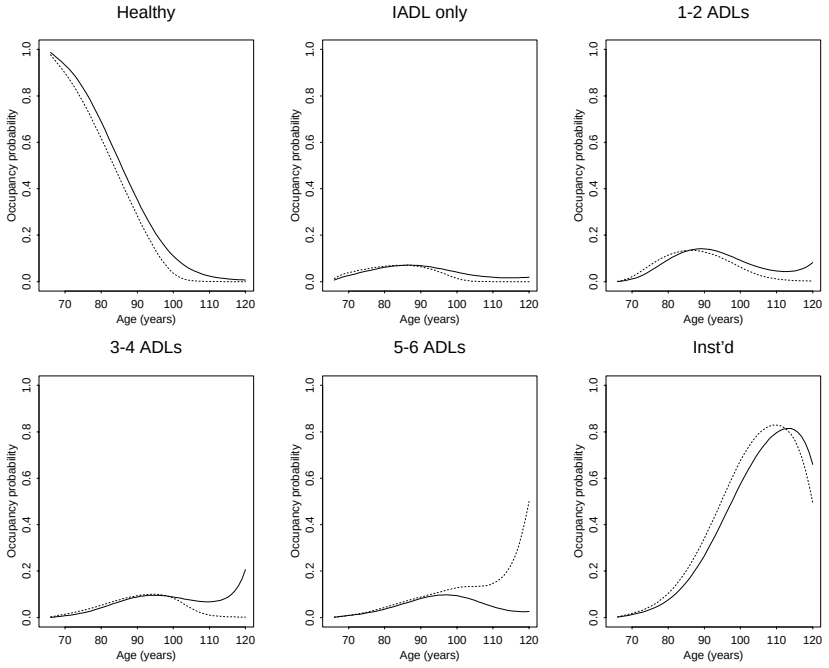

Figure 6.65: Model occupancy probabilities, conditional on being alive, for the 1984–89 (5) models (solid line) and the 1984–89 (10) models (dotted line), for females.

2. the occupancy probabilities for the disability states: IADL only, 1–2 ADLs, 3–4 ADLs and 5–6 ADLs, follow the same trend — they start at zero at age 65, slowly increase up to ages 90–100 years, then decrease back to zero;

3. the occupancy probabilities for the institutionalised state start at zero, then steadily increase to 1 by age 120 years.

Although not all of the occupancy probabilities follow exactly the trends described above, the majority do, which suggests that overall mortality from the disability model using the mortality estimates from the 1982–84 NLTCS and otherwise parameterized using the 1984–89 NLTCS should be reasonably consistent with the logistic model (since overall model mortality using the 1982–84 NLTCS was).

This is confirmed in Figure 6.69, which compares overall mortality for the adjusted parametric transition intensities fitted to the 1984–89 NLTCS, with the mortality transition intensities from the 1982–84 NLTCS, (which I will refer to as the adjusted 1984–89 (5) models and the adjusted 1984–89 (10) models), with that of the logistic model. At younger ages, overall mortality for all the adjusted 1984–89

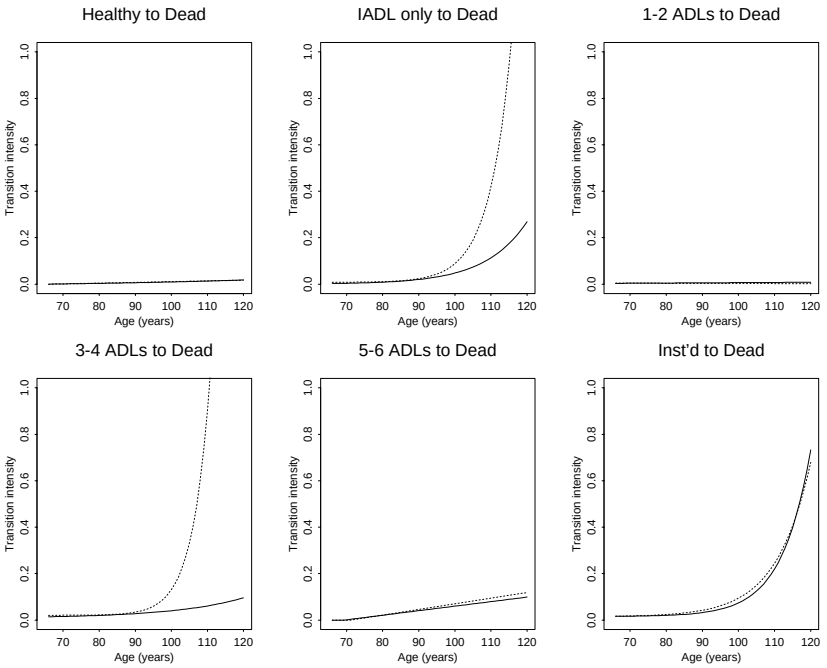

Figure 6.66: The parametric force of mortality for the 1984–89 (5) models (solid line) and the 1984–89 (10) models (dotted line), for females.

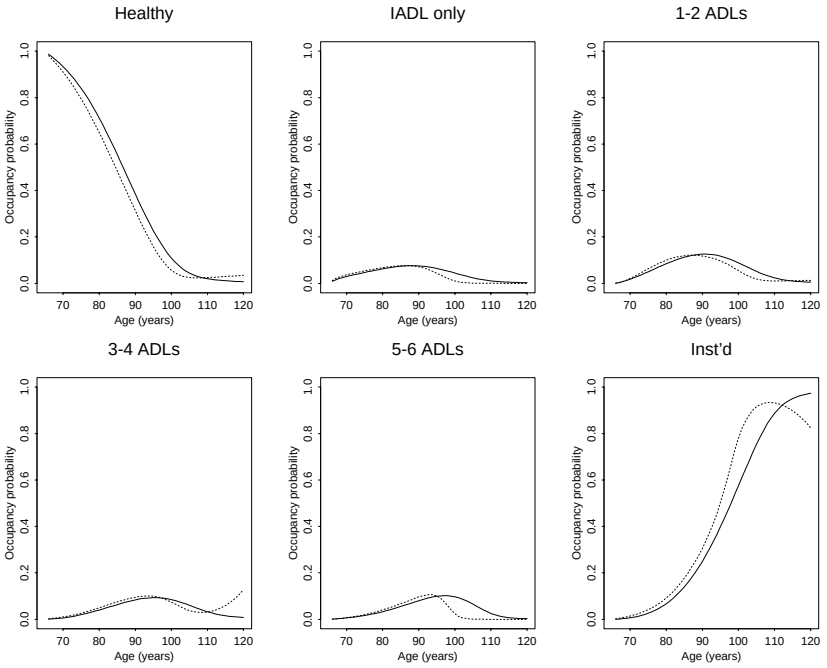

Figure 6.67: Model occupancy probabilities, conditional on being alive, for the 1984–89 (5) models (solid line) and the 1984–89 (10) models (dotted line), for males and females combined.

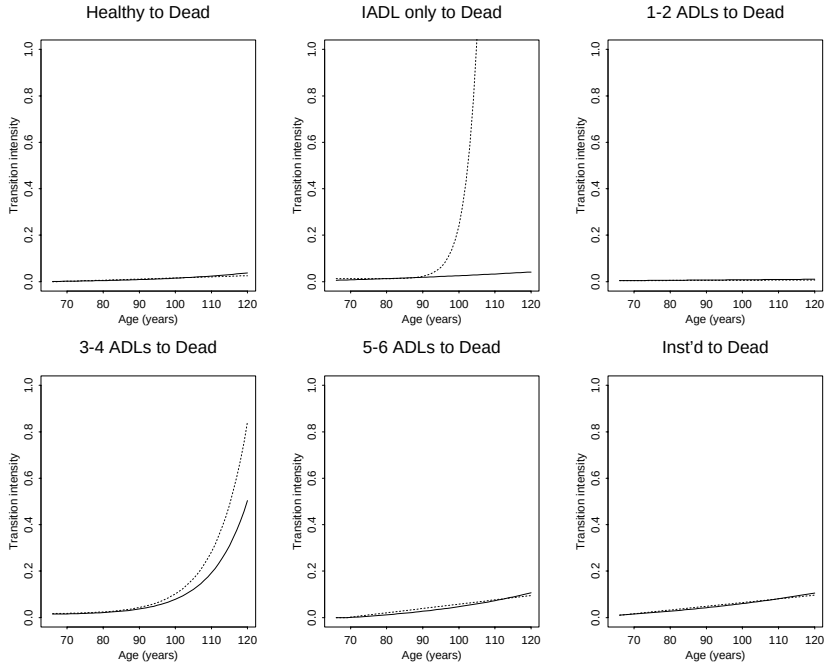

Figure 6.68: The parametric force of mortality for the 1984–89 (5) models (solid line) and the 1984–89 (10) models (dotted line), for males and females combined.

models is very close to that of the logistic model, but by age 90, they drop below those of the logistic model and, in general, the difference increases with increasing age.

The reason why overall mortality of the adjusted 1984–89 models is lower than that of the 1982–84 models (even though they both have the same mortality transition intensities) can be investigated by looking at the separate components of overall mortality — the occupancy probabilities of each state (conditional on being alive) and the force of mortality for each non-dead state. The force of mortality from each non-dead state was given in the previous section (as they are taken from the 1982–84 NLTCS), and the occupancy probabilities are given in Figures 6.70, 6.71 and 6.72, for males, females and in aggregate, respectively.

At younger ages, for both the 1982–84 models and the adjusted 1984–89 models, the state with the largest occupancy probability is the same, that for the Healthy state — it starts at one at age 65 and deceases to about 1/2 by age 90. So it is not surprising that, at younger ages, overall mortality in the adjusted 1984–89 models is very similar to that of the 1982–84 models. However, at older ages the patterns of

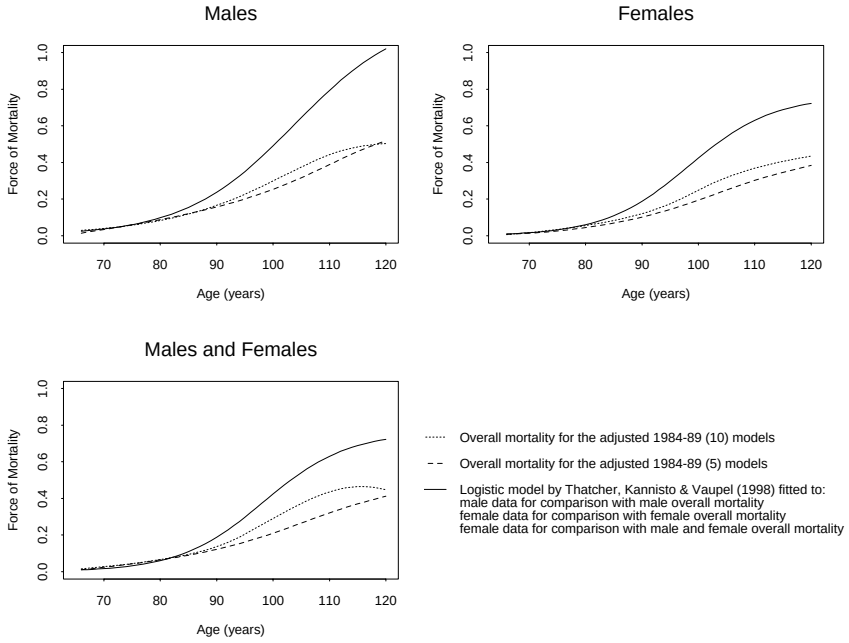

Figure 6.69: Overall model mortality for the adjusted 1984–89 (5) models and the adjusted 1984–89 (10) models compared with a logistic model of mortality by Thatcher, Kannisto & Vaupel (1998).

the occupancy probabilities are different in respect of the 1982–84 models and the adjusted 1984–89 models:

1. the occupancy probabilities of the institutionalised state in the 1982–84 models, generally dominate (apart from for males, where the occupancy probability of the 1–2 ADL state dominates) at older ages (and often increases to almost 1 by age 120); whereas

2. for the adjusted 1984–89 models there is a strong tendency for the occupancy probabilities of the 3–4 ADLs state to monotonically increase with age, remaining at a similar level to those of the institutionalised state.

Now, looking at the parametric force of mortality from the 3–4 ADLs state in the 1982–84 models (Figures 6.57, 6.59 and 6.61), which are small and linearly increase slowly (and for one data set even decrease) with age, it is clear why the overall forces of mortality in the 1984–89 models drop below those of the 1982–84 models (and thus below that of the logistic model). So even using the same transition intensities

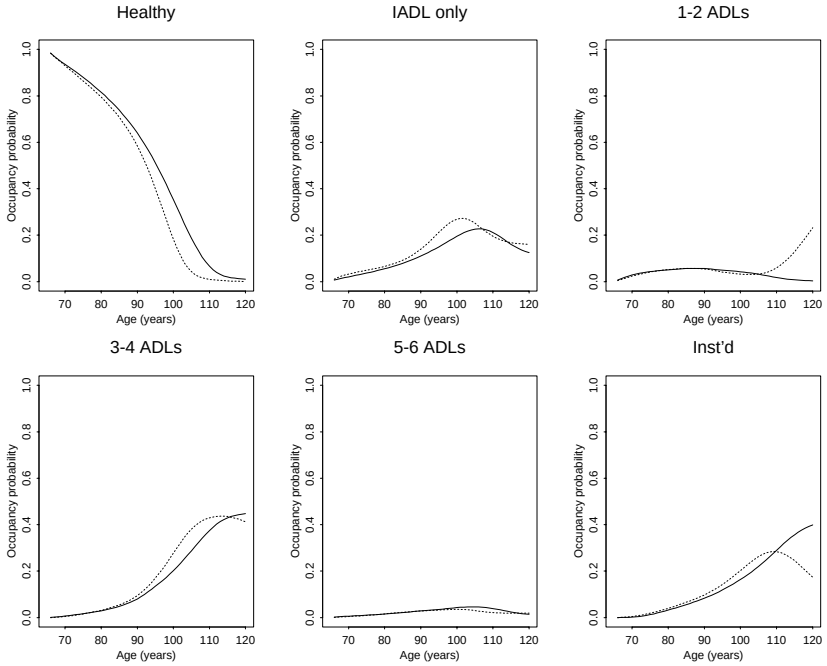

Figure 6.70: Model occupancy probabilities, conditional on being alive, for the adjusted 1984–89 (5) models (solid line) and the adjusted 1984–89 (10) models (dotted line), for males.

for the force of mortality from each non-dead state, there is no guarantee that the overall force of mortality of two models otherwise parameterized differently, will be similar. This also suggests that an important factor of overall model mortality is the force of disability and recovery within the model and not just the force of mortality from each state, especially at older ages — another factor to take into account when adjusting the level of overall model mortality.

Table 6.70 gives the survival probabilities (as percentages) for each of the adjusted 1984–89 models, assuming that a person is alive and in the healthy state at age 65. They are slightly higher than for the 1982–84 models — at age 65, they are about 1% higher, and the difference increases to about 3% by age 90 (apart from the adjusted 1984–89 (5) model for females, where the difference is almost 7%). The differences then decrease with age, which is expected since they are tending towards 0%. So, while the survival probabilities are higher for the adjusted 1984–89 models than the 1982–84 models, the probability of survival to 100 years is still low (less than 8%) and to 110 years it is less than 0.29%.

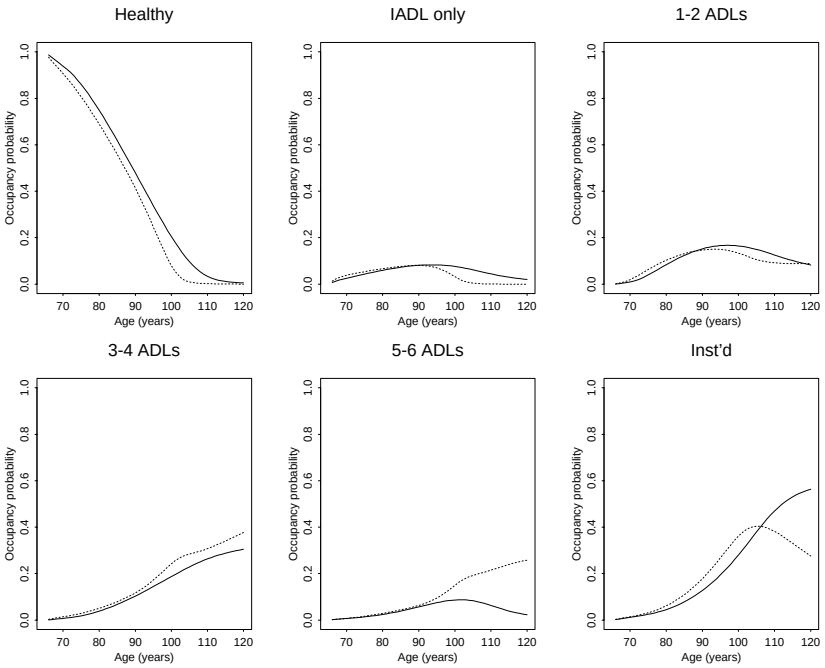

Figure 6.71: Model occupancy probabilities, conditional on being alive, for the adjusted 1984–89 (5) models (solid line) and the adjusted 1984–89 (10) models (dotted line), for females.

Table 6.70: Model survival probabilities for lives in the healthy state at age 65 calculated from the 1984–89 (10) models and the 1984–89 (5) models, for males, females and in aggregate (agg).

	Survival probability from age 65 for the adjusted:					
	1984–89 (10) model			1984–89 (5) model		
Age	males	females	agg	males	females	agg
	%	%	%	%	%	%
70	84.117	94.164	89.679	87.948	94.742	91.442
75	65.401	81.990	74.389	68.266	85.219	76.521
80	45.366	64.740	55.947	46.287	70.884	57.916
85	26.860	45.161	37.028	26.939	52.815	38.926
90	12.844	26.733	20.439	13.176	33.952	22.630
95	4.648	12.569	8.432	5.266	18.034	10.981
100	1.195	4.248	2.274	1.651	7.564	4.259
105	0.210	0.974	0.398	0.386	2.418	1.266
110	0.026	0.164	0.048	0.064	0.587	0.283

Should further adjustments be made to the 1984–89 models, to make overall model mortality more consistent with that from the logistic model? I would argue 'no' for the following reasons:

1. as discussed in the previous section, many arbitrary choices would need to be made about how to adjust mortality;

2. as discussed earlier in this section, the transition intensities between disability states themselves can have a large impact on overall mortality (especially at older ages), and so should also be considered when adjusting overall model mortality;

3. furthermore, mortality improvement (decreasing force of mortality) over time is a well documented observation (Macdonald *et al.* (1998)), and so it would be expected that overall mortality in the 1984–89 NLTCS should be lighter than in the 1982–84 NLTCS, on average — so some mortality improvement should be allowed for;

4. at younger ages, the overall force of mortality is reasonably consistent with that from the logistic model, and at older ages the transition intensities will be subject to sensitivity analyses (as described in the previous section).

So, given the adjustments already made to the 1984–89 models, and the points discussed above, I think that further adjustments to mortality (or otherwise) in the models are not justified.

In this section I have analysed and made some major adjustments to the 1984–89 models (replacing all the mortality transition intensities with those from the 1982–84 NLTCS). The adjustments made, improved the overall force of mortality for all of these models (closer to mortality from the logistic model), and while I have argued that this is a reasonable approach to use, there are many other ways that mortality could have been adjusted. With such major adjustments made, it would be difficult to justify attributing much weight to the absolute value of any application of these models, however, they will be very useful in providing estimates to compare with estimates from the 1982–84 models. Again, I give the same caution as for the 1982–84 models — for use in an insurance application, careful consideration would need

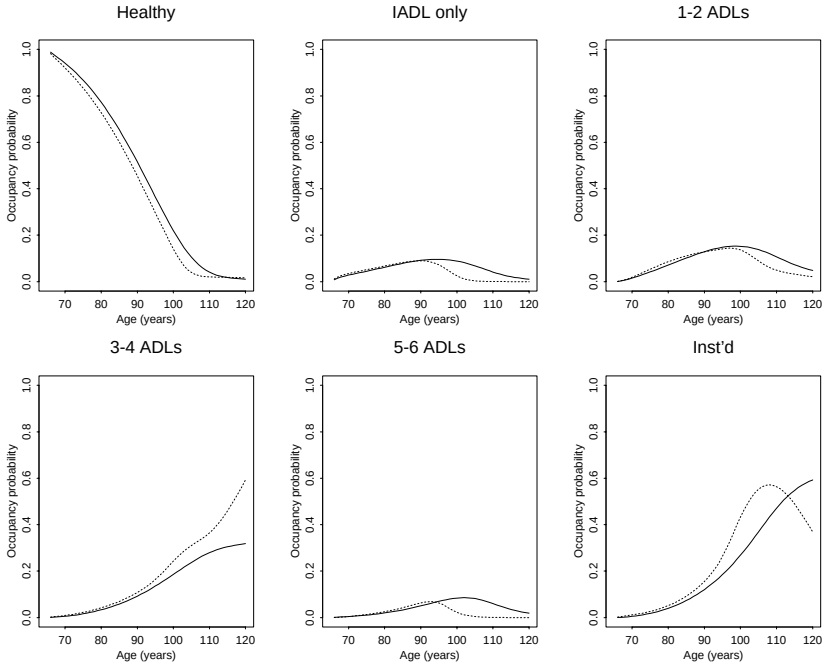

Figure 6.72: Model occupancy probabilities, conditional on being alive, for the adjusted 1984–89 (5) models (solid line) and the adjusted 1984–89 (10) models (dotted line), for males and females combined.

to be made as to how models could be adjusted appropriately.

6.5 Overall Model Mortality, Using the Fitted Parametric Transition Intensities from the 1989–94 NLTCS

In this section I compare the overall force of mortality in the disability models using the parametric transition intensities fitted to the 1989–94 NLTCS (which I will refer to, using the notation of the Section 6.3, as the 1989–94 (10) models and the 1989–94 (5) models, depending on the age bands used to group the original data) with the same benchmark force of mortality as in Section 6.3. Figure 6.73 shows the overall model force of mortality in the 1989–94 models (as defined in Equation 6.62) for males, females and combined compared with a logistic model of mortality (described in Section 6.3).

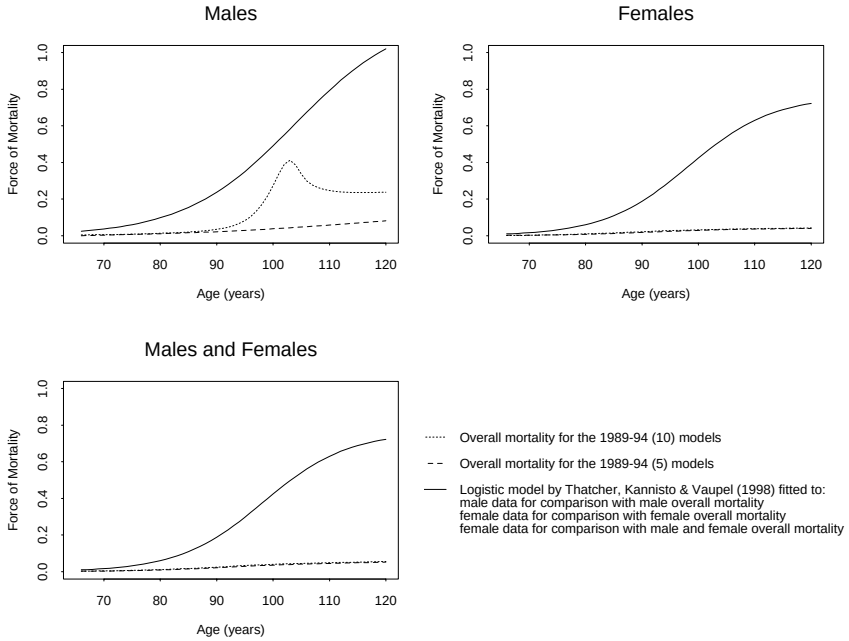

Figure 6.73: Overall model mortality for the 1989–94 (5) models and the 1989–94 (10) models compared with a logistic model of mortality by Thatcher, Kannisto & Vaupel (1998).

The overall forces of model mortality in the 1989–94 models (except for the anomalous 1989–94 (10) model for males) are very similar those of the 1984–89 models, and the general comments given in the previous section, on why they are substantially lighter than those given by the logistic model, apply equally.

The reason that overall model mortality is so low becomes clear when it is decomposed it into its separate components — the occupancy probabilities of each state (conditional on being alive) and the force of mortality for each non-dead state. For males, these are illustrated in Figures 6.74 and 6.75, for females in Figures 6.76 and 6.77 and for males and females combined in Figures 6.78 and 6.79. For all of the sets of parametric transition intensities (except those for the 1989–94 (10) model for males) the reason why overall mortality is so low is clear — the force of mortality from every state is very low, so overall mortality must also be low. Even though the forces of mortality are very low, it is worth noting that the patterns of the occupancy probabilities are very similar to those of the 1982–84 models.

For the 1989–94 (10) model for males, overall mortality behaves unexpectedly:

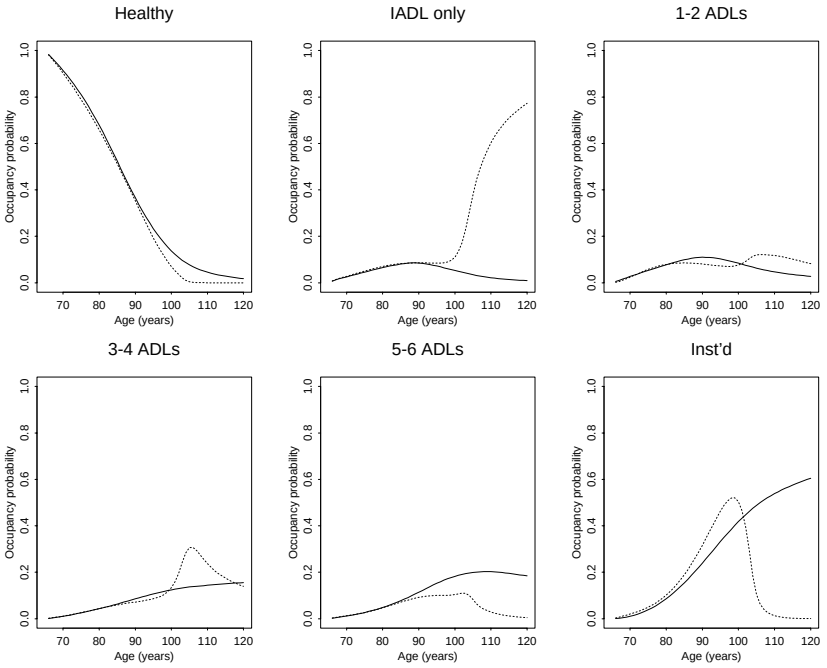

Figure 6.74: Model occupancy probabilities, conditional on being alive, for the 1989–94 (5) models (solid line) and the 1989–94 (10) models (dotted line), for males.

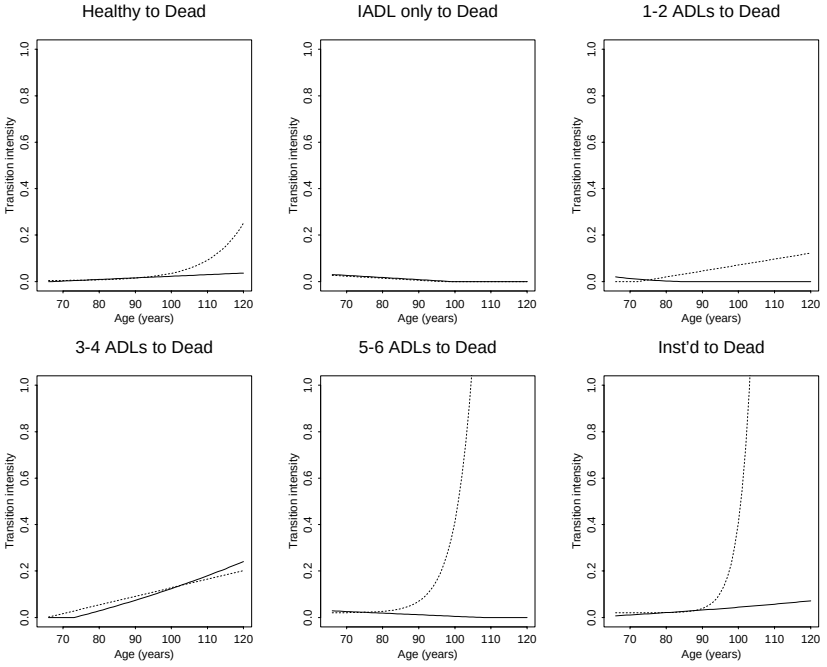

Figure 6.75: The parametric force of mortality for the 1989–94 (5) models (solid line) and the 1989–94 (10) models (dotted line), for males.

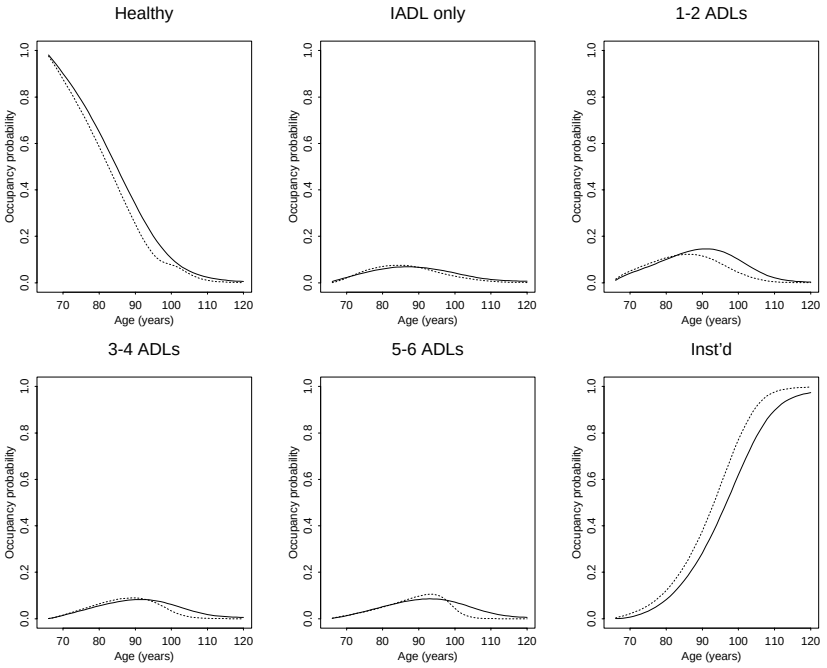

Figure 6.76: Model occupancy probabilities, conditional on being alive, for the 1989–94 (5) models (solid line) and the 1989–94 (10) models (dotted line), for females.

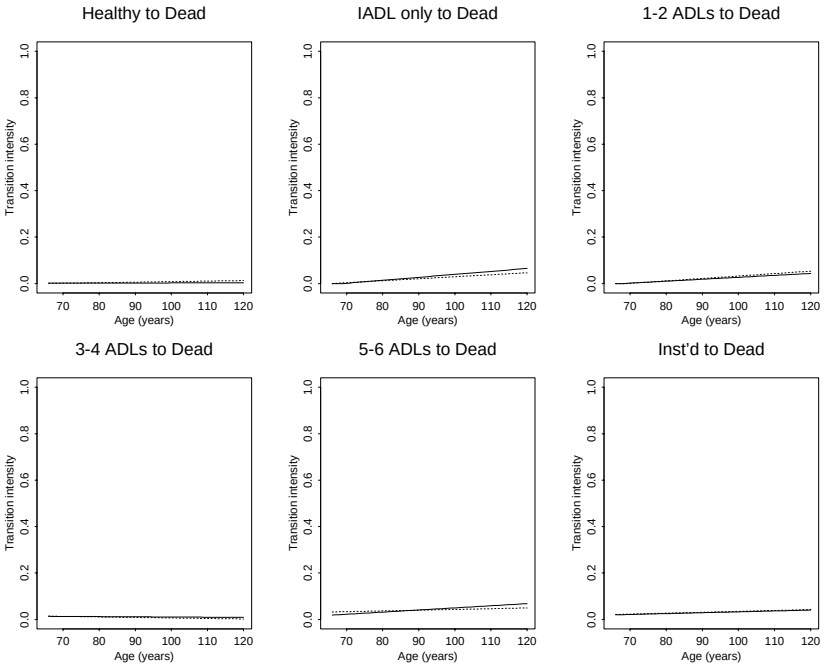

Figure 6.77: The parametric force of mortality for the 1989–94 (5) models (solid line) and 1989–94 (10) models (dotted line), for females.

it remains low until about age 90, then it increases exponentially until about age 102, when it decreases rapidly for a few years and then remains almost constant up to age 120. Most of this can be explained by looking at the occupancy probability (conditional on being alive) and mortality of the instutionalized state. The occupancy probability of the institutionalised state starts at zero at age 65 and exponentially increases until age 90 and the force of mortality in this state is very low until age 90, when it starts to exponentially increase (which is a large element of overall mortality, since the occupancy probability is quite high at these ages), causing overall mortality to increase exponentially also. The force of mortality of the institutionalised state continues to increase rapidly, exceeding 1.0 by age 100, thus increasing the probability of dying from this state (relative to other states), and reducing its occupancy probability (conditional on being alive). Also, at about age 100, the occupancy probability of the IADL only state starts to increase rapidly, which has a very low force of mortality that linearly decreases with age (by age 100, it is zero and remains zero) — these effects combined act to reduce overall mortality. By about age 110, the only other two states that contribute significantly to overall mortality are the 1–2 ADLs state and the 3–4 ADLs state, both of which have fairly low and linearly increasing mortality, the combined effect of which is a levelling out of overall mortality.

It is clear, as with the 1984–89 models, that some adjustment to the forces of mortality in the 1989–94 models is necessary, to make overall model mortality more consistent with the benchmark mortality — that from the logistic model. I will adjust the models in the same way and for the same reasons as the 1984–89 models were adjusted, by substituting the mortality transition intensities from the 1982–84 models (see previous section for more detail). Figure 6.80 compares overall mortality for the adjusted parametric transition intensities fitted to the 1989–94 NLTCS, with the mortality transition intensities from the 1982–84 NLTCS, (which I will refer to as the adjusted 1989–94 (5) models and the adjusted 1989–94 (10) models), with that of the logistic model.

At younger ages, overall mortality for all the adjusted 1989–94 models is very close to that of the logistic model, but by age 90, it drops below those of the logistic

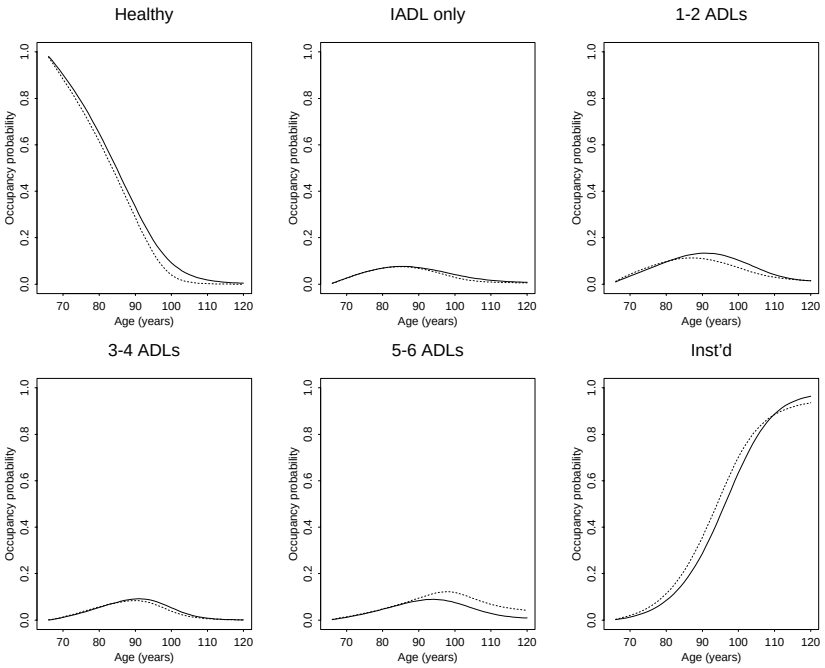

Figure 6.78: Model occupancy probabilities, conditional on being alive, for the 1989–94 (5) models (solid line) and the 1989–94 (10) models (dotted line), for males and females combined.

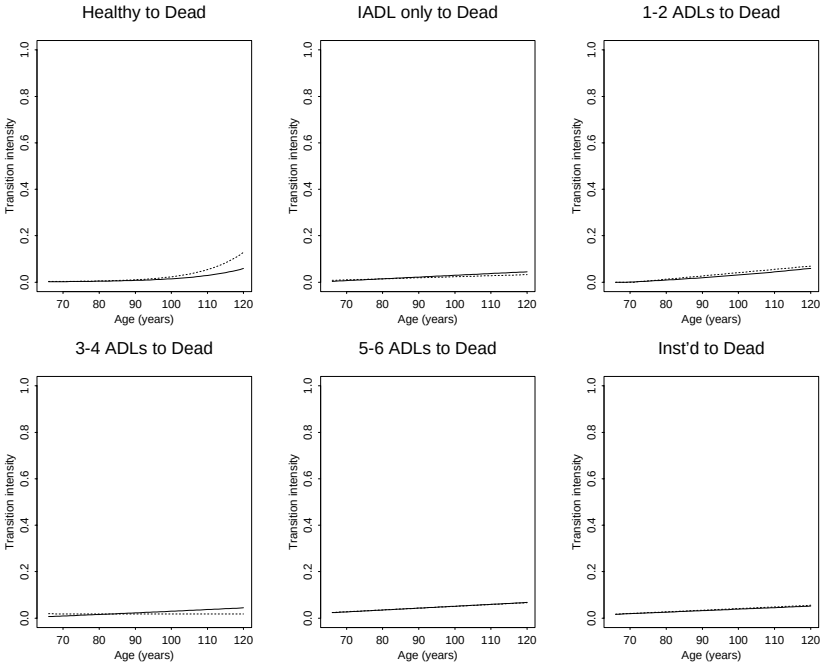

Figure 6.79: The parametric force of mortality for the 1989–94 (5) models (solid line) and the 1989–94 (10) models (dotted line), for males and females combined.

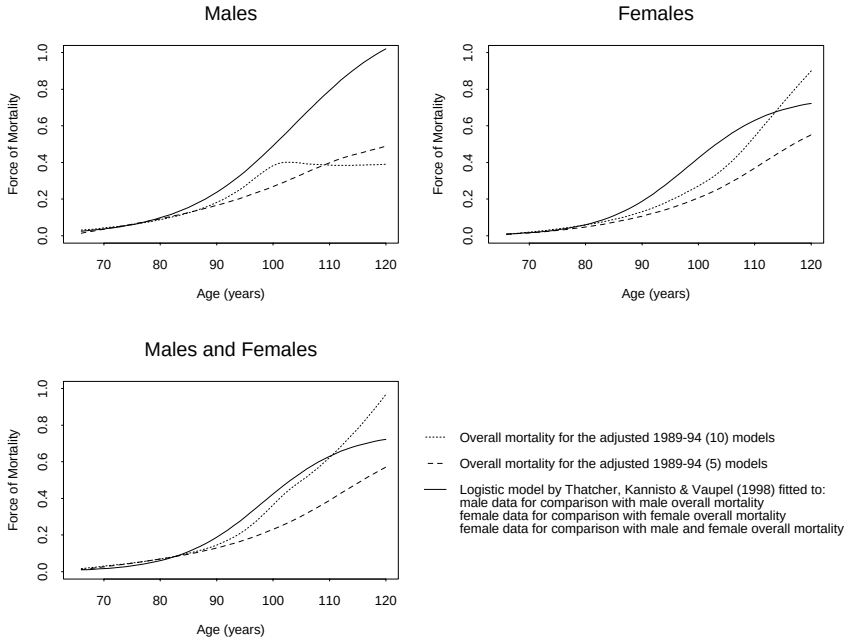

Figure 6.80: Overall model mortality for the adjusted 1989–94 (5) models and the adjusted 1989–94 (10) models compared with a logistic model of mortality by Thatcher, Kannisto & Vaupel (1998).

model and, in general, the difference between the two increases with increasing age. The overall mortality for the adjusted 1989–94 models lies, in general, between those of the 1982–84 models and the adjusted 1984–89 models (see next section for comparison of overall mortality in the 1982–84, 1984–89 and 1989–94 models) and is very similar in shape to the 1982–84 models — except for the adjusted 1989–94 (10) model for males, which partly retains the shape of overall mortality from the unadjusted 1989–94 model.

The occupancy probabilities of each state (conditional on being alive) for the adjusted 1989–94 models are given in Figures 6.81, 6.82 and 6.83, for males, females and in aggregate, respectively. From Figures 6.82 and 6.83 the reason why overall model mortality, for the female and aggregate adjusted 1989–94 models, is similar to the 1982–84 models is clear – the occupancy probabilities are very similar (and the force of mortality from each state is identical). For males, the occupancy probabilities of the 1982–84 models and the adjusted 1989–94 models are similar at younger ages (65–90 years), but are very different for older ages (90–120 years). For

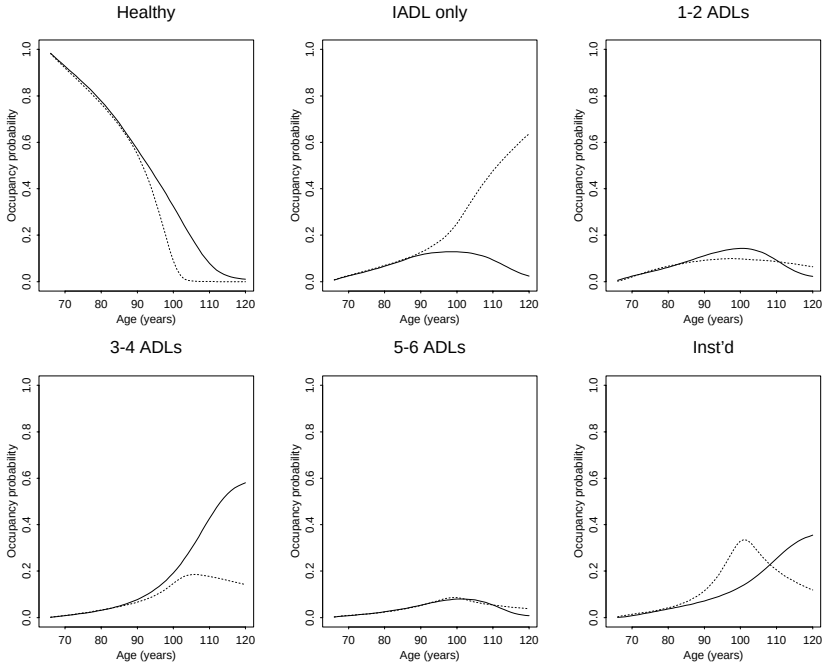

Figure 6.81: Model occupancy probabilities, conditional on being alive, for the adjusted 1989–94 (5) models (solid line) and the adjusted 1989–94 (5) models (dotted line), for males.

the 1982–84 males models, the occupancy probabilites for the 1–2 ADLs state and the institutionalised state dominate by age 100 and continue to increase with age, whereas:

1. for the adjusted 1989–94 (10) model, by age 100 the occupancy probabilities for the IADL only state, the 3–4 ADLs state and the institutionalised state dominate, but only those for the IADL only state continue to increase with age — the other two decrease thereafter; and

2. for the adjusted 1989–94 (5) model, by age 100 the occupancy probabilities are spread out fairly evenly among all states, but only those for the 3–4 ADLs state and the institutionalised state continue to increase with age, clearly dominating by age 110.

The first point above explains why overall mortality for the adjusted 1989–94 (10) model for males levels off so suddenly at about age 100 — by this age the occupancy probability of the IADL state dominates and continues to increase with age, giving

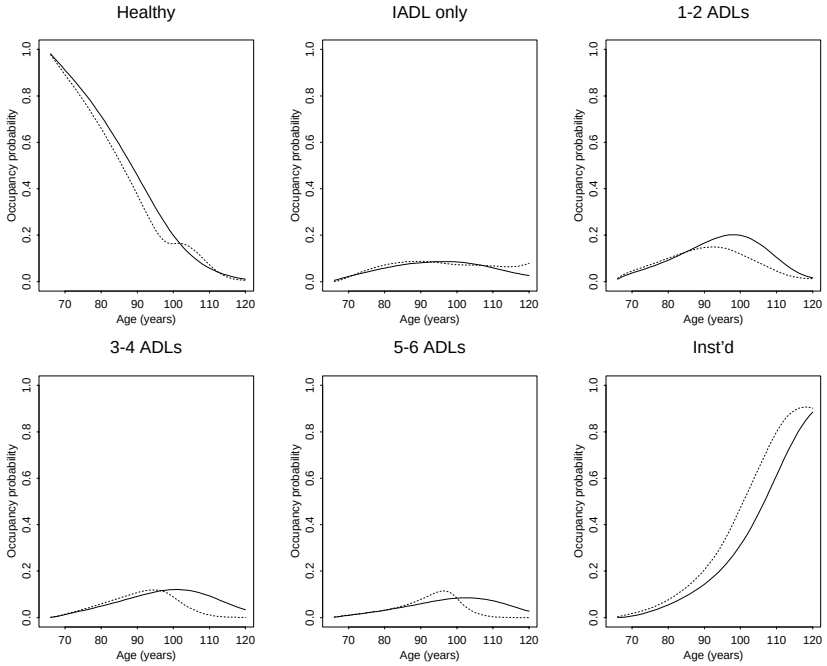

Figure 6.82: Model occupancy probabilities, conditional on being alive, for the adjusted 1989–94 (5) models (solid line) and the adjusted 1989–94 (10) models (dotted line), for females.

increasing weight to the force of mortality from this state, which is very low and linearly increases (with a low gradient) with age, the effect of which is a levelling-off of overall mortality. The second point above indicates why overall mortality for the adjusted 1989–94 (5) model is below that of the 1982–84 model, since the force of mortality for the 3–4 ADLs state (which has a high occupancy probability at older ages in the adjusted 1989–94 model) is below that of the 1–2 ADL state (which has a high occupancy probability at older ages in the 1982–84 model) and increases at a slower rate — and the other state with high occupancy probability at older ages is the same for both models, the institutionalised state.

Table 6.71 gives the survival probabilities (as percentages) for each of the adjusted 1989–94 models, assuming that a person is alive and in the healthy state at age 65. They are very similar to those of the 1982–84 models (see Tables 6.69) — at all ages and for all models (except that for females with transition intensities fitted to the data in 5-year age bands) the survival probabilities do not differ by more than 2% (even for the male models) from the 1982–84 models and in general the

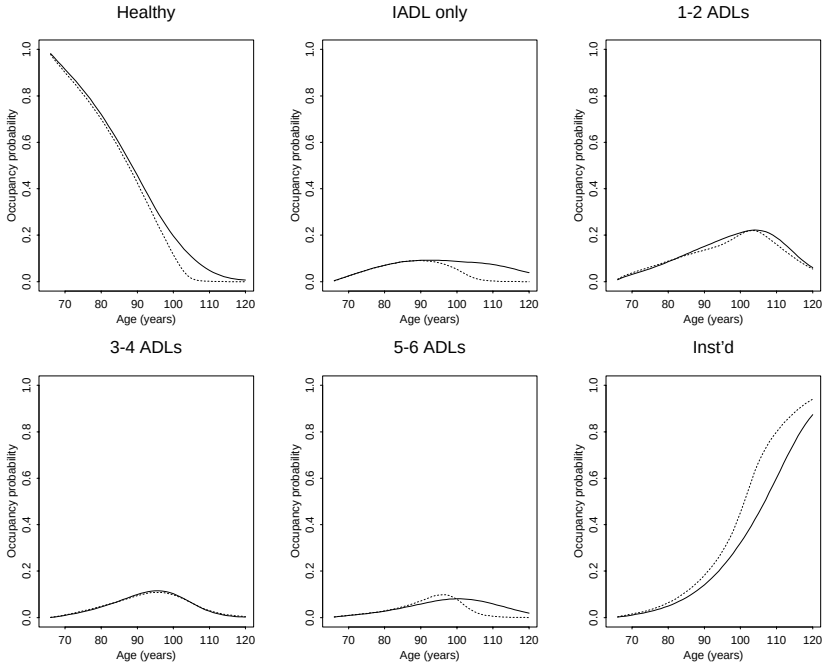

Figure 6.83: Model occupancy probabilities, conditional on being alive, for the adjusted 1989–94 (5) models (solid line) and the adjusted 1989–94 (10) models (dotted line), for males and females combined.

survival probabilities are slightly higher than for the 1982–84 models. Given that overall mortality for the 1989–94 models is closer to those given by the logistic fit than overall mortality in the 1984–89 models and that survival to older ages is low (less than 7% by age 100 and less than 0.4% by age 110), I argue as for the 1984–89 models — that further adjustments to mortality (or otherwise) in the models are not justified (see previous section). However, when I estimate the costs of disability in long-term care insurance in Section 7, I will look at the sensitivity of these costs to arbitrary adjustments in the force of mortality at older ages (95+ years), as well as isolating the model costs attributable to disabled lives over age 95.

In this section I have analysed and made some major adjustments to the 1989–94 models (replacing all the mortality transition intensities with those from the 1982–84 NLTCS). These improved the overall force of mortality for all of these models (making it closer to mortality from the logistic model), and while I have argued that this is a reasonable approach to use, there are many other ways that mortality could have been adjusted. With such major adjustments made, it would be difficult

Table 6.71: Model survival probabilities for lives in the healthy state at age 65 for the adjusted 1989–94 (5) models and the adjusted 1989–94 (10) models, for males, females and in aggregate (agg).

	Survival probability from age 65 for the adjusted:					
	1989–94 (10) model			1989–94 (5) model		
Age	males	females	agg	males	females	agg
	%	%	%	%	%	%
70	83.455	93.485	88.897	87.716	94.316	90.772
75	63.815	80.425	72.639	67.285	83.887	74.715
80	43.384	62.626	53.741	44.695	68.691	55.342
85	25.007	42.763	34.832	25.311	50.147	36.218
90	11.338	24.372	18.590	11.925	31.458	20.342
95	3.540	10.684	7.124	4.518	16.233	9.384
100	0.638	3.231	1.479	1.319	6.519	3.350
105	0.082	0.607	0.140	0.283	1.892	0.862
110	0.011	0.056	0.007	0.043	0.362	0.148

to justify attributing much weight to the absolute value of any application of these models, however, they will be very useful in providing estimates to compare with estimates from the 1982–84 models. Again, I give the same caution as for the 1982–84 and 1984–89 models — for use in an insurance application, careful consideration would need to be made as to how models could be adjusted appropriately.

6.6 Comparison of Overall Mortality in the 1982–84, 1984–89 and 1989–94 Disability Models

The aim of this section is first to compare overall mortality from the 1982–84, the adjusted 1984–89 and the adjusted 1989–94 models and then to discuss their application to estimating the costs of disability in a long-term care insurance contract.

Figures 6.84 and 6.85 compare the overall forces of mortality of the 1982–84, the adjusted 1984–89 and the adjusted 1989–94 models, with the transition intensities fitted to the data in 10-year age bands and 5-year age bands, respectively. The force of mortality from the logistic model is also shown for comparison. A few points are worth noting:

1. for all of the disability models, overall mortality is lighter than that given by

the logistic model, even for males and females combined (since the logistic fit here is for females, that for males and females would give a higher force of mortality);

2. the overall forces of mortality for all models are very close, to each other and to that of the logistic model, up to (at least) ages 90–95 years, after which they tend to diverge;

3. in general, the overall forces of mortality of the 1982–84 models are the highest, followed by those of the adjusted 1989–94 models and then those of the adjusted 1984–89 models (the notable exception to this is the overall force of mortality for the adjusted 1989–94 (5) model for males and females combined, which is exceptionally high);

4. there is less difference, in general, between the overall force of mortality for the models fitted to the data in 5-year age bands across survey periods, than for those fitted to the data in 10-year age bands.

The first point above suggests that any costs of disability estimated using these models may be overstated, as mortality is understated and lives, in the models, will be living longer than expected, increasing the estimates of the costs of disability — this can, however, be seen to be an implicit allowance for improving mortality over time. This will especially be the case at older ages, where the differences between overall mortality in the models and that of the logistic model are the greatest. This, together with the second observation above, suggests that more reliance should be given to applications of the model to younger ages (< 90 years) and less to older ages. For example, when estimating the costs of disability, sensitivity of the costs to the mortality assumptions at older ages could be carried out in a similar manner to that in the Alzheimer's model (see Section 2.5):

1. to estimate an upper bound on the costs, assume that after age 95 mortality remains constant at the level it was at age 95, in all states; and

2. to estimate a lower bound on the costs, assume that after age 95, mortality continues to increase exponentially, in all states;

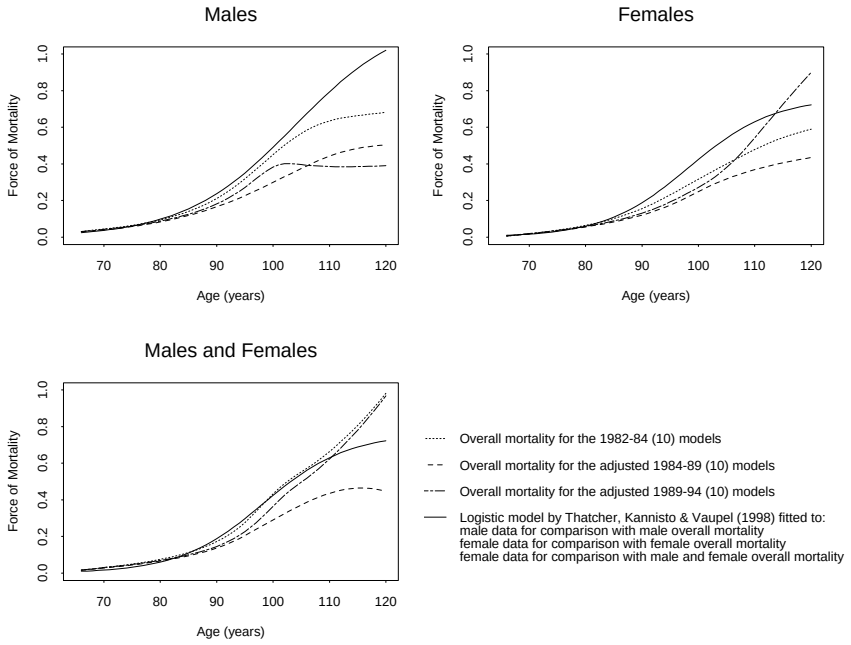

Figure 6.84: Overall mortality from the 1982–84 (10), the adjusted 1984–89 (10) and the adjusted 1989–94 (10) models compared with a logistic model of mortality by Thatcher, Kannisto, & Vaupel, (1998).

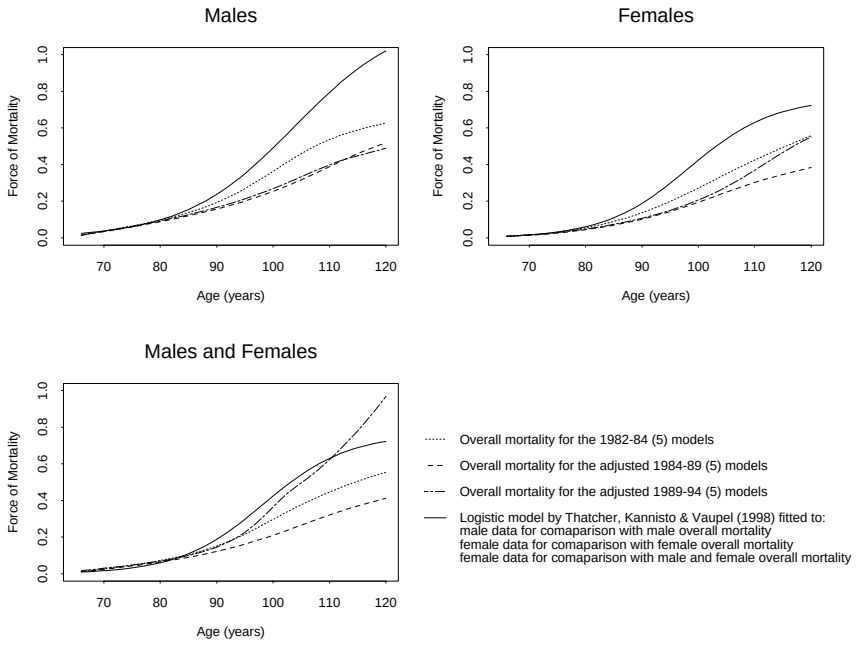

Figure 6.85: Overall mortality from the 1982–84 (5), the adjusted 1984–89 (5) and the adjusted 1989–94 (5) models compared with a logistic model of mortality by Thatcher, Kannisto, & Vaupel (1998).

In Section 6.3, I looked at the overall force of mortality in the 1982–84 models and compared them to a logistic model of mortality that was parameterized using a very large dataset. I then argued that, while further adjustments could be made to mortality in the disability models (to make them closer to that of the logistic model), it would be difficult to justify as a number of choices would need to be made, some of which would necessarily be arbitrary. Rather than making adjustments to the models, I suggested that sensitivity analyses should be carried out for any application of the models — especially for any application of the model to older ages (> 90 years). The justification for this is two-fold:

1. the parametric transition intensities were fitted to point estimates that had been assigned to single ages, the oldest of which is just over 90 years (91 years for the 1982–84 models and 92.5 years for the 1984–89 and 1989–94 models), so after this age the transition intensities are extrapolations; and

2. at younger ages (< 95 years), overall mortality in the models is fairly consistent with that of the benchmark force of mortality (from the logistic model), but it becomes less consistent at older ages.

Then in Sections 6.4 and 6.5, I looked at the overall force of mortality in the 1984–89 and 1989–94 models, respectively, and compared them to the same logistic model of mortality as in Section 6.3. For both sets of models, overall mortality was very low, which may be due to the addition of an extra classification in the original data — all lives in the 1982 and 1984 NLTCS were accounted for, but in the 1989 and 1994 NLTCS a new classification, 'Not in Survey Year' was introduced, leaving 9,524 lives unaccounted for in the 1989 NLTCS and 12,730 lives unaccounted for in the 1994 NLTCS. In answer to the question posed in Section 3.4, 'Is there any evidence in the data to suggest what may have happened to these lives?', I would argue that there is strong evidence that suggests a substantial proportion of these lives unaccounted for, were unaccounted for because of death — which would justify adjusting mortality in these models. I outlined various methods for adjusting mortality in the models to make them more consistent with the benchmark force of mortality (from the logistic model) and argued the case that the forces of mortality from each non-dead

state in the models should be replaced with those from the 1982–84 models, which I call the adjusted 1984–89 and the adjusted 1989–94 models. Overall mortality in these adjusted models is much more consistent with the logistic model and I argued that further adjustments were not justified, but rather sensitivity analyses should be carried out on any application of the models.

In the next chapter, I will use these parameterized models to estimate the costs of disability in a long-term care contract. Of the three sets of models (the 1982–84, the adjusted 1984–89 and the adjusted 1989–94), I place most reliance on the results of the 1982–84 models, even though it is oldest data set since:

1. it is based on the largest data set (see Section 3.4) and all lives were accounted for in this survey period;

2. overall mortality from these models is fairly consistent with the benchmark force of mortality; and

3. no adjustments were made to these models (unlike the adjusted 1984–89 and the adjusted 1989–94 models)

For these reasons, when estimating costs in the next chapter, I will focus on the results from the 1982–84 models. I will also use the adjusted 1984–89 and adjusted 1989–94 models, but the results from these will mainly be for comparison with the results from the 1982–84 models — they will also be useful in estimating ranges of feasible costs.

Chapter 7

Model Costs of Disability and Adverse Selection in Long-Term Care Insurance Revisited

7.1 Introduction

In Chapter 2, in order to look at the potential costs of adverse selection arising from variants of the APOE gene, the cost of other events in the ageing process (mainly disability) that trigger benefits were very simply assumed to be a multiple of those costs arising from Alzheimer's disease.

The aim of this chapter is to use the model described in Chapter 3, and parameterized in Chapters 4 to 6 to estimate (independently from, but including Alzheimer's disease) the costs of disability in long-term care insurance. These estimates can then be combined with estimates of the genotype-specific costs of Alzheimer's disease, given in Chapter 2, to estimate the potential costs of adverse selection — without having to make such a simplifying assumption. This will provide a better basis for estimating the potential for adverse selection, as well as providing an independent check on how reasonable the assumptions were in Chapter 2.

In Section 7.2, I use the disability model to estimate the costs of disability in a long-term care contract and look at trends of the costs by gender and age at entry — also providing a first check of the assumptions made in Chapter 2. Then

in Section 7.3, I look at the sensitivity of the costs of disability to some of the model assumptions. Using the genotype-specific costs of Alzheimer's disease (from Chapter 2), in Section 7.4, I look at the potential costs of adverse selection arising from variants of the APOE gene. Finally, I provide conclusions in Section 7.5.

The notation introduced in the previous section I will continue to use (slightly adjusted for brevity) in this section and it is as follows: the disability models using the parametric transition intensities fitted to the 1982–84 NLTCS, I will refer to as the 1982–84 models; those using the parametric transition intensities fitted to the 1984–89 NLTCS and 1989–94 NLTCS, adjusted for mortality (see Section 6.4 and 6.5), I will simply refer to as the 1984–89 models and 1989–94 models, respectively. For each pair of NLTCS and for each gender classification, two sets of parametric transition intensities were fitted: those with the data grouped into 10-year age bands; and those with the data grouped into 5-year age bands. For brevity and to avoid confusion I will refer to the former as the '1982–84 (10) models' and the latter as the '1982–84 (5)' models, for the 1982–84 NLTCS, and I will use similar notation for the 1984–89 models and the 1989–94 models.

Of the three sets of models (the 1982–84, the 1984–89 and the 1989–94 models), I will place most reliance on the results of the 1982–84 models, even though it is from the oldest data set since:

1. it is based on the largest data set (see Section 3.4) and all lives were accounted for in this survey period;

2. overall mortality from these models is fairly consistent with the benchmark force of mortality (see Section 6.3); and

3. no adjustments were made to these models (unlike the 1984–89 and the 1989–94 models)

Furthermore, using 10 year age groups (providing 3 data points for graduation) caused the fitting procedure to be unstable in some cases as the trend of the transition intensities with age was not clear (see Section 5.5). When using 5 year age bands (providing 5 data points for graduation), this instability in the fitting procedure did not occur, as there were enough estimates to use a more robust method (weighted

least squares) when fitting a Makeham curve. Given that the process of graduating the transition intensities with data in 5 year age groups used a more stable fitting procedure, I will place more reliance on the graduated transition intensities that used data grouped into 5 year age bands.

For the reasons outline above, the models that I will concentrate on in this chapter are the 1982–84 (5) models — I will also use the other models, but the results from these will mainly be for comparison with the results from the 1982–84 (5) models and to help estimate ranges of feasible costs.

7.2 The Costs of Disability in Long-Term Care Insurance

The disability model was specified in Chapter 3 and parameterized in Chapters 4 to 6 and is shown in Figure 3.19. (The states in the model I denote as follows: Healthy — state 1, IADL only — state 2, 1–2 ADLs — state 3, 3–4 ADLs — state 4, 5–6 ADLs — state 5, instutionalized — state 6 and dead — state 7.) The notation I use is defined in Sections 1.3.1 and 2.3.

I assume that the LTC contract has a single premium paid at outset, so the only policy cash-flows thereafter are the benefits. Benefits are payable continuously, while in a claiming state — which, as discussed in Section 3.2, are the states that represent the loss of 3 or more ADLs (states 4, 5 and 6).

The quantum of benefit is £1 per annum at inception of the policy, increasing continuously at rate δ_b per year. This is represented in the model by $b_t^j = 1$ if $j = 4, 5$ or 6 (level benefit) or by $b_t^j = e^{\delta_b t}$ if $j = 4, 5$ or 6 (benefit escalating at rate δ_b per year). The latter is the default, since this is a feature of most LTC policies (Dullaway & Elliot, 1998). I only use level benefits in some sensitivity tests in the next section — in this section I assume that $\delta_b = 0.05$, representing indexation to earnings. I use a force of interest $\delta = 0.05$ throughout. The present value of the benefit is the random variable whose expected value and higher moments are obtained by solving Norberg's equations (see Section 2.3), the expected value being the relevant quantity for use in the traditional actuarial equation of value. I look at policies with inception

ages of 60, 65, 70 and 75. Note that the data used to parameterize the disability models started from age 65, so using the parametric transition intensities back to age 60 is extrapolation — I did this to allow comparison with the Alzheimer's disease model which was parameterized from age 60.

Tables 7.72 to 7.74 give the first 3 moments of the present value of benefits using the 1982–84 models for lives entering at ages 60, 65, 70 and 75 in each state in the model, for males, females and both combined, respectively. General trends accross all three tables are:

1. the EPV of benefits increases with lives entering with increasing disability level, even for the non-claiming disability states (states 2 and 3), indicating that disability may be a predictor of further disability;

2. the increase in the EPV of benefits is the greatest between lives starting in state 3 and those starting in state 4, which is as expected since lives starting in state 4 immediately start claiming benefits;

3. the variance of the present value of benefits also tends to increase with lives entering with increasing disability level, indicating greater uncertainty for the costs of these lives;

4. the variance of the present value of benefits generally decreases for lives starting at older ages, which is also as expected since these lives have a shorter expected future life time and so there is less uncertainty of their costs;

5. the variance and skewness of the present value of benefits are much greater for females than for males, which may be from a combination of females' longer lives and their tendency to suffer prolonged disability (as opposed to males' quick decline).

It is worth noting here that the models for males and females combined were parameterized using aggregate (across genders) data and are not an aggregation of the separate models for males and females — so there is no guarantee that the EPV of benefits for males and females combined need be consistent with those for males and females calculated separately. Even so, there is almost complete consistency

Table 7.72: Mean, variance and skewness (q = 1, 2 and 3) of the present value of disability claims costs for a life starting in each model state, unit benefit increasing continuously ($\delta_b = 0.05$), for males using the 1982–84 models.

			State at start of contract					
	Entry			IADL	1–2	3–4	5–6	
Model	age	q	Healthy	only	ADLs	ADLs	ADLs	Inst'd
1982–84 (5)	60	1	1.2062	1.3187	1.8603	2.3550	3.2400	7.0200
		2	5.492	6.221	8.238	8.347	11.801	23.512
		3	36.415	44.099	58.941	58.094	83.316	107.723
	65	1	1.1267	1.3340	1.8979	2.6738	3.1980	5.2636
		2	5.032	6.114	8.050	8.683	10.431	16.960
		3	32.060	40.618	51.805	52.972	62.795	80.390
	70	1	1.0726	1.3992	1.8939	2.9764	3.1409	4.1032
		2	4.545	6.027	7.625	8.962	9.406	12.037
		3	27.083	36.638	44.326	47.896	49.891	55.573
	75	1	1.0364	1.4598	1.8007	3.1012	3.0060	3.2959
		2	4.060	5.668	6.750	8.412	8.225	8.585
		3	22.165	30.637	35.433	38.952	38.886	37.139
1982–84 (10)	60	1	1.0627	1.2248	1.9148	2.4599	2.8785	3.0011
		2	4.630	5.383	7.373	7.553	8.472	9.201
		3	29.591	35.499	46.272	45.545	49.936	54.876
	65	1	1.0691	1.3166	1.9907	2.7708	3.0161	3.5940
		2	4.536	5.698	7.811	8.370	9.036	11.414
		3	27.821	36.216	47.247	47.665	51.192	64.168
	70	1	1.0624	1.4041	1.9316	2.9328	3.0124	3.5538
		2	4.274	5.739	7.327	8.356	8.637	10.536
		3	24.550	33.718	40.954	42.988	44.704	52.869
	75	1	1.0270	1.4639	1.7813	2.9449	2.8841	3.1837
		2	3.812	5.408	6.342	7.686	7.618	8.449
		3	19.991	28.275	32.184	34.870	35.208	37.957

between EPV of the benefits between gender classification, for all starting states, in that the EPV of benefits is greatest for females, then for males and females combined, and the lowest, that for males (the only exception being for the 1982–94 (10) model for males, where the EPV of benefits for lives entering at age 60 in the 3–4 ADL state is greater than that for males and females combined).

In Table 7.75, the EPV of benefits for lives entering at ages 60, 65, 70 and 75 in the healthy state only (which is of primary interest) are given for all models and gender classifications. It is again noticeable that there is consistency of the EPV of benefits between gender classifications, as discussed above. In fact, for any given starting age, across all 6 models:

Table 7.73: Mean, variance and skewness ($q = 1$, 2 and 3) of the present value of disability claims costs for a life starting in each model state, unit benefit increasing continuously ($\delta_b = 0.05$), for females using the 1982–84 models.

			State at start of contract					
Model	Entry age	q	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd
1982–84 (5)	60	1	2.5600	3.2462	3.1062	4.6016	4.8299	10.2584
		2	13.647	17.632	17.716	21.081	23.172	41.379
		3	99.436	139.430	144.930	165.939	195.143	214.538
	65	1	2.5982	3.1725	3.2607	4.9122	4.8583	7.9417
		2	13.535	17.382	18.200	21.582	22.549	32.683
		3	95.913	131.516	139.762	154.083	171.723	182.072
	70	1	2.6052	3.0869	3.3515	5.0287	4.6986	6.2800
		2	12.931	15.928	17.226	19.940	20.056	24.717
		3	86.859	110.550	118.982	125.560	136.238	141.989
	75	1	2.5433	2.9698	3.3136	4.9506	4.4228	5.0890
		2	11.749	13.971	15.262	17.367	17.102	18.495
		3	73.392	87.994	94.143	97.225	104.500	104.726
1982–84 (10)	60	1	2.6288	3.0560	3.0353	4.5973	4.6616	5.2376
		2	13.433	16.134	16.608	19.544	20.504	24.788
		3	94.979	122.832	130.840	144.753	160.218	199.394
	65	1	2.5676	3.0796	3.1872	4.8115	4.6264	5.7377
		2	12.959	16.244	17.112	19.878	20.370	26.004
		3	89.492	120.051	128.784	138.173	151.875	185.646
	70	1	2.5258	3.0485	3.2652	4.9041	4.4776	5.4624
		2	12.234	15.291	16.376	18.811	18.727	22.542
		3	80.134	104.476	112.049	117.788	127.589	143.519
	75	1	2.4749	2.9406	3.2335	4.8132	4.2283	4.8681
		2	11.159	13.535	14.635	16.575	16.229	18.035
		3	67.387	83.363	88.875	91.993	99.241	104.133

Table 7.74: Mean, variance and skewness ($q = 1$, 2 and 3) of the present value of disability claims costs for a life starting in each model state, unit benefit increasing continuously ($\delta_b = 0.05$), for males and females combined using the 1982–84 models.

			State at start of contract					
Model	Entry age	q	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd
1982–84 (5)	60	1	1.9986	2.1463	2.5246	3.6596	3.8504	7.2711
		2	10.399	11.496	13.155	14.874	16.289	31.180
		3	76.511	90.090	104.908	114.024	131.132	196.274
	65	1	1.9526	2.2783	2.6692	3.9193	3.9298	6.2260
		2	10.018	12.084	13.734	15.608	16.240	25.009
		3	71.879	91.554	103.526	110.932	120.269	150.968
	70	1	1.9397	2.3823	2.7396	4.0996	3.8970	5.2410
		2	9.517	11.878	13.248	15.209	15.117	19.492
		3	64.646	82.764	90.761	96.534	100.925	113.449
	75	1	1.9451	2.4165	2.7322	4.1575	3.7682	4.4207
		2	8.885	10.983	12.095	13.928	13.465	15.031
		3	55.846	68.967	74.400	78.203	80.904	83.364
1982–84 (10)	60	1	1.8667	1.8998	2.2284	2.7237	3.2374	4.2109
		2	9.456	9.907	11.184	11.097	12.305	17.620
		3	68.072	75.578	86.939	81.742	93.720	135.053
	65	1	1.8944	2.1407	2.5580	3.5677	3.5823	4.7174
		2	9.385	11.020	12.723	13.804	14.126	19.385
		3	65.285	81.676	94.184	96.547	103.448	135.465
	70	1	1.9136	2.3247	2.7160	4.0141	3.7462	4.6496
		2	9.032	11.314	12.843	14.579	14.341	17.720
		3	59.015	77.226	86.424	91.227	95.330	110.654
	75	1	1.9099	2.4049	2.7251	4.1289	3.7444	4.2679
		2	8.331	10.608	11.780	13.590	13.225	14.714
		3	49.692	64.108	69.966	73.837	77.154	82.350

Table 7.75: Expected present value of disability claims costs for a life starting in the healty state, unit benefit increasing continuously ($\delta_b = 0.05$), for males, females and combined using all disability models.

		EPV using:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5)	(10)	(5)	(10)	(5)	(10)
M & F	60	1.9986	1.8667	1.7891	1.8678	1.9441	2.0039
	65	1.9526	1.8944	1.7549	1.8488	1.8330	1.9404
	70	1.9397	1.9136	1.7524	1.8569	1.7744	1.8975
	75	1.9451	1.9099	1.7708	1.8579	1.7439	1.8603
F	60	2.5600	2.6288	2.7357	2.8789	2.7668	2.9927
	65	2.5982	2.5676	2.6377	2.6784	2.6478	2.7604
	70	2.6052	2.5258	2.5612	2.5365	2.5159	2.5894
	75	2.5433	2.4749	2.4661	2.4407	2.3770	2.4677
M	60	1.2062	1.0627	0.9624	0.8855	1.0680	1.0593
	65	1.1267	1.0691	0.8824	0.9339	1.0059	1.0500
	70	1.0726	1.0624	0.8848	1.0005	0.9577	1.0381
	75	1.0364	1.0270	0.9080	1.0415	0.9285	1.0189

1. the lowest EPV of benefits for females is higher than the highest for males and females combined; and

2. the lowest EPV of benefits for males and females combined is higher than the highest for males alone.

Given that the three sets of models (1982–84, 1984–89 and 1989–94) were parameterized using different data sets (with the exception of the forces of mortality in the 1984–89 and 1989–94 models, which use the forces of mortality from the 1982–84 models), the estimates of the EPV of benefits are reasonably consistent between the models. For example, for males and females combined, the greatest difference between the maximum and minimum estimate of the EPV of benefits (as a percentage of the mid-point), for all starting ages is 11.33% — for females it is 15.59% and for males it is 30.66%.

There is no clear pattern of the EPV of benefits with starting age. For many cases, the EPV of benefits is almost constant across starting ages. This may be caused by, with increasing starting age, shorter expected future lifetime reducing expected costs, being compensated for by shorter expected time to disability increasing costs.

It was assumed in Chapter 2 (Section 2.6.3) that the costs arising from

Table 7.76: Comparison of the EPV of disability claims costs (total LTC costs) with the EPV of Alzheimer's disease (AD) claims costs for a life starting in the healty state, unit benefit increasing continuously ($\delta_b = 0.05$), for females.

Entry age	AD claims costs[1]	EPV of: Total LTC costs Highest[2]	Lowest[2]	AD claims costs as a % of total LTC costs
60	1.1534	2.9927	2.5600	38.54–45.05
65	1.1634	2.7604	2.5676	42.15–45.31
70	1.1739	2.6052	2.5159	45.06–46.66
75	1.1738	2.5433	2.3770	46.15–49.38

(1) From Table 2.10.
(2) Highest and lowest estimates from the 6 disability models in Table 7.75.

Alzheimer's disease (AD) were between 40% and 50% of total long-term care costs. The values in Table 7.75 are estimates of total long-term care costs and can be compared with the costs of AD, given in Table 2.10, to check the reasonableness of this assumption. Table 7.76 compares the costs arising from AD with total long-term care costs for females, since the AD claims costs in Table 2.10 were calculated using female mortality (Makeham approximation to 65% AF80, since the majority of elderly people will be women) and aggregate incidence of AD.

Table 7.76 gives strong support of the assumption made in Chapter 2 that the costs arising from Alzheimer's disease (AD) are indeed between 40% and 50% of total long-term care costs — the costs arising from the model of Alzheimer's disease as a percentage of those arising from the disability models are between 38.54% and 49.38%. However, it is not clear whether the two models are comparable in terms of the mortality assumptions. Figure 7.86 shows that overall mortality (as defined in Section 6.1) in the Alzheimer's disease model is fairly consistent with overall mortality in the disability models. This figure shows:

1. overall mortality for the 1982–84(10) model for females (which, for females, has the highest overall mortality of the 6 models);

2. overall mortality for the 1984–89 (5) model for females (which, for females, has the lowest overall morality of the 6 models); and

3. overall mortality for the Alzheimer's disease model using 65% AF80 as baseline mortality (see Section 2.4 for more detail).

Overall mortality in the Alzheimer's disease model is very close to that of the 1984–89 (5) model (representing a lower bound on overall mortality in the disability models for females) until just after age 100. Overall mortality in the disability models then starts to level off with age, whereas in the Alzheimer's disease model it continue to grow exponentially, becoming greater than overall mortality in the 1982–84 (10) model (representing an upper bound on mortality in the disability models for females) by just after age 110. Baseline mortality in the Alzheimer's disease model was taken as a Makeham approximation to 65% AF80, and this results in overall mortality in the model that is nearly exponential — which, as illustrated in Section 6.2, may not be appropriate at older ages. Overall, however, it seems reasonable to compare the EPV of benefits from the Alzheimer's disease model with those from the disability models.

Care is required in interpreting the effect on the EPV of benefits from any of the disability models from their overall force of mortality. It may be expected that a model with a high overall force of mortality would give lower estimates of the EPV of benefits (using the argument that if people are expected to die sooner then the costs will be lower). However, this argument does not always hold — the EPVs of benefits from the 1984–89 (5) model for females, which has the highest overall mortality, are not the lowest for entry at any age and, moreover, for entry at age 75, it gives a higher estimate than does the 1982–84 (10) model, which has the lowest overall force of mortality. Though, of course, adjusting the mortality in any given model will affect the EPVs of benefits in the way described above.

The EPV of benefits given in Table 7.75 can be analysed in more detail, by looking at the EPV of benefits attributable to a given time period of the contract and specific claiming state. For example, using the 1982–84 (5) model, the EPV of benefits for a females aged 60 is 2.5600, but how much of this is from claiming in the 3–4 ADL state in the time period 0 to 5? (i.e. between the ages 60 and 65). This can easily be calculated in the model by setting:

$$b^i_{60+t} = \begin{cases} e^{0.05t} & \text{if } i = 4 \text{ and } 0 \leq t \leq 5 \\ 0 & \text{otherwise} \end{cases} \tag{7.69}$$

This can be done for the other claiming states and for any given time period in a

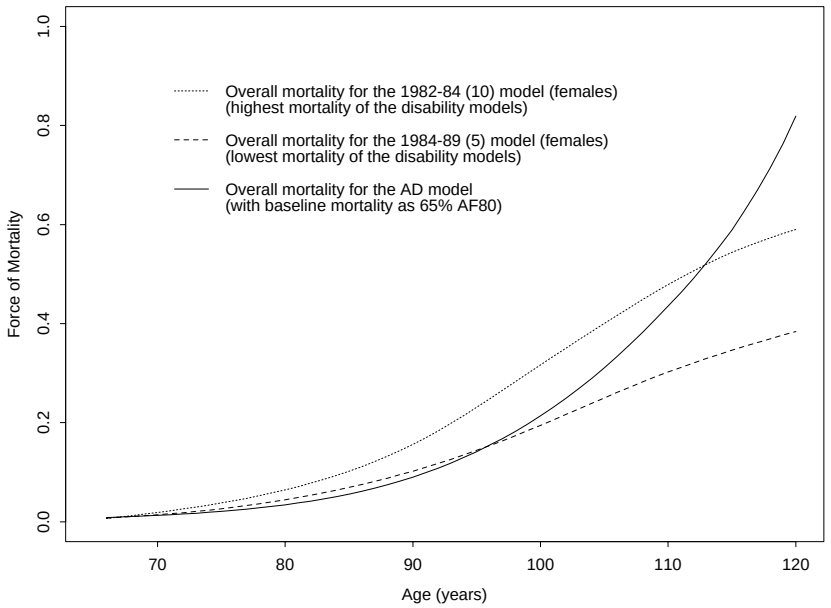

Figure 7.86: Comparison of the overall force of mortality in the disability models to the overall force of mortality in the Alzheimer's disease model, for females.

similar manner. For illustration, in Table 7.77, I look at the EPVs of benefits at age 60 from the 1982–84(5) model for males and females combined and separately, in 5-year time periods (upto age 90 and for 90+ years) and for each of the claiming states — the total (across all claiming states) for each time period and the total (across all time periods) for each claiming state are also given. It is worth noting some of the general trends from this table:

1. over all time periods, the EPV of the benefits paid for the institutionalised state is greater than for the 5–6 ADLs state, which in turn, is greater than for the 3–4 ADLs state for all gender classifications;

2. for early time periods (0–5 years and 5–10 years), the EPV of benefits is greatest while in the 3–4 ADLs state or the 5–6 ADLs state, while after this period, the EPV of benefits is greatest for the institutionalised state, possibly as disability becomes more likely with age (and all lives are assumed to be healthy at age 60); and

3. the EPV of benefits increases with time periods that cover older ages up to

a maximum and then decrease — for males this maximum is the time period 15–20 years, whereas for females and both genders combined it is the time period 20–25 years (it may be expected that the EPV of benefits for females continues to increase at older ages than males, as disability becomes more likely with age, and females are expected to live longer than males)

So, although the total EPVs of benefits vary greatly by gender classification (for females, it is over double that of males), the patterns by claiming state and time period show consistency between gender classifications.

In this section I have used the disability model to estimate the costs of disability in a long-term care contract and have looked at trends of the costs by gender, age at entry, claiming state and by time period in the contract. I have also used these costs of disability to check the crude assumption made in Chapter 2, that the costs of Alzheimer's disease make up 40–50% of total long-term care costs and showed that the two models are comparable and that the disability model produces results that are in good agreement with this assumption. In the next section I look at the sensitivity of these costs of disability to some of the model assumptions.

7.3 Sensitivity Analysis

In this section I look at the sensitivity of the model costs of disability to some of the key assumptions, namely:

1. the rate of benefit increase — I have so far assumed that benefits increase at a rate of $\delta_b = 0.05$, which I compare with $\delta_b = 0.025$ and level benefits;

2. the effect of benefits commencing at a lesser disability level; and

3. the mortality assumptions at older ages (> 90 years).

I assume throughout that the discount rate is $\delta = 0.05$. The effect of the above changes are reported as percentage changes to the EPVs in Tables 7.75, which I refer to as the baseline EPVs. For the baseline EPVs:

1. the discount rate, $\delta = 0.05$;

Table 7.77: Expected present value of disability claims costs, by time period and claiming state, for a life starting in the healty state at age 60, unit benefit increasing continuously ($\delta_b = 0.05$), for males, females and combined using the 1982-84 (5) models.

		EPV of claims from state:			
Gender	Time period	3–4 ADLs	5–6 ADLs	Inst'd	All 3 States
M & F	0–5	0.01561	0.01720	0.00558	0.03838
	5–10	0.05313	0.04911	0.03787	0.14011
	10–15	0.08339	0.07773	0.10240	0.26352
	15–20	0.10122	0.09924	0.17764	0.37810
	20–25	0.10001	0.10529	0.22520	0.43050
	25–30	0.07807	0.08985	0.21242	0.38033
	30+	0.06041	0.08527	0.22197	0.36765
	All	0.49183	0.52368	0.98307	1.9986
F	0–5	0.00526	0.01719	0.00088	0.02334
	5–10	0.03247	0.03901	0.01913	0.09061
	10–15	0.08203	0.07024	0.10138	0.25364
	15–20	0.12512	0.10886	0.23305	0.46703
	20–25	0.13687	0.13184	0.33364	0.60235
	25–30	0.10870	0.11920	0.32915	0.55706
	30+	0.08522	0.11375	0.36695	0.56592
	All	0.57566	0.60010	1.38418	2.5599
M	0–5	0.02915	0.01367	0.01069	0.05351
	5–10	0.05176	0.04928	0.04917	0.15022
	10–15	0.06286	0.07358	0.09125	0.22769
	15–20	0.06618	0.08308	0.11098	0.26025
	20–25	0.05844	0.07591	0.10144	0.23579
	25–30	0.04057	0.05408	0.07132	0.16596
	30+	0.02589	0.03640	0.05047	0.11276
	All	0.33484	0.38601	0.48532	1.2062

2. unit benefits at policy inception increase continuously at a rate of $\delta_b = 0.05$, payable while in states 4, 5 and 6 (the 3–4 ADLs, 5–6 ADLs and institutionalised state, respectively); and

3. the parameters of the models are as given in Chapters 5 and 6 (the 1982–84 models and the adjusted 1984–89 and 1989–94 models, as then described).

It is a common policy feature for benefits to increase at some rate, as discussed in Section 2.2. I chose a rate of 0.05, to represent indexation to earnings and to be consistent with the assumption made in Chapter 2, when estimating the costs of Alzheimer's disease in a LTC contract. In Tables 7.78 and 7.79 I look at the EPVs of benefits increasing continuously at $\delta_b = 0.025$ and level benefits ($\delta_b = 0.0$), respectively, as a percentage of the EPVs of benefits with $\delta_b = 0.05$. From these tables:

1. it is clear that the rate of benefit escalation has a large impact on the EPV of benefits: with $\delta_b = 0.025$, the EPVs of benefits are between 55% and 78% of those with $\delta_b = 0.05$, and with $\delta_b = 0.0$, they are between 32% and 62%;

2. the effect of the rate of benefit escalation is much greater at younger entry ages: with $\delta_b = 0.025$ compared to $\delta_b = 0.05$, for every 5-year period that entry into insurance is delayed, the difference in the EPV of benefits is reduced by approximately 5% (this effect is slightly greater with $\delta_b = 0.0$ compared with $\delta_b = 0.05$);

3. the rate of benefit escalation has a greater impact on EPV of benefits for females than for males (between 5% and 10% difference).

The third point above may be expected, considering the timing of benefit payments given in Table 7.77 — the benefit payments to females peak at older ages than for males, which are then discounted over a longer time period, producing a greater effect for a given difference between discount rate and rate of benefit escalation. The effect on males and females combined for a change in the rate of benefit escalation is closer to that of females than males (as expected, since females made up the majority of the aggregate data).

Table 7.78: EPV of disability claims costs with benefits increasing continuously ($\delta_b = 0.025$) as a percentage of EPV of disability claims costs with benefits increasing continuously ($\delta_b = 0.05$, baseline) for a life starting in the healthy state for males, females and combined using all disability models.

		EPV of benefits with $\delta_b = 0.025$ as a % of EPV of benefits with $\delta_b = 0.05$ for:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
M & F	60	59.05	59.51	55.93	57.02	59.36	59.85
	65	64.82	65.14	61.69	61.83	64.21	64.23
	70	70.01	70.34	66.82	66.89	68.84	68.75
	75	74.63	75.11	71.32	71.56	73.17	73.32
F	60	56.42	57.37	55.81	56.72	57.68	58.73
	65	62.92	63.25	60.67	61.11	62.65	62.97
	70	68.92	68.84	65.66	65.74	67.38	67.54
	75	73.91	73.79	70.21	70.19	71.82	72.15
M	60	64.25	65.36	60.82	58.67	62.26	63.65
	65	69.43	69.60	64.83	63.58	67.75	67.54
	70	73.91	73.89	69.89	69.08	72.05	71.45
	75	77.79	77.79	74.21	73.79	75.55	75.21

Adjusting the rate of benefit escalation has a similar effect on the EPV of benefits in both the disability model and the Alzheimer's disease model (see Section 2.5), though overall the EPV of benefits in the Alzheimer's disease model seem slightly more sensitive.

It has been assumed so far that benefits are payable while in states 4, 5 and 6 (3–4 ADLs, 5–6 ADLs and institutionalised). For some LTC contracts there is the option of benefits commencing earlier than this — for example on the failure of 2 ADLs, half or all of the annual sum assured may be payable. To look at the extra cost of a proportion of benefits commencing on the loss of exactly 2 ADLs, the disability model would need to be redefined (with separate states for loss of 1 ADL and loss of 2 ADLs) and then parameterized again. Instead, I look the the extra costs of benefits commencing in state 3 (1–2 ADLs), which can easily be estimated by setting $b^3_{x+t} = 1/2$ or 1 reflecting a contract that pays half or all of the annual sum assured, respectively, on the loss of 1 or 2 ADLS. Tables 7.80 and 7.81 give the EPV of disability claims costs with additional benefits of 1/2 and 1 increasing continuously ($\delta_b = 0.05$) in state 3, respectively, as a percentage of EPV of disability claims costs

Table 7.79: EPV of disability claims costs with level benefits as a percentage of EPV of disability claims costs with benefits increasing continuously ($\delta_b = 0.05$, baseline) for a life starting in the healthy state for males, females and combined using all disability models.

		EPV of level benefits as a % of EPV of benefits with $\delta_b = 0.05$ for:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
M & F	60	36.46	37.05	32.93	34.35	37.11	37.76
	65	43.57	43.95	39.77	39.89	42.95	42.92
	70	50.41	50.78	46.27	46.24	48.91	48.67
	75	56.86	57.45	52.30	52.50	54.85	54.91
F	60	33.13	34.42	33.00	34.16	35.04	36.41
	65	40.98	41.47	38.55	39.12	40.95	41.33
	70	48.85	48.74	44.72	44.80	46.97	47.07
	75	55.80	55.59	50.73	50.63	52.98	53.30
M	60	43.02	44.66	38.94	36.16	40.54	42.67
	65	49.77	50.05	43.68	42.00	47.67	47.44
	70	55.94	55.91	50.39	49.22	53.49	52.55
	75	61.57	61.54	56.44	55.77	58.41	57.75

with no benefits in state 3 (baseline). It is easily seen that the percentage increases in Table 7.81 (for unit benefits payable in state 3) are twice those in Table 7.80 (for 1/2 benefits payable in state 3). I include them both for illustration.

The increases in costs range from 136% to 174% for an extra unit benefit payable while in state 3. For females and both genders combined the extra costs are roughly 50%, slightly more at younger entry ages and less at older entry ages (except for females in the 1982–84 (10) model), though the difference between them is always less than 10%. For males, the increase in costs is slightly greater, between 60% and 70% (except in the 1989–94 (10) model) and there is no clear trend by entry age.

In Section 6.6, when looking at overall mortality in the disability models, it was noted that the oldest data point used for any of the models was just over 90 years and that after this age the transition intensities were extrapolations. Also, overall mortality in the disability models were very consistent with the benchmark force of mortality used upto about the same age (see Chapter 6 for more detail), after which they became less consistent. With this motivation, I aim to look at the sensitivity of the EPVs of benefits to the force of mortality after age 90.

Table 7.80: EPV of disability claims costs with additional benefits of 1/2 increasing continuously ($\delta_b = 0.05$) in state 3 as a percentage of EPV of disability claims costs ($\delta = 0.05$) with no benefits in state 3 (baseline) for a life starting in the healthy state for males, females and combined using all disability models.

		EPV of benefits with 1/2 benefits in state 3 as a % of EPV of benefits with no benefits in state 3 for:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
M & F	60	128.17	127.99	129.40	127.50	131.63	129.23
	65	126.29	126.96	128.45	126.49	130.87	128.38
	70	124.51	125.77	126.95	125.87	130.08	127.13
	75	122.94	124.55	125.01	124.12	129.02	125.42
F	60	126.30	127.28	126.34	125.74	130.23	127.24
	65	125.56	126.27	126.89	125.33	129.99	126.48
	70	123.82	124.88	127.41	125.41	129.52	125.25
	75	121.59	123.31	126.63	124.35	128.55	123.35
M	60	133.45	133.35	137.43	129.80	134.59	125.40
	65	131.67	132.08	131.26	126.81	132.65	127.58
	70	131.69	131.89	124.60	122.43	131.73	128.42
	75	133.27	133.61	119.25	118.24	131.41	127.21

Table 7.81: EPV of disability claims costs with the addition of unit benefits increasing continuously ($\delta_b = 0.05$) in state 3 as a percentage of EPV of disability claims costs ($\delta = 0.05$) with no benefits in state 3 (baseline) for a life starting in the healthy state for males, females and combined using all disability models.

		EPV of benefits with unit benefits in state 3 as a % of EPV of benefits with no benefits in state 3 for:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
M & F	60	156.35	155.98	158.80	155.00	163.26	158.47
	65	152.58	153.91	156.89	152.98	161.74	156.75
	70	149.02	151.55	153.90	151.74	160.16	154.27
	75	145.87	149.11	150.02	148.25	158.04	150.84
F	60	152.60	154.56	152.69	151.48	160.47	154.47
	65	151.13	152.54	153.79	150.66	159.99	152.96
	70	147.65	149.75	154.83	150.82	159.03	150.51
	75	143.17	146.63	153.25	148.70	157.11	146.70
M	60	166.91	166.71	174.87	159.61	169.18	150.79
	65	163.34	164.17	162.52	153.61	165.31	155.15
	70	163.38	163.78	149.19	144.87	163.46	156.84
	75	166.55	167.21	138.49	136.47	162.82	154.42

For a lower bound on the force of mortality, an obvious choice is to assume that the force of mortality from each state after age 90 remains at the same level as it was at age 90. To ensure that mortality is not actually being increased in any state using this assumption (for example, if mortality in a state is actually decreasing), after age 90 I take the minimum of the force of mortality at age 90 and the original force of mortality. (This modification only affected the mortality in 3 models, all for males and females combined — the 1982–84 (5) model, the 1984–89 (5) model and the 1989–94 (5) model, all of which have, necessarily identical, slightly decreasing mortality in the 3–4 ADL state.) Or more specifically, if μ^{i7}_{x+t} is the force of mortality in state i for a person aged $x+t$ then define a lower bound on the forces of mortality after age 90, ${}^{l}\mu^{i7}_{x+t}$, as:

$$ {}^{l}\mu^{i7}_{x+t} = \begin{cases} \mu^{i7}_{x+t} & \text{if } x+t \leq 90 \\ \min\left(\mu^{i7}_{90},\, \mu^{i7}_{x+t}\right) & \text{otherwise} \end{cases} \tag{7.70} $$

Upper bounds on the forces of mortality can be defined in a similar way — by assuming that the force of mortality after age 90 continues to increase exponentially. For males I assume that the forces of mortality increase at least as fast as does the Makeham fit to AM80 mortality, ${}^{AM80}\mu_{x+t}$, given in equation 1.2. For females and both genders combined, I assume the forces of mortality increase at least as fast as does the Makeham fit to AF80 mortality, ${}^{AF80}\mu_{x+t}$, also given in equation 1.2. For example, for males define an upper bound on the force of mortality after age 90, ${}^{u}\mu^{i7}_{x+t}$, as:

$$ {}^{u}\mu^{i7}_{x+t} = \begin{cases} \mu^{i7}_{x+t} & \text{if } x+t \leq 90 \\ \max\left(\left[\mu^{i7}_{90} - {}^{AM80}\mu_{90} + {}^{AM80}\mu_{x+t}\right],\, \mu^{i7}_{x+t}\right) & \text{otherwise} \end{cases} \tag{7.71} $$

For females and for both genders combined, an upper bound can be defined by replacing AM80 mortality with AF80 mortality in the above equation.

In a few of the disability models, the force of mortality in some states increased faster than the Makeham fit to AM80 or AF80 (for males and both genders combined in the 1982–84 (10) model, the 1984–89 (10) model and the 1989–94 (10) model; and for females in the 1984–89 (10) model). Though it is noticeable that this only occurred in the models that were parameterized to data grouped in 10-year age

Table 7.82: EPV of disability claims costs with mortality from all states adjusted to be constant after age 90 as a percentage of EPV of disability claims costs with no mortality adjustments (baseline) for benefits increasing continuously ($\delta_b = 0.05$) for a life starting in the healthy state for males, females and combined using all disability models.

		EPV of benefits with level mortality (> 90 yrs) as a % of EPV of benefits with original mortality for:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
M & F	60	102.24	104.03	103.84	106.30	103.00	105.46
	65	102.43	104.38	104.11	107.00	103.38	106.27
	70	102.86	105.15	104.74	108.08	104.08	107.58
	75	103.77	106.80	106.03	110.20	105.40	109.91
F	60	103.33	103.43	105.23	105.60	104.46	104.54
	65	103.46	103.66	105.77	106.34	105.01	105.25
	70	103.89	104.17	106.63	107.42	105.98	106.29
	75	105.00	105.33	108.34	109.33	107.73	108.07
M	60	101.27	101.62	102.45	103.94	102.13	102.91
	65	101.47	101.94	102.89	104.41	102.40	103.47
	70	101.89	102.50	103.46	105.09	103.05	104.39
	75	102.77	103.65	104.61	106.56	104.40	106.14

bands. This is not surprising since it was noted in Section 5.5 that some of the Makeham fits using these age groupings were unstable and, in particular, produced unusually high estimates of the exponential parameter.

Tables 7.82 and 7.83 give the EPV of benefits using the lower and upper bound forces of mortality, as defined above, respectively, as a percentage of the EPV of benefits using the original forces of mortality. From these Tables:

1. using the lower bound forces of mortality, the EPV of benefits are increased by between 1% and 10% and the increase is greater at older entry ages (102% to 110% at entry age 75) and less at younger entry ages (101% to 106% at entry age 60);

2. using the upper bound forces of mortality, the EPV of benefits are reduced by between 1% and 10% and the reduction is greater at older entry ages (90% to 99% at entry age 75) and less at younger entry ages (94% to 99% at entry age 60);

Table 7.83: EPV of disability claims costs with mortality after age 90 from all states adjusted to exponential (males — AM80, females and combined — AF80) as a percentage of EPV of disability claims costs with no mortality adjustments (baseline) for benefits increasing continuously ($\delta_b = 0.05$) for a life starting in the healthy state for males, females and combined using all disability models.

		EPV of benefits with exponential mortality (> 90 yrs) as a % of EPV of benefits with original mortality for:					
	Entry	1982–84 models		1984–89 models		1989–94 models	
Gender	age	(5) %	(10) %	(5) %	(10) %	(5) %	(10) %
M & F	60	98.12	99.20	94.60	96.53	96.99	98.36
	65	97.97	99.13	94.23	96.15	96.61	98.12
	70	97.60	98.98	93.33	95.55	95.91	97.72
	75	96.82	98.65	91.49	94.35	94.57	97.01
F	60	97.62	98.06	94.08	95.09	95.91	97.25
	65	97.53	97.93	93.46	94.44	95.41	96.82
	70	97.22	97.63	92.48	93.47	94.51	96.18
	75	96.41	96.95	90.54	91.73	92.89	95.07
M	60	99.15	99.41	96.26	96.07	97.39	98.37
	65	99.01	99.30	95.58	95.60	97.06	98.05
	70	98.73	99.09	94.71	94.92	96.26	97.53
	75	98.13	98.67	92.94	93.45	94.60	96.55

It is expected that the percentage changes to the EPV of benefits will be largest for the oldest entry ages, since the changes to the force of mortality are at ages over 90, which affect the benefits at ages over 90, which, in turn, make up a greater proportion of costs for a contract starting at age 75, as opposed to one starting at age 60 (which also includes the EPV of benefits payable in the period 60 to 75 years). This effect is also compounded by discounting over a longer period for contracts starting at age 60. Overall, it seems that the mortality assumptions over age 90 years have a relatively small effect on the EPV of benefits — even for contracts starting at older ages.

7.4 Impact of Adverse Selection on Long-Term Care Insurance Revisited

In this section, the costs of disability from the disability models are combined with the costs of Alzheimer's disease (from Section 2.6.2) to revisit the potential costs

of adverse selection in LTC insurance. I use the same definitions and terminology as introduced in Chapters 1 and 2 — for example, by adverse selection here I am referring to lives that know they have ApoE genotypes that predispose them to Alzheimer's disease (the $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes) being more likely to take out long-term care insurance than lives with other genotypes.

In Chapter 2, the costs of adverse selection were first calculated based on the costs of Alzheimer's disease alone (i.e. the extra costs from adverse selection were given as a percentage of the baseline costs of AD only). The 'baseline' asssumptions were:

1. mortality: 65% AM80 for males and 65% AF80 for females and both genders combined;
2. incidence of AD: genotype-specific aggregate (across genders) incidence rate of AD (defined in equations 1.3 and 1.13) used for all gender classifications;
3. level of relative risk, $m = 1$, 0.5, 0.25;
4. rate of benefit escalation, $\delta_b = 0.05$;
5. rate of discount, $\delta = 0.05$; and
6. claiming commences $w = 0$ years before institutionalization.

Adverse selection was then modelled by assuming that lives with high-risk genotypes were k ($k = 2$, 4, 10 and 100) times more likely to insure than lives with other genotypes (see Section 2.6 for more detail) and the associated costs were given as a percentage of the baseline costs of AD. Then to allow for other costs of claiming in a LTC contract, it was very simply assumed that AD was responsible for between 40% and 50% of all long-term care costs (for all age groups and gender classifications). This translated directly into the total percentage costs of adverse selection being 40–50% of the costs of adverse selection as a percentage of AD-related costs alone. With estimates of total long-term care costs from the disability models (the EPV of benefits, given in the previous two sections), it is now possible to estimate the proportion of total long-term care costs attributable to Alzheimer's disease by gender and starting age. These proportions can then be applied to the costs of adverse

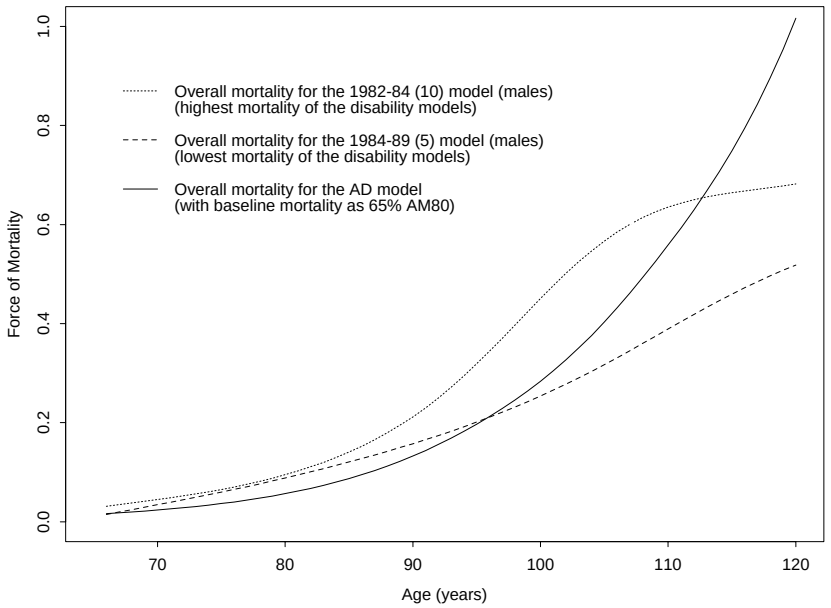

Figure 7.87: Comparison of the overall force of mortality in the disability models to the overall force of mortality in the Alzheimer's disease model, for males.

selection (as a percentage of AD-related costs) to give estimates of total percentage costs of adverse selection.

As lives with Alzheimer's disease were not excluded nor identifiable in the NLTCS, I assume that the EPV of benefits calculated from the disability models are estimates of total long-term care costs (i.e. they include the costs of claiming from lives with Alzheimer's disease, as these lives in the survey would become disabled from Alzheimer's disease). It is now a simple matter to calculate the proportion of costs in a long-term care contract attributable to Alzheimer's disease — by dividing the aggregated (by genotype) costs of AD given in Table 2.13 by the total long-term care costs given in Table 7.75. However, before comparing the EPV of benefits between the two models, I look at overall mortality in both models to check for consistency. For females, this was done in Section 7.2 and for males and both genders combined the forces of mortality are compared in Figures 7.87 and 7.88, respectively. These figures show:

1. the highest overall mortality for the 6 disability models (for males and both genders combined this is from the 1982–84 (10) model);

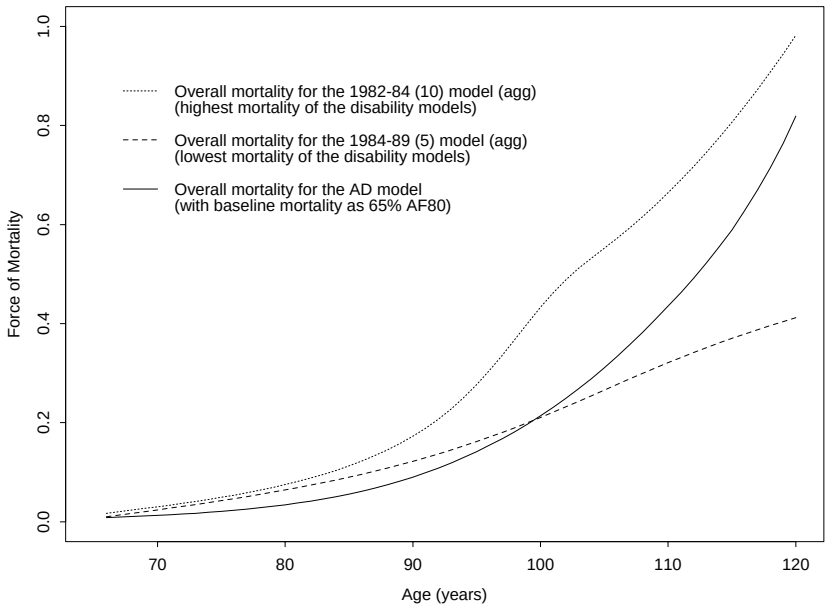

Figure 7.88: Comparison of the overall force of mortality in the disability models to the overall force of mortality in the Alzheimer's disease model, for males and females combined.

2. the lowest overall mortality for the 6 disability models (for males and both genders combined this is from the 1984–89 (5) model); and

3. overall mortality for the Alzheimer's disease model with baseline mortality as 65% AM80 for males and as 65% AF80 for both genders combined (see Section 2.4 for more detail).

For both of these gender classifications, the trends are very similar to those for females. However, for these two gender classifications, the difference is greater at younger ages (60 – 90 years) between the overall forces of mortality in the AD models and that from the disability model with the lowest mortality. The trends at older ages are similar for all three models, at about age 100 (age 95 for males), the forces of mortality in the AD models increase above the force of mortality from the 1984–89 (5) models (representing a lower bound), which are then starting to level off. After this, overall mortality in the AD models continue to increase exponentially: for males, going above the upper limit of mortality in the disability models (from the 1982–84 (10) model) by just after age 110; and for both genders combined, at

a similar rate but at a smaller magnitude to the upper limit of mortality in the disability models (from the 1982–84 (10) model).

From the previous section and Section 2.5, it was seen that the EPVs of benefits were not very sensitive, in either the AD model nor the disability model, to the mortality assumptions at older ages (> 90 years) — however, the difference in mortality in younger ages may be significant, and it is noticeable that, for males and both genders combined, the forces of mortality at younger ages in the AD models are below those in the disability models. This may result in relatively higher estimates of EPVs of benefits in the AD models compared to the disability models, which, in turn, could cause estimates of the proportion of total LTC cost attributable to AD to be artificially high.

Table 7.84 gives these proportions using the EPV of benefits from the 1982–84 (5) model and in Table 7.85 ranges of proportions are calculated using the EPV of benefits from all of the disability models. The figures in Table 7.85 for females differ slightly from those given in Table 7.76, since different estimates of the costs of AD are used — in the former the costs of AD are calculated from aggregating the genotype specific costs, whereas in the latter the aggregate costs of AD are calculated directly.

From these tables, for females the proportion of total LTC costs attributable to AD is between 39.25% and 49.31%. As noted in Section 7.2, the assumption used in Chapter 2 that the costs of AD make up between 40% and 50% of total LTC costs is strongly supported for females. The figures from the 1982–84 (5) model are almost constant, at about 45%, for all ages at entry into insurance and for all levels of relative risk. Looking at Table 7.85 using all disability models, there is a slight trend for the range of proportions to increase with age at entry: at entry age 60 the range is 39.26%–46.80%; at entry age 65 it is 42.75%–46.31%; at entry age 70 it is 44.91%–47.02%; and at entry age 75 it is 44.59%–49.31%.

For males, the proportion of total LTC costs are are much higher than for females — from the 1982–84 (5) model, the proportions are about 55% at entry age 60, increasing to about 65% at entry age 75. The ranges for the proportions are much wider for males as well (ranges of 20% in some cases), which is a direct result of the

Table 7.84: EPV of benefits from Alzheimer's disease, with proportion of relative risk, $m = 1.00$, 0.5 and 0.25 (from Table 2.13) as a percentage of EPV of benefits from disability using the 1982–84 (5) model (from Table 7.75), with benefits increasing at $\delta_b = 0.05$, for males, females and combined.

Gender	Entry age	AD costs as a % of disability costs with proportion of relative risk: $m = 1.00$ %	$m = 0.50$ %	$m = 0.025$ %
M & F	60	61.19	59.74	58.49
	65	62.28	61.20	60.12
	70	61.76	61.35	60.63
	75	59.95	60.36	60.10
F	60	46.80	46.45	45.90
	65	45.80	45.76	45.42
	70	44.91	45.41	45.33
	75	44.59	45.77	46.08
M	60	57.29	55.46	54.55
	65	61.68	60.26	59.55
	70	64.61	64.05	63.68
	75	65.90	66.67	66.67

Table 7.85: EPV of benefits from Alzheimer's disease, with proportion of relative risk, $m = 1.00$, 0.5 and 0.25 (from Table 2.13) as a range of percentages of EPV of benefits from disability using all the disability models (from Table 7.75), with benefits increasing at $\delta_b = 0.05$, for males, females and combined.

Gender	Entry age	AD costs as a % of disability costs with proportion of relative risk: $m = 1.00$ % – %	$m = 0.50$ % – %	$m = 0.025$ % – %
M & F	60	61.03 – 68.36	59.58 – 66.74	58.34 – 65.34
	65	62.28 – 69.29	61.20 – 68.10	60.12 – 66.90
	70	61.76 – 68.36	61.35 – 67.91	60.63 – 67.11
	75	59.95 – 66.86	60.36 – 67.32	60.10 – 67.03
F	60	40.03 – 46.80	39.73 – 46.45	39.26 – 45.90
	65	43.11 – 46.35	43.07 – 46.31	42.75 – 45.96
	70	44.91 – 46.50	45.41 – 47.02	45.33 – 46.94
	75	44.59 – 47.71	45.77 – 48.97	46.08 – 49.31
M	60	57.29 – 78.04	55.46 – 75.55	54.55 – 74.31
	65	61.68 – 78.76	60.26 – 76.95	59.55 – 76.04
	70	64.61 – 78.32	64.05 – 77.64	63.68 – 77.19
	75	65.58 – 75.22	66.35 – 76.10	66.35 – 76.10

Table 7.86: Costs of adverse selection as a percentage of total LTC insurance costs (proportions from Table 7.85), with $\varepsilon 2/\varepsilon 4$, $\varepsilon 3/\varepsilon 4$ and $\varepsilon 4/\varepsilon 4$ genotypes k times as likely to insure as low-risk genotypes, for benefits increasing continuously ($\delta_b = 0.05$) and commencing on institutionalisation, for males and females.

Gender	Likelihood of high risk genotypes insuring, k	Prop'n of relative risk, m	Cost of adverse selection at age: 60	65	70	75
			%	%	%	%
F	2	1.00	3.9 – 4.6	4.0 – 4.3	3.6 – 3.8	2.9 – 3.1
		0.25	1.1 – 1.3	1.2 – 1.2	1.1 – 1.1	0.8 – 0.9
M	2	1.00	2.3 – 3.2	2.2 – 2.8	1.8 – 2.2	1.5 – 1.7
		0.25	0.5 – 0.7	0.5 – 0.6	0.4 – 0.5	0.3 – 0.4
F	4	1.00	8.4 – 9.8	8.5 – 9.1	7.8 – 8.1	6.3 – 6.7
		0.25	2.4 – 2.9	2.5 – 2.6	2.2 – 2.3	1.8 – 1.9
M	4	1.00	5.0 – 6.8	4.6 – 5.9	3.9 – 4.7	3.1 – 3.6
		0.25	1.1 – 1.5	1.0 – 1.3	0.8 – 1.0	0.7 – 0.7
F	10	1.00	13.5 – 15.8	13.7 – 14.7	12.7 – 13.2	10.3 – 11.0
		0.25	3.9 – 4.6	4.0 – 4.3	3.6 – 3.8	2.9 – 3.1
M	10	1.00	8.0 – 10.9	7.4 – 9.5	6.2 – 7.5	5.0 – 5.8
		0.25	1.8 – 2.4	1.6 – 2.1	1.3 – 1.6	1.0 – 1.2
F	100	1.00	18.6 – 21.7	19.0 – 20.4	17.8 – 18.4	14.6 – 15.6
		0.25	5.4 – 6.3	5.5 – 5.9	5.0 – 5.2	4.0 – 4.3
M	100	1.00	11.0 – 15.0	10.3 – 13.1	8.6 – 10.4	7.0 – 8.0
		0.25	2.4 – 3.3	2.3 – 2.9	1.9 – 2.3	1.4 – 1.7

wider spread of cost estimates of disability for males (which in turn may be a result of there being less data for males when parameterizing the original disability models). This implies that, for males, the assumption that AD costs constitute 40%–50% of total LTC costs is an underestimate — the figures in Table 7.85 suggest that the lower bound on the proportion of total LTC costs attributable to AD should be about 55% at entry age 60, increasing to just below 65% at entry age 75 and that an upper bound of just below 80% would be appropriate for all ages at entry. This means that the upper bound on the costs of adverse selection as a percentage of total LTC costs could have been underestimated by as much as 30% in Chapter 2.

The proportions for both genders combined are, at first, quite surprising. They are at a similar level to those for males, though there does not seem to be any trend by age at entry, and the ranges are much smaller — over all ages at entry and models the range of proportions is 58.34%–69.29%. It would be expected that the

proportions would lie between those for males separately and females separately, as do the estimates of total LTC costs (EPV of benefits in the disability models). This anomaly comes from the fact that the estimates of the costs of AD for males and females combined were very similar to those for females alone, which is a consequence of how the AD model for males and females combined was parameterized. While the relative risks used were for both genders combined, the baseline mortality was assumed to be 65% of a Makeham fit to AF80 (the same assumption that was used for the female AD model) — the justification for this was that the majority of older people will be females, due to the usual difference in mortality between genders. I have given the proportions for both genders combined for illustration, but, for the above reason, I will place more reliance on the results for males and females separately.

These proportions can be applied to the costs of adverse selection as percentages of AD-related costs given in Section 2.4 to give new estimates of the total percentage costs of adverse selection. For illustration, Table 7.86 gives ranges of total percentage costs using the proportions from Table 7.85 and the costs of adverse selection from Section 2.4. Comparing this table to the original estimates of ranges of total percentage costs given in Table 2.30:

1. for females there is very little difference between the two and, in fact, the revised ranges of costs all lie within those originally calculated; and

2. for males, the revised costs are substantially higher, the new lower bounds always exceed the upper bounds from the original estimates — however the costs for males are still only about 2/3 of those for females.

While I have included the revised costs of adverse selection for males, it is worth noting that the relative risks used for males were found to be anomalous in Section 2.6.1 (in that 'high risk' genotypes seemed to confer protection in some cases), and they were adjusted to ensure that 'high risk' genotypes did actually confer risk — however this was a subjective adjustment. This does not affect the validity of the estimates of the proprotion of total LTC costs attributable to AD for males though, and these could be used in future to estimate total percentage costs of AD for men (given a consistent set of relative risks for males).

Table 7.86 gives the range of costs of adverse selection for the baseline models. However, other costs can easily be calculated using other tables. For example, for females taking out insurance at age 70 and under the following assumptions:

1. claiming from Alzheimer's disease commences 2 years before institutionalisation;

2. the extra costs of Alzheimer's disease from the above assumption do not increase total long-term care costs (which may be a reasonable assumption, since lives suffering from Alzheimer's disease would start claiming before institutionalisation in the disability model);

3. there is an additional benefit of 1/2 in state 3 (1–2 ADL state) in the disability model;

4. there is an increased likelihood of $k = 10$ of high-risk genotypes insuring;

5. the proportion of relative risk $m = 1.00$;

6. all benefits increase continuously at a rate $\delta_b = 0.05$; and

7. the discount rate $\delta = 0.05$.

an upper bound on the cost of adverse selection can be calculated as (from Tables 2.27, 7.80 and 7.85) :

$$\frac{0.2565 \times 0.4650}{1.2382} = 9.63\% \tag{7.72}$$

and a lower bound as:

$$\frac{0.2565 \times 0.4491}{1.2952} = 8.89\% \tag{7.73}$$

7.5 Summary and Conclusions

In Chapter 2, in order to look at the potential costs of adverse selection arising from variants of the APOE gene, the cost of other events in the ageing process (mainly disability) that trigger benefits were very simply assumed to be a multiple of those costs arising from Alzheimer's disease.

In this chapter I have used the disability model described in Chapter 3, and parameterized in Chapters 4 to 6 to estimate (independently from Alzheimer's disease) the costs of disability (EPV of benefits) in long-term care insurance.

The sensitivity analysis, on the key model assumptions, indicated that: the EPVs of benefits are sensitive to the rate of benefit escalation and to the addition of extra benefits at lesser disability levels; and that they are not very sensitive to mortality assumptions after age 90 — this was investigated since the overall forces of mortality in the disability models were very consistent with the benchmark forces of mortality up to about age 90, but became less consistent at older ages (see Section 6.6).

I then compared overall mortality in the AD model with that in the disability model to check for consistency. It was clear that: at younger ages (60–90 years), for females, mortality between the two models was very similar; but for males and both genders combined, the overall force of mortality in the AD model was clearly below that of the disability models. At older ages, the differences between the forces of mortality of the two models were similar for all gender classifications. Overall:

1. for females, mortality between the two models seemed comparable; but

2. for males and both genders combined, lighter mortality at younger ages in the AD model may cause the estimates of AD-related costs to increase relative to total LTC costs, artificially increasing estimates of the proportion of total LTC costs attributable to AD.

The proportions of total LTC costs attributable to AD were then estimated for each gender classification and starting age (60, 65, 70 and 75), using the EPV of benefits from the AD model and from the disability model (assumed to be estimates of total LTC costs), using the baseline assumptions for both models, to ensure consistency. In Chapter 2, it was assumed that the costs of AD made up 40%–50% of total LTC costs for all genders classifications and starting ages. In this chapter the range of estimated proportions was estimated as:

1. for females, between 39.26% and 49.31%, with a slight tendancy for the proportion to increase with increasing starting age — though overall, this is in strong agreement with the assumption made in Chapter 2;

2. for males, between 54.55% and 78.76%, with a tendacy for the proportions to increase with starting age, with a fairly stable upper bound of about 80% for all starting ages; and

3. for both genders combined, between 58.34% and 69.29%, with no clear trend by starting age. For the 1982–84 (5) model, the proportions were about 60%.

For illustration, I applied these proportions to the costs of adverse selection as percentages of AD-related costs (from Section 2.4), to give ranges of estimates of total percentage costs of adverse selection, for a given set of assumptions about the level of adverse selection. Also, for example, I illustrated how a range of total percentage costs of adverse selection could be estimated, for other types of contracts/assumptions, using the results from the sensitivity analyses of the AD model and the disability model.

The results differ for each gender classifications and I conclude for each separately. For females:

1. When parameterizing both models, there was either more data for females (disability model), or more reliable data (AD model), suggesting that the results for females will be the most reliable of the three gender classifications.

2. Furthermore, the overall forces of model mortality for females in the AD model and the disability models were very similar (and the closest of the 3 genders classifications) at younger ages (60–90 years), suggesting that it would be reasonable to compare results from the AD model and the disability model.

3. The proportions estimated using the EPV of benefits from the AD model and the disability model were in almost total agreement with the assumption made in Chapter 2 — providing very strong support to the results and conclusions of that chapter (see Section 2.8 for conclusions).

For males:

1. In Chapter 2, the costs of adverse selection for males appeared to be negative (the $\varepsilon 2$ allele confered such protection that the $\varepsilon 3/\varepsilon 3$ genotype was high-risk at many ages). For the study of adverse selection in this chapter, the relative

risks were adjusted so that the $\varepsilon 4$ allele never conferred lower risk, but it was noted that no reliance should be placed on these results.

2. Even so, this does not affect the overall estimates of the costs of AD for males, since the genotype-specific incidence rates of AD were adjusted so that the overall incidence of AD was consistent with the benchmark incidence of AD — suggesting that it is still reasonable to use these overall cost estimates of AD to calculate the proportion of total LTC costs attributable to AD.

3. The overall force of mortality in the AD model was found to be slightly lighter than the overall forces of mortality in the disability models at younger ages (60–95 years). After age 95, mortality between the two models was roughly comparable. This suggests that the costs of AD may be slightly inflated in comparison to the total LTC costs estimated from the disability models.

4. Even after allowing for the above point, it still seems very likely that the proportion of total LTC costs attributable to AD for males is substantially greater than that assumed in Chapter 2 (40%–50%). For example, assuming that total LTC costs had been underestimated by 10% (from mortality differences), the estimated range of proportions would still be 45% – 70%.

5. The main findings for males are: estimates of total LTC costs; and the suggestion that the proportion of total LTC costs attributable to AD is likely to be in excess of 50%. Application of the proportions found in this chapter resulted in estimates of total percentage costs of adverse selection for males to be about 2/3 of those for females — though no reliance can be placed on these results because of the anomalous relative risks.

And for both genders combined:

1. It is not really necessary to look at both genders combined, since the results for males and females separately can be combined to get the same result (using the appropriate proportions). The main advantage of looking at both genders combined is that there will be more data for parameterizing models, especially important when there is a scarcity of data.

2. In the Alzheimer's disease model baseline mortality was taken to be 65% AF80 (the same as for females) — the only difference between the model for females and both genders combined was the relative risks for AD (the main focus in this chapter). However, as the relative risks were adjusted to be consistent with a benchmark incidence of AD (for both genders), the aggregate (over genotypes) costs of AD were very similar for females and both genders combined.

3. In the disability model, the results for both genders combined were very consistent with the results for males and females separately (even though the models were parameterized separately) — they lay in between the results for males and females, closer to the results for females (as expected, since there were more females in the original data sets).

4. It would then be expected that the estimated proportions of AD-related LTC costs would be overestimated (as the costs of AD would be overestimated).

5. This was confirmed when comparing the overall force of mortality in the AD model with the overall forces of mortality in the disability models — the force of mortality in the AD model was lighter than that for the disability model with the lightest force of mortality, at younger ages (60–100 years). The difference was greater than that reported above for males.

6. Given this inconsistency between models it is very difficult to draw any firm conclusions — combining the results for males and females separately, and the proportions calculated for both genders combined, it may be that the proportion of total LTC costs attributable to AD for both genders is slighter greater than that assumed in Chapter 2 (40%–50%).

Chapter 8

Areas for Further Research

I will first discuss areas of further research for the two models that I have proposed in this thesis separately (the Alzheimer's disease model, described in Chapter 1, and the disability model, described in Chapter 3) and then discuss areas common to both. For the Alzheimer's disease model:

1. There was no single study that would allow all the intensities in the model to be estimated simultaneously. The estimation was based on a number of different studies, some quite small, of different populations, with different research protocols and methods of analysis, and very likely different definitions of 'onset of AD' and 'institutionalization'. Use of a complete data set of lives with Alzheimer's disease would remove many uncertainties in the estimation process as well as allowing confidence intervals to be estimated and, depending on the data, claiming could be represented using definitions closer to those used in LTC insurance (i.e. on reaching a certain level of cognitive impairment, or loss of ADLs).

2. The relative risks of the APOE genotypes were based on case-based studies, not prospective population studies, and the risks associated with the $\varepsilon 4$ allele are almost certain to be lower than those estimated to date. I was unable to do more than to show what effect this might have. It would be very useful to update the relative risks of AD by APOE genotype as more prospective population studies are done. Ideally, the incidence of AD by APOE genotype could be directly estimated, rather than having to adjust an aggregate (by

genotype) incidence rate — though this will depend on the data and research that become available.

3. In the AD model, all of the transition intensities other than the incidence of AD were assumed to be same for each APOE genotype (the data was not available to assume otherwise). As more genetic studies are undertaken, it may be possible to allow (or at least check) for the dependency of the other transition intensities on APOE genotype (i.e. overall mortality may be different between different genotypes).

In general, with the great speed at which research on human genetics is advancing, revisiting the model and updating it in the light of new knowledge will be necessary to keep it credible. While some of the refinements described above will depend on the research that is done in the future and the data gathered, others could be approached given access to present data sets. For example, access to the CERAD (Consortium to Establish a Registry for Alzheimer's Disease) data set would enable research to be done on issues raised in the first point above.
For the disability model:

1. The data set I used to parameterise the disability model has been used by previous researchers to investigate trends of disability (using different methodology). The disability model I have described here could be put to similar uses and the results compared with their results.

2. I have applied the disability model in this thesis to estimating the costs of disability in a LTC insurance contract. However, it could easily be applied to estimating health care costs from disability in general, or for projecting disability levels.

3. The data set used, the NLTCS, is from the U.S.A. and experience in the U.K. may be different. Similar data from the U.K. could be analysed using the methodology set out in this thesis, and the trends of disability compared between countries, as well as then having a model more relevant to the U.K.. Of course, longitudinal data would be ideal, but is rarely available since collection of such data is very costly and time consuming.

Areas in general and those that affect both models:

1. It would be ideal to be able to combine disability and Alzheimer's disease in a single model that describes both processes. However, this would require data that describe at an individual level progress through ADLs and onset and progress of Alzheimer's disease.

2. Throughout this thesis, I assumed a constant discount rate of 0.05. Incorporation of a financial model of interest rates would be particularly useful for modelling products that have index linked benefits/premiums.

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Appendix A

Overview of the 1982, 1984, 1989 and 1994 National Long-Term Care Surveys — transitions between states

The tables in this appendix summarise the number of transitions between the classification of Figure 3.20. The first table, Table A.87 is the key for the following 3 tables — it defines what classifications the letters A–K represent in each survey year. Take, for example, the classification '1994 aged-in population', then from Table A.87:

1. in the 1982 survey, the letter K is used to represent '1994 aged-in population' (lives aged 65–70 years in 1994 and lives aged over 95 years in 1994) in Table A.88;

2. in 1984, the letter H is used (in Tables A.88 and A.89) for the same group of lives;

3. in 1989, for the 1994 aged-in lives, J1 and J2 are used (in Table A.90) to represent lives aged 65–70 years and those over 95 years in 1994, respectively, and J is used to represent both (in Table A.89); and

Table A.87: Key to classifications in each Nation Long-Term Care Survey Year.

Classification		Classification in survey year: 1982	1984	1989	1994
Screener only	complete	A	A	A	A
	non-response dead	B		B	B
	non-response other	C	C[(1)]	C	C
Community detail	complete	D	B	D	D
	non-response dead	E		E	E
	non-response other	F	C[(1)]	F	F
Institutionalized	before 1 April 1982	G			
	after 1 April 1982	H			
	complete		D	G	G
	non-response dead			H	H
	non-response other		E	I	I
Dead			F		
1984 aged-in population		I			
1989 aged-in population		J	G		
1994 aged-in population 65-70 yrs:		K	H	J1[(2)]	
1994 aged-in population 95+ yrs:		K	H	J2[(2)]	
Not in Survey Year				K	J

(1) Non-responders to the screener questionnaire and the community detail questionnaire in 1984 are grouped together in a single classification due to data not being available in 1982.

(2) Classification J is used in Table A.89 to represent both J1 and J2 in 1989.

4. in 1994, this group of lives is part of the survey and are included in the other states.

Table A.88: Transitions between classifications in the 1982 and 1984 National Long-Term Care Surveys.

1982 Status[1]	1984 Status[1]								
	A	B	C	D	E	F	G	H	Total
A	9404	1091	194	152	6	723	0	0	11570
B	6	0	4	2	0	203	0	0	215
C	113	29	106	20	3	44	0	0	315
D	0	4182	210	414	22	1260	0	0	6088
E	0	0	0	0	0	67	0	0	67
F	0	92	46	41	3	56	0	0	238
G	0	60	7	908	45	688	0	0	1708
H	0	40	4	114	4	122	0	0	284
I	4263	440	118	39	0	56	0	0	4916
J	0	0	0	0	0	0	4907	0	4907
K	0	0	0	0	0	0	0	5540	5540
Total	13786	5934	689	1690	83	3219	4907	5540	35848

(1) See Table A.87 for description of classifications.

Table A.89: Transitions between classifications in the 1984 and 1989 National Long-Term Care Surveys.

1984 Status[1]	1989 Status[1]											
	A	B	C	D	E	F	G	H	I	J	K	Total
A	6038	195	139	1105	47	77	277	7	5	0	5896	13786
B	8	191	89	2771	60	126	503	35	5	0	2146	5934
C	90	25	51	99	4	34	32	3	4	0	351	689
D	1	73	9	19	9	3	476	23	1	0	1073	1690
E	0	7	1	2	0	3	11	0	0	0	58	83
F	0	0	0	0	0	0	0	0	0	0	3219	3219
G	4193	38	101	467	11	40	55	0	2	0	0	4907
H	0	0	0	0	0	0	0	0	0	5540	0	5540
Total	10330	529	390	4463	131	283	1354	68	17	5540	12743	35848

(1) See Table A.87 for description of classifications.

Table A.90: Transitions between classifications in the 1989 and 1994 National Long-Term Care Surveys.

1989 Status[1]	1994 Status[1]										
	A	B	C	D	E	F	G	H	I	J	Total
A	4315	202	211	1474	23	195	304	8	8	3590	10330
B	13	18	15	9	0	1	6	2	0	465	529
C	88	10	46	47	1	6	26	0	0	166	390
D	0	202	152	1800	23	123	392	8	1	1762	4463
E	0	0	0	0	0	0	0	0	0	131	131
F	0	0	0	0	0	0	0	0	0	283	283
G	0	45	21	13	3	15	290	7	1	959	1354
H	0	0	0	0	0	0	0	0	0	68	68
I	0	0	0	0	0	0	0	0	0	17	17
J1	3692	57	202	872	11	114	51	0	1	0	5000
J2	58	115	70	127	9	14	139	6	2	0	540
K	2308	77	134	747	11	105	122	1	2	9236	12743
Total	10474	726	851	5089	81	573	1330	32	15	16677	35848

(1) See Table A.87 for description of classifications.

Appendix B

Numbers of transitions between disability states of the 1982 and 1984 NLTCS by gender and age group

The tables in this appendix give the number of transitions between disability states for the 1982–84 NLTCS for separate genders. This data has been adjusted for censored data (see Section 3.7), which is why the numbers of transition are not integer numbers of lives. Tables B.91 and B.92 give the data split into 10 year age groups (65–74 years, 75–84 years and 85+ years) for females and males, respectively. Tables B.93 and B.94 give the data split into 5 year age groups (65–69 years, 70–74 years, 75–79 years, 80–84 years and 85+ years) for females and males, respectively. The same data over all ages (>65 years) are given in Table 3.36.

Table B.91: Transitions for females between disability states in the 1982 and 1984 National Long-Term Care Surveys using 10 year age groupings, adjusted for censored data.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74	4140.92	155.90	111.18	36.52	32.94	33.90	168.27
	75–84	1587.47	134.50	124.87	41.47	29.08	72.36	186.36
	85+	193.85	33.63	46.00	8.90	16.68	40.39	65.95
	All	5922.24	324.03	282.05	86.89	78.70	146.65	420.58
IADL only	65–74	130.90	161.26	98.24	27.18	14.29	22.41	43.20
	75–84	56.67	141.95	120.51	20.94	17.21	37.52	72.72
	85+	3.71	44.18	40.23	12.10	12.20	18.91	34.29
	All	191.28	347.39	258.98	60.22	43.70	78.84	150.21
1–2 ADLs	65–74	68.91	75.38	185.35	48.25	24.36	24.55	61.44
	75–84	48.26	71.79	210.54	71.39	32.72	58.52	103.00
	85+	9.22	23.75	94.93	43.42	26.55	38.64	60.53
	All	126.39	170.92	490.82	163.06	83.63	121.71	224.97
3–4 ADLs	65–74	11.14	10.77	54.71	58.40	32.43	10.80	23.43
	75–84	7.69	9.83	49.07	55.40	49.52	30.08	48.66
	85+	1.75	5.21	9.25	29.11	33.17	23.55	35.47
	All	20.58	25.81	113.03	142.91	115.12	64.43	107.56
5–6 ADLs	65–74	11.66	11.84	15.80	18.44	62.47	15.88	57.36
	75–84	10.24	8.90	16.16	23.44	79.56	32.16	71.58
	85+	2.99	1.37	10.45	15.20	51.26	25.73	83.40
	All	24.89	22.11	42.41	57.08	193.29	73.77	212.34
Inst'd	65–74	8.77	1.47	2.19	4.21	2.22	124.37	45.30
	75–84	5.89	4.47	3.66	3.16	5.30	315.71	171.67
	85+	3.04	2.24	0.29	5.13	4.21	307.78	271.36
	All	17.70	8.18	6.14	12.50	11.73	747.86	488.33

Table B.92: Transitions for males between disability states in the 1982 and 1984 National Long-Term Care Surveys using 10 year age groupings, adjusted for censored data.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74	3097.98	86.51	65.45	25.42	27.05	28.25	283.24
	75–84	1022.67	70.83	56.86	12.75	20.61	27.09	192.56
	85+	86.24	17.71	19.55	6.39	3.43	8.20	45.29
	All	4206.89	175.05	141.86	44.56	51.09	63.54	521.09
IADL only	65–74	98.37	101.92	40.22	4.68	14.65	7.44	57.85
	75–84	35.84	74.81	33.89	13.40	22.53	18.03	51.76
	85+	1.47	23.05	9.15	5.03	3.03	4.23	22.56
	All	135.68	199.78	83.26	23.11	40.21	29.70	132.17
1–2 ADLs	65–74	33.77	28.69	88.94	41.57	21.51	15.19	63.20
	75–84	14.44	23.50	68.50	20.23	19.34	6.67	86.11
	85+	5.93	4.61	20.85	11.34	15.38	17.91	37.67
	All	54.14	56.80	178.29	73.14	56.23	39.77	186.98
3–4 ADLs	65–74	15.23	2.62	24.79	29.42	21.41	3.88	38.16
	75–84	3.06	4.24	8.24	17.11	15.16	9.31	36.12
	85+	0.21	2.02	2.07	6.01	11.01	7.10	24.70
	All	18.50	8.88	35.10	52.54	47.58	20.29	98.98
5–6 ADLs	65–74	13.15	8.61	17.79	18.41	53.41	10.87	64.06
	75–84	4.92	7.47	6.54	12.24	41.30	12.59	67.02
	85+	0.20	0.02	2.06	3.01	12.01	3.10	28.66
	All	18.27	16.10	26.39	33.66	106.72	26.56	159.74
Inst'd	65–74	7.17	1.25	3.36	1.26	2.21	71.54	45.48
	75–84	2.91	2.20	1.17	2.08	1.15	85.29	86.42
	85+	0.34	0.03	0.11	0.02	0.02	51.17	83.12
	All	10.42	3.48	4.64	3.36	3.38	208.00	215.02

Table B.93: Transitions for females between disability states in the 1982 and 1984 National Long-Term Care Surveys using 5 year age groupings, adjusted for censored data.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	2375.92	67.90	59.50	14.26	13.74	9.05	76.97
	70–74	1765.01	88.00	51.69	22.26	19.20	24.85	91.29
	75–79	1101.46	86.92	67.02	22.80	16.66	32.24	98.71
	80–84	486.01	47.58	57.85	18.67	12.42	40.12	87.66
	85+	193.85	33.63	46.00	8.90	16.68	40.39	65.95
	All	5922.25	324.03	282.06	86.89	78.70	146.65	420.58
IADL only	65–69	68.46	70.88	38.76	15.43	5.38	5.82	16.65
	70–74	62.43	90.38	59.48	11.75	8.91	16.60	26.56
	75–79	36.59	76.10	58.50	12.56	7.70	19.63	37.99
	80–84	20.08	65.86	62.01	8.38	9.50	17.90	34.73
	85+	3.71	44.18	40.23	12.10	12.20	18.91	34.29
	All	191.27	347.40	258.98	60.22	43.69	78.86	150.22
1–2 ADLs	65–69	39.58	37.14	89.06	22.57	8.53	6.11	26.77
	70–74	29.33	38.24	96.29	25.68	15.83	18.43	34.67
	75–79	30.31	36.43	116.92	35.74	15.91	26.05	45.14
	80–84	17.95	35.36	93.62	35.65	16.81	32.48	57.86
	85+	9.22	23.75	94.93	43.42	26.55	38.64	60.53
	All	126.39	170.92	490.82	163.06	83.63	121.71	224.97
3–4 ADLs	65–69	4.62	6.30	32.23	27.14	11.11	4.26	10.73
	70–74	6.52	4.47	22.48	31.26	21.32	6.54	12.70
	75–79	4.34	5.50	30.68	27.25	23.32	14.73	26.89
	80–84	3.35	4.34	18.40	28.14	26.20	15.35	21.77
	85+	1.75	5.21	9.25	29.12	33.17	23.55	35.47
	All	20.58	25.82	113.04	142.91	115.12	64.43	107.56
5–6 ADLs	65–69	6.82	5.46	8.42	6.23	33.21	4.44	21.70
	70–74	4.84	6.38	7.38	12.21	29.26	11.44	35.66
	75–79	5.02	4.33	7.47	10.16	44.22	15.53	32.85
	80–84	5.23	4.57	8.69	13.28	35.34	16.63	38.74
	85+	2.99	1.37	10.45	15.20	51.26	25.73	83.40
	All	24.90	22.11	42.41	57.08	193.29	73.77	212.35
Inst'd	65–69	6.18	0.23	2.05	2.08	0.02	58.11	8.18
	70–74	2.59	1.24	0.14	2.13	2.19	66.26	37.12
	75–79	2.52	2.23	1.40	1.09	4.17	118.51	65.43
	80–84	3.37	2.23	2.26	2.06	1.13	197.19	106.24
	85+	3.04	2.24	0.29	5.13	4.21	307.78	271.36
	All	17.70	8.17	6.14	12.49	11.72	747.85	488.33

Table B.94: Transitions for males between disability states in the 1982 and 1984 National Long-Term Care Surveys using 5 year age groupings, adjusted for censored data.

		1984 Status						
1982 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	1853.76	45.23	30.64	13.47	12.66	13.18	112.97
	70–74	1244.21	41.28	34.80	11.95	14.39	15.07	170.27
	75–79	732.68	42.10	30.58	8.17	12.97	14.17	123.49
	80–84	289.98	28.73	26.29	4.58	7.64	12.92	69.07
	85+	86.24	17.71	19.55	6.39	3.43	8.20	45.29
	All	4206.87	175.05	141.86	44.56	51.09	63.54	521.09
IADL only	65–69	51.64	52.49	22.68	2.36	11.38	2.75	26.65
	70–74	46.73	49.44	17.54	2.33	3.27	4.69	31.20
	75–79	27.70	51.62	19.77	8.35	14.47	11.83	34.60
	80–84	8.14	23.19	14.12	5.06	8.06	6.21	17.16
	85+	1.47	23.05	9.15	5.03	3.03	4.23	22.56
	All	135.68	199.79	83.26	23.13	40.21	29.71	132.17
1–2 ADLs	65–69	18.92	11.25	40.40	15.23	13.25	4.48	26.77
	70–74	14.85	17.44	48.54	26.34	8.27	10.70	36.44
	75–79	9.63	17.23	34.28	9.13	13.23	0.36	43.79
	80–84	4.81	6.28	34.22	11.10	6.11	6.31	42.32
	85+	5.93	4.61	20.85	11.34	15.38	17.91	37.67
	All	54.14	56.81	178.29	73.14	56.24	39.76	186.99
3–4 ADLs	65–69	7.00	1.28	13.38	10.19	10.21	3.41	19.73
	70–74	8.23	1.34	11.41	19.23	11.21	0.48	18.43
	75–79	2.83	1.19	4.23	12.10	8.15	6.26	23.16
	80–84	0.23	3.05	4.01	5.01	7.01	3.05	12.97
	85+	0.21	2.02	2.07	6.01	11.01	7.10	24.70
	All	18.50	8.88	35.10	52.54	47.59	20.30	98.99
5–6 ADLs	65–69	5.21	4.44	6.57	9.28	23.30	5.59	30.70
	70–74	7.93	4.18	11.22	9.13	30.11	5.28	33.36
	75–79	2.81	3.19	2.23	5.10	24.15	7.26	33.10
	80–84	2.11	4.29	4.31	7.14	17.15	5.33	33.92
	85+	0.20	0.02	2.06	3.01	12.01	3.10	28.66
	All	18.26	16.12	26.39	33.66	106.72	26.56	159.74
Inst'd	65–69	4.41	1.14	0.23	1.14	0.15	43.29	13.18
	70–74	2.76	0.11	3.13	0.12	2.06	28.25	32.30
	75–79	2.55	2.13	1.15	1.07	0.14	47.21	41.87
	80–84	0.37	0.08	0.02	1.01	1.01	38.08	44.56
	85+	0.34	0.03	0.11	0.02	0.02	51.17	83.12
	All	10.43	3.49	4.64	3.36	3.38	208.00	215.03

Appendix C

Numbers of transitions between disability states of the 1984 and 1989 NLTCS by gender and age group

The tables in this appendix give the number of transitions between disability states for the 1984–89 NLTCS for separate genders. This data has been adjusted for censored data (see Section 3.8), which is why the numbers of transition are not integer numbers of lives. Tables C.95 and C.96 give the data split into 10 year age groups (65–74 years, 75–84 years and 85+ years) for females and males, respectively. Tables C.97 and C.98 give the data split into 5 year age groups (65–69 years, 70–74 years, 75–79 years, 80–84 years and 85+ years) for females and males, respectively. The same data over all ages (>65 years) are given in Table 3.38.

Table C.95: Transitions for females between disability states in the 1984 and 1989 National Long-Term Care Surveys using 10 year age groupings, adjusted for censored data.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74	2662.36	146.43	157.67	61.05	51.95	82.55	47.63
	75–84	1220.66	119.35	191.79	81.05	64.35	145.00	73.63
	85+	108.62	19.10	27.38	21.17	16.83	59.09	20.63
	All	3991.64	284.88	376.84	163.27	133.13	286.64	141.89
IADL only	65–74	88.12	76.71	90.55	22.33	15.69	24.29	12.40
	75–84	31.47	68.44	74.90	34.47	22.82	48.85	19.40
	85+	2.95	16.62	22.91	7.69	14.62	25.47	17.40
	All	122.54	161.77	188.36	64.49	53.13	98.61	49.20
1–2 ADLs	65–74	47.71	28.55	145.74	53.38	22.89	31.33	12.44
	75–84	27.85	28.92	105.18	69.75	35.86	86.31	20.44
	85+	5.20	6.13	31.69	26.14	16.99	42.37	21.44
	All	80.76	63.60	282.61	149.27	75.74	160.01	54.32
3–4 ADLs	65–74	11.04	10.67	31.50	49.49	23.21	14.82	11.17
	75–84	5.63	4.05	16.67	39.06	18.78	34.65	15.17
	85+	2.77	1.50	5.75	12.49	9.43	17.02	12.17
	All	19.44	16.22	53.92	101.04	51.42	66.49	38.51
5–6 ADLs	65–74	6.32	4.62	18.89	20.59	26.46	18.05	4.13
	75–84	6.27	2.45	8.74	8.52	32.35	17.67	14.13
	85+	0.91	0.28	2.42	3.29	19.26	19.61	11.13
	All	13.50	7.35	30.05	32.40	78.07	55.33	29.39
Inst'd	65–74	4.23	0.64	3.89	1.66	2.50	92.09	10.23
	75–84	5.17	2.83	5.35	0.94	0.64	175.23	28.23
	85+	2.92	0.69	1.03	0.68	2.58	129.40	42.23
	All	12.32	4.16	10.27	3.28	5.72	396.72	80.69

Table C.96: Transitions for males between disability states in the 1984 and 1989 National Long-Term Care Surveys using 10 year age groupings, adjusted for censored data.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74	2034.79	99.74	87.26	32.47	23.77	46.45	60.63
	75–84	731.01	65.01	60.97	38.94	31.30	66.01	76.63
	85+	47.71	11.65	7.01	5.28	4.18	14.56	12.90
	All	2813.51	176.40	155.24	76.69	59.25	127.02	150.16
IADL only	65–74	45.00	43.48	33.87	12.06	10.81	16.28	12.40
	75–84	17.75	20.89	16.58	16.92	17.57	13.54	12.40
	85+	2.19	2.27	3.41	6.30	3.27	7.61	7.32
	All	64.94	66.64	53.86	35.28	31.65	37.43	32.12
1–2 ADLs	65–74	27.75	23.97	50.58	27.45	18.13	12.09	8.44
	75–84	12.46	10.98	28.73	10.01	10.62	13.68	11.44
	85+	0.47	3.06	4.09	5.14	5.13	7.27	6.35
	All	40.68	38.01	83.40	42.60	33.88	33.04	26.23
3–4 ADLs	65–74	7.14	4.56	20.69	16.39	11.30	9.85	5.17
	75–84	0.73	0.36	8.64	6.38	10.23	12.62	8.17
	85+	0.18	0.02	0.03	2.05	0.05	4.11	3.14
	All	8.05	4.94	29.36	24.82	21.58	26.58	16.48
5–6 ADLs	65–74	4.18	4.29	9.31	7.18	25.13	8.42	4.13
	75–84	0.49	2.20	6.38	7.23	11.12	8.35	3.13
	85+	0.14	0.02	0.03	1.04	4.04	4.08	2.11
	All	4.81	6.51	15.72	15.45	40.29	20.85	9.37
Inst'd	65–74	1.95	2.56	0.63	1.36	0.26	46.82	7.23
	75–84	2.17	0.16	0.36	1.23	0.08	33.29	20.23
	85+	1.47	0.17	0.25	0.18	0.16	20.37	5.18
	All	5.59	2.89	1.24	2.77	0.50	100.48	32.64

Table C.97: Transitions for females between disability states in the 1984 and 1989 National Long-Term Care Surveys using 5 year age groupings, adjusted for censored data.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	1350.66	56.48	52.30	19.52	21.39	30.37	19.73
	70–74	1311.70	89.95	105.38	41.52	30.56	52.19	27.90
	75–79	894.18	79.04	113.64	45.11	34.24	76.16	40.18
	80–84	326.47	40.31	78.15	35.94	30.10	68.83	33.45
	85+	108.62	19.10	27.38	21.17	16.83	59.09	20.63
	All	3991.63	284.88	376.85	163.26	133.12	286.64	141.89
IADL only	65–69	49.34	34.31	45.32	9.97	6.74	10.88	4.08
	70–74	38.78	42.40	45.23	12.36	8.95	13.41	8.32
	75–79	22.24	39.52	42.65	16.41	12.32	24.27	8.24
	80–84	9.23	28.92	32.25	18.06	10.50	24.58	11.16
	85+	2.95	16.62	22.91	7.69	14.62	25.47	17.40
	All	122.54	161.77	188.36	64.49	53.13	98.61	49.20
1–2 ADLs	65–69	33.85	14.25	56.66	21.08	9.01	16.08	5.09
	70–74	13.86	14.30	89.09	32.30	13.87	15.25	7.35
	75–79	13.37	13.94	59.32	35.77	23.63	30.94	10.26
	80–84	14.48	14.98	45.86	33.98	12.23	55.37	10.18
	85+	5.20	6.13	31.69	26.14	16.99	42.37	21.44
	All	80.76	63.60	282.62	149.27	75.73	160.01	54.32
3–4 ADLs	65–69	4.75	8.84	20.19	19.71	13.64	7.40	5.03
	70–74	6.29	1.83	11.31	29.78	9.57	7.42	6.14
	75–79	3.70	2.42	8.78	20.41	8.41	16.59	6.10
	80–84	1.93	1.62	7.89	18.65	10.37	18.06	9.07
	85+	2.77	1.50	5.75	12.49	9.43	17.02	12.17
	All	19.44	16.21	53.92	101.04	51.42	66.49	38.51
5–6 ADLs	65–69	5.43	1.31	12.40	10.27	13.26	6.52	3.03
	70–74	0.89	3.30	6.49	10.32	13.20	11.53	1.11
	75–79	1.68	2.17	7.37	4.19	14.21	9.19	7.08
	80–84	4.59	0.28	1.37	4.33	18.15	8.48	7.05
	85+	0.91	0.28	2.42	3.29	19.26	19.61	11.13
	All	13.50	7.34	30.05	32.40	78.08	55.33	29.40
Inst'd	65–69	2.36	0.31	1.33	1.27	1.29	31.50	4.05
	70–74	1.87	0.34	2.56	0.39	1.22	60.59	6.18
	75–79	2.17	0.30	4.64	0.33	0.35	77.33	11.14
	80–84	3.00	2.53	0.71	0.61	0.29	97.90	17.09
	85+	2.92	0.69	1.03	0.68	2.58	129.40	42.23
	All	12.32	4.17	10.27	3.28	5.73	396.72	80.69

Table C.98: Transitions for males between disability states in the 1984 and 1989 National Long-Term Care Surveys using 5 year age groupings, adjusted for censored data.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	1080.98	45.91	49.76	15.29	13.73	15.06	29.90
	70–74	953.81	53.82	37.50	17.18	10.04	31.38	30.73
	75–79	558.93	43.33	34.78	23.14	19.63	40.67	47.45
	80–84	172.08	21.68	26.20	15.80	11.67	25.34	29.18
	85+	47.71	11.65	7.01	5.28	4.18	14.56	12.90
	All	2813.51	176.39	155.25	76.69	59.25	127.01	150.16
IADL only	65–69	33.15	20.82	20.34	4.69	5.57	6.54	6.32
	70–74	11.85	22.66	13.53	7.37	5.24	9.74	6.08
	75–79	13.52	12.78	10.45	7.90	10.55	5.41	6.16
	80–84	4.23	8.11	6.13	9.03	7.02	8.13	6.24
	85+	2.19	2.27	3.41	6.30	3.27	7.61	7.32
	All	64.94	66.64	53.86	35.29	31.65	37.43	32.12
1–2 ADLs	65–69	12.28	15.83	21.37	14.70	9.58	5.58	3.35
	70–74	15.47	8.14	29.22	12.75	8.55	6.51	5.09
	75–79	7.44	6.64	17.26	8.80	8.45	6.17	7.18
	80–84	5.02	4.34	11.47	1.21	2.17	7.51	4.26
	85+	0.47	3.06	4.09	5.14	5.13	7.27	6.35
	All	40.68	38.01	83.41	42.60	33.88	33.04	26.23
3–4 ADLs	65–69	4.48	1.40	9.65	6.34	6.28	2.75	2.14
	70–74	2.66	3.16	11.03	10.05	5.02	7.11	3.03
	75–79	0.47	0.17	6.36	4.24	7.12	5.32	5.07
	80–84	0.25	0.19	2.28	2.13	3.11	7.30	3.10
	85+	0.18	0.02	0.03	2.05	0.05	4.11	3.14
	All	8.04	4.94	29.35	24.81	21.58	26.59	16.48
5–6 ADLs	65–69	2.44	2.03	3.08	3.03	10.02	4.11	2.11
	70–74	1.74	2.26	6.24	4.16	15.11	4.31	2.03
	75–79	0.41	2.16	4.33	3.22	10.11	5.30	3.05
	80–84	0.08	0.04	2.04	4.01	1.01	3.04	0.08
	85+	0.14	0.02	0.03	1.04	4.04	4.08	2.11
	All	4.81	6.51	15.72	15.46	40.29	20.84	9.38
Inst'd	65–69	0.91	1.20	0.36	0.17	0.14	28.43	3.18
	70–74	1.04	1.36	0.27	1.19	0.12	18.39	4.05
	75–79	1.04	0.10	0.29	0.22	0.07	22.21	12.09
	80–84	1.13	0.06	0.07	1.01	0.01	11.08	8.14
	85+	1.47	0.17	0.25	0.18	0.16	20.37	5.18
	All	5.59	2.89	1.24	2.77	0.50	100.48	32.64

Appendix D

Numbers of transitions between disability states of the 1989 and 1994 NLTCS by gender and age group

The tables in this appendix give the number of transitions between disability states for the 1989–94 NLTCS for separate genders. This data has been adjusted for censored data (see Section 3.9), which is why the numbers of transition are not integer numbers of lives. Tables D.99 and D.100 give the data split into 10 year age groups (65–74 years, 75–84 years and 85+ years) for females and males, respectively. Tables D.101 and D.102 give the data split into 5 year age groups (65–69 years, 70–74 years, 75–79 years, 80–84 years and 85+ years) for females and males, respectively. The same data over all ages (>65 years) are given in Table 3.40.

Table D.99: Transitions for females between disability states in the 1989 and 1994 National Long-Term Care Surveys using 10 year age groupings, adjusted for censored data.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74	1984.58	104.13	158.11	51.75	42.10	81.23	39.00
	75–84	1152.14	130.17	186.76	89.88	67.87	194.10	79.34
	85+	95.52	18.24	30.91	17.40	28.05	70.48	19.12
	All	3232.24	252.54	375.78	159.04	138.02	345.81	137.47
IADL only	65–74	45.95	36.45	29.03	16.19	9.06	7.45	5.00
	75–84	35.54	31.36	47.38	18.99	17.88	33.48	19.17
	85+	5.34	9.78	13.18	13.80	7.71	17.84	6.23
	All	86.83	77.59	89.59	48.98	34.65	58.77	30.39
1–2 ADLs	65–74	36.27	21.85	55.56	40.51	20.34	12.09	8.00
	75–84	35.99	23.34	72.78	63.82	34.65	60.33	27.23
	85+	6.48	8.15	19.74	17.16	12.03	42.67	15.31
	All	78.75	53.33	148.09	121.49	67.02	115.09	50.54
3–4 ADLs	65–74	9.61	4.52	10.20	22.28	9.13	16.68	6.00
	75–84	7.83	6.39	18.99	32.17	22.11	35.64	10.10
	85+	7.58	1.67	9.01	4.65	10.58	27.47	17.13
	All	25.02	12.57	38.20	59.11	41.82	79.78	33.23
5–6 ADLs	65–74	4.15	1.55	3.79	6.46	19.41	5.95	6.00
	75–84	4.03	1.69	3.98	9.58	21.55	20.31	11.05
	85+	3.42	0.82	2.24	2.75	11.66	11.64	6.07
	All	11.61	4.06	10.00	18.79	52.62	37.90	23.13
Inst'd	65–74	6.11	0.97	3.36	0.80	0.71	46.65	6.00
	75–84	4.96	1.73	2.98	2.59	2.60	90.41	19.11
	85+	4.25	0.89	1.36	0.86	0.76	100.91	17.14
	All	15.32	3.59	7.70	4.25	4.07	237.97	42.25

Table D.100: Transitions for males between disability states in the 1989 and 1994 National Long-Term Care Surveys using 10 year age groupings, adjusted for censored data.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74	1490.55	74.51	79.05	33.55	30.03	44.28	57.78
	75–84	711.60	77.67	76.93	37.95	35.09	70.84	65.56
	85+	57.55	3.21	13.62	3.07	8.71	27.60	19.78
	All	2259.69	155.39	169.61	74.57	73.83	142.72	143.12
IADL only	65–74	32.41	16.11	13.66	7.15	2.87	7.13	6.06
	75–84	11.84	20.75	12.05	7.67	4.65	10.57	7.11
	85+	0.75	2.09	7.19	3.08	4.13	0.34	3.06
	All	45.00	38.95	32.90	17.90	11.64	18.04	16.23
1–2 ADLs	65–74	20.49	8.72	28.06	12.87	5.57	9.43	4.08
	75–84	7.53	4.72	12.97	9.64	10.64	22.57	12.15
	85+	1.60	0.30	3.53	1.26	1.31	4.79	3.08
	All	29.62	13.74	44.56	23.78	17.52	36.79	19.31
3–4 ADLs	65–74	7.27	2.66	5.98	8.68	12.51	5.26	5.03
	75–84	2.69	1.18	0.23	6.16	6.18	10.44	7.07
	85+	0.70	1.08	3.15	0.07	1.09	1.24	3.03
	All	10.66	4.92	9.36	14.91	19.79	16.95	15.13
5–6 ADLs	65–74	2.10	1.09	3.13	0.14	8.08	5.20	2.02
	75–84	3.53	0.15	1.20	2.13	10.14	6.33	4.04
	85+	0.04	0.01	0.03	0.01	1.02	2.07	2.02
	All	5.67	1.25	4.36	2.28	19.24	13.60	8.07
Inst'd	65–74	3.61	1.45	0.66	0.51	0.35	22.88	3.04
	75–84	1.53	0.39	1.53	1.34	0.34	26.83	4.07
	85+	1.23	0.49	0.78	0.43	0.41	11.02	6.04
	All	6.36	2.33	2.97	2.28	1.11	60.72	13.14

Table D.101: Transitions for females between disability states in the 1989 and 1994 National Long-Term Care Surveys using 5 year age groupings, adjusted for censored data.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	961.09	41.79	61.56	19.09	20.45	26.76	21.00
	70–74	1023.49	62.34	96.55	32.66	21.65	54.48	18.00
	75–79	777.76	68.77	109.23	38.77	36.65	95.15	31.78
	80–84	374.37	61.41	77.54	51.11	31.22	98.95	47.56
	85+	95.52	18.24	30.91	17.40	28.05	70.48	19.12
	All	3232.24	252.54	375.78	159.04	138.02	345.81	137.47
IADL only	65–69	29.66	16.60	6.97	8.53	7.46	2.12	0.00
	70–74	16.29	19.85	22.06	7.66	1.59	5.33	5.00
	75–79	23.81	20.18	23.68	10.05	10.99	10.27	8.06
	80–84	11.73	11.18	23.70	8.94	6.89	23.21	11.11
	85+	5.34	9.78	13.18	13.80	7.71	17.84	6.23
	All	86.83	77.59	89.59	48.98	34.65	58.77	30.39
1–2 ADLs	65–69	15.11	5.27	18.49	12.24	6.21	2.51	2.00
	70–74	21.16	16.57	37.07	28.27	14.13	9.58	6.00
	75–79	26.53	12.34	41.88	43.20	17.14	17.59	13.08
	80–84	9.46	10.99	30.91	20.63	17.51	42.74	14.15
	85+	6.48	8.15	19.74	17.16	12.03	42.67	15.31
	All	78.75	53.33	148.09	121.49	67.02	115.09	50.54
3–4 ADLs	65–69	3.22	3.42	1.67	7.37	2.32	7.78	2.00
	70–74	6.39	1.10	8.53	14.91	6.81	8.89	4.00
	75–79	4.88	4.54	10.75	17.48	12.46	13.04	6.03
	80–84	2.96	1.85	8.24	14.69	9.64	22.59	4.07
	85+	7.58	1.67	9.01	4.65	10.58	27.47	17.13
	All	25.02	12.57	38.20	59.11	41.82	79.78	33.23
5–6 ADLs	65–69	1.18	0.20	0.33	2.18	8.16	4.38	2.00
	70–74	2.98	1.35	3.46	4.28	11.25	1.57	4.00
	75–79	2.40	0.37	2.52	4.33	10.31	6.70	3.02
	80–84	1.63	1.32	1.46	5.25	11.24	13.61	8.04
	85+	3.42	0.82	2.24	2.75	11.66	11.64	6.07
	All	11.61	4.06	10.00	18.79	52.62	37.90	23.13
Inst'd	65–69	2.17	0.27	1.45	0.24	0.21	11.51	1.00
	70–74	3.94	0.70	1.91	0.56	0.50	35.14	5.00
	75–79	3.12	0.35	1.45	1.31	0.32	41.69	12.04
	80–84	1.84	1.38	1.53	1.28	2.28	48.73	7.07
	85+	4.25	0.89	1.36	0.86	0.76	100.91	17.14
	All	15.32	3.59	7.70	4.25	4.07	237.97	42.25

Table D.102: Transitions for males between disability states in the 1989 and 1994 National Long-Term Care Surveys using 5 year age groupings, adjusted for censored data.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	745.64	26.24	30.27	11.57	12.19	17.92	15.00
	70–74	744.91	48.27	48.78	21.98	17.83	26.36	42.78
	75–79	489.08	55.57	47.42	24.92	22.74	36.20	32.78
	80–84	222.51	22.10	29.51	13.04	12.34	34.65	32.78
	85+	57.55	3.21	13.62	3.07	8.71	27.60	19.78
	All	2259.69	155.39	169.61	74.57	73.83	142.72	143.12
IADL only	65–69	11.64	6.27	3.32	3.24	0.16	1.45	3.00
	70–74	20.77	9.84	10.34	3.91	2.71	5.68	3.06
	75–79	10.29	10.55	6.84	3.54	2.43	5.14	5.06
	80–84	1.55	10.20	5.21	4.12	2.22	5.43	2.06
	85+	0.75	2.09	7.19	3.08	4.13	0.34	3.06
	All	45.00	38.95	32.90	17.90	11.64	18.04	16.23
1–2 ADLs	65–69	8.01	1.28	10.31	3.25	1.15	3.45	2.00
	70–74	12.48	7.45	17.75	9.62	4.42	5.98	2.08
	75–79	5.67	2.36	10.55	5.39	8.28	14.82	3.08
	80–84	1.86	2.36	2.42	4.25	2.37	7.75	9.08
	85+	1.60	0.30	3.53	1.26	1.31	4.79	3.08
	All	29.62	13.74	44.56	23.78	17.52	36.79	19.31
3–4 ADLs	65–69	5.71	1.50	4.71	4.44	5.36	1.90	1.00
	70–74	1.55	1.16	1.27	4.24	7.15	3.36	4.03
	75–79	1.48	1.15	0.22	3.16	3.11	7.34	1.03
	80–84	1.22	0.04	0.00	3.00	3.07	3.10	6.03
	85+	0.70	1.08	3.15	0.07	1.09	1.24	3.03
	All	10.66	4.92	9.36	14.91	19.79	16.95	15.13
5–6 ADLs	65–69	0.89	0.08	2.10	0.08	3.05	2.14	1.00
	70–74	1.21	1.01	1.03	0.06	5.03	3.06	1.02
	75–79	2.79	0.07	0.10	1.08	3.05	4.16	3.02
	80–84	0.74	0.08	1.10	1.06	7.08	2.17	1.02
	85+	0.04	0.01	0.03	0.01	1.02	2.07	2.02
	All	5.67	1.25	4.36	2.28	19.24	13.60	8.07
Inst'd	65–69	0.53	1.04	0.01	0.04	0.01	10.05	1.00
	70–74	3.08	0.41	0.65	0.46	0.35	12.83	2.04
	75–79	0.50	0.15	1.22	0.17	0.11	12.35	2.04
	80–84	1.03	0.24	0.31	1.18	0.23	14.48	2.04
	85+	1.23	0.49	0.78	0.43	0.41	11.02	6.04
	All	6.36	2.33	2.97	2.28	1.11	60.72	13.14

Appendix E

5-year transition probabilities between disability states calculated from the 1984, 1989 and 1994 National Long-Term Care Surveys

The tables in this appendix give the transition probabilities between disability states as a percentage for men, women and in aggregate, calculated using Equation 3.32 and based on data grouped in 5-year age bands. They are given for the 1984–89 NLTCS in Tables E.103 to E.105 and for the 1989–94 NLTCS in Tables E.106 to E.108.

Table E.103: The 5-year transition probabilities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys as a percentage, for males using 5 year age groupings.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	86.43	3.67	3.98	1.22	1.10	1.20	2.39
	70–74	84.08	4.74	3.31	1.51	0.88	2.77	2.71
	75–79	72.78	5.64	4.53	3.01	2.56	5.30	6.18
	80–84	56.99	7.18	8.68	5.23	3.86	8.39	9.66
	85+	46.19	11.28	6.79	5.11	4.05	14.09	12.49
IADL only	65–69	34.02	21.37	20.88	4.81	5.72	6.71	6.49
	70–74	15.50	29.63	17.69	9.64	6.85	12.74	7.95
	75–79	20.25	19.15	15.65	11.83	15.80	8.10	9.23
	80–84	8.66	16.59	12.53	18.46	14.36	16.63	12.76
	85+	6.77	7.00	10.54	19.48	10.09	23.51	22.61
1–2 ADLs	65–69	14.85	19.15	25.84	17.78	11.59	6.74	4.05
	70–74	18.05	9.50	34.08	14.87	9.97	7.59	5.93
	75–79	12.01	10.73	27.86	14.21	13.64	9.97	11.58
	80–84	13.96	12.05	31.88	3.36	6.04	20.86	11.85
	85+	1.50	9.70	12.98	16.30	16.28	23.08	20.16
3–4 ADLs	65–69	13.56	4.24	29.22	19.18	19.01	8.32	6.47
	70–74	6.32	7.51	26.23	23.90	11.92	16.89	7.21
	75–79	1.65	0.59	22.13	14.76	24.75	18.50	17.63
	80–84	1.39	1.05	12.41	11.61	16.93	39.74	16.88
	85+	1.91	0.23	0.36	21.42	0.52	42.84	32.72
5–6 ADLs	65–69	9.10	7.58	11.48	11.29	37.37	15.32	7.86
	70–74	4.85	6.31	17.40	11.60	42.16	12.03	5.66
	75–79	1.42	7.57	15.15	11.26	35.37	18.55	10.68
	80–84	0.77	0.36	19.83	38.92	9.78	29.56	0.78
	85+	1.26	0.15	0.24	9.09	35.24	35.63	18.39
Inst'd	65–69	2.64	3.50	1.04	0.49	0.41	82.67	9.25
	70–74	3.94	5.15	1.03	4.52	0.47	69.59	15.31
	75–79	2.89	0.27	0.80	0.61	0.19	61.67	33.57
	80–84	5.28	0.29	0.34	4.72	0.06	51.49	37.83
	85+	5.28	0.59	0.92	0.66	0.58	73.31	18.65

Table E.104: The 5-year transition probabilities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys as a percentage, for females using 5 year age groupings.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	87.11	3.64	3.37	1.26	1.38	1.96	1.27
	70–74	79.06	5.42	6.35	2.50	1.84	3.15	1.68
	75–79	69.72	6.16	8.86	3.52	2.67	5.94	3.13
	80–84	53.24	6.57	12.74	5.86	4.91	11.22	5.45
	85+	39.81	7.00	10.04	7.76	6.17	21.66	7.56
IADL only	65–69	30.71	21.36	28.21	6.21	4.20	6.77	2.54
	70–74	22.89	25.02	26.69	7.29	5.28	7.92	4.91
	75–79	13.43	23.86	25.75	9.91	7.44	14.65	4.97
	80–84	6.85	21.47	23.94	13.41	7.80	18.25	8.28
	85+	2.74	15.44	21.28	7.15	13.58	23.66	16.16
1–2 ADLs	65–69	21.70	9.13	36.31	13.51	5.78	10.31	3.26
	70–74	7.45	7.69	47.89	17.36	7.46	8.20	3.95
	75–79	7.14	7.44	31.68	19.11	12.62	16.52	5.48
	80–84	7.74	8.01	24.52	18.17	6.53	29.60	5.44
	85+	3.47	4.09	21.13	17.43	11.33	28.26	14.30
3–4 ADLs	65–69	5.97	11.11	25.37	24.78	17.14	9.30	6.33
	70–74	8.70	2.53	15.64	41.16	13.22	10.26	8.48
	75–79	5.57	3.65	13.23	30.73	12.66	24.98	9.19
	80–84	2.85	2.40	11.67	27.59	15.35	26.72	13.42
	85+	4.54	2.45	9.41	20.43	15.42	27.85	19.91
5–6 ADLs	65–69	10.40	2.52	23.74	19.67	25.39	12.48	5.80
	70–74	1.90	7.05	13.86	22.03	28.18	24.61	2.36
	75–79	3.67	4.74	16.06	9.13	30.95	20.03	15.42
	80–84	10.37	0.63	3.10	9.79	41.01	19.16	15.94
	85+	1.59	0.49	4.25	5.79	33.84	34.47	19.57
Inst'd	65–69	5.60	0.73	3.16	3.02	3.06	74.82	9.61
	70–74	2.55	0.46	3.50	0.53	1.66	82.84	8.45
	75–79	2.25	0.31	4.82	0.34	0.37	80.34	11.57
	80–84	2.46	2.07	0.58	0.50	0.23	80.16	13.99
	85+	1.63	0.38	0.58	0.38	1.44	72.08	23.52

Table E.105: The 5-year transition probabilities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys as a percentage, for males and females using 5 year age groupings.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	86.81	3.66	3.64	1.24	1.25	1.62	1.77
	70–74	81.09	5.15	5.11	2.10	1.45	2.99	2.10
	75–79	70.87	5.97	7.24	3.33	2.63	5.70	4.27
	80–84	54.47	6.77	11.40	5.65	4.56	10.29	6.84
	85+	41.57	8.18	9.14	7.03	5.59	19.58	8.92
IADL only	65–69	31.96	21.36	25.44	5.68	4.77	6.75	4.03
	70–74	20.59	26.46	23.89	8.02	5.77	9.42	5.85
	75–79	15.39	22.51	22.85	10.46	9.84	12.77	6.20
	80–84	7.33	20.17	20.90	14.76	9.54	17.82	9.48
	85+	3.67	13.49	18.80	10.00	12.77	23.62	17.65
1–2 ADLs	65–69	19.33	12.60	32.69	14.99	7.79	9.07	3.54
	70–74	10.79	8.26	43.53	16.58	8.25	8.01	4.58
	75–79	8.35	8.26	30.73	17.89	12.88	14.89	7.00
	80–84	8.75	8.66	25.70	15.78	6.46	28.19	6.47
	85+	3.12	5.06	19.72	17.23	12.19	27.36	15.32
3–4 ADLs	65–69	8.20	9.09	26.50	23.14	17.69	9.01	6.37
	70–74	7.83	4.36	19.53	34.82	12.75	12.70	8.02
	75–79	4.38	2.72	15.92	25.91	16.31	23.02	11.74
	80–84	2.54	2.11	11.83	24.17	15.69	29.50	14.16
	85+	4.18	2.15	8.18	20.57	13.40	29.88	21.64
5–6 ADLs	65–69	9.96	4.23	19.58	16.83	29.46	13.45	6.49
	70–74	3.18	6.73	15.39	17.50	34.24	19.16	3.79
	75–79	2.81	5.82	15.71	9.95	32.65	19.46	13.60
	80–84	8.55	0.58	6.26	15.29	35.11	21.12	13.08
	85+	1.54	0.43	3.57	6.34	34.08	34.67	19.37
Inst'd	65–69	4.27	1.97	2.21	1.88	1.87	78.35	9.45
	70–74	2.92	1.70	2.84	1.59	1.35	79.33	10.27
	75–79	2.42	0.30	3.72	0.41	0.32	75.26	17.56
	80–84	2.88	1.80	0.55	1.13	0.21	75.87	17.56
	85+	2.12	0.41	0.62	0.41	1.32	72.24	22.87

Table E.106: The 5-year transition probabilities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys as a percentage, for males using 5 year age groupings.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	86.82	3.05	3.53	1.35	1.42	2.09	1.75
	70–74	78.34	5.08	5.13	2.31	1.88	2.77	4.50
	75–79	69.01	7.84	6.69	3.52	3.21	5.11	4.63
	80–84	60.64	6.02	8.04	3.55	3.36	9.44	8.93
	85+	43.09	2.41	10.20	2.30	6.53	20.67	14.81
IADL only	65–69	40.03	21.55	11.41	11.15	0.56	4.98	10.32
	70–74	36.89	17.48	18.36	6.95	4.80	10.09	5.43
	75–79	23.46	24.07	15.60	8.08	5.53	11.73	11.53
	80–84	5.03	33.12	16.93	13.39	7.21	17.64	6.68
	85+	3.64	10.13	34.86	14.92	20.00	1.63	14.81
1–2 ADLs	65–69	27.20	4.34	34.99	11.05	3.92	11.72	6.79
	70–74	20.88	12.46	29.70	16.10	7.39	10.01	3.47
	75–79	11.31	4.71	21.03	10.75	16.51	29.56	6.13
	80–84	6.17	7.83	8.06	14.12	7.87	25.76	30.18
	85+	10.07	1.89	22.26	7.96	8.24	30.18	19.40
3–4 ADLs	65–69	23.20	6.09	19.12	18.04	21.76	7.72	4.06
	70–74	6.83	5.08	5.59	18.61	31.42	14.75	17.71
	75–79	8.44	6.56	1.28	18.08	17.79	41.95	5.90
	80–84	7.40	0.21	0.02	18.23	18.63	18.85	36.65
	85+	6.74	10.40	30.40	0.66	10.54	12.00	29.25
5–6 ADLs	65–69	9.54	0.90	22.48	0.81	32.66	22.91	10.71
	70–74	9.71	8.14	8.33	0.51	40.48	24.62	8.20
	75–79	19.54	0.48	0.72	7.55	21.39	29.17	21.15
	80–84	5.61	0.61	8.27	7.97	53.47	16.39	7.68
	85+	0.78	0.17	0.60	0.15	19.70	39.80	38.80
Inst'd	65–69	4.15	8.22	0.08	0.32	0.04	79.29	7.89
	70–74	15.54	2.06	3.30	2.34	1.76	64.73	10.27
	75–79	3.01	0.90	7.41	1.00	0.69	74.69	12.31
	80–84	5.30	1.22	1.57	6.04	1.16	74.28	10.44
	85+	6.02	2.41	3.82	2.11	2.02	54.02	29.59

Table E.107: The 5-year transition probabilities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys as a percentage, for females using 5 year age groupings.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	83.45	3.63	5.35	1.66	1.78	2.32	1.82
	70–74	78.18	4.76	7.37	2.50	1.65	4.16	1.37
	75–79	67.16	5.94	9.43	3.35	3.16	8.22	2.74
	80–84	50.44	8.27	10.45	6.89	4.21	13.33	6.41
	85+	34.15	6.52	11.05	6.22	10.03	25.20	6.84
IADL only	65–69	41.57	23.27	9.77	11.95	10.46	2.97	0.00
	70–74	20.94	25.52	28.36	9.85	2.05	6.85	6.43
	75–79	22.24	18.86	22.12	9.39	10.27	9.59	7.53
	80–84	12.12	11.55	24.49	9.24	7.12	23.99	11.48
	85+	7.23	13.24	17.84	18.68	10.44	24.15	8.43
1–2 ADLs	65–69	24.44	8.53	29.90	19.79	10.04	4.06	3.23
	70–74	15.94	12.48	27.92	21.29	10.64	7.22	4.52
	75–79	15.45	7.19	24.38	25.15	9.98	10.24	7.61
	80–84	6.46	7.51	21.11	14.09	11.96	29.20	9.67
	85+	5.33	6.70	16.24	14.12	9.90	35.11	12.59
3–4 ADLs	65–69	11.59	12.31	6.02	26.52	8.36	28.00	7.20
	70–74	12.62	2.17	16.85	29.46	13.45	17.56	7.90
	75–79	7.05	6.56	15.54	25.27	18.01	18.85	8.72
	80–84	4.62	2.89	12.86	22.94	15.06	35.28	6.35
	85+	9.70	2.14	11.54	5.96	13.55	35.18	21.94
5–6 ADLs	65–69	6.39	1.11	1.78	11.83	44.26	23.77	10.85
	70–74	10.31	4.67	11.97	14.82	38.95	5.44	13.85
	75–79	8.10	1.24	8.50	14.59	34.77	22.62	10.18
	80–84	3.83	3.11	3.44	12.35	26.42	31.97	18.88
	85+	8.85	2.11	5.79	7.13	30.22	30.16	15.73
Inst'd	65–69	12.88	1.62	8.60	1.42	1.24	68.30	5.94
	70–74	8.25	1.46	4.01	1.17	1.04	73.59	10.47
	75–79	5.18	0.57	2.41	2.18	0.53	69.16	19.97
	80–84	2.86	2.16	2.38	1.99	3.56	76.01	11.03
	85+	3.37	0.71	1.08	0.68	0.60	79.98	13.59

Table E.108: The 5-year transition probabilities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys as a percentage, for males and females using 5 year age groupings.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	84.89	3.38	4.57	1.53	1.62	2.22	1.79
	70–74	78.24	4.89	6.43	2.42	1.75	3.58	2.69
	75–79	67.86	6.66	8.39	3.41	3.18	7.04	3.46
	80–84	53.82	7.53	9.65	5.78	3.93	12.05	7.24
	85+	37.04	5.19	10.78	4.95	8.90	23.73	9.41
IADL only	65–69	41.13	22.77	10.25	11.72	7.59	3.55	2.99
	70–74	27.64	22.14	24.16	8.63	3.21	8.21	6.01
	75–79	22.60	20.37	20.23	9.01	8.89	10.21	8.69
	80–84	10.41	16.76	22.67	10.24	7.14	22.46	10.32
	85+	6.44	12.56	21.56	17.86	12.53	19.23	9.82
1–2 ADLs	65–69	25.33	7.18	31.54	16.97	8.07	6.53	4.38
	70–74	17.47	12.47	28.47	19.68	9.63	8.08	4.19
	75–79	14.51	6.63	23.63	21.90	11.46	14.61	7.28
	80–84	6.41	7.57	18.89	14.10	11.26	28.61	13.16
	85+	5.88	6.15	16.94	13.41	9.71	34.54	13.38
3–4 ADLs	65–69	17.05	9.39	12.17	22.54	14.66	18.47	5.72
	70–74	10.83	3.07	13.35	26.09	19.02	16.69	10.94
	75–79	7.33	6.56	12.66	23.82	17.97	23.52	8.15
	80–84	5.19	2.34	10.24	21.98	15.79	31.92	12.55
	85+	9.35	3.10	13.75	5.34	13.20	32.46	22.79
5–6 ADLs	65–69	7.45	1.04	8.74	8.12	40.36	23.48	10.80
	70–74	10.13	5.72	10.88	10.52	39.41	11.20	12.15
	75–79	11.82	0.99	5.97	12.31	30.42	24.74	13.74
	80–84	4.26	2.52	4.58	11.31	32.84	28.27	16.22
	85+	7.89	1.88	5.18	6.30	28.97	31.31	18.47
Inst'd	65–69	9.13	4.45	4.94	0.95	0.73	73.02	6.77
	70–74	10.39	1.64	3.80	1.52	1.25	70.99	10.41
	75–79	4.71	0.64	3.49	1.92	0.56	70.35	18.32
	80–84	3.43	1.94	2.19	2.94	3.00	75.61	10.89
	85+	3.74	0.95	1.46	0.88	0.80	76.37	15.81

Appendix F

Maximum likelihood estimates of the annual transition intensities between disability states calculated from the 1984, 1989 and 1994 National Long-Term Care Surveys

The tables in this appendix give the maximum likelihood estimates of the annual transition intensities between disability states for men, women and in aggregate (as described in Section 4.2), based on data grouped in 5-year age bands. They are given for the 1984–89 NLTCS in Tables F.109 – F.111 and for the 1989–94 NLTCS in Tables F.112 to F.114. These estimates are the result of direct transformations of the corresponding transition probabilities and so there is no constraint on the transition intensities to be positive.

For some categories these estimates do not lie in the real plane (the matrix of transition intensities is complex) and they are marked '-' in these tables. However, in Section 4.5, I calculate approximate MLEs for these categores, which are given in Tables 4.48 and 4.49, respectively.

Table F.109: The MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males using 5 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		-	-	-	-	-	-
	70–74		0.0173	0.0079	0.0021	0.0008	0.0050	0.0045
	75–79		0.0275	0.0119	0.0072	0.0008	0.0127	0.0095
	80–84		-	-	-	-	-	-
	85+		-	-	-	-	-	-
IADL	65–69	-		-	-	-	-	-
only	70–74	0.0458		0.0985	0.0438	0.0223	0.0426	0.0180
	75–79	0.0986		0.0704	0.1170	0.0607	0.0028	0.0127
	80–84	-		-	-	-	-	-
	85+	-		-	-	-	-	-
1–2	65–69	-	-		-	-	-	-
ADLs	70–74	0.0651	0.0527		0.1168	0.0406	0.0065	0.0118
	75–79	0.0351	0.1452		0.1677	0.0004	0.0169	0.0182
	80–84	-	-		-	-	-	-
	85+	-	-		-	-	-	-
3–4	65–69	-	-	-		-	-	-
ADLs	70–74	−0.0074	0.0237	0.2147		0.0608	0.0731	0.0131
	75–79	0.0088	−0.1668	0.3077		0.3036	0.0778	0.0533
	80–84	-	-	-		-	-	-
	85+	-	-	-		-	-	-
5–6	65–69	-	-	-	-		-	-
ADLs	70–74	0.0021	0.0199	0.0728	0.0549		0.0320	0.0089
	75–79	−0.0143	0.0620	0.0636	0.0825		0.0664	0.0078
	80–84	-	-	-	-		-	-
	85+	-	-	-	-		-	-
Inst'd	65–69	-	-	-	-	-		-
	70–74	0.0088	0.0217	−0.0096	0.0227	−0.0018		0.0353
	75–79	0.0083	0.0007	0.0023	0.0032	−0.0011		0.0841
	80–84	-	-	-	-	-		-
	85+	-	-	-	-	-		-

Table F.110: The MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for females using 5 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0154	0.0043	0.0024	0.0039	0.0035	0.0021
	70–74		0.0221	0.0134	0.0041	0.0044	0.0052	0.0025
	75–79		0.0265	0.0266	0.0049	0.0043	0.0095	0.0050
	80–84		0.0298	0.0596	0.0054	0.0146	0.0157	0.0093
	85+		0.0508	0.0383	0.0367	0.0093	0.0528	0.0027
IADL	65–69	0.1032		0.2245	−0.0172	0.0262	0.0117	0.0046
only	70–74	0.0954		0.1559	0.0040	0.0227	0.0192	0.0133
	75–79	0.0541		0.1998	0.0087	0.0159	0.0371	0.0081
	80–84	0.0169		0.2380	0.0300	0.0327	0.0182	0.0208
	85+	0.0113		0.2705	−0.0575	0.0926	0.0501	0.0389
1–2	65–69	0.0709	0.0473		0.1089	0.0003	0.0334	0.0041
ADLs	70–74	0.0126	0.0429		0.0766	0.0251	0.0153	0.0055
	75–79	0.0195	0.0508		0.1335	0.0637	0.0297	0.0029
	80–84	0.0390	0.0750		0.1593	0.0031	0.1017	−0.0055
	85+	0.0099	0.0399		0.2009	0.0333	0.0863	0.0146
3–4	65–69	−0.0297	0.1137	0.1007		0.1697	0.0176	0.0175
ADLs	70–74	0.0296	−0.0039	0.0663		0.0806	0.0181	0.0245
	75–79	0.0163	0.0128	0.0631		0.0766	0.0823	0.0171
	80–84	−0.0062	0.0007	0.1070		0.0965	0.0813	0.0331
	85+	0.0272	0.0160	0.0837		0.1121	0.0875	0.0501
5–6	65–69	0.0288	−0.0487	0.1527	0.1602		0.0400	0.0125
ADLs	70–74	−0.0150	0.0522	0.0384	0.1328		0.0911	−0.0058
	75–79	0.0055	0.0224	0.0944	0.0312		0.0622	0.0456
	80–84	0.0441	−0.0042	−0.0002	0.0602		0.0532	0.0392
	85+	0.0034	−0.0017	0.0266	0.0352		0.1266	0.0412
Inst'd	65–69	0.0123	0.0003	0.0060	0.0094	0.0108		0.0214
	70–74	0.0060	−0.0002	0.0106	−0.0018	0.0062		0.0184
	75–79	0.0053	−0.0012	0.0205	−0.0041	−0.0011		0.0256
	80–84	0.0073	0.0094	−0.0031	0.0012	−0.0003		0.0307
	85+	0.0059	0.0012	0.0011	0.0003	0.0051		0.0546

Table F.111: The MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1984		1989 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0149	0.0066	0.0026	0.0028	0.0026	0.0031
	70–74		0.0202	0.0108	0.0037	0.0027	0.0052	0.0033
	75–79		0.0264	0.0206	0.0058	0.0036	0.0102	0.0071
	80–84		0.0324	0.0494	0.0069	0.0131	0.0151	0.0125
	85+		0.0639	0.0302	0.0275	0.0078	0.0453	0.0073
IADL	65–69	0.1198		0.2537	−0.0459	0.0312	0.0147	0.0111
only	70–74	0.0767		0.1412	0.0150	0.0212	0.0273	0.0147
	75–79	0.0646		0.1786	0.0237	0.0376	0.0294	0.0084
	80–84	0.0218		0.1879	0.0738	0.0432	0.0184	0.0213
	85+	0.0194		0.2671	−0.0076	0.0753	0.0559	0.0445
1–2	65–69	0.0545	0.1042		0.1551	0.0050	0.0276	0.0030
ADLs	70–74	0.0265	0.0449		0.0854	0.0287	0.0137	0.0071
	75–79	0.0252	0.0664		0.1414	0.0515	0.0270	0.0068
	80–84	0.0428	0.0824		0.1328	0.0015	0.0938	−0.0028
	85+	0.0071	0.0655		0.1974	0.0499	0.0730	0.0163
3–4	65–69	−0.0119	0.0469	0.1998		0.1633	0.0144	0.0172
ADLs	70–74	0.0205	0.0089	0.0965		0.0718	0.0344	0.0216
	75–79	0.0118	−0.0065	0.1034		0.1109	0.0814	0.0241
	80–84	−0.0101	−0.0028	0.1110		0.1223	0.1048	0.0354
	85+	0.0236	0.0095	0.0802		0.0936	0.1071	0.0571
5–6	65–69	0.0245	−0.0143	0.1004	0.1270		0.0450	0.0151
ADLs	70–74	−0.0040	0.0352	0.0509	0.0957		0.0614	0.0017
	75–79	−0.0009	0.0320	0.0864	0.0421		0.0611	0.0340
	80–84	0.0379	−0.0082	0.0162	0.1134		0.0526	0.0277
	85+	0.0025	−0.0027	0.0218	0.0419		0.1259	0.0400
Inst'd	65–69	0.0080	0.0077	0.0017	0.0069	0.0052		0.0207
	70–74	0.0058	0.0057	0.0068	0.0034	0.0036		0.0225
	75–79	0.0059	−0.0013	0.0169	−0.0030	−0.0006		0.0401
	80–84	0.0088	0.0084	−0.0027	0.0040	−0.0012		0.0395
	85+	0.0076	0.0009	0.0015	0.0002	0.0045		0.0529

Table F.112: The MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males using 5 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0126	0.0093	0.0019	0.0047	0.0031	0.0022
	70–74		0.0233	0.0140	0.0051	0.0030	0.0040	0.0087
	75–79		0.0362	0.0241	0.0088	0.0047	0.0034	0.0073
	80–84		0.0195	0.0547	−0.0047	0.0083	0.0142	0.0122
	85+		-	-	-	-	-	-
IADL	65–69	0.1516		0.0698	0.1034	−0.0429	0.0182	0.0381
only	70–74	0.1770		0.1822	0.0070	0.0213	0.0388	0.0138
	75–79	0.1082		0.1421	0.0483	−0.0262	0.0066	0.0415
	80–84	0.0072		0.2319	0.0244	0.0139	0.0131	−0.0334
	85+	-		-	-	-	-	-
1–2	65–69	0.0818	0.0118		0.0913	−0.0107	0.0422	0.0185
ADLs	70–74	0.0456	0.1214		0.1403	−0.0266	0.0267	−0.0098
	75–79	0.0203	0.0294		0.0871	0.1252	0.0765	−0.0048
	80–84	0.0220	0.1018		0.2213	0.0078	0.1396	0.1094
	85+	-	-		-	-	-	-
3–4	65–69	0.0651	0.0609	0.0791		0.1897	−0.0084	−0.0043
ADLs	70–74	0.0105	0.0088	0.0144		0.2246	0.0245	0.0595
	75–79	−0.0158	0.0802	−0.0405		0.2224	0.1767	−0.0273
	80–84	0.0355	0.0072	−0.0620		0.1193	0.0928	0.1556
	85+	-	-	-		-	-	-
5–6	65–69	0.0122	−0.0107	0.1431	−0.0267		0.0746	0.0268
ADLs	70–74	0.0038	0.0537	0.0286	−0.0153		0.0900	0.0190
	75–79	0.1038	−0.0251	−0.0116	0.0883		0.1158	0.0775
	80–84	0.0135	−0.0103	0.0821	0.0198		0.0320	−0.0023
	85+	-	-	-	-		-	-
Inst'd	65–69	0.0009	0.0392	−0.0050	−0.0060	0.0032		0.0153
	70–74	0.0408	0.0052	0.0107	0.0079	0.0023		0.0233
	75–79	0.0069	0.0010	0.0355	−0.0014	−0.0067		0.0286
	80–84	0.0138	0.0025	0.0102	0.0273	−0.0029		0.0155
	85+	-	-	-	-	-		-

Table F.113: The MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for females using 5 year age groupings.

1989		1994 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0145	0.0189	−0.0016	0.0026	0.0061	0.0036
	70–74		0.0157	0.0254	0.0001	0.0030	0.0093	0.0013
	75–79		0.0297	0.0376	−0.0058	0.0076	0.0201	0.0016
	80–84		0.0716	0.0101	0.0306	0.0083	0.0196	0.0096
	85+		-	-	-	-	-	-
IADL	65–69	0.1732		0.0644	0.0693	0.0519	−0.0188	−0.0096
only	70–74	0.0712		0.3037	−0.0360	−0.0248	0.0250	0.0221
	75–79	0.0854		0.2496	−0.0876	0.0813	0.0261	0.0154
	80–84	0.0911		0.5310	−0.0467	−0.0218	0.0618	0.0367
	85+	-		-	-	-	-	-
1–2	65–69	0.0792	0.0299		0.1365	0.0389	−0.0276	0.0028
ADLs	70–74	0.0401	0.1391		0.1993	0.0516	−0.0028	−0.0004
	75–79	0.0722	0.0326		0.2993	−0.0129	0.0027	0.0170
	80–84	0.0152	0.1445		0.1481	0.0818	0.0836	0.0157
	85+	-	-		-	-	-	-
3–4	65–69	−0.0028	0.1077	0.0120		0.0354	0.1285	0.0209
ADLs	70–74	0.0354	−0.0358	0.1598		0.0716	0.0761	0.0174
	75–79	−0.0046	0.0624	0.1230		0.1361	0.0662	0.0129
	80–84	0.0169	−0.0383	0.1836		0.1079	0.1202	−0.0037
	85+	-	-	-		-	-	-
5–6	65–69	0.0153	−0.0128	−0.0022	0.0741		0.0683	0.0263
ADLs	70–74	0.0225	0.0193	0.0456	0.0770		0.0056	0.0387
	75–79	0.0247	−0.0123	0.0391	0.0912		0.0784	0.0191
	80–84	0.0094	0.0470	−0.0523	0.1235		0.1128	0.0605
	85+	-	-	-	-		-	-
Inst'd	65–69	0.0275	0.0037	0.0358	−0.0056	0.0006		0.0137
	70–74	0.0200	0.0017	0.0167	−0.0016	0.0019		0.0238
	75–79	0.0141	−0.0001	0.0090	0.0062	−0.0006		0.0472
	80–84	0.0071	0.0118	0.0023	0.0043	0.0135		0.0229
	85+	-	-	-	-	-		-

Table F.114: The MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1989		1994 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0134	0.0156	−0.0008	0.0036	0.0046	0.0031
	70–74		0.0188	0.0197	0.0022	0.0037	0.0072	0.0044
	75–79		0.0328	0.0301	0.0004	0.0066	0.0146	0.0038
	80–84		0.0442	0.0337	0.0170	0.0064	0.0190	0.0118
	85+		-	-	-	-	-	-
IADL	65–69	0.1650		0.0522	0.0915	0.0259	−0.0081	0.0042
only	70–74	0.1109		0.2462	−0.0217	−0.0003	0.0312	0.0192
	75–79	0.0900		0.2047	−0.0355	0.0522	0.0210	0.0203
	80–84	0.0524		0.3368	0.0018	0.0025	0.0426	0.0150
	85+	-		-	-	-	-	-
1–2	65–69	0.0756	0.0288		0.1367	0.0171	0.0015	0.0086
ADLs	70–74	0.0410	0.1295		0.1818	0.0166	0.0031	−0.0027
	75–79	0.0610	0.0296		0.2348	0.0179	0.0200	0.0141
	80–84	0.0206	0.1071		0.1526	0.0590	0.0964	0.0327
	85+	-	-		-	-	-	-
3–4	65–69	0.0297	0.0819	0.0739		0.0951	0.0724	0.0107
ADLs	70–74	0.0273	−0.0186	0.1040		0.1256	0.0674	0.0283
	75–79	−0.0051	0.0645	0.0903		0.1442	0.0827	0.0050
	80–84	0.0182	−0.0153	0.1265		0.1140	0.1124	0.0242
	85+	-	-	-		-	-	-
5–6	65–69	0.0128	−0.0125	0.0394	0.0468		0.0789	0.0281
ADLs	70–74	0.0195	0.0268	0.0454	0.0511		0.0317	0.0313
	75–79	0.0470	−0.0150	0.0225	0.0930		0.0888	0.0347
	80–84	0.0121	0.0156	0.0035	0.0866		0.0902	0.0426
	85+	-	-	-	-		-	-
Inst'd	65–69	0.0151	0.0197	0.0181	−0.0051	0.0005		0.0151
	70–74	0.0257	0.0029	0.0140	0.0011	0.0028		0.0238
	75–79	0.0119	0.0001	0.0157	0.0027	−0.0007		0.0429
	80–84	0.0092	0.0085	0.0029	0.0104	0.0086		0.0229
	85+	-	-	-	-	-		-

Appendix G

Constrained maximum likelihood estimates of the annual transition intensities between disability states calculated from the 1982, 1984, 1989 and 1994 National Long-Term Care Surveys

The tables in this appendix give the constrained maximum likelihood estimates of the annual transition intensities between disability states for men, women and in aggregate (as described in Section 4.5). They are given for all pairs of surveys in 10-year age bands (65–74 years, 75–84 years and 85+ years) in Tables G.115 to G.123 and in 5-year age bands (65–69 years, 70–74 years, 75–79 years, 80–84 years and 85+ years) in Tables G.124 to G.129 (except for those from the 1982–84 NLTCS, which are given in Tables 4.50 to 4.52).

Table G.115: The constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males and females using 10 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0247	0.0125	0.0039	0.0042	0.0036	0.0250
	75–84		0.0551	0.0335	0.0084	0.0062	0.0146	0.0477
	85+		0.1069	0.1115	0.0000	0.0243	0.0579	0.0807
IADL	65–74	0.2405		0.2619	0.0085	0.0353	0.0262	0.0617
only	75–84	0.1175		0.3687	0.0000	0.0686	0.0603	0.0683
	85+	0.0122		0.3886	0.0323	0.0517	0.0637	0.1220
1–2	65–74	0.0714	0.2019		0.2043	0.0365	0.0436	0.0869
ADLs	75–84	0.0560	0.1907		0.2303	0.0164	0.0553	0.1329
	85+	0.0469	0.1026		0.2980	0.0463	0.1186	0.0945
3–4	65–74	0.0348	0.0000	0.3907		0.2869	0.0189	0.0754
ADLs	75–84	0.0056	0.0000	0.3389		0.4066	0.1036	0.0972
	85+	0.0042	0.0692	0.0563		0.5877	0.1808	0.0798
5–6	65–74	0.0332	0.0603	0.0718	0.1672		0.0723	0.2409
ADLs	75–84	0.0292	0.0555	0.0238	0.1897		0.1132	0.2511
	85+	0.0130	0.0000	0.0847	0.1570		0.1337	0.3663
Inst'd	65–74	0.0310	0.0051	0.0108	0.0175	0.0107		0.1736
	75–84	0.0084	0.0100	0.0020	0.0086	0.0076		0.2384
	85+	0.0044	0.0022	0.0000	0.0107	0.0027		0.3368

Table G.116: The constrained MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males and females using 10 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0175	0.0089	0.0031	0.0028	0.0039	0.0032
	75–84		0.0281	0.0283	0.0061	0.0062	0.0116	0.0086
	85+		0.0638	0.0307	0.0269	0.0080	0.0453	0.0073
IADL	65–74	0.1000		0.1765	0.0000	0.0205	0.0225	0.0126
only	75–84	0.0471		0.1810	0.0453	0.0404	0.0248	0.0140
	85+	0.0191		0.2603	0.0000	0.0740	0.0555	0.0440
1–2	65–74	0.0377	0.0663		0.1038	0.0236	0.0187	0.0053
ADLs	75–84	0.0304	0.0727		0.1354	0.0282	0.0575	0.0028
	85+	0.0073	0.0646		0.1927	0.0506	0.0733	0.0166
3–4	65–74	0.0106	0.0245	0.1331		0.1037	0.0261	0.0197
ADLs	75–84	0.0033	0.0000	0.0978		0.1148	0.0924	0.0289
	85+	0.0237	0.0075	0.0812		0.0933	0.1069	0.0571
5–6	65–74	0.0081	0.0178	0.0686	0.1089		0.0539	0.0078
ADLs	75–84	0.0143	0.0122	0.0611	0.0701		0.0555	0.0318
	85+	0.0022	0.0000	0.0191	0.0422		0.1258	0.0398
Inst'd	65–74	0.0067	0.0064	0.0052	0.0045	0.0044		0.0217
	75–84	0.0068	0.0034	0.0071	0.0005	0.0000		0.0399
	85+	0.0076	0.0008	0.0016	0.0002	0.0045		0.0529

Table G.117: The constrained MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males and females using 10 year age groupings.

1989		1994 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0158	0.0181	0.0006	0.0036	0.0059	0.0038
	75–84		0.0359	0.0325	0.0056	0.0064	0.0160	0.0067
	85+		0.0340	0.0645	0.0000	0.0417	0.0578	0.0139
IADL only	65–74	0.1333		0.1449	0.0342	0.0099	0.0143	0.0109
	75–84	0.0752		0.2319	0.0000	0.0274	0.0345	0.0194
	85+	0.0058		0.2640	0.1718	0.0314	0.0000	0.0000
1–2 ADLs	65–74	0.0501	0.0819		0.1553	0.0171	0.0049	0.0033
	75–84	0.0419	0.0610		0.1859	0.0390	0.0505	0.0195
	85+	0.0000	0.0887		0.3329	0.0000	0.1105	0.0000
3–4 ADLs	65–74	0.0313	0.0265	0.0795		0.1139	0.0664	0.0202
	75–84	0.0080	0.0305	0.0965		0.1287	0.0966	0.0146
	85+	0.0906	0.0000	0.2898		0.2054	0.1367	0.1034
5–6 ADLs	65–74	0.0151	0.0102	0.0473	0.0464		0.0521	0.0299
	75–84	0.0268	0.0000	0.0152	0.0876		0.0906	0.0390
	85+	0.0368	0.0046	0.0261	0.0533		0.1048	0.0445
Inst'd	65–74	0.0225	0.0083	0.0144	0.0000	0.0022		0.0211
	75–84	0.0102	0.0039	0.0098	0.0070	0.0039		0.0324
	85+	0.0133	0.0036	0.0032	0.0016	0.0006		0.0350

Table G.118: The constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for females using 10 year age groupings.

1982 status	Age group	1984 Status Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0279	0.0126	0.0036	0.0043	0.0030	0.0159
	75–84		0.0595	0.0339	0.0124	0.0059	0.0170	0.0369
	85+		0.1052	0.0985	0.0000	0.0352	0.0731	0.0626
IADL	65–74	0.2212		0.2877	0.0462	0.0155	0.0344	0.0386
only	75–84	0.1048		0.4225	0.0000	0.0333	0.0530	0.0747
	85+	0.0143		0.4450	0.0000	0.0950	0.0844	0.0866
1–2	65–74	0.0797	0.2286		0.1476	0.0350	0.0371	0.0733
ADLs	75–84	0.0627	0.1945		0.2287	0.0067	0.0765	0.0905
	85+	0.0352	0.1232		0.2886	0.0112	0.0976	0.0902
3–4	65–74	0.0037	0.0000	0.4167		0.2620	0.0265	0.0194
ADLs	75–84	0.0000	0.0000	0.3665		0.3997	0.0881	0.0697
	85+	0.0065	0.0662	0.0570		0.5585	0.1695	0.0334
5–6	65–74	0.0307	0.0719	0.0474	0.1513		0.0791	0.2269
ADLs	75–84	0.0334	0.0403	0.0333	0.1856		0.1271	0.2013
	85+	0.0166	0.0000	0.0817	0.1506		0.1500	0.3401
Inst'd	65–74	0.0286	0.0052	0.0013	0.0241	0.0070		0.1437
	75–84	0.0073	0.0086	0.0023	0.0057	0.0094		0.2090
	85+	0.0047	0.0026	0.0000	0.0118	0.0034		0.3093

Table G.119: The constrained MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for females using 10 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0187	0.0091	0.0033	0.0042	0.0043	0.0023
	75–84		0.0275	0.0357	0.0052	0.0073	0.0114	0.0061
	85+		0.0503	0.0412	0.0331	0.0101	0.0528	0.0029
IADL	65–74	0.1024		0.1810	0.0000	0.0210	0.0168	0.0090
only	75–84	0.0393		0.2156	0.0177	0.0243	0.0285	0.0134
	85+	0.0090		0.2246	0.0000	0.0801	0.0498	0.0358
1–2	65–74	0.0361	0.0464		0.0873	0.0172	0.0229	0.0048
ADLs	75–84	0.0248	0.0628		0.1422	0.0332	0.0637	0.0000
	85+	0.0113	0.0376		0.1693	0.0400	0.0872	0.0164
3–4	65–74	0.0080	0.0389	0.0883		0.1139	0.0169	0.0212
ADLs	75–84	0.0073	0.0073	0.0801		0.0852	0.0832	0.0243
	85+	0.0265	0.0152	0.0771		0.1084	0.0866	0.0491
5–6	65–74	0.0089	0.0099	0.0845	0.1435		0.0668	0.0036
ADLs	75–84	0.0233	0.0072	0.0493	0.0444		0.0547	0.0425
	85+	0.0033	0.0000	0.0259	0.0329		0.1266	0.0412
Inst'd	65–74	0.0079	0.0003	0.0095	0.0013	0.0082		0.0195
	75–84	0.0058	0.0045	0.0069	0.0000	0.0000		0.0285
	85+	0.0059	0.0011	0.0012	0.0003	0.0050		0.0546

Table G.120: The constrained MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for females using 10 year age groupings.

1989		1994 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0148	0.0222	0.0000	0.0028	0.0073	0.0023
	75–84		0.0384	0.0357	0.0057	0.0064	0.0201	0.0047
	85+		0.0461	0.0694	0.0000	0.0509	0.0597	0.0061
IADL only	65–74	0.1179		0.1519	0.0307	0.0149	0.0069	0.0058
	75–84	0.0855		0.2600	0.0000	0.0360	0.0414	0.0214
	85+	0.0161		0.2375	0.2106	0.0017	0.0000	0.0000
1–2 ADLs	65–74	0.0479	0.0843		0.1414	0.0413	0.0000	0.0038
	75–84	0.0452	0.0687		0.1867	0.0361	0.0393	0.0178
	85+	0.0000	0.1002		0.3196	0.0000	0.1021	0.0000
3–4 ADLs	65–74	0.0262	0.0258	0.0784		0.0619	0.0804	0.0166
	75–84	0.0052	0.0300	0.1221		0.1241	0.0889	0.0025
	85+	0.0789	0.0000	0.1895		0.2017	0.1776	0.0946
5–6 ADLs	65–74	0.0172	0.0078	0.0291	0.0732		0.0317	0.0336
	75–84	0.0176	0.0056	0.0084	0.1026		0.0994	0.0415
	85+	0.0469	0.0000	0.0423	0.0514		0.0896	0.0379
Inst'd	65–74	0.0220	0.0022	0.0196	0.0000	0.0014		0.0211
	75–84	0.0103	0.0051	0.0067	0.0052	0.0060		0.0348
	85+	0.0123	0.0018	0.0025	0.0010	0.0002		0.0299

Table G.121: The constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males using 10 year age groupings.

1982		1984 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0206	0.0129	0.0036	0.0043	0.0045	0.0369
	75–84		0.0484	0.0339	0.0014	0.0064	0.0118	0.0636
	85+		0.1145	0.1448	0.0000	0.0000	0.0168	0.1218
IADL	65–74	0.2707		0.1737	0.0000	0.0520	0.0137	0.1039
	75–84	0.1402		0.2263	0.0534	0.1225	0.0733	0.0496
	85+	0.0087		0.2708	0.0724	0.0000	0.0145	0.1971
1–2	65–74	0.0552	0.1497		0.2621	0.0553	0.0552	0.1069
ADLs	75–84	0.0409	0.1626		0.1713	0.0760	0.0000	0.2465
	85+	0.0882	0.0429		0.4066	0.0925	0.2068	0.0866
3–4	65–74	0.0890	0.0000	0.3155		0.3106	0.0047	0.1762
ADLs	75–84	0.0187	0.0383	0.1789		0.3831	0.1284	0.2251
	85+	0.0000	0.0859	0.0431		0.7191	0.2653	0.1722
5–6	65–74	0.0338	0.0493	0.1169	0.1699		0.0650	0.2475
ADLs	75–84	0.0202	0.0730	0.0342	0.1854		0.0921	0.3240
	85+	0.0000	0.0000	0.0918	0.1486		0.0576	0.4924
Inst'd	65–74	0.0355	0.0059	0.0265	0.0040	0.0172		0.2211
	75–84	0.0115	0.0142	0.0026	0.0176	0.0013		0.3330
	85+	0.0029	0.0000	0.0008	0.0000	0.0001		0.4809

Table G.122: The constrained MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males using 10 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0155	0.0094	0.0020	0.0011	0.0032	0.0044
	75–84		0.0298	0.0178	0.0098	0.0006	0.0129	0.0112
	85+		0.0965	0.0157	0.0000	0.0021	0.0299	0.0182
IADL only	65–74	0.0954		0.1592	0.0026	0.0198	0.0331	0.0193
	75–84	0.0769		0.0697	0.1579	0.0620	0.0000	0.0198
	85+	0.0491		0.2636	0.1636	0.0509	0.0435	0.0525
1–2 ADLs	65–74	0.0437	0.1119		0.1606	0.0385	0.0049	0.0064
	75–84	0.0365	0.1020		0.0550	0.0339	0.0394	0.0193
	85+	0.0000	0.2103		0.1114	0.1367	0.0139	0.0300
3–4 ADLs	65–74	0.0124	0.0000	0.2486		0.0893	0.0497	0.0168
	75–84	0.0000	0.0000	0.0883		0.3225	0.1644	0.0354
	85+	0.0000	0.0000	0.0001		0.0022	0.1953	0.0998
5–6 ADLs	65–74	0.0080	0.0208	0.0613	0.0615		0.0391	0.0130
	75–84	0.0000	0.0000	0.1087	0.2476		0.0343	0.0000
	85+	0.0000	0.0000	0.0000	0.0644		0.1134	0.0331
Inst'd	65–74	0.0048	0.0156	0.0000	0.0073	0.0000		0.0261
	75–84	0.0104	0.0000	0.0005	0.0096	0.0000		0.0901
	85+	0.0168	0.0000	0.0033	0.0010	0.0010		0.0420

Table G.123: The constrained MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males using 10 year age groupings.

1989		1994 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		0.0176	0.0120	0.0037	0.0036	0.0037	0.0056
	75–84		0.0305	0.0313	0.0056	0.0053	0.0077	0.0094
	85+		0.0131	0.0575	0.0000	0.0213	0.0677	0.0200
IADL only	65–74	0.1633		0.1311	0.0434	0.0000	0.0278	0.0202
	75–84	0.0598		0.1515	0.0453	0.0000	0.0114	0.0114
	85+	0.0000		0.0000	0.5805	0.0000	0.0000	0.0000
1–2 ADLs	65–74	0.0568	0.0741		0.1064	0.0000	0.0329	0.0028
	75–84	0.0286	0.0472		0.1294	0.0670	0.1024	0.0332
	85+	0.0538	0.0248		0.0899	0.2037	0.0000	0.0134
3–4 ADLs	65–74	0.0417	0.0294	0.0605		0.1819	0.0115	0.0301
	75–84	0.0196	0.0190	0.0000		0.1381	0.1299	0.0671
	85+	0.0000	0.1643	0.7154		0.0000	0.0000	0.1001
5–6 ADLs	65–74	0.0121	0.0148	0.0670	0.0000		0.0838	0.0218
	75–84	0.0455	0.0000	0.0150	0.0493		0.0714	0.0332
	85+	0.0000	0.0000	0.0000	0.0000		0.2885	0.1228
Inst'd	65–74	0.0225	0.0221	0.0017	0.0037	0.0018		0.0206
	75–84	0.0103	0.0017	0.0175	0.0120	0.0000		0.0231
	85+	0.0221	0.0138	0.0033	0.0082	0.0055		0.0741

Table G.124: The constrained MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1984		1989 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0148	0.0072	0.0019	0.0031	0.0026	0.0031
	70–74		0.0202	0.0108	0.0037	0.0027	0.0052	0.0033
	75–79		0.0264	0.0206	0.0058	0.0036	0.0102	0.0071
	80–84		0.0323	0.0489	0.0073	0.0129	0.0151	0.0124
	85+		0.0638	0.0307	0.0269	0.0080	0.0453	0.0073
IADL	65–69	0.1233		0.2151	0.0000	0.0137	0.0159	0.0104
only	70–74	0.0762		0.1410	0.0151	0.0212	0.0273	0.0146
	75–79	0.0644		0.1756	0.0255	0.0375	0.0292	0.0083
	80–84	0.0224		0.1826	0.0767	0.0417	0.0192	0.0203
	85+	0.0191		0.2603	0.0000	0.0740	0.0555	0.0440
1–2	65–69	0.0487	0.1033		0.1154	0.0205	0.0265	0.0036
ADLs	70–74	0.0263	0.0450		0.0854	0.0287	0.0137	0.0071
	75–79	0.0255	0.0621		0.1364	0.0514	0.0276	0.0070
	80–84	0.0391	0.0775		0.1258	0.0048	0.0922	0.0000
	85+	0.0073	0.0646		0.1927	0.0506	0.0733	0.0166
3–4	65–69	0.0000	0.0315	0.1864		0.1474	0.0155	0.0169
ADLs	70–74	0.0190	0.0098	0.0966		0.0714	0.0344	0.0215
	75–79	0.0109	0.0000	0.0966		0.1093	0.0810	0.0240
	80–84	0.0000	0.0000	0.0969		0.1131	0.1050	0.0328
	85+	0.0237	0.0075	0.0812		0.0933	0.1069	0.0571
5–6	65–69	0.0197	0.0000	0.0896	0.1246		0.0448	0.0149
ADLs	70–74	0.0000	0.0321	0.0505	0.0941		0.0612	0.0018
	75–79	0.0000	0.0291	0.0876	0.0417		0.0610	0.0340
	80–84	0.0311	0.0000	0.0131	0.1093		0.0525	0.0279
	85+	0.0022	0.0000	0.0191	0.0422		0.1258	0.0398
Inst'd	65–69	0.0079	0.0075	0.0023	0.0062	0.0054		0.0207
	70–74	0.0057	0.0057	0.0068	0.0034	0.0036		0.0225
	75–79	0.0058	0.0000	0.0124	0.0000	0.0000		0.0399
	80–84	0.0080	0.0065	0.0000	0.0025	0.0000		0.0391
	85+	0.0076	0.0008	0.0016	0.0002	0.0045		0.0529

Table G.125: The constrained MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0131	0.0153	0.0000	0.0035	0.0043	0.0030
	70–74		0.0188	0.0198	0.0021	0.0037	0.0072	0.0044
	75–79		0.0325	0.0306	0.0000	0.0066	0.0147	0.0038
	80–84		0.0440	0.0341	0.0168	0.0064	0.0190	0.0118
	85+		0.0340	0.0645	0.0000	0.0417	0.0578	0.0139
IADL only	65–69	0.1629		0.0550	0.0782	0.0247	0.0000	0.0042
	70–74	0.1092		0.2143	0.0000	0.0000	0.0284	0.0170
	75–79	0.0937		0.1812	0.0000	0.0412	0.0183	0.0210
	80–84	0.0518		0.3241	0.0056	0.0030	0.0431	0.0152
	85+	0.0058		0.2640	0.1718	0.0314	0.0000	0.0000
1–2 ADLs	65–69	0.0753	0.0302		0.1279	0.0187	0.0030	0.0086
	70–74	0.0431	0.1117		0.1626	0.0164	0.0056	0.0000
	75–79	0.0575	0.0344		0.2081	0.0265	0.0223	0.0134
	80–84	0.0215	0.0959		0.1476	0.0585	0.0959	0.0325
	85+	0.0000	0.0887		0.3329	0.0000	0.1105	0.0000
3–4 ADLs	65–69	0.0323	0.0723	0.0711		0.0938	0.0645	0.0107
	70–74	0.0247	0.0000	0.0803		0.1248	0.0656	0.0257
	75–79	0.0000	0.0519	0.0885		0.1356	0.0811	0.0058
	80–84	0.0168	0.0000	0.1058		0.1140	0.1126	0.0246
	85+	0.0906	0.0000	0.2898		0.2054	0.1367	0.1034
5–6 ADLs	65–69	0.0097	0.0000	0.0384	0.0372		0.0762	0.0281
	70–74	0.0204	0.0202	0.0517	0.0487		0.0320	0.0314
	75–79	0.0416	0.0000	0.0187	0.0801		0.0896	0.0340
	80–84	0.0125	0.0109	0.0096	0.0846		0.0901	0.0425
	85+	0.0368	0.0046	0.0261	0.0533		0.1048	0.0445
Inst'd	65–69	0.0159	0.0161	0.0159	0.0000	0.0000		0.0150
	70–74	0.0257	0.0029	0.0139	0.0013	0.0028		0.0237
	75–79	0.0119	0.0000	0.0155	0.0024	0.0000		0.0428
	80–84	0.0093	0.0079	0.0036	0.0102	0.0086		0.0229
	85+	0.0133	0.0036	0.0032	0.0016	0.0006		0.0350

Table G.126: The constrained MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for females using 5 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0148	0.0049	0.0022	0.0040	0.0035	0.0021
	70–74		0.0220	0.0135	0.0041	0.0043	0.0052	0.0025
	75–79		0.0265	0.0266	0.0049	0.0043	0.0095	0.0050
	80–84		0.0296	0.0590	0.0061	0.0144	0.0159	0.0090
	85+		0.0503	0.0412	0.0331	0.0101	0.0528	0.0029
IADL	65–69	0.1008		0.2046	0.0000	0.0178	0.0122	0.0043
only	70–74	0.0939		0.1539	0.0056	0.0220	0.0194	0.0130
	75–79	0.0540		0.1973	0.0105	0.0163	0.0368	0.0080
	80–84	0.0183		0.2267	0.0364	0.0314	0.0201	0.0183
	85+	0.0090		0.2246	0.0000	0.0801	0.0498	0.0358
1–2	65–69	0.0637	0.0521		0.0949	0.0065	0.0330	0.0043
ADLs	70–74	0.0124	0.0432		0.0754	0.0253	0.0154	0.0055
	75–79	0.0196	0.0494		0.1286	0.0624	0.0305	0.0032
	80–84	0.0360	0.0732		0.1504	0.0044	0.0990	0.0000
	85+	0.0113	0.0376		0.1693	0.0400	0.0872	0.0164
3–4	65–69	0.0000	0.0658	0.1018		0.1551	0.0179	0.0170
ADLs	70–74	0.0244	0.0000	0.0644		0.0773	0.0188	0.0223
	75–79	0.0162	0.0127	0.0632		0.0764	0.0820	0.0170
	80–84	0.0000	0.0000	0.0983		0.0934	0.0828	0.0281
	85+	0.0265	0.0152	0.0771		0.1084	0.0866	0.0491
5–6	65–69	0.0099	0.0000	0.1357	0.1330		0.0402	0.0128
ADLs	70–74	0.0000	0.0375	0.0433	0.1155		0.0871	0.0000
	75–79	0.0055	0.0226	0.0935	0.0316		0.0620	0.0455
	80–84	0.0397	0.0000	0.0000	0.0576		0.0526	0.0399
	85+	0.0033	0.0000	0.0259	0.0329		0.1266	0.0412
Inst'd	65–69	0.0121	0.0000	0.0063	0.0095	0.0106		0.0214
	70–74	0.0054	0.0002	0.0097	0.0000	0.0051		0.0177
	75–79	0.0052	0.0000	0.0147	0.0000	0.0000		0.0254
	80–84	0.0067	0.0075	0.0000	0.0003	0.0000		0.0300
	85+	0.0059	0.0011	0.0012	0.0003	0.0050		0.0546

Table G.127: The constrained MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for females using year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0133	0.0189	0.0000	0.0028	0.0048	0.0034
	70–74		0.0161	0.0248	0.0003	0.0031	0.0093	0.0013
	75–79		0.0281	0.0344	0.0000	0.0061	0.0197	0.0017
	80–84		0.0646	0.0221	0.0262	0.0067	0.0199	0.0097
	85+		0.0461	0.0694	0.0000	0.0509	0.0597	0.0061
IADL only	65–69	0.1616		0.0619	0.0489	0.0415	0.0000	0.0000
	70–74	0.0694		0.2160	0.0000	0.0000	0.0200	0.0184
	75–79	0.0910		0.1856	0.0000	0.0539	0.0196	0.0159
	80–84	0.0835		0.4002	0.0000	0.0000	0.0578	0.0350
	85+	0.0161		0.2375	0.2106	0.0017	0.0000	0.0000
1–2 ADLs	65–69	0.0722	0.0445		0.1051	0.0350	0.0000	0.0031
	70–74	0.0424	0.0999		0.1669	0.0312	0.0020	0.0030
	75–79	0.0629	0.0434		0.1990	0.0161	0.0103	0.0166
	80–84	0.0187	0.1087		0.1248	0.0726	0.0858	0.0159
	85+	0.0000	0.1002		0.3196	0.0000	0.1021	0.0000
3–4 ADLs	65–69	0.0087	0.0813	0.0167		0.0385	0.0863	0.0149
	70–74	0.0327	0.0000	0.1006		0.0808	0.0725	0.0153
	75–79	0.0019	0.0481	0.0945		0.1142	0.0615	0.0130
	80–84	0.0132	0.0000	0.1192		0.1070	0.1181	0.0000
	85+	0.0789	0.0000	0.1895		0.2017	0.1776	0.0946
5–6 ADLs	65–69	0.0115	0.0000	0.0000	0.0613		0.0583	0.0206
	70–74	0.0230	0.0104	0.0564	0.0719		0.0062	0.0386
	75–79	0.0213	0.0000	0.0424	0.0660		0.0793	0.0189
	80–84	0.0120	0.0156	0.0000	0.0992		0.1130	0.0579
	85+	0.0469	0.0000	0.0423	0.0514		0.0896	0.0379
Inst'd	65–69	0.0275	0.0015	0.0319	0.0000	0.0006		0.0136
	70–74	0.0200	0.0021	0.0148	0.0000	0.0020		0.0237
	75–79	0.0139	0.0000	0.0094	0.0050	0.0000		0.0473
	80–84	0.0071	0.0120	0.0023	0.0043	0.0132		0.0226
	85+	0.0123	0.0018	0.0025	0.0010	0.0002		0.0299

Table G.128: The constrained MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males using 5 year age groupings.

1984		1989 Status						
status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0127	0.0122	0.0008	0.0015	0.0017	0.0043
	70–74		0.0173	0.0079	0.0021	0.0008	0.0050	0.0045
	75–79		0.0269	0.0123	0.0071	0.0008	0.0126	0.0095
	80–84		0.0384	0.0312	0.0184	0.0024	0.0125	0.0171
	85+		0.0965	0.0157	0.0000	0.0021	0.0299	0.0182
IADL	65–69	0.1501		0.1837	0.0000	0.0135	0.0223	0.0207
only	70–74	0.0459		0.0951	0.0460	0.0217	0.0422	0.0180
	75–79	0.0981		0.0909	0.1050	0.0678	0.0034	0.0130
	80–84	0.0427		0.0563	0.0321	0.2079	0.0195	0.0360
	85+	0.0491		0.2636	0.1636	0.0509	0.0435	0.0525
1–2	65–69	0.0063	0.1717		0.1586	0.0399	0.0107	0.0009
ADLs	70–74	0.0618	0.0522		0.1107	0.0407	0.0072	0.0117
	75–79	0.0308	0.0971		0.1402	0.0161	0.0180	0.0189
	80–84	0.0496	0.0934		0.0000	0.0000	0.0878	0.0087
	85+	0.0000	0.2103		0.1114	0.1367	0.0139	0.0300
3–4	65–69	0.0385	0.0000	0.2251		0.1287	0.0160	0.0169
ADLs	70–74	0.0000	0.0250	0.1914		0.0596	0.0712	0.0133
	75–79	0.0000	0.0000	0.1254		0.2545	0.0760	0.0512
	80–84	0.0000	0.0000	0.0774		0.2134	0.2334	0.0000
	85+	0.0000	0.0000	0.0001		0.0022	0.1953	0.0998
5–6	65–69	0.0163	0.0197	0.0485	0.0699		0.0501	0.0185
ADLs	70–74	0.0016	0.0200	0.0740	0.0540		0.0320	0.0089
	75–79	0.0000	0.0116	0.1012	0.0717		0.0645	0.0083
	80–84	0.0000	0.0000	0.0592	0.5061		0.0000	0.0000
	85+	0.0000	0.0000	0.0000	0.0644		0.1134	0.0331
Inst'd	65–69	0.0025	0.0152	0.0000	0.0009	0.0005		0.0197
	70–74	0.0077	0.0185	0.0000	0.0145	0.0000		0.0352
	75–79	0.0075	0.0000	0.0032	0.0019	0.0000		0.0841
	80–84	0.0144	0.0000	0.0000	0.0192	0.0000		0.1000
	85+	0.0168	0.0000	0.0033	0.0010	0.0010		0.0420

Table G.129: The constrained MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males using 5 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		0.0121	0.0096	0.0027	0.0038	0.0033	0.0023
	70–74		0.0232	0.0140	0.0052	0.0030	0.0040	0.0086
	75–79		0.0358	0.0242	0.0090	0.0045	0.0035	0.0072
	80–84		0.0208	0.0486	0.0000	0.0073	0.0144	0.0130
	85+		0.0131	0.0575	0.0000	0.0213	0.0677	0.0200
IADL only	65–69	0.1486		0.0597	0.0631	0.0000	0.0119	0.0348
	70–74	0.1767		0.1782	0.0211	0.0084	0.0409	0.0092
	75–79	0.1019		0.1263	0.0371	0.0000	0.0081	0.0353
	80–84	0.0094		0.1755	0.0132	0.0192	0.0235	0.0000
	85+	0.0000		0.0000	0.5805	0.0000	0.0000	0.0000
1–2 ADLs	65–69	0.0827	0.0143		0.0779	0.0000	0.0403	0.0175
	70–74	0.0461	0.1142		0.1089	0.0000	0.0224	0.0000
	75–79	0.0257	0.0262		0.0948	0.1084	0.0784	0.0000
	80–84	0.0226	0.0793		0.2070	0.0118	0.1323	0.0743
	85+	0.0538	0.0248		0.0899	0.2037	0.0000	0.0134
3–4 ADLs	65–69	0.0661	0.0462	0.0986		0.1436	0.0000	0.0000
	70–74	0.0092	0.0126	0.0184		0.1992	0.0306	0.0493
	75–79	0.0029	0.0423	0.0000		0.1610	0.1603	0.0000
	80–84	0.0288	0.0000	0.0000		0.1031	0.0800	0.1464
	85+	0.0000	0.1643	0.7154		0.0000	0.0000	0.1001
5–6 ADLs	65–69	0.0091	0.0000	0.1129	0.0000		0.0710	0.0253
	70–74	0.0043	0.0535	0.0200	0.0000		0.0873	0.0181
	75–79	0.0831	0.0000	0.0000	0.0617		0.1098	0.0599
	80–84	0.0144	0.0000	0.0437	0.0290		0.0373	0.0032
	85+	0.0000	0.0000	0.0000	0.0000		0.2885	0.1228
Inst'd	65–69	0.0023	0.0280	0.0000	0.0000	0.0000		0.0159
	70–74	0.0408	0.0051	0.0111	0.0067	0.0027		0.0228
	75–79	0.0066	0.0018	0.0224	0.0000	0.0000		0.0250
	80–84	0.0145	0.0027	0.0050	0.0252	0.0000		0.0167
	85+	0.0221	0.0138	0.0033	0.0082	0.0055		0.0741

Appendix H

Log-likelihood values for the maximum likelihood estimates, adjusted maximum likelihood estimates and the constrained maximum likelihood estimates of the transition intensities calculated from the 1984–89 and 1989–94 NLTCS

Tables H.130 and H.131 show for the transition intensities between disability states calculated from the 1984–89 and 1989–94 NLTCS, respectively, the log-likelihood (Equation 4.35) for: the unconstrained maximum likelihood estimates (transformed from transition probabilities in Section 4.3) and the constrained (real) MLEs (I refer to these as original MLEs); the unconstrained MLEs (and constrined (real) MLEs) with all negative transition intensities set to zero (I refer to these as adjusted MLEs); and for the constrained (positive) MLEs (given in Section 4.7). The same likelihood

Table H.130: Comparison of log-likelihood values for the unconstrained MLEs (and constrained (real) MLEs), adjusted MLEs and the constrained (positive) MLEs of the transition intensities calculated from the 1984-1989 NLTCS.

Gender	Age group	Log-likelihood value for: Original MLE	Adjusted MLE	Constrained MLE
Males	65–69	−2975.92	−2980.42	−2976.64
and	70–74	−3551.98	−3552.12	−3552.07
Females	65–74	−6566.07	−6566.08	−6566.07
	75–79	−3561.64	−3562.47	−3562.05
	80–84	−2446.15	−2447.63	−2446.87
	75–84	−6069.67	−6069.88	−6069.77
	85+	−1592.23	−1592.30	−1592.26
Females	65–69	−1736.24	−1740.28	−1737.64
	70–74	−2266.43	−2267.63	−2267.12
	65–74	−4043.75	−4043.78	−4043.75
	75–79	−2345.92	−2346.80	−2346.44
	80–84	−1762.77	−1763.70	−1763.20
	75–84	−4153.31	−4153.51	−4153.44
	85+	−1245.85	−1247.47	−1246.27
Males	65–69	−1223.94	−1224.02	−1224.01
	70–74	−1255.02	−1255.53	−1255.26
	65–74	−2493.78	−2496.47	−2494.29
	75–79	−1185.60	−1192.50	−1187.13
	80–84	−658.30	−659.70	−659.38
	75–84	−1867.18	−1886.68	−1869.33
	85+	−332.13	−332.41	−332.34

values for the 1982–84 NLTCS are given in Table 4.53.

Table H.131: Comparison of log-likelihood values for the unconstrained MLEs (and constrained (real) MLEs), adjusted MLEs and the constrained (positive) MLEs of the transition intensities calculated from the 1989-1994 NLTCS.

Gender	Age group	Log-likelihood value for: Original MLE	Adjusted MLE	Constrained MLE
Males	65–69	−1862.61	−1862.50	−1862.03
and	70–74	−2872.68	−2873.63	−2872.91
Females	65–74	−4765.33	−4765.33	−4765.33
	75–79	−3200.60	−3202.39	−3201.06
	80–84	−2522.97	−2523.29	−2523.05
	75–84	−5784.29	−5785.67	−5784.45
	85+	−1462.64	−1475.80	−1464.12
Females	65–69	−1148.09	−1154.55	−1151.00
	70–74	−1710.73	−1714.46	−1711.80
	65–74	−2887.95	−2888.85	−2888.20
	75–79	−2119.99	−2129.32	−2121.20
	80–84	−1830.69	−1833.06	−1830.99
	75–84	−4009.23	−4017.13	−4009.95
	85+	−1121.22	−1128.02	−1122.36
Males	65–69	−693.96	−696.37	−695.12
	70–74	−1136.63	−1138.28	−1137.11
	65–74	−1853.20	−1854.26	−1853.78
	75–79	−1056.70	−1059.11	−1057.53
	80–84	−663.75	−666.37	−664.89
	75–84	−1745.57	−1746.96	−1746.21
	85+	−319.13	−360.93	−321.90

Appendix I

5-year transition probabilities between disability states calculated from the constrained transition intensities of the 1984, 1989 and 1994 National Long-Term Care Surveys

The tables in this appendix give the 5-year transition probabilities between disability states calculated from the constrained (positive) MLEs of the transition intensities, based on data grouped in 5-year age bands (see Section 4.5) as a percentage for men, women and in aggregate, calculated using Equation 3.32. They are given for the 1984–89 NLTCS in Tables I.132 to I.134 and for the 1989–94 NLTCS in Tables I.135 to I.137.

Table I.132: The 5-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys as a percentage, for males using 5 year age groupings.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	86.43	3.67	3.99	1.22	1.10	1.20	2.39
	70–74	84.08	4.74	3.30	1.51	0.88	2.77	2.71
	75–79	72.79	5.64	4.53	3.01	2.55	5.29	6.18
	80–84	57.03	7.17	8.66	5.22	3.86	8.39	9.66
	85+	46.46	10.39	6.63	5.99	3.97	14.07	12.48
IADL only	65–69	33.94	22.07	18.73	6.58	5.53	6.69	6.47
	70–74	15.48	29.78	17.49	9.75	6.80	12.75	7.96
	75–79	20.19	19.28	15.57	11.84	15.80	8.09	9.22
	80–84	8.71	16.54	12.49	17.24	15.67	16.60	12.75
	85+	5.77	9.64	10.60	16.62	10.58	23.95	22.84
1–2 ADLs	65–69	14.98	17.12	29.73	15.36	11.92	6.81	4.08
	70–74	17.73	9.52	34.44	14.79	9.98	7.60	5.94
	75–79	10.80	10.48	29.19	14.19	13.63	10.06	11.65
	80–84	12.89	11.08	33.78	4.69	4.88	21.02	11.66
	85+	2.58	8.31	13.54	16.48	16.27	22.66	20.16
3–4 ADLs	65–69	13.31	9.38	22.75	21.56	18.47	8.12	6.41
	70–74	7.13	7.43	25.00	24.52	11.82	16.87	7.22
	75–79	3.03	4.00	16.65	15.71	24.84	18.22	17.56
	80–84	3.39	2.17	11.11	20.44	11.58	37.25	14.06
	85+	1.79	0.25	0.32	22.84	0.50	42.20	32.09
5–6 ADLs	65–69	9.14	6.55	12.52	10.93	37.60	15.41	7.87
	70–74	4.79	6.31	17.50	11.55	42.17	12.03	5.66
	75–79	3.21	4.59	16.95	10.91	35.40	18.30	10.65
	80–84	2.92	2.38	13.25	26.47	17.46	29.35	8.17
	85+	1.36	0.18	0.24	9.26	35.02	35.56	18.38
Inst'd	65–69	2.63	3.30	1.26	0.45	0.41	82.69	9.25
	70–74	3.83	4.57	2.29	3.45	0.85	69.74	15.27
	75–79	2.61	0.25	0.83	0.49	0.31	61.91	33.60
	80–84	3.98	0.36	0.93	2.98	1.03	53.52	37.20
	85+	4.93	0.81	0.87	0.65	0.59	73.49	18.67

Table I.133: The 5-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys as a percentage, for females using 5 year age groupings.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	87.11	3.64	3.37	1.26	1.38	1.96	1.28
	70–74	79.06	5.42	6.35	2.50	1.84	3.15	1.68
	75–79	69.72	6.16	8.86	3.51	2.67	5.94	3.13
	80–84	53.28	6.56	12.72	5.87	4.91	11.23	5.44
	85+	39.82	7.00	10.04	7.75	6.17	21.66	7.56
IADL only	65–69	29.78	22.81	27.43	6.61	4.09	6.73	2.55
	70–74	22.76	25.22	26.58	7.35	5.28	7.93	4.88
	75–79	13.43	23.88	25.72	9.92	7.44	14.65	4.97
	80–84	6.88	21.76	23.57	13.55	7.80	18.31	8.13
	85+	2.70	15.71	19.56	8.93	13.37	23.61	16.12
1–2 ADLs	65–69	21.21	8.98	37.22	13.08	5.86	10.36	3.27
	70–74	7.40	7.73	48.02	17.27	7.49	8.19	3.91
	75–79	7.16	7.37	32.13	18.79	12.50	16.55	5.49
	80–84	7.59	7.91	24.82	17.86	6.44	29.22	6.16
	85+	3.50	4.02	22.14	16.23	11.43	28.34	14.34
3–4 ADLs	65–69	8.95	9.18	23.48	26.00	16.93	9.15	6.31
	70–74	8.06	2.66	15.66	42.02	13.18	10.29	8.13
	75–79	5.55	3.66	13.12	30.88	12.68	24.93	9.17
	80–84	3.55	2.31	11.31	28.17	15.25	26.99	12.42
	85+	4.52	2.42	9.18	20.99	15.32	27.74	19.83
5–6 ADLs	65–69	9.02	5.18	23.80	17.83	25.82	12.57	5.79
	70–74	4.25	5.75	13.64	20.23	28.85	23.85	3.43
	75–79	3.66	4.75	15.96	9.17	31.02	20.02	15.41
	80–84	9.59	1.08	3.17	9.65	41.43	19.07	16.01
	85+	1.56	0.64	4.30	5.44	34.02	34.47	19.57
Inst'd	65–69	5.53	0.72	3.16	3.02	3.06	74.89	9.62
	70–74	2.41	0.47	3.31	0.89	1.50	83.20	8.21
	75–79	2.20	0.47	3.77	1.01	0.61	80.39	11.55
	80–84	2.34	1.75	0.98	0.46	0.27	80.53	13.67
	85+	1.63	0.37	0.58	0.37	1.44	72.08	23.52

Table I.134: The 5-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys as a percentage, for males and females using 5 year age groupings.

		1989 Status						
1984 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	86.81	3.65	3.65	1.24	1.25	1.62	1.77
	70–74	81.10	5.15	5.11	2.10	1.45	2.99	2.10
	75–79	70.87	5.96	7.24	3.33	2.63	5.70	4.27
	80–84	54.51	6.76	11.39	5.66	4.56	10.29	6.84
	85+	41.57	8.18	9.14	7.03	5.59	19.58	8.92
IADL only	65–69	31.87	21.59	24.40	6.81	4.58	6.73	4.03
	70–74	20.51	26.51	23.91	8.03	5.77	9.42	5.86
	75–79	15.37	22.55	22.81	10.47	9.84	12.76	6.19
	80–84	7.35	20.38	20.68	14.89	9.46	17.85	9.40
	85+	3.67	13.54	18.60	10.19	12.75	23.62	17.65
1–2 ADLs	65–69	19.10	12.58	33.91	13.71	8.04	9.11	3.55
	70–74	10.74	8.27	43.56	16.58	8.25	8.01	4.58
	75–79	8.36	8.11	31.13	17.68	12.80	14.92	7.01
	80–84	8.56	8.42	26.09	15.61	6.46	28.01	6.85
	85+	3.13	5.02	19.86	17.09	12.20	27.37	15.32
3–4 ADLs	65–69	9.15	8.34	25.90	23.96	17.33	8.97	6.35
	70–74	7.62	4.41	19.59	34.91	12.74	12.71	8.02
	75–79	4.34	3.14	15.52	26.07	16.22	23.00	11.72
	80–84	3.55	2.35	11.10	24.65	15.19	29.43	13.73
	85+	4.20	2.07	8.20	20.64	13.39	29.87	21.63
5–6 ADLs	65–69	9.58	5.01	19.27	16.43	29.76	13.44	6.50
	70–74	3.98	6.37	15.15	17.28	34.35	19.08	3.79
	75–79	2.93	5.56	15.73	9.94	32.79	19.44	13.60
	80–84	7.59	1.40	5.85	15.26	35.76	21.04	13.10
	85+	1.49	0.63	3.45	6.30	34.11	34.66	19.37
Inst'd	65–69	4.26	1.95	2.22	1.87	1.87	78.38	9.45
	70–74	2.87	1.71	2.85	1.59	1.34	79.36	10.28
	75–79	2.37	0.46	3.01	0.83	0.50	75.30	17.54
	80–84	2.71	1.50	0.88	0.95	0.41	76.17	17.38
	85+	2.13	0.40	0.62	0.41	1.32	72.24	22.87

Table I.135: The 5-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys as a percentage, for males using 5 year age groupings.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	86.84	3.05	3.52	1.35	1.41	2.09	1.75
	70–74	78.34	5.08	5.13	2.31	1.87	2.77	4.49
	75–79	69.04	7.82	6.67	3.52	3.20	5.11	4.63
	80–84	60.69	6.01	7.79	3.81	3.35	9.41	8.93
	85+	43.11	2.40	10.19	2.31	6.49	20.69	14.80
IADL only	65–69	39.40	22.65	11.32	7.96	3.40	5.00	10.27
	70–74	37.01	17.57	18.35	7.14	4.64	10.17	5.11
	75–79	23.47	24.59	14.73	7.61	6.45	11.76	11.39
	80–84	4.93	33.87	15.68	10.26	7.23	17.30	10.73
	85+	4.85	12.04	32.42	13.74	12.60	7.46	16.89
1–2 ADLs	65–69	27.36	4.14	35.69	10.16	4.12	11.78	6.75
	70–74	20.65	12.29	30.24	13.91	8.43	9.77	4.71
	75–79	11.30	4.64	21.62	11.06	15.45	29.39	6.54
	80–84	6.30	7.25	10.55	14.63	7.95	26.69	26.62
	85+	8.83	3.46	24.39	5.06	17.63	20.17	20.45
3–4 ADLs	65–69	23.60	5.92	19.02	20.73	18.59	7.82	4.32
	70–74	6.98	5.21	5.54	21.74	29.58	15.22	15.73
	75–79	8.26	4.87	3.32	19.21	16.14	39.83	8.37
	80–84	6.53	0.65	1.87	19.06	16.79	18.47	36.62
	85+	6.64	5.56	28.33	7.72	15.30	12.66	23.79
5–6 ADLs	65–69	9.20	2.05	19.26	3.19	34.42	21.70	10.18
	70–74	9.54	7.89	7.03	2.09	41.07	24.04	8.33
	75–79	17.88	2.13	2.39	6.63	23.84	28.75	18.38
	80–84	5.68	1.46	5.87	7.82	55.23	16.45	7.48
	85+	2.65	1.13	1.45	0.98	13.64	41.54	38.60
Inst'd	65–69	4.07	6.23	1.03	0.84	0.22	79.72	7.90
	70–74	15.58	2.07	3.40	2.14	1.74	65.02	10.05
	75–79	3.11	0.90	4.93	1.04	1.30	77.44	11.28
	80–84	5.40	1.10	1.39	5.43	1.65	74.70	10.35
	85+	5.58	2.13	3.56	1.97	2.02	55.20	29.54

Table I.136: The 5-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys as a percentage, for females using 5 year age groupings.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	83.50	3.61	5.34	1.67	1.77	2.31	1.80
	70–74	78.18	4.76	7.38	2.49	1.65	4.16	1.38
	75–79	67.22	5.90	9.14	3.67	3.12	8.20	2.74
	80–84	50.45	8.27	10.45	6.88	4.20	13.33	6.41
	85+	34.22	6.34	10.86	6.58	9.99	25.18	6.84
IADL only	65–69	40.15	24.40	9.86	10.01	9.16	4.63	1.79
	70–74	20.80	26.27	25.38	10.44	3.91	6.81	6.39
	75–79	21.96	19.86	20.50	10.70	10.00	9.50	7.48
	80–84	12.05	12.00	23.26	9.71	7.60	23.90	11.48
	85+	6.89	14.12	18.57	15.63	10.08	24.45	10.26
1–2 ADLs	65–69	24.02	9.11	30.34	17.51	9.53	6.62	2.87
	70–74	16.05	11.89	29.73	21.08	9.44	7.26	4.55
	75–79	15.50	7.31	26.74	22.13	10.28	10.34	7.70
	80–84	6.50	7.23	21.89	13.77	11.70	29.30	9.60
	85+	6.07	6.32	17.18	13.40	9.94	34.91	12.17
3–4 ADLs	65–69	12.10	11.27	5.95	33.00	9.39	22.19	6.10
	70–74	12.42	3.84	14.88	29.67	13.86	17.47	7.86
	75–79	7.00	6.32	14.46	27.50	17.45	18.66	8.61
	80–84	4.56	3.37	11.89	23.57	14.69	34.94	6.98
	85+	8.71	2.92	9.38	8.70	14.02	35.43	20.84
5–6 ADLs	65–69	6.43	2.29	2.06	12.06	48.43	20.00	8.73
	70–74	10.30	4.11	12.34	14.67	39.38	5.41	13.78
	75–79	7.93	2.18	8.97	12.18	35.90	22.64	10.20
	80–84	3.85	2.47	4.65	11.62	26.94	32.05	18.42
	85+	9.05	1.95	6.87	5.92	30.15	30.17	15.89
Inst'd	65–69	12.48	1.44	7.77	1.93	1.15	69.27	5.96
	70–74	8.27	1.37	3.94	1.34	1.00	73.61	10.47
	75–79	5.16	0.56	2.53	1.91	0.61	69.19	20.03
	80–84	2.87	2.21	2.31	2.01	3.56	76.16	10.88
	85+	3.34	0.69	1.10	0.66	0.60	79.97	13.64

Table I.137: The 5-year transition probabilities, calculated from the constrained (positive) MLEs of the transition intensities, between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys as a percentage, for males and females using 5 year age groupings.

		1994 Status						
1989 Status	Age group	Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69	84.89	3.37	4.54	1.56	1.62	2.22	1.79
	70–74	78.25	4.89	6.43	2.42	1.75	3.58	2.69
	75–79	67.87	6.65	8.39	3.42	3.18	7.03	3.46
	80–84	53.82	7.53	9.65	5.78	3.93	12.05	7.24
	85+	37.08	5.11	10.44	5.40	8.85	23.72	9.41
IADL only	65–69	40.98	23.12	10.41	10.79	7.20	4.51	2.98
	70–74	27.58	22.46	23.10	9.20	3.52	8.21	5.93
	75–79	22.57	20.54	19.16	10.14	8.72	10.19	8.67
	80–84	10.40	16.84	22.61	10.25	7.14	22.45	10.32
	85+	6.12	13.67	20.99	14.24	11.57	22.67	10.74
1–2 ADLs	65–69	25.39	7.11	31.97	16.57	8.17	6.40	4.39
	70–74	17.50	12.19	29.23	19.27	9.35	8.07	4.39
	75–79	14.51	6.59	24.33	21.09	11.54	14.64	7.30
	80–84	6.43	7.36	19.05	14.08	11.28	28.63	13.17
	85+	6.30	5.89	18.92	12.45	10.04	33.10	13.30
3–4 ADLs	65–69	17.10	9.04	12.07	23.61	14.96	17.49	5.73
	70–74	10.75	3.89	12.54	26.42	19.11	16.71	10.58
	75–79	7.52	6.18	12.36	24.63	17.72	23.45	8.13
	80–84	5.11	2.92	9.79	22.08	15.72	31.83	12.54
	85+	9.12	3.27	11.76	8.66	13.64	31.43	22.12
5–6 ADLs	65–69	7.36	2.34	8.56	7.43	40.96	22.54	10.81
	70–74	10.16	5.19	11.22	10.49	39.61	11.21	12.13
	75–79	11.32	2.18	5.93	11.12	30.93	24.76	13.74
	80–84	4.28	2.35	4.67	11.28	32.89	28.30	16.23
	85+	7.96	1.86	6.06	5.07	29.43	31.14	18.47
Inst'd	65–69	9.12	3.87	4.54	1.59	0.71	73.43	6.74
	70–74	10.40	1.59	3.85	1.51	1.24	70.99	10.42
	75–79	4.70	0.60	3.50	1.81	0.70	70.37	18.31
	80–84	3.44	1.88	2.21	2.94	3.00	75.64	10.89
	85+	3.70	0.95	1.48	0.84	0.80	76.47	15.77

Appendix J

Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1982, 1984, 1989 and 1994 National Long-Term Care Surveys

The tables in this appendix give the variance estiamtes of the constrained (positive) MLEs of the annual transition intensities between disability states for men, women and in aggregate (as described in Section 5.4). They are given for all pairs of surveys in 10-year age bands (65–74 years, 75–84 years and 85+ years) in Tables J.138 – J.146 and in 5-year age bands (65–69 years, 70–74 years, 75–79 years and 80–84 years) in Tables J.147 – J.152 (except for those from the 1982–84 NLTCS, which are given in Tables 5.61 – 5.63).

Table J.138: Variance estimates of the MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males and females using 10 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		3.88×10^{-6}	3.24×10^{-6}	1.42×10^{-6}	1.00×10^{-6}	4.67×10^{-7}	2.00×10^{-6}
	75–84		2.46×10^{-5}	2.50×10^{-5}	1.12×10^{-5}	6.58×10^{-6}	5.22×10^{-6}	1.12×10^{-5}
	85+		3.48×10^{-4}	5.09×10^{-4}	2.55×10^{-4}	2.11×10^{-4}	1.72×10^{-4}	1.97×10^{-4}
IADL only	65–74	3.56×10^{-4}		9.63×10^{-4}	3.40×10^{-4}	1.91×10^{-4}	6.99×10^{-5}	1.36×10^{-4}
	75–84	2.55×10^{-4}		1.83×10^{-3}	8.04×10^{-4}	4.66×10^{-4}	2.06×10^{-4}	2.63×10^{-4}
	85+	2.23×10^{-4}		4.80×10^{-3}	3.12×10^{-3}	2.21×10^{-3}	9.70×10^{-4}	1.11×10^{-3}
1–2 ADLs	65–74	2.09×10^{-4}	7.43×10^{-4}		1.08×10^{-3}	4.37×10^{-4}	1.10×10^{-4}	2.02×10^{-4}
	75–84	1.37×10^{-4}	7.67×10^{-4}		1.55×10^{-3}	6.65×10^{-4}	2.00×10^{-4}	2.98×10^{-4}
	85+	2.15×10^{-4}	8.45×10^{-4}		3.87×10^{-3}	3.02×10^{-3}	9.17×10^{-4}	8.47×10^{-4}
3–4 ADLs	65–74	3.63×10^{-4}	1.13×10^{-3}	4.67×10^{-3}		2.97×10^{-3}	2.95×10^{-4}	7.22×10^{-4}
	75–84	2.27×10^{-4}	1.32×10^{-3}	5.11×10^{-3}		5.52×10^{-3}	1.02×10^{-3}	1.26×10^{-3}
	85+	2.38×10^{-4}	1.52×10^{-3}	3.22×10^{-3}		1.74×10^{-2}	3.43×10^{-3}	3.90×10^{-3}
5–6 ADLs	65–74	1.92×10^{-4}	5.34×10^{-4}	1.18×10^{-3}	1.53×10^{-3}		2.85×10^{-4}	7.35×10^{-4}
	75–84	1.32×10^{-4}	4.91×10^{-4}	9.48×10^{-4}	2.03×10^{-3}		5.05×10^{-4}	8.32×10^{-4}
	85+	1.35×10^{-4}	2.72×10^{-4}	1.22×10^{-3}	3.30×10^{-3}		1.28×10^{-3}	2.24×10^{-3}
Inst'd	65–74	7.50×10^{-5}	4.04×10^{-5}	9.30×10^{-5}	1.09×10^{-4}	6.82×10^{-5}		3.61×10^{-4}
	75–84	1.14×10^{-5}	2.40×10^{-5}	2.15×10^{-5}	3.06×10^{-5}	2.45×10^{-5}		2.34×10^{-4}
	85+	6.68×10^{-6}	6.91×10^{-6}	6.36×10^{-6}	3.30×10^{-5}	2.43×10^{-5}		3.41×10^{-4}

Table J.139: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males and females using 10 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		2.35×10^{-6}	2.18×10^{-6}	1.00×10^{-6}	5.78×10^{-7}	3.08×10^{-7}	1.88×10^{-7}
	75–84		9.56×10^{-6}	1.38×10^{-5}	7.44×10^{-6}	3.51×10^{-6}	2.66×10^{-6}	1.23×10^{-6}
	85+		2.59×10^{-4}	3.32×10^{-4}	2.02×10^{-4}	7.22×10^{-5}	9.13×10^{-5}	2.98×10^{-5}
IADL only	65–74	1.38×10^{-4}		6.46×10^{-4}	2.91×10^{-4}	1.32×10^{-4}	4.43×10^{-5}	1.88×10^{-5}
	75–84	1.07×10^{-4}		1.09×10^{-3}	6.56×10^{-4}	2.79×10^{-4}	1.32×10^{-4}	4.11×10^{-5}
	85+	2.80×10^{-4}		8.27×10^{-3}	4.72×10^{-3}	1.56×10^{-3}	1.07×10^{-3}	4.29×10^{-4}
1–2 ADLs	65–74	6.12×10^{-5}	2.28×10^{-4}		4.00×10^{-4}	1.55×10^{-4}	3.67×10^{-5}	1.32×10^{-5}
	75–84	6.58×10^{-5}	3.34×10^{-4}		7.93×10^{-4}	2.90×10^{-4}	1.59×10^{-4}	3.71×10^{-5}
	85+	2.04×10^{-4}	1.46×10^{-3}		4.48×10^{-3}	1.18×10^{-3}	9.14×10^{-4}	3.17×10^{-4}
3–4 ADLs	65–74	9.90×10^{-5}	4.37×10^{-4}	1.18×10^{-3}		7.36×10^{-4}	1.28×10^{-4}	5.68×10^{-5}
	75–84	7.43×10^{-5}	3.07×10^{-4}	1.33×10^{-3}		9.41×10^{-4}	4.34×10^{-4}	1.45×10^{-4}
	85+	3.93×10^{-4}	9.77×10^{-4}	3.48×10^{-3}		1.90×10^{-3}	1.59×10^{-3}	6.80×10^{-4}
5–6 ADLs	65–74	8.52×10^{-5}	3.49×10^{-4}	8.93×10^{-4}	1.12×10^{-3}		1.97×10^{-4}	4.65×10^{-5}
	75–84	9.09×10^{-5}	2.62×10^{-4}	8.61×10^{-4}	9.13×10^{-4}		3.03×10^{-4}	1.35×10^{-4}
	85+	7.62×10^{-5}	1.32×10^{-4}	5.97×10^{-4}	9.15×10^{-4}		9.68×10^{-4}	3.53×10^{-4}
Inst'd	65–74	1.32×10^{-5}	2.50×10^{-5}	3.02×10^{-5}	2.43×10^{-5}	1.71×10^{-5}		2.88×10^{-5}
	75–84	8.21×10^{-6}	1.07×10^{-5}	2.02×10^{-5}	8.20×10^{-6}	2.68×10^{-6}		3.40×10^{-5}
	85+	1.42×10^{-5}	8.59×10^{-6}	1.27×10^{-5}	7.72×10^{-6}	1.14×10^{-5}		6.21×10^{-5}

Table J.140: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males and females using 10 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		3.98×10^{-6}	5.07×10^{-6}	3.00×10^{-6}	9.93×10^{-7}	6.82×10^{-7}	3.30×10^{-7}
	75–84		1.68×10^{-5}	2.89×10^{-5}	1.54×10^{-5}	5.58×10^{-6}	4.31×10^{-6}	1.50×10^{-6}
	85+		2.43×10^{-4}	1.41×10^{-3}	2.71×10^{-3}	3.06×10^{-4}	1.33×10^{-4}	5.23×10^{-5}
IADL only	65–74	4.29×10^{-4}		1.31×10^{-3}	8.98×10^{-4}	2.06×10^{-4}	9.34×10^{-5}	4.02×10^{-5}
	75–84	3.09×10^{-4}		2.97×10^{-3}	1.74×10^{-3}	5.21×10^{-4}	2.62×10^{-4}	8.51×10^{-5}
	85+	1.68×10^{-3}		4.51×10^{-2}	8.65×10^{-2}	7.16×10^{-3}	2.17×10^{-3}	1.06×10^{-3}
1–2 ADLs	65–74	2.27×10^{-4}	6.78×10^{-4}		1.38×10^{-3}	3.60×10^{-4}	1.22×10^{-4}	3.68×10^{-5}
	75–84	1.66×10^{-4}	6.43×10^{-4}		2.21×10^{-3}	7.03×10^{-4}	3.03×10^{-4}	7.54×10^{-5}
	85+	3.97×10^{-3}	1.06×10^{-2}		2.96×10^{-1}	2.18×10^{-2}	3.31×10^{-3}	2.79×10^{-3}
3–4 ADLs	65–74	3.28×10^{-4}	7.78×10^{-4}	1.60×10^{-3}		1.12×10^{-3}	4.50×10^{-4}	1.42×10^{-4}
	75–84	1.99×10^{-4}	7.10×10^{-4}	2.27×10^{-3}		1.52×10^{-3}	6.69×10^{-4}	1.55×10^{-4}
	85+	1.03×10^{-2}	2.14×10^{-2}	2.86×10^{-1}		5.32×10^{-2}	8.10×10^{-3}	6.98×10^{-3}
5–6 ADLs	65–74	2.07×10^{-4}	3.99×10^{-4}	9.24×10^{-4}	1.01×10^{-3}		3.79×10^{-4}	1.76×10^{-4}
	75–84	2.03×10^{-4}	2.49×10^{-4}	8.35×10^{-4}	1.58×10^{-3}		6.00×10^{-4}	2.10×10^{-4}
	85+	9.69×10^{-4}	9.02×10^{-4}	5.85×10^{-3}	1.18×10^{-2}		1.59×10^{-3}	6.76×10^{-4}
Inst'd	65–74	7.86×10^{-5}	7.58×10^{-5}	1.15×10^{-4}	4.91×10^{-5}	1.88×10^{-5}		5.38×10^{-5}
	75–84	2.48×10^{-5}	2.84×10^{-5}	6.88×10^{-5}	5.68×10^{-5}	2.53×10^{-5}		5.06×10^{-5}
	85+	3.87×10^{-5}	2.86×10^{-5}	6.83×10^{-5}	8.61×10^{-5}	1.40×10^{-5}		5.81×10^{-5}

Table J.141: Variance estimates of the MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for females using 10 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		7.99×10^{-6}	6.07×10^{-6}	2.26×10^{-6}	1.58×10^{-6}	7.46×10^{-7}	2.34×10^{-6}
	75–84		4.43×10^{-5}	4.71×10^{-5}	2.17×10^{-5}	9.83×10^{-6}	9.59×10^{-6}	1.49×10^{-5}
	85+		5.35×10^{-4}	6.77×10^{-4}	2.99×10^{-4}	3.20×10^{-4}	2.79×10^{-4}	2.49×10^{-4}
IADL only	65–74	5.61×10^{-4}		1.77×10^{-3}	5.98×10^{-4}	2.51×10^{-4}	1.32×10^{-4}	1.66×10^{-4}
	75–84	3.77×10^{-4}		3.35×10^{-3}	1.32×10^{-3}	5.05×10^{-4}	3.16×10^{-4}	3.46×10^{-4}
	85+	3.45×10^{-4}		8.18×10^{-3}	4.51×10^{-3}	3.78×10^{-3}	1.65×10^{-3}	1.54×10^{-3}
1–2 ADLs	65–74	3.21×10^{-4}	1.31×10^{-3}		1.05×10^{-3}	4.47×10^{-4}	1.46×10^{-4}	2.29×10^{-4}
	75–84	2.00×10^{-4}	1.12×10^{-3}		2.09×10^{-3}	7.84×10^{-4}	3.08×10^{-4}	3.08×10^{-4}
	85+	2.27×10^{-4}	1.38×10^{-3}		4.33×10^{-3}	3.35×10^{-3}	1.04×10^{-3}	9.00×10^{-4}
3–4 ADLs	65–74	4.14×10^{-4}	2.11×10^{-3}	7.00×10^{-3}		3.81×10^{-3}	4.77×10^{-4}	7.49×10^{-4}
	75–84	3.12×10^{-4}	1.85×10^{-3}	7.63×10^{-3}		6.94×10^{-3}	1.34×10^{-3}	1.30×10^{-3}
	85+	3.36×10^{-4}	2.08×10^{-3}	3.74×10^{-3}		1.93×10^{-2}	4.09×10^{-3}	3.98×10^{-3}
5–6 ADLs	65–74	3.18×10^{-4}	1.12×10^{-3}	1.83×10^{-3}	2.19×10^{-3}		5.51×10^{-4}	1.24×10^{-3}
	75–84	2.16×10^{-4}	6.46×10^{-4}	1.56×10^{-3}	2.96×10^{-3}		8.22×10^{-4}	1.04×10^{-3}
	85+	1.92×10^{-4}	4.00×10^{-4}	1.40×10^{-3}	3.51×10^{-3}		1.65×10^{-3}	2.55×10^{-3}
Inst'd	65–74	1.09×10^{-4}	5.92×10^{-5}	1.04×10^{-4}	1.90×10^{-4}	8.65×10^{-5}		4.84×10^{-4}
	75–84	1.31×10^{-5}	2.77×10^{-5}	2.71×10^{-5}	3.02×10^{-5}	3.09×10^{-5}		2.68×10^{-4}
	85+	8.51×10^{-6}	1.01×10^{-5}	8.55×10^{-6}	4.03×10^{-5}	3.24×10^{-5}		3.74×10^{-4}

Table J.142: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for females using 10 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		4.18×10^{-6}	3.51×10^{-6}	1.79×10^{-6}	1.53×10^{-6}	6.03×10^{-7}	2.57×10^{-7}
	75–84		1.43×10^{-5}	2.58×10^{-5}	1.05×10^{-5}	4.84×10^{-6}	4.31×10^{-6}	1.53×10^{-6}
	85+		2.28×10^{-4}	3.46×10^{-4}	2.63×10^{-4}	1.08×10^{-4}	1.39×10^{-4}	3.61×10^{-5}
IADL only	65–74	2.12×10^{-4}		7.88×10^{-4}	3.64×10^{-4}	2.38×10^{-4}	6.26×10^{-5}	2.19×10^{-5}
	75–84	1.29×10^{-4}		1.67×10^{-3}	6.98×10^{-4}	2.43×10^{-4}	1.75×10^{-4}	4.61×10^{-5}
	85+	2.29×10^{-4}		5.65×10^{-3}	3.34×10^{-3}	1.61×10^{-3}	1.09×10^{-3}	4.06×10^{-4}
1–2 ADLs	65–74	6.94×10^{-5}	2.00×10^{-4}		3.70×10^{-4}	1.98×10^{-4}	4.63×10^{-5}	1.44×10^{-5}
	75–84	7.58×10^{-5}	3.58×10^{-4}		8.39×10^{-4}	2.81×10^{-4}	2.00×10^{-4}	4.06×10^{-5}
	85+	2.20×10^{-4}	7.54×10^{-4}		3.45×10^{-3}	1.11×10^{-3}	8.63×10^{-4}	2.82×10^{-4}
3–4 ADLs	65–74	1.24×10^{-4}	5.49×10^{-4}	1.05×10^{-3}		1.34×10^{-3}	1.66×10^{-4}	7.73×10^{-5}
	75–84	9.36×10^{-5}	2.79×10^{-4}	1.16×10^{-3}		6.85×10^{-4}	4.23×10^{-4}	1.39×10^{-4}
	85+	4.96×10^{-4}	8.75×10^{-4}	3.25×10^{-3}		2.49×10^{-3}	1.70×10^{-3}	7.06×10^{-4}
5–6 ADLs	65–74	1.76×10^{-4}	6.49×10^{-4}	1.59×10^{-3}	2.65×10^{-3}		4.52×10^{-4}	8.04×10^{-5}
	75–84	1.53×10^{-4}	2.29×10^{-4}	8.35×10^{-4}	6.70×10^{-4}		3.41×10^{-4}	1.99×10^{-4}
	85+	9.86×10^{-5}	1.18×10^{-4}	6.93×10^{-4}	9.03×10^{-4}		1.16×10^{-3}	4.27×10^{-4}
Inst'd	65–74	2.06×10^{-5}	1.19×10^{-5}	4.61×10^{-5}	3.00×10^{-5}	4.40×10^{-5}		3.90×10^{-5}
	75–84	8.82×10^{-6}	1.44×10^{-5}	2.60×10^{-5}	7.53×10^{-6}	2.44×10^{-6}		2.98×10^{-5}
	85+	1.31×10^{-5}	7.39×10^{-6}	1.13×10^{-5}	7.57×10^{-6}	1.44×10^{-5}		7.40×10^{-5}

Table J.143: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for females using 10 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		6.47×10^{-6}	1.03×10^{-5}	4.68×10^{-6}	1.60×10^{-6}	1.32×10^{-6}	4.28×10^{-7}
	75–84		3.49×10^{-5}	5.51×10^{-5}	2.83×10^{-5}	1.02×10^{-5}	7.80×10^{-6}	2.23×10^{-6}
	85+		4.69×10^{-4}	1.55×10^{-3}	2.95×10^{-3}	4.70×10^{-4}	2.39×10^{-4}	6.17×10^{-5}
IADL only	65–74	5.72×10^{-4}		2.08×10^{-3}	1.07×10^{-3}	2.89×10^{-4}	1.19×10^{-4}	4.33×10^{-5}
	75–84	5.86×10^{-4}		6.20×10^{-3}	3.55×10^{-3}	1.05×10^{-3}	4.54×10^{-4}	1.43×10^{-4}
	85+	2.15×10^{-3}		3.71×10^{-2}	7.98×10^{-2}	8.18×10^{-3}	3.39×10^{-3}	1.10×10^{-3}
1–2 ADLs	65–74	3.06×10^{-4}	9.48×10^{-4}		1.60×10^{-3}	4.27×10^{-4}	1.83×10^{-4}	5.13×10^{-5}
	75–84	2.34×10^{-4}	1.20×10^{-3}		3.11×10^{-3}	9.51×10^{-4}	3.58×10^{-4}	8.38×10^{-5}
	85+	2.77×10^{-3}	9.51×10^{-3}		1.40×10^{-1}	1.40×10^{-2}	4.42×10^{-3}	1.64×10^{-3}
3–4 ADLs	65–74	3.79×10^{-4}	8.53×10^{-4}	1.94×10^{-3}		8.13×10^{-4}	6.40×10^{-4}	1.59×10^{-4}
	75–84	2.58×10^{-4}	1.34×10^{-3}	3.85×10^{-3}		2.08×10^{-3}	8.36×10^{-4}	1.60×10^{-4}
	85+	6.39×10^{-3}	1.32×10^{-2}	8.77×10^{-2}		2.77×10^{-2}	8.44×10^{-3}	3.49×10^{-3}
5–6 ADLs	65–74	2.77×10^{-4}	4.49×10^{-4}	1.09×10^{-3}	1.60×10^{-3}		4.17×10^{-4}	2.71×10^{-4}
	75–84	2.43×10^{-4}	6.17×10^{-4}	1.56×10^{-3}	2.79×10^{-3}		9.47×10^{-4}	3.17×10^{-4}
	85+	1.35×10^{-3}	1.07×10^{-3}	5.39×10^{-3}	1.05×10^{-2}		1.72×10^{-3}	6.47×10^{-4}
Inst'd	65–74	1.09×10^{-4}	6.53×10^{-5}	2.04×10^{-4}	6.85×10^{-5}	2.55×10^{-5}		7.95×10^{-5}
	75–84	3.32×10^{-5}	4.74×10^{-5}	8.04×10^{-5}	6.32×10^{-5}	3.89×10^{-5}		6.97×10^{-5}
	85+	4.17×10^{-5}	2.26×10^{-5}	4.91×10^{-5}	6.01×10^{-5}	1.16×10^{-5}		5.55×10^{-5}

Table J.144: Variance estimates of the MLEs of the annual transition intensities between disability states calculated from the 1982 and 1984 National Long-Term Care Surveys, for males using 10 year age groupings.

1982 status	Age group	1984 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		6.97×10^{-6}	6.42×10^{-6}	3.65×10^{-6}	2.62×10^{-6}	1.26×10^{-6}	6.72×10^{-6}
	75–84		5.25×10^{-5}	4.65×10^{-5}	1.91×10^{-5}	2.01×10^{-5}	1.16×10^{-5}	3.80×10^{-5}
	85+		1.03×10^{-3}	2.30×10^{-3}	1.82×10^{-3}	8.87×10^{-4}	4.99×10^{-4}	8.90×10^{-4}
IADL only	65–74	9.45×10^{-4}		1.50×10^{-3}	7.25×10^{-4}	5.52×10^{-4}	1.22×10^{-4}	4.59×10^{-4}
	75–84	7.65×10^{-4}		2.69×10^{-3}	1.90×10^{-3}	2.01×10^{-3}	6.45×10^{-4}	9.77×10^{-4}
	85+	6.48×10^{-4}		1.14×10^{-2}	1.21×10^{-2}	6.16×10^{-3}	2.56×10^{-3}	4.32×10^{-3}
1–2 ADLs	65–74	5.59×10^{-4}	1.40×10^{-3}		4.40×10^{-3}	1.82×10^{-3}	3.70×10^{-4}	8.27×10^{-4}
	75–84	3.83×10^{-4}	1.98×10^{-3}		3.56×10^{-3}	2.36×10^{-3}	3.81×10^{-4}	1.61×10^{-3}
	85+	1.66×10^{-3}	2.34×10^{-3}		3.84×10^{-2}	2.81×10^{-2}	8.36×10^{-3}	8.35×10^{-3}
3–4 ADLs	65–74	1.41×10^{-3}	1.86×10^{-3}	1.06×10^{-2}		9.35×10^{-3}	6.67×10^{-4}	2.94×10^{-3}
	75–84	8.21×10^{-4}	4.06×10^{-3}	8.52×10^{-3}		1.90×10^{-2}	4.10×10^{-3}	7.11×10^{-3}
	85+	7.73×10^{-4}	6.76×10^{-3}	2.36×10^{-2}		1.34×10^{-1}	2.48×10^{-2}	3.85×10^{-2}
5–6 ADLs	65–74	4.76×10^{-4}	9.16×10^{-4}	2.83×10^{-3}	4.04×10^{-3}		5.99×10^{-4}	1.75×10^{-3}
	75–84	3.34×10^{-4}	1.61×10^{-3}	1.74×10^{-3}	5.52×10^{-3}		1.40×10^{-3}	3.15×10^{-3}
	85+	3.09×10^{-4}	7.82×10^{-4}	7.61×10^{-3}	2.31×10^{-2}		5.10×10^{-3}	1.53×10^{-2}
Inst'd	65–74	2.29×10^{-4}	1.19×10^{-4}	3.72×10^{-4}	2.14×10^{-4}	2.39×10^{-4}		1.20×10^{-3}
	75–84	6.74×10^{-5}	1.40×10^{-4}	8.81×10^{-5}	2.30×10^{-4}	1.03×10^{-4}		1.40×10^{-3}
	85+	2.69×10^{-5}	4.95×10^{-6}	2.27×10^{-5}	1.35×10^{-5}	6.13×10^{-6}		3.01×10^{-3}

Table J.145: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males using 10 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		5.44×10^{-6}	7.17×10^{-6}	3.35×10^{-6}	8.16×10^{-7}	6.48×10^{-7}	5.69×10^{-7}
	75–84		3.18×10^{-5}	2.18×10^{-5}	8.39×10^{-5}	4.64×10^{-5}	1.10×10^{-5}	5.33×10^{-6}
	85+		4.24×10^{-3}	2.74×10^{-3}	7.36×10^{-4}	2.85×10^{-4}	3.76×10^{-4}	1.71×10^{-4}
IADL only	65–74	3.86×10^{-4}		3.41×10^{-3}	1.76×10^{-3}	3.12×10^{-4}	1.59×10^{-4}	7.81×10^{-5}
	75–84	5.67×10^{-4}		2.57×10^{-3}	2.12×10^{-2}	1.27×10^{-2}	1.44×10^{-3}	3.16×10^{-4}
	85+	3.85×10^{-3}		1.26×10^{-1}	3.21×10^{-2}	1.23×10^{-2}	1.37×10^{-2}	5.12×10^{-3}
1–2 ADLs	65–74	3.65×10^{-4}	2.33×10^{-3}		5.87×10^{-3}	8.43×10^{-4}	2.21×10^{-4}	8.39×10^{-5}
	75–84	3.43×10^{-4}	1.78×10^{-3}		5.96×10^{-3}	3.49×10^{-3}	6.59×10^{-4}	2.16×10^{-4}
	85+	1.62×10^{-3}	8.99×10^{-2}		1.99×10^{-2}	1.09×10^{-2}	9.00×10^{-3}	3.28×10^{-3}
3–4 ADLs	65–74	5.53×10^{-4}	3.75×10^{-3}	1.77×10^{-2}		2.41×10^{-3}	7.38×10^{-4}	2.47×10^{-4}
	75–84	8.81×10^{-4}	8.44×10^{-3}	1.93×10^{-2}		1.53×10^{-1}	1.50×10^{-2}	2.46×10^{-3}
	85+	8.74×10^{-4}	1.45×10^{-3}	1.12×10^{-3}		3.19×10^{-4}	1.14×10^{-2}	5.32×10^{-3}
5–6 ADLs	65–74	1.65×10^{-4}	7.95×10^{-4}	2.63×10^{-3}	2.15×10^{-3}		3.29×10^{-4}	1.15×10^{-4}
	75–84	5.10×10^{-4}	4.51×10^{-3}	1.09×10^{-2}	1.08×10^{-1}		6.77×10^{-3}	9.95×10^{-4}
	85+	3.65×10^{-4}	4.76×10^{-4}	3.86×10^{-4}	4.63×10^{-3}		5.67×10^{-3}	1.94×10^{-3}
Inst'd	65–74	3.67×10^{-5}	1.52×10^{-4}	1.27×10^{-4}	1.12×10^{-4}	1.72×10^{-5}		1.02×10^{-4}
	75–84	6.24×10^{-5}	2.17×10^{-5}	3.92×10^{-5}	2.97×10^{-4}	1.44×10^{-4}		4.25×10^{-4}
	85+	2.23×10^{-4}	2.51×10^{-4}	1.85×10^{-4}	6.37×10^{-5}	3.87×10^{-5}		3.72×10^{-4}

Table J.146: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males using 10 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–74		1.11×10^{-5}	8.06×10^{-6}	5.12×10^{-6}	2.38×10^{-6}	1.49×10^{-6}	1.07×10^{-6}
	75–84		2.72×10^{-5}	5.48×10^{-5}	2.95×10^{-5}	1.30×10^{-5}	1.14×10^{-5}	5.17×10^{-6}
	85+		1.29×10^{-3}	8.83×10^{-3}	1.24×10^{-2}	1.38×10^{-3}	7.32×10^{-4}	2.78×10^{-4}
IADL only	65–74	1.60×10^{-3}		2.94×10^{-3}	2.11×10^{-3}	7.06×10^{-4}	3.87×10^{-4}	1.88×10^{-4}
	75–84	5.91×10^{-4}		3.76×10^{-3}	2.20×10^{-3}	7.94×10^{-4}	6.67×10^{-4}	2.34×10^{-4}
	85+	6.19×10^{-2}		1.76×10^{1}	2.55×10^{1}	1.24×10^{0}	2.18×10^{-1}	2.05×10^{-1}
1–2 ADLs	65–74	6.60×10^{-4}	1.68×10^{-3}		2.07×10^{-3}	8.11×10^{-4}	3.25×10^{-4}	9.53×10^{-5}
	75–84	6.05×10^{-4}	1.14×10^{-3}		4.81×10^{-3}	2.34×10^{-3}	1.78×10^{-3}	5.62×10^{-4}
	85+	6.20×10^{-3}	5.67×10^{-2}		5.48×10^{-1}	7.44×10^{-2}	3.35×10^{-2}	1.04×10^{-2}
3–4 ADLs	65–74	1.20×10^{-3}	2.96×10^{-3}	4.05×10^{-3}		4.92×10^{-3}	1.24×10^{-3}	5.09×10^{-4}
	75–84	1.00×10^{-3}	1.00×10^{-3}	2.21×10^{-3}		5.34×10^{-3}	3.42×10^{-3}	1.37×10^{-3}
	85+	1.12×10^{-1}	2.89×10^{0}	2.48×10^{1}		2.05×10^{0}	4.47×10^{-1}	3.08×10^{-1}
5–6 ADLs	65–74	7.28×10^{-4}	1.72×10^{-3}	2.76×10^{-3}	1.10×10^{-3}		1.75×10^{-3}	4.37×10^{-4}
	75–84	8.86×10^{-4}	2.98×10^{-4}	1.17×10^{-3}	2.36×10^{-3}		1.54×10^{-3}	6.14×10^{-4}
	85+	4.47×10^{-3}	1.93×10^{-2}	4.84×10^{-2}	9.34×10^{-2}		5.71×10^{-2}	1.96×10^{-2}
Inst'd	65–74	2.81×10^{-4}	4.81×10^{-4}	1.70×10^{-4}	1.42×10^{-4}	6.35×10^{-5}		1.65×10^{-4}
	75–84	1.08×10^{-4}	6.80×10^{-5}	3.62×10^{-4}	3.09×10^{-4}	9.23×10^{-5}		1.77×10^{-4}
	85+	5.39×10^{-4}	1.22×10^{-3}	2.64×10^{-3}	4.54×10^{-3}	6.97×10^{-4}		1.12×10^{-3}

Table J.147: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		4.63×10^{-6}	4.90×10^{-6}	2.10×10^{-6}	1.19×10^{-6}	4.17×10^{-7}	3.22×10^{-7}
	70–74		4.92×10^{-6}	3.95×10^{-6}	1.92×10^{-6}	1.16×10^{-6}	8.40×10^{-7}	4.37×10^{-7}
	75–79		9.14×10^{-6}	1.21×10^{-5}	6.08×10^{-6}	3.31×10^{-6}	2.18×10^{-6}	1.08×10^{-6}
	80–84		4.50×10^{-5}	7.82×10^{-5}	4.71×10^{-5}	1.93×10^{-5}	1.66×10^{-5}	6.54×10^{-6}
IADL only	65–69	4.16×10^{-4}		2.98×10^{-3}	1.38×10^{-3}	4.76×10^{-4}	9.65×10^{-5}	3.75×10^{-5}
	70–74	1.84×10^{-4}		6.75×10^{-4}	3.09×10^{-4}	1.78×10^{-4}	8.65×10^{-5}	3.84×10^{-5}
	75–79	1.95×10^{-4}		1.60×10^{-3}	8.36×10^{-4}	4.56×10^{-4}	1.66×10^{-4}	5.29×10^{-5}
	80–84	2.19×10^{-4}		2.61×10^{-3}	1.94×10^{-3}	6.46×10^{-4}	4.13×10^{-4}	1.20×10^{-4}
1–2 ADLs	65–69	2.42×10^{-4}	1.31×10^{-3}		1.93×10^{-3}	7.03×10^{-4}	1.10×10^{-4}	3.26×10^{-5}
	70–74	6.46×10^{-5}	1.92×10^{-4}		3.66×10^{-4}	1.65×10^{-4}	5.21×10^{-5}	2.14×10^{-5}
	75–79	1.11×10^{-4}	5.32×10^{-4}		1.31×10^{-3}	6.32×10^{-4}	2.15×10^{-4}	6.60×10^{-5}
	80–84	1.88×10^{-4}	7.86×10^{-4}		1.74×10^{-3}	4.70×10^{-4}	4.52×10^{-4}	8.50×10^{-5}
3–4 ADLs	65–69	3.88×10^{-4}	2.65×10^{-3}	7.36×10^{-3}		3.65×10^{-3}	3.20×10^{-4}	1.44×10^{-4}
	70–74	1.27×10^{-4}	3.08×10^{-4}	9.22×10^{-4}		6.74×10^{-4}	2.13×10^{-4}	9.41×10^{-5}
	75–79	1.21×10^{-4}	4.77×10^{-4}	2.05×10^{-3}		1.44×10^{-3}	5.78×10^{-4}	2.02×10^{-4}
	80–84	2.35×10^{-4}	6.09×10^{-4}	2.58×10^{-3}		1.98×10^{-3}	1.10×10^{-3}	3.39×10^{-4}
5–6 ADLs	65–69	2.95×10^{-4}	1.22×10^{-3}	3.90×10^{-3}	4.29×10^{-3}		3.83×10^{-4}	1.33×10^{-4}
	70–74	9.54×10^{-5}	5.20×10^{-4}	9.12×10^{-4}	1.27×10^{-3}		3.97×10^{-4}	6.29×10^{-5}
	75–79	8.84×10^{-5}	6.56×10^{-4}	1.82×10^{-3}	1.20×10^{-3}		4.86×10^{-4}	2.32×10^{-4}
	80–84	3.77×10^{-4}	2.84×10^{-4}	1.16×10^{-3}	2.88×10^{-3}		7.58×10^{-4}	3.07×10^{-4}
Inst'd	65–69	3.67×10^{-5}	7.65×10^{-5}	8.82×10^{-5}	8.42×10^{-5}	5.56×10^{-5}		6.35×10^{-5}
	70–74	1.93×10^{-5}	3.53×10^{-5}	4.42×10^{-5}	2.97×10^{-5}	2.25×10^{-5}		5.24×10^{-5}
	75–79	2.45×10^{-5}	1.71×10^{-5}	9.71×10^{-5}	3.30×10^{-5}	1.36×10^{-5}		1.23×10^{-4}
	80–84	1.93×10^{-5}	2.96×10^{-5}	2.02×10^{-5}	1.94×10^{-5}	5.65×10^{-6}		6.41×10^{-5}

Table J.148: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males and females using 5 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		5.82×10^{-6}	6.20×10^{-6}	5.38×10^{-6}	1.80×10^{-6}	1.09×10^{-6}	5.31×10^{-7}
	70–74		1.07×10^{-5}	1.44×10^{-5}	6.12×10^{-6}	1.96×10^{-6}	1.61×10^{-6}	7.77×10^{-7}
	75–79		2.06×10^{-5}	3.36×10^{-5}	2.33×10^{-5}	9.12×10^{-6}	5.26×10^{-6}	1.82×10^{-6}
	80–84		7.96×10^{-5}	1.52×10^{-4}	5.27×10^{-5}	1.84×10^{-5}	1.89×10^{-5}	6.45×10^{-6}
IADL only	65–69	1.07×10^{-3}		1.02×10^{-3}	1.78×10^{-3}	4.96×10^{-4}	1.66×10^{-4}	5.65×10^{-5}
	70–74	7.40×10^{-4}		4.85×10^{-3}	2.11×10^{-3}	4.03×10^{-4}	2.49×10^{-4}	1.02×10^{-4}
	75–79	6.02×10^{-4}		2.99×10^{-3}	2.33×10^{-3}	8.45×10^{-4}	2.78×10^{-4}	1.23×10^{-4}
	80–84	6.44×10^{-4}		1.78×10^{-2}	5.49×10^{-3}	1.53×10^{-3}	1.17×10^{-3}	2.84×10^{-4}
1–2 ADLs	65–69	6.18×10^{-4}	8.98×10^{-4}		2.47×10^{-3}	5.62×10^{-4}	2.27×10^{-4}	7.59×10^{-5}
	70–74	3.61×10^{-4}	1.80×10^{-3}		2.60×10^{-3}	7.41×10^{-4}	2.30×10^{-4}	7.43×10^{-5}
	75–79	3.48×10^{-4}	8.66×10^{-4}		4.00×10^{-3}	1.34×10^{-3}	4.15×10^{-4}	9.98×10^{-5}
	80–84	3.15×10^{-4}	3.02×10^{-3}		4.51×10^{-3}	1.53×10^{-3}	1.01×10^{-3}	2.51×10^{-4}
3–4 ADLs	65–69	1.08×10^{-3}	2.53×10^{-3}	2.53×10^{-3}		2.10×10^{-3}	1.09×10^{-3}	2.29×10^{-4}
	70–74	4.39×10^{-4}	1.22×10^{-3}	3.20×10^{-3}		2.09×10^{-3}	6.92×10^{-4}	2.79×10^{-4}
	75–79	4.79×10^{-4}	1.43×10^{-3}	3.58×10^{-3}		3.12×10^{-3}	1.11×10^{-3}	2.49×10^{-4}
	80–84	3.49×10^{-4}	1.55×10^{-3}	5.88×10^{-3}		2.58×10^{-3}	1.54×10^{-3}	3.68×10^{-4}
5–6 ADLs	65–69	4.04×10^{-4}	5.32×10^{-4}	1.36×10^{-3}	1.81×10^{-3}		1.21×10^{-3}	3.87×10^{-4}
	70–74	3.99×10^{-4}	1.07×10^{-3}	2.16×10^{-3}	1.80×10^{-3}		4.54×10^{-4}	3.14×10^{-4}
	75–79	6.58×10^{-4}	5.61×10^{-4}	1.71×10^{-3}	3.34×10^{-3}		1.30×10^{-3}	4.46×10^{-4}
	80–84	2.29×10^{-4}	7.00×10^{-4}	1.86×10^{-3}	2.50×10^{-3}		1.09×10^{-3}	3.86×10^{-4}
Inst'd	65–69	2.35×10^{-4}	3.32×10^{-4}	3.09×10^{-4}	1.78×10^{-4}	3.83×10^{-5}		1.25×10^{-4}
	70–74	1.21×10^{-4}	8.26×10^{-5}	1.80×10^{-4}	7.42×10^{-5}	3.20×10^{-5}		8.74×10^{-5}
	75–79	5.80×10^{-5}	2.52×10^{-5}	1.50×10^{-4}	9.82×10^{-5}	2.37×10^{-5}		1.37×10^{-4}
	80–84	4.51×10^{-5}	9.52×10^{-5}	1.47×10^{-4}	1.21×10^{-4}	7.28×10^{-5}		7.21×10^{-5}

Table J.149: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for females using 5 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		9.51×10^{-6}	7.31×10^{-6}	3.24×10^{-6}	3.06×10^{-6}	1.48×10^{-6}	6.16×10^{-7}
	70–74		1.54×10^{-5}	2.42×10^{-5}	9.31×10^{-6}	5.99×10^{-6}	3.93×10^{-6}	1.46×10^{-6}
	75–79		6.37×10^{-5}	1.53×10^{-4}	6.51×10^{-5}	2.37×10^{-5}	2.58×10^{-5}	8.15×10^{-6}
	80–84		2.28×10^{-4}	3.46×10^{-4}	2.63×10^{-4}	1.08×10^{-4}	1.39×10^{-4}	3.61×10^{-5}
IADL only	65–69	3.31×10^{-4}		9.76×10^{-4}	3.68×10^{-4}	3.28×10^{-4}	1.11×10^{-4}	4.89×10^{-5}
	70–74	2.58×10^{-4}		2.44×10^{-3}	9.44×10^{-4}	5.64×10^{-4}	2.64×10^{-4}	6.57×10^{-5}
	75–79	2.86×10^{-4}		4.76×10^{-3}	2.08×10^{-3}	5.09×10^{-4}	5.19×10^{-4}	1.34×10^{-4}
	80–84	2.29×10^{-4}		5.65×10^{-3}	3.34×10^{-3}	1.61×10^{-3}	1.09×10^{-3}	4.06×10^{-4}
1–2 ADLs	65–69	6.42×10^{-5}	2.30×10^{-4}		3.68×10^{-4}	2.32×10^{-4}	6.41×10^{-5}	2.37×10^{-5}
	70–74	1.37×10^{-4}	5.89×10^{-4}		1.33×10^{-3}	8.13×10^{-4}	3.03×10^{-4}	7.69×10^{-5}
	75–79	2.40×10^{-4}	9.94×10^{-4}		2.28×10^{-3}	4.88×10^{-4}	5.82×10^{-4}	1.06×10^{-4}
	80–84	2.20×10^{-4}	7.54×10^{-4}		3.45×10^{-3}	1.11×10^{-3}	8.63×10^{-4}	2.82×10^{-4}
3–4 ADLs	65–69	1.79×10^{-4}	2.70×10^{-4}	7.72×10^{-4}		1.13×10^{-3}	2.39×10^{-4}	1.25×10^{-4}
	70–74	1.64×10^{-4}	4.46×10^{-4}	1.57×10^{-3}		1.16×10^{-3}	6.29×10^{-4}	1.81×10^{-4}
	75–79	2.50×10^{-4}	5.51×10^{-4}	2.92×10^{-3}		1.41×10^{-3}	9.61×10^{-4}	3.23×10^{-4}
	80–84	4.96×10^{-4}	8.75×10^{-4}	3.25×10^{-3}		2.49×10^{-3}	1.70×10^{-3}	7.06×10^{-4}
5–6 ADLs	65–69	2.37×10^{-4}	1.12×10^{-3}	1.52×10^{-3}	2.65×10^{-3}		1.02×10^{-3}	1.24×10^{-4}
	70–74	1.62×10^{-4}	7.90×10^{-4}	2.63×10^{-3}	1.32×10^{-3}		7.20×10^{-4}	4.04×10^{-4}
	75–79	4.71×10^{-4}	1.76×10^{-4}	6.48×10^{-4}	1.26×10^{-3}		5.89×10^{-4}	3.64×10^{-4}
	80–84	9.86×10^{-5}	1.18×10^{-4}	6.93×10^{-4}	9.03×10^{-4}		1.16×10^{-3}	4.27×10^{-4}
Inst'd	65–69	2.13×10^{-5}	1.39×10^{-5}	5.51×10^{-5}	1.98×10^{-5}	3.84×10^{-5}		5.44×10^{-5}
	70–74	3.93×10^{-5}	2.70×10^{-5}	1.89×10^{-4}	5.44×10^{-5}	2.86×10^{-5}		1.38×10^{-4}
	75–79	1.88×10^{-5}	3.63×10^{-5}	2.89×10^{-5}	9.59×10^{-6}	3.19×10^{-6}		5.60×10^{-5}
	80–84	1.31×10^{-5}	7.39×10^{-6}	1.13×10^{-5}	7.57×10^{-6}	1.44×10^{-5}		7.40×10^{-5}

Table J.150: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for females using 5 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		1.05×10^{-5}	1.19×10^{-5}	6.02×10^{-6}	2.70×10^{-6}	2.21×10^{-6}	9.02×10^{-7}
	70–74		1.43×10^{-5}	2.80×10^{-5}	1.16×10^{-5}	3.18×10^{-6}	3.00×10^{-6}	8.14×10^{-7}
	75–79		2.89×10^{-5}	5.04×10^{-5}	3.10×10^{-5}	1.07×10^{-5}	9.02×10^{-6}	2.51×10^{-6}
	80–84		4.07×10^{-4}	5.80×10^{-4}	1.38×10^{-4}	5.99×10^{-5}	4.10×10^{-5}	1.14×10^{-5}
IADL only	65–69	1.44×10^{-3}		1.16×10^{-3}	1.22×10^{-3}	5.97×10^{-4}	2.49×10^{-4}	4.92×10^{-5}
	70–74	7.57×10^{-4}		7.33×10^{-3}	3.00×10^{-3}	5.00×10^{-4}	3.01×10^{-4}	1.56×10^{-4}
	75–79	8.74×10^{-4}		4.53×10^{-3}	3.08×10^{-3}	1.10×10^{-3}	3.75×10^{-4}	1.58×10^{-4}
	80–84	2.11×10^{-3}		5.63×10^{-2}	1.15×10^{-2}	5.02×10^{-3}	2.53×10^{-3}	6.33×10^{-4}
1–2 ADLs	65–69	8.52×10^{-4}	1.44×10^{-3}		1.99×10^{-3}	5.91×10^{-4}	3.78×10^{-4}	7.65×10^{-5}
	70–74	4.56×10^{-4}	2.04×10^{-3}		3.97×10^{-3}	8.84×10^{-4}	3.32×10^{-4}	1.04×10^{-4}
	75–79	4.35×10^{-4}	1.16×10^{-3}		4.13×10^{-3}	1.11×10^{-3}	3.61×10^{-4}	1.21×10^{-4}
	80–84	4.82×10^{-4}	7.34×10^{-3}		4.48×10^{-3}	2.30×10^{-3}	1.17×10^{-3}	2.35×10^{-4}
3–4 ADLs	65–69	1.06×10^{-3}	3.20×10^{-3}	1.34×10^{-3}		1.10×10^{-3}	1.74×10^{-3}	2.89×10^{-4}
	70–74	6.17×10^{-4}	1.20×10^{-3}	4.97×10^{-3}		1.85×10^{-3}	9.15×10^{-4}	2.79×10^{-4}
	75–79	4.71×10^{-4}	1.64×10^{-3}	4.22×10^{-3}		2.56×10^{-3}	9.84×10^{-4}	2.68×10^{-4}
	80–84	4.52×10^{-4}	3.93×10^{-3}	8.67×10^{-3}		3.64×10^{-3}	2.02×10^{-3}	2.99×10^{-4}
5–6 ADLs	65–69	4.51×10^{-4}	6.55×10^{-4}	4.29×10^{-4}	2.17×10^{-3}		1.47×10^{-3}	3.98×10^{-4}
	70–74	5.58×10^{-4}	1.05×10^{-3}	3.61×10^{-3}	3.43×10^{-3}		3.50×10^{-4}	5.15×10^{-4}
	75–79	5.87×10^{-4}	6.81×10^{-4}	2.74×10^{-3}	3.69×10^{-3}		1.48×10^{-3}	4.25×10^{-4}
	80–84	3.98×10^{-4}	2.37×10^{-3}	3.84×10^{-3}	4.18×10^{-3}		2.01×10^{-3}	7.07×10^{-4}
Inst'd	65–69	5.75×10^{-4}	2.37×10^{-4}	9.64×10^{-4}	2.74×10^{-4}	9.24×10^{-5}		2.04×10^{-4}
	70–74	1.28×10^{-4}	8.12×10^{-5}	2.53×10^{-4}	8.83×10^{-5}	3.37×10^{-5}		1.20×10^{-4}
	75–79	8.24×10^{-5}	3.17×10^{-5}	1.36×10^{-4}	1.10×10^{-4}	2.24×10^{-5}		1.94×10^{-4}
	80–84	5.53×10^{-5}	2.32×10^{-4}	2.29×10^{-4}	1.07×10^{-4}	1.35×10^{-4}		9.43×10^{-5}

Table J.151: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1984 and 1989 National Long-Term Care Surveys, for males using 5 year age groupings.

1984 status	Age group	1989 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		1.27×10^{-5}	1.62×10^{-5}	6.82×10^{-6}	2.03×10^{-6}	7.00×10^{-7}	9.90×10^{-7}
	70–74		9.62×10^{-6}	9.08×10^{-6}	5.90×10^{-6}	1.52×10^{-6}	2.30×10^{-6}	1.34×10^{-6}
	75–79		2.43×10^{-5}	1.92×10^{-5}	3.07×10^{-5}	1.43×10^{-5}	5.71×10^{-6}	4.02×10^{-6}
	80–84		1.71×10^{-4}	1.09×10^{-4}	1.29×10^{-3}	7.45×10^{-4}	1.67×10^{-4}	3.23×10^{-5}
IADL only	65–69	1.45×10^{-3}		1.12×10^{-2}	5.15×10^{-3}	9.83×10^{-4}	2.34×10^{-4}	1.65×10^{-4}
	70–74	3.90×10^{-4}		2.23×10^{-3}	1.67×10^{-3}	4.35×10^{-4}	4.12×10^{-4}	1.52×10^{-4}
	75–79	7.93×10^{-4}		3.75×10^{-3}	7.32×10^{-3}	3.82×10^{-3}	5.18×10^{-4}	2.85×10^{-4}
	80–84	1.23×10^{-3}		6.74×10^{-3}	2.58×10^{-1}	1.41×10^{-1}	2.54×10^{-2}	1.39×10^{-3}
1–2 ADLs	65–69	1.42×10^{-3}	1.20×10^{-2}		1.44×10^{-2}	2.88×10^{-3}	3.53×10^{-4}	1.76×10^{-4}
	70–74	4.59×10^{-4}	1.01×10^{-3}		3.17×10^{-3}	7.22×10^{-4}	3.24×10^{-4}	1.20×10^{-4}
	75–79	4.77×10^{-4}	2.85×10^{-3}		9.67×10^{-3}	4.44×10^{-3}	6.42×10^{-4}	3.66×10^{-4}
	80–84	9.56×10^{-4}	3.51×10^{-3}		9.15×10^{-3}	6.67×10^{-3}	2.46×10^{-3}	5.74×10^{-4}
3–4 ADLs	65–69	2.41×10^{-3}	1.74×10^{-2}	4.25×10^{-2}		8.77×10^{-3}	9.95×10^{-4}	5.57×10^{-4}
	70–74	6.29×10^{-4}	2.21×10^{-3}	1.05×10^{-2}		2.18×10^{-3}	1.52×10^{-3}	3.78×10^{-4}
	75–79	8.52×10^{-4}	7.98×10^{-3}	2.37×10^{-2}		3.11×10^{-2}	4.70×10^{-3}	2.26×10^{-3}
	80–84	2.41×10^{-3}	1.15×10^{-2}	1.96×10^{-2}		4.00×10^{-1}	8.61×10^{-2}	6.03×10^{-3}
5–6 ADLs	65–69	6.56×10^{-4}	3.18×10^{-3}	6.71×10^{-3}	5.78×10^{-3}		8.62×10^{-4}	3.46×10^{-4}
	70–74	2.09×10^{-4}	8.08×10^{-4}	2.90×10^{-3}	2.48×10^{-3}		5.66×10^{-4}	1.65×10^{-4}
	75–79	3.32×10^{-4}	2.35×10^{-3}	6.72×10^{-3}	1.01×10^{-2}		1.82×10^{-3}	6.69×10^{-4}
	80–84	4.71×10^{-3}	2.74×10^{-2}	4.87×10^{-2}	2.03×10^{0}		2.09×10^{-1}	9.88×10^{-3}
Inst'd	65–69	5.78×10^{-5}	2.86×10^{-4}	1.47×10^{-4}	5.37×10^{-5}	1.92×10^{-5}		1.31×10^{-4}
	70–74	1.08×10^{-4}	3.82×10^{-4}	2.81×10^{-4}	4.46×10^{-4}	5.58×10^{-5}		3.38×10^{-4}
	75–79	7.15×10^{-5}	3.12×10^{-5}	7.96×10^{-5}	9.63×10^{-5}	3.38×10^{-5}		6.33×10^{-4}
	80–84	2.77×10^{-4}	1.04×10^{-4}	1.42×10^{-4}	2.11×10^{-3}	9.31×10^{-4}		1.37×10^{-3}

Table J.152: Variance estimates of the constrained (positive) MLEs of the annual transition intensities between disability states calculated from the 1989 and 1994 National Long-Term Care Surveys, for males using 5 year age groupings.

1989 status	Age group	1994 Status: Healthy	IADL only	1–2 ADLs	3–4 ADLs	5–6 ADLs	Inst'd	Dead
Healthy	65–69		1.11×10^{-5}	9.41×10^{-6}	7.12×10^{-6}	3.72×10^{-6}	2.07×10^{-6}	1.36×10^{-6}
	70–74		3.86×10^{-5}	2.49×10^{-5}	1.12×10^{-5}	5.76×10^{-6}	3.97×10^{-6}	2.96×10^{-6}
	75–79		5.47×10^{-5}	5.85×10^{-5}	3.82×10^{-5}	3.33×10^{-5}	1.44×10^{-5}	6.35×10^{-6}
	80–84		6.95×10^{-5}	3.50×10^{-4}	1.82×10^{-4}	3.01×10^{-5}	5.68×10^{-5}	3.85×10^{-5}
IADL only	65–69	3.17×10^{-3}		3.05×10^{-3}	3.97×10^{-3}	1.15×10^{-3}	4.35×10^{-4}	5.97×10^{-4}
	70–74	3.49×10^{-3}		8.58×10^{-3}	3.50×10^{-3}	1.54×10^{-3}	9.86×10^{-4}	2.76×10^{-4}
	75–79	1.69×10^{-3}		4.41×10^{-3}	2.86×10^{-3}	2.43×10^{-3}	9.86×10^{-4}	4.62×10^{-4}
	80–84	4.80×10^{-4}		1.45×10^{-2}	6.46×10^{-3}	9.92×10^{-4}	1.78×10^{-3}	9.53×10^{-4}
1–2 ADLs	65–69	1.55×10^{-3}	1.09×10^{-3}		3.32×10^{-3}	9.72×10^{-4}	6.83×10^{-4}	2.87×10^{-4}
	70–74	1.27×10^{-3}	5.32×10^{-3}		3.19×10^{-3}	1.68×10^{-3}	6.24×10^{-4}	1.96×10^{-4}
	75–79	1.08×10^{-3}	1.30×10^{-3}		4.65×10^{-3}	5.61×10^{-3}	2.30×10^{-3}	4.11×10^{-4}
	80–84	1.86×10^{-3}	5.52×10^{-3}		2.82×10^{-2}	4.34×10^{-3}	6.82×10^{-3}	5.34×10^{-3}
3–4 ADLs	65–69	2.97×10^{-3}	3.53×10^{-3}	9.01×10^{-3}		7.80×10^{-3}	1.45×10^{-3}	5.01×10^{-4}
	70–74	1.59×10^{-3}	7.69×10^{-3}	4.34×10^{-3}		9.20×10^{-3}	3.08×10^{-3}	1.29×10^{-3}
	75–79	2.72×10^{-3}	4.10×10^{-3}	3.89×10^{-3}		1.76×10^{-2}	8.45×10^{-3}	1.44×10^{-3}
	80–84	1.54×10^{-3}	4.84×10^{-4}	5.11×10^{-3}		5.17×10^{-3}	3.76×10^{-3}	4.86×10^{-3}
5–6 ADLs	65–69	1.72×10^{-3}	1.50×10^{-3}	9.30×10^{-3}	3.05×10^{-3}		3.49×10^{-3}	1.22×10^{-3}
	70–74	1.47×10^{-3}	5.70×10^{-3}	3.17×10^{-3}	1.01×10^{-3}		3.22×10^{-3}	6.57×10^{-4}
	75–79	3.91×10^{-3}	1.68×10^{-3}	2.24×10^{-3}	6.98×10^{-3}		5.55×10^{-3}	2.23×10^{-3}
	80–84	5.78×10^{-4}	3.64×10^{-4}	3.96×10^{-3}	3.04×10^{-3}		1.48×10^{-3}	5.95×10^{-4}
Inst'd	65–69	2.63×10^{-4}	1.10×10^{-3}	1.51×10^{-4}	1.96×10^{-4}	3.78×10^{-5}		3.17×10^{-4}
	70–74	7.10×10^{-4}	5.08×10^{-4}	4.95×10^{-4}	3.27×10^{-4}	1.80×10^{-4}		3.16×10^{-4}
	75–79	1.64×10^{-4}	1.27×10^{-4}	7.58×10^{-4}	2.11×10^{-4}	2.32×10^{-4}		3.65×10^{-4}
	80–84	2.67×10^{-4}	1.09×10^{-4}	4.16×10^{-4}	8.83×10^{-4}	1.23×10^{-4}		3.41×10^{-4}

Appendix K

Tables of parameter values for the parametric transition intensities fitted to data grouped in 10-year age bands, calculated from the 1982, 1984, 1989 and 1994 NLTCS

The tables in this appendix give the parameter values for the parametric transition intensities ($\mathring{\mu}^{ij}_{x+t}$) fitted to the NLTCS in 10-year age bands, where, for $65 \leq x+t \leq 120$:

$$\mathring{\mu}^{ij}_{x+t} = \begin{cases} A_{ij} + B_{ij}\, e^{C_{ij}((x-(70+M))+t)} & \text{if } (\overline{\mu}^{ij}_{70+M} - \overline{\mu}^{ij}_{80+M}) < (\overline{\mu}^{ij}_{80+M} - \overline{\mu}^{ij}_{90+M})\ , \\ & \overline{\mu}^{ij}_{70+M} \neq \overline{\mu}^{ij}_{80+M} \neq \overline{\mu}^{ij}_{90+M} \text{ and } |C_{ij}| < 0.5 \\ A_{ij} + D_{ij}\,(x+t) & \text{otherwise} \end{cases}$$

and with a lower bound of zero on all intensities at all ages and where $M = 1$ for the 1982–84 NLTCS and $M = 2.5$ for the 1984-89 and 1989–94 NLTCS (see Section 5.5 for more details). Tables K.153 and K.154 give the parameter values for the 1982–84 NLTCS for males and females—in aggregate the values are given in Table 5.65. The values for males, female and in aggregate for the 1984–89 and 1989–94 NLTCS are given in Tables K.155 to K.160.

Table K.153: Parameter values for the parametric transition intensities for males, calculated from the 1982 and 1984 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	1.95×10^{-4}	2.03×10^{-2}	8.63×10^{-2}	-
	1–2 ADLs	7.89×10^{-3}	4.94×10^{-3}	1.66×10^{-1}	-
	3–4 ADLs	-1.82×10^{-3}	5.45×10^{-3}	-5.47×10^{-2}	-
	5–6 ADLs	-7.80×10^{-3}	-	-	1.70×10^{-4}
	Inst'd	-4.66×10^{-2}	-	-	7.20×10^{-4}
	Dead	1.43×10^{-2}	2.26×10^{-2}	7.80×10^{-2}	-
IADL only	Healthy	1.20×10^{0}	-	-	-1.31×10^{-2}
	1–2 ADLs	-1.82×10^{-1}	-	-	5.02×10^{-3}
	3–4 ADLs	-3.24×10^{-1}	-	-	4.58×10^{-3}
	5–6 ADLs	-1.71×10^{-2}	-	-	1.07×10^{-3}
	Inst'd	-1.81×10^{-1}	-	-	2.79×10^{-3}
	Dead	8.06×10^{-2}	-	-	1.77×10^{-4}
1–2 ADLs	Healthy	-6.97×10^{-3}	-	-	7.46×10^{-4}
	IADL only	5.06×10^{-1}	-	-	-4.80×10^{-3}
	3–4 ADLs	4.14×10^{-1}	-	-	-2.50×10^{-3}
	5–6 ADLs	-8.67×10^{-2}	-	-	2.00×10^{-3}
	Inst'd	2.32×10^{-1}	-	-	-2.62×10^{-3}
	Dead	-3.17×10^{-1}	-	-	6.19×10^{-3}
3–4 ADLs	Healthy	-6.78×10^{-3}	9.58×10^{-2}	-1.32×10^{-1}	-
	IADL only	-1.58×10^{-1}	1.58×10^{-1}	2.18×10^{-2}	-
	1–2 ADLs	-2.31×10^{1}	2.34×10^{1}	-5.85×10^{-4}	-
	5–6 ADLs	2.91×10^{-1}	2.00×10^{-2}	1.53×10^{-1}	-
	Inst'd	-1.16×10^{0}	1.17×10^{0}	1.01×10^{-2}	-
	Dead	1.13×10^{-2}	-	-	2.38×10^{-3}
5–6 ADLs	Healthy	1.57×10^{-1}	-	-	-1.71×10^{-3}
	IADL only	2.41×10^{-1}	-	-	-2.55×10^{-3}
	1–2 ADLs	3.20×10^{-1}	-	-	-3.18×10^{-3}
	3–4 ADLs	1.78×10^{-1}	-	-	-5.49×10^{-5}
	Inst'd	7.40×10^{-3}	-	-	8.57×10^{-4}
	Dead	1.84×10^{-1}	6.38×10^{-2}	7.88×10^{-2}	-
Inst'd	Healthy	-1.91×10^{-3}	3.73×10^{-2}	-1.03×10^{-1}	-
	IADL only	4.37×10^{-2}	-	-	-4.79×10^{-4}
	1–2 ADLs	6.59×10^{-4}	2.58×10^{-2}	-2.58×10^{-1}	-
	3–4 ADLs	4.32×10^{-2}	-	-	-4.70×10^{-4}
	5–6 ADLs	-7.43×10^{-5}	1.72×10^{-2}	-2.54×10^{-1}	-
	Dead	-1.25×10^{-1}	3.46×10^{-1}	2.80×10^{-2}	-

Table K.154: Parameter values for the parametric transition intensities for females, calculated from the 1982 and 1984 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-4.25×10^{-2}	7.04×10^{-2}	3.70×10^{-2}	-
	1–2 ADLs	2.18×10^{-3}	1.04×10^{-2}	1.11×10^{-1}	-
	3–4 ADLs	-4.12×10^{-2}	-	-	6.32×10^{-4}
	5–6 ADLs	4.17×10^{-3}	9.64×10^{-5}	2.89×10^{-1}	-
	Inst'd	-1.67×10^{-3}	4.67×10^{-3}	1.39×10^{-1}	-
	Dead	-7.85×10^{-2}	9.43×10^{-2}	2.01×10^{-2}	-
IADL only	Healthy	-3.04×10^{-1}	5.25×10^{-1}	-2.51×10^{-2}	-
	1–2 ADLs	-3.72×10^{-1}	-	-	9.41×10^{-3}
	3–4 ADLs	2.69×10^{-1}	-	-	-3.18×10^{-3}
	5–6 ADLs	8.29×10^{-3}	7.20×10^{-3}	1.24×10^{-1}	-
	Inst'd	7.40×10^{-3}	2.70×10^{-2}	5.24×10^{-2}	-
	Dead	-1.68×10^{-1}	-	-	2.93×10^{-3}
1–2 ADLs	Healthy	2.42×10^{-1}	-	-	-2.26×10^{-3}
	IADL only	6.08×10^{-1}	-	-	-5.25×10^{-3}
	3–4 ADLs	-3.69×10^{-1}	-	-	7.30×10^{-3}
	5–6 ADLs	1.66×10^{-1}	-	-	-1.88×10^{-3}
	Inst'd	-2.01×10^{-1}	-	-	3.36×10^{-3}
	Dead	-2.55×10^{-3}	-	-	1.09×10^{-3}
3–4 ADLs	Healthy	-9.79×10^{-3}	-	-	1.60×10^{-4}
	IADL only	-2.48×10^{-1}	-	-	3.32×10^{-3}
	1–2 ADLs	1.81×10^{0}	-	-	-1.90×10^{-2}
	5–6 ADLs	-6.34×10^{-1}	8.96×10^{-1}	1.43×10^{-2}	-
	Inst'd	-1.64×10^{-1}	1.91×10^{-1}	2.80×10^{-2}	-
	Dead	-1.24×10^{-1}	-	-	2.11×10^{-3}
5–6 ADLs	Healthy	9.16×10^{-2}	-	-	-7.95×10^{-4}
	IADL only	3.35×10^{-1}	-	-	-3.67×10^{-3}
	1–2 ADLs	-9.66×10^{-2}	-	-	1.86×10^{-3}
	3–4 ADLs	1.44×10^{-1}	-	-	2.24×10^{-4}
	Inst'd	-1.88×10^{-1}	-	-	3.79×10^{-3}
	Dead	-1.05×10^{-1}	-	-	4.31×10^{-3}
Inst'd	Healthy	4.29×10^{-3}	2.43×10^{-2}	-2.08×10^{-1}	-
	IADL only	2.69×10^{-2}	-	-	-2.62×10^{-4}
	1–2 ADLs	1.22×10^{-2}	-	-	-1.33×10^{-4}
	3–4 ADLs	1.71×10^{-2}	-	-	-8.91×10^{-5}
	5–6 ADLs	2.91×10^{-2}	-	-	-2.70×10^{-4}
	Dead	2.23×10^{-2}	1.21×10^{-1}	4.30×10^{-2}	-

Table K.155: Parameter values for the parametric transition intensities for males, calculated from the 1984 and 1989 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	1.16×10^{-2}	3.90×10^{-3}	1.54×10^{-1}	-
	1–2 ADLs	-5.02×10^{-2}	-	-	8.23×10^{-4}
	3–4 ADLs	-3.44×10^{-2}	-	-	5.02×10^{-4}
	5–6 ADLs	2.20×10^{-3}	-	-	-1.47×10^{-5}
	Inst'd	-9.83×10^{-3}	1.30×10^{-2}	5.57×10^{-2}	-
	Dead	-2.99×10^{-1}	3.03×10^{-1}	2.23×10^{-3}	-
IADL only	Healthy	2.45×10^{-1}	-	-	-2.06×10^{-3}
	1–2 ADLs	6.75×10^{-1}	-	-	-7.21×10^{-3}
	3–4 ADLs	-7.18×10^{-1}	-	-	9.97×10^{-3}
	5–6 ADLs	-1.29×10^{-1}	-	-	2.06×10^{-3}
	Inst'd	1.86×10^{-1}	-	-	-2.12×10^{-3}
	Dead	1.93×10^{-2}	7.96×10^{-6}	4.17×10^{-1}	-
1–2 ADLs	Healthy	1.66×10^{-1}	-	-	-1.64×10^{-3}
	IADL only	1.30×10^{-1}	-	-	-2.83×10^{-4}
	3–4 ADLs	4.87×10^{-1}	-	-	-4.76×10^{-3}
	5–6 ADLs	-1.69×10^{-1}	-	-	2.81×10^{-3}
	Inst'd	-1.87×10^{-1}	-	-	2.67×10^{-3}
	Dead	-8.52×10^{-2}	-	-	1.26×10^{-3}
3–4 ADLs	Healthy	5.88×10^{-2}	-	-	-6.60×10^{-4}
	IADL only	-4.06×10^{-8}	4.43×10^{-8}	1.07×10^{-1}	-
	1–2 ADLs	-1.07×10^{-1}	3.56×10^{-1}	-5.98×10^{-2}	-
	5–6 ADLs	4.14×10^{-1}	-	-	-4.45×10^{-3}
	Inst'd	-5.22×10^{-1}	-	-	7.90×10^{-3}
	Dead	9.14×10^{-3}	7.63×10^{-3}	1.24×10^{-1}	-
5–6 ADLs	Healthy	3.85×10^{-2}	-	-	-4.29×10^{-4}
	IADL only	9.41×10^{-2}	-	-	-1.02×10^{-3}
	1–2 ADLs	3.20×10^{-1}	-	-	-3.44×10^{-3}
	3–4 ADLs	4.53×10^{-2}	-	-	2.49×10^{-4}
	Inst'd	-1.82×10^{-1}	-	-	3.04×10^{-3}
	Dead	-1.04×10^{-2}	-	-	3.10×10^{-4}
Inst'd	Healthy	-3.59×10^{-2}	4.06×10^{-2}	1.30×10^{-2}	-
	IADL only	7.86×10^{-2}	-	-	-9.36×10^{-4}
	1–2 ADLs	-8.65×10^{-5}	8.66×10^{-5}	1.84×10^{-1}	-
	3–4 ADLs	3.28×10^{-2}	-	-	-3.39×10^{-4}
	5–6 ADLs	-3.44×10^{-3}	-	-	4.71×10^{-5}
	Dead	-7.66×10^{-2}	-	-	1.49×10^{-3}

Table K.156: Parameter values for the parametric transition intensities for females, calculated from the 1984 and 1989 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	1.31×10^{-2}	5.58×10^{-3}	9.48×10^{-2}	-
	1–2 ADLs	-1.65×10^{-1}	-	-	2.40×10^{-3}
	3–4 ADLs	3.16×10^{-3}	1.39×10^{-4}	2.68×10^{-1}	-
	5–6 ADLs	-1.81×10^{-2}	-	-	3.08×10^{-4}
	Inst'd	2.86×10^{-3}	1.45×10^{-3}	1.77×10^{-1}	-
	Dead	-2.13×10^{-2}	-	-	3.26×10^{-4}
IADL only	Healthy	-1.91×10^{-2}	1.21×10^{-1}	-7.33×10^{-2}	-
	1–2 ADLs	-9.61×10^{-3}	-	-	2.65×10^{-3}
	3–4 ADLs	-5.68×10^{-2}	-	-	8.11×10^{-4}
	5–6 ADLs	2.08×10^{-2}	2.09×10^{-4}	2.82×10^{-1}	-
	Inst'd	2.59×10^{-3}	1.42×10^{-2}	5.99×10^{-2}	-
	Dead	7.97×10^{-3}	1.06×10^{-3}	1.63×10^{-1}	-
1–2 ADLs	Healthy	1.24×10^{-1}	-	-	-1.21×10^{-3}
	IADL only	4.67×10^{-2}	-	-	4.38×10^{-5}
	3–4 ADLs	-2.52×10^{-1}	-	-	4.69×10^{-3}
	5–6 ADLs	-7.70×10^{-2}	-	-	1.31×10^{-3}
	Inst'd	-2.40×10^{-1}	-	-	3.63×10^{-3}
	Dead	6.94×10^{-3}	-	-	-3.86×10^{-5}
3–4 ADLs	Healthy	-3.49×10^{-2}	-	-	5.57×10^{-4}
	IADL only	1.34×10^{-1}	-	-	-1.44×10^{-3}
	1–2 ADLs	7.54×10^{-2}	1.29×10^{-2}	-1.01×10^{-1}	-
	5–6 ADLs	1.56×10^{-1}	-	-	-7.28×10^{-4}
	Inst'd	-3.31×10^{-1}	-	-	4.84×10^{-3}
	Dead	2.08×10^{-2}	4.38×10^{-4}	2.08×10^{-1}	-
5–6 ADLs	Healthy	4.69×10^{-2}	-	-	-4.29×10^{-4}
	IADL only	5.27×10^{-2}	-	-	-5.66×10^{-4}
	1–2 ADLs	-2.06×10^{-2}	1.05×10^{-1}	-4.08×10^{-2}	-
	3–4 ADLs	3.14×10^{-2}	1.12×10^{-1}	-2.16×10^{-1}	-
	Inst'd	-9.05×10^{-2}	-	-	1.99×10^{-3}
	Dead	-1.69×10^{-1}	-	-	2.42×10^{-3}
Inst'd	Healthy	1.34×10^{-2}	-	-	-8.55×10^{-5}
	IADL only	2.53×10^{-5}	-	-	1.98×10^{-5}
	1–2 ADLs	4.30×10^{-2}	-	-	-4.50×10^{-4}
	3–4 ADLs	2.42×10^{-3}	-	-	-2.51×10^{-5}
	5–6 ADLs	-9.74×10^{-3}	-	-	1.30×10^{-4}
	Dead	1.47×10^{-2}	4.78×10^{-3}	1.06×10^{-1}	-

Table K.157: Parameter values for the parametric transition intensities for males and females, calculated from the 1984 and 1989 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	1.31×10^{-2}	4.45×10^{-3}	1.22×10^{-1}	-
	1–2 ADLs	-1.23×10^{-1}	-	-	1.82×10^{-3}
	3–4 ADLs	2.52×10^{-3}	5.37×10^{-4}	1.91×10^{-1}	-
	5–6 ADLs	-2.10×10^{-2}	-	-	3.28×10^{-4}
	Inst'd	1.50×10^{-3}	2.35×10^{-3}	1.46×10^{-1}	-
	Dead	-3.18×10^{-2}	-	-	4.83×10^{-4}
IADL only	Healthy	-1.22×10^{-2}	1.12×10^{-1}	-6.38×10^{-2}	-
	1–2 ADLs	1.76×10^{-1}	2.79×10^{-4}	2.85×10^{-1}	-
	3–4 ADLs	-1.87×10^{-1}	-	-	2.62×10^{-3}
	5–6 ADLs	-8.88×10^{-3}	2.94×10^{-2}	5.19×10^{-2}	-
	Inst'd	2.23×10^{-2}	1.87×10^{-4}	2.59×10^{-1}	-
	Dead	1.25×10^{-2}	7.16×10^{-5}	3.04×10^{-1}	-
1–2 ADLs	Healthy	1.34×10^{-1}	-	-	-1.31×10^{-3}
	IADL only	5.24×10^{-2}	-	-	2.08×10^{-4}
	3–4 ADLs	6.46×10^{-2}	3.92×10^{-2}	5.92×10^{-2}	-
	5–6 ADLs	2.25×10^{-2}	1.16×10^{-3}	1.59×10^{-1}	-
	Inst'd	-2.25×10^{-1}	-	-	3.37×10^{-3}
	Dead	1.60×10^{-3}	-	-	4.54×10^{-5}
3–4 ADLs	Healthy	-2.88×10^{-3}	-	-	1.38×10^{-4}
	IADL only	1.05×10^{-1}	-	-	-1.18×10^{-3}
	1–2 ADLs	6.65×10^{-2}	6.66×10^{-2}	-7.53×10^{-2}	-
	5–6 ADLs	1.24×10^{-1}	-	-	-2.24×10^{-4}
	Inst'd	-3.44×10^{-1}	-	-	5.13×10^{-3}
	Dead	1.54×10^{-2}	4.38×10^{-3}	1.13×10^{-1}	-
5–6 ADLs	Healthy	3.38×10^{-2}	-	-	-3.13×10^{-4}
	IADL only	8.72×10^{-2}	-	-	-9.36×10^{-4}
	1–2 ADLs	2.62×10^{-1}	-	-	-2.58×10^{-3}
	3–4 ADLs	-2.86×10^{-2}	1.37×10^{-1}	-3.31×10^{-2}	-
	Inst'd	5.39×10^{-2}	3.95×10^{-5}	3.75×10^{-1}	-
	Dead	-1.26×10^{-1}	-	-	1.86×10^{-3}
Inst'd	Healthy	6.67×10^{-3}	1.27×10^{-5}	2.15×10^{-1}	-
	IADL only	-1.58×10^{-2}	2.21×10^{-2}	-1.45×10^{-2}	-
	1–2 ADLs	2.36×10^{-2}	-	-	-2.30×10^{-4}
	3–4 ADLs	1.59×10^{-4}	4.37×10^{-3}	-2.44×10^{-1}	-
	5–6 ADLs	-4.64×10^{-3}	-	-	7.11×10^{-5}
	Dead	-9.34×10^{-2}	-	-	1.60×10^{-3}

Table K.158: Parameter values for the parametric transition intensities for males, calculated from the 1989 and 1994 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-6.62×10^{-2}	-	-	1.16×10^{-3}
	1–2 ADLs	-4.25×10^{-2}	5.45×10^{-2}	3.04×10^{-2}	-
	3–4 ADLs	-9.58×10^{-3}	-	-	1.83×10^{-4}
	5–6 ADLs	3.35×10^{-3}	2.12×10^{-4}	2.22×10^{-1}	-
	Inst'd	3.45×10^{-3}	2.82×10^{-4}	2.71×10^{-1}	-
	Dead	3.47×10^{-3}	2.14×10^{-3}	1.02×10^{-1}	-
IADL only	Healthy	-8.16×10^{-2}	2.45×10^{-1}	-5.50×10^{-2}	-
	1–2 ADLs	-1.61×10^{-2}	-	-	2.03×10^{-3}
	3–4 ADLs	2.85×10^{-2}	-	-	2.05×10^{-4}
	5–6 ADLs	-1.05×10^{-8}	1.36×10^{-8}	1.62×10^{-1}	-
	Inst'd	-2.65×10^{-2}	5.43×10^{-2}	-3.58×10^{-2}	-
	Dead	8.38×10^{-2}	-	-	-8.78×10^{-4}
1–2 ADLs	Healthy	1.79×10^{-1}	-	-	-1.74×10^{-3}
	IADL only	-8.74×10^{-2}	1.61×10^{-1}	-1.82×10^{-2}	-
	3–4 ADLs	-5.04×10^{-2}	-	-	2.17×10^{-3}
	5–6 ADLs	-6.43×10^{-2}	6.43×10^{-2}	7.14×10^{-2}	-
	Inst'd	-3.52×10^{-1}	-	-	5.32×10^{-3}
	Dead	-1.83×10^{-1}	-	-	2.56×10^{-3}
3–4 ADLs	Healthy	-1.54×10^{-1}	1.96×10^{-1}	-1.20×10^{-2}	-
	IADL only	1.03×10^{-1}	-	-	-1.01×10^{-3}
	1–2 ADLs	4.97×10^{-1}	-	-	-6.02×10^{-3}
	5–6 ADLs	5.04×10^{-1}	-	-	-4.44×10^{-3}
	Inst'd	-8.14×10^{-1}	-	-	1.14×10^{-2}
	Dead	-2.38×10^{-1}	-	-	3.70×10^{-3}
5–6 ADLs	Healthy	-6.00×10^{-2}	-	-	1.09×10^{-3}
	IADL only	1.10×10^{-1}	-	-	-1.33×10^{-3}
	1–2 ADLs	-6.04×10^{-3}	7.30×10^{-2}	-1.25×10^{-1}	-
	3–4 ADLs	-3.17×10^{-1}	-	-	4.38×10^{-3}
	Inst'd	4.43×10^{-2}	-	-	4.60×10^{-4}
	Dead	2.02×10^{-2}	1.65×10^{-3}	2.06×10^{-1}	-
Inst'd	Healthy	3.66×10^{-2}	-	-	-2.68×10^{-4}
	IADL only	8.84×10^{-2}	-	-	-1.02×10^{-3}
	1–2 ADLs	-6.62×10^{-2}	-	-	9.53×10^{-4}
	3–4 ADLs	-4.52×10^{-2}	-	-	6.79×10^{-4}
	5–6 ADLs	2.46×10^{-3}	-	-	-1.53×10^{-5}
	Dead	2.05×10^{-2}	1.22×10^{-4}	3.04×10^{-1}	-

Table K.159: Parameter values for the parametric transition intensities for females, calculated from the 1989 and 1994 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-1.42×10^{-1}	-	-	2.16×10^{-3}
	1–2 ADLs	1.30×10^{-2}	9.14×10^{-3}	9.09×10^{-2}	-
	3–4 ADLs	-3.99×10^{-2}	-	-	5.50×10^{-4}
	5–6 ADLs	2.45×10^{-3}	3.17×10^{-4}	2.52×10^{-1}	-
	Inst'd	1.26×10^{-3}	6.09×10^{-3}	1.13×10^{-1}	-
	Dead	-1.41×10^{-2}	-	-	2.26×10^{-4}
IADL only	Healthy	4.49×10^{-1}	-	-	-4.51×10^{-3}
	1–2 ADLs	-4.13×10^{-1}	-	-	7.85×10^{-3}
	3–4 ADLs	1.00×10^{-1}	-	-	-9.93×10^{-4}
	5–6 ADLs	-6.14×10^{-2}	-	-	1.07×10^{-3}
	Inst'd	-1.35×10^{-1}	-	-	1.98×10^{-3}
	Dead	-5.55×10^{-2}	-	-	8.57×10^{-4}
1–2 ADLs	Healthy	1.30×10^{-1}	-	-	-1.09×10^{-3}
	IADL only	1.19×10^{-1}	-	-	-5.25×10^{-4}
	3–4 ADLs	1.18×10^{-1}	2.35×10^{-2}	1.07×10^{-1}	-
	5–6 ADLs	1.09×10^{-1}	-	-	-9.19×10^{-4}
	Inst'd	-6.54×10^{-2}	6.54×10^{-2}	4.70×10^{-2}	-
	Dead	-7.16×10^{-2}	-	-	1.05×10^{-3}
3–4 ADLs	Healthy	9.54×10^{-2}	-	-	-1.02×10^{-3}
	IADL only	4.20×10^{-2}	-	-	-2.03×10^{-4}
	1–2 ADLs	-2.06×10^{-3}	8.04×10^{-2}	4.34×10^{-2}	-
	5–6 ADLs	-1.88×10^{-1}	2.50×10^{-1}	2.22×10^{-2}	-
	Inst'd	7.95×10^{-2}	8.85×10^{-4}	2.35×10^{-1}	-
	Dead	2.86×10^{-2}	-	-	-2.20×10^{-4}
5–6 ADLs	Healthy	1.72×10^{-2}	3.66×10^{-6}	4.50×10^{-1}	-
	IADL only	3.44×10^{-2}	-	-	-3.62×10^{-4}
	1–2 ADLs	4.35×10^{-2}	-	-	-2.62×10^{-4}
	3–4 ADLs	4.59×10^{-2}	-	-	4.54×10^{-4}
	Inst'd	-2.35×10^{-1}	-	-	3.75×10^{-3}
	Dead	1.20×10^{-2}	-	-	3.17×10^{-4}
Inst'd	Healthy	3.99×10^{-2}	-	-	-3.21×10^{-4}
	IADL only	8.16×10^{-3}	-	-	-6.29×10^{-5}
	1–2 ADLs	3.90×10^{-4}	1.92×10^{-2}	-1.11×10^{-1}	-
	3–4 ADLs	-1.21×10^{-3}	-	-	3.98×10^{-5}
	5–6 ADLs	9.81×10^{-3}	-	-	-9.73×10^{-5}
	Dead	-3.68×10^{-3}	-	-	3.90×10^{-4}

Table K.160: Parameter values for the parametric transition intensities for males and females, calculated from the 1989 and 1994 NLTCS, grouped in 10-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-1.11×10^{-1}	-	-	1.75×10^{-3}
	1–2 ADLs	6.33×10^{-3}	1.18×10^{-2}	8.00×10^{-2}	-
	3–4 ADLs	-3.43×10^{-2}	-	-	4.83×10^{-4}
	5–6 ADLs	3.40×10^{-3}	2.36×10^{-4}	2.54×10^{-1}	-
	Inst'd	2.75×10^{-3}	3.19×10^{-3}	1.42×10^{-1}	-
	Dead	1.73×10^{-3}	2.06×10^{-3}	8.86×10^{-2}	-
IADL only	Healthy	5.82×10^{-1}	-	-	-6.17×10^{-3}
	1–2 ADLs	-4.35×10^{-1}	-	-	8.02×10^{-3}
	3–4 ADLs	2.10×10^{-1}	-	-	-2.46×10^{-3}
	5–6 ADLs	-1.04×10^{-1}	-	-	1.57×10^{-3}
	Inst'd	-5.72×10^{-2}	-	-	1.01×10^{-3}
	Dead	-2.09×10^{-2}	-	-	4.49×10^{-4}
1–2 ADLs	Healthy	1.39×10^{-1}	-	-	-1.21×10^{-3}
	IADL only	1.82×10^{-1}	-	-	-1.41×10^{-3}
	3–4 ADLs	1.47×10^{-1}	8.04×10^{-3}	1.57×10^{-1}	-
	5–6 ADLs	-1.10×10^{-1}	-	-	1.76×10^{-3}
	Inst'd	-1.39×10^{-1}	1.44×10^{-1}	2.75×10^{-2}	-
	Dead	-9.87×10^{-2}	-	-	1.41×10^{-3}
3–4 ADLs	Healthy	1.48×10^{-1}	-	-	-1.65×10^{-3}
	IADL only	2.04×10^{-2}	-	-	9.92×10^{-5}
	1–2 ADLs	7.79×10^{-2}	1.62×10^{-3}	2.44×10^{-1}	-
	5–6 ADLs	1.10×10^{-1}	3.59×10^{-3}	1.64×10^{-1}	-
	Inst'd	-2.64×10^{-2}	9.28×10^{-2}	2.82×10^{-2}	-
	Dead	1.95×10^{-2}	-	-	-1.37×10^{-5}
5–6 ADLs	Healthy	-6.53×10^{-2}	-	-	1.11×10^{-3}
	IADL only	3.99×10^{-2}	-	-	-4.45×10^{-4}
	1–2 ADLs	1.97×10^{-1}	-	-	-2.11×10^{-3}
	3–4 ADLs	-1.28×10^{-1}	-	-	2.47×10^{-3}
	Inst'd	-1.62×10^{-1}	-	-	2.98×10^{-3}
	Dead	-2.63×10^{-2}	-	-	7.81×10^{-4}
Inst'd	Healthy	3.98×10^{-2}	-	-	-3.16×10^{-4}
	IADL only	3.61×10^{-3}	4.68×10^{-3}	-2.82×10^{-1}	-
	1–2 ADLs	5.60×10^{-2}	-	-	-5.67×10^{-4}
	3–4 ADLs	-8.86×10^{-3}	-	-	1.45×10^{-4}
	5–6 ADLs	9.44×10^{-3}	-	-	-9.00×10^{-5}
	Dead	-2.84×10^{-2}	-	-	7.03×10^{-4}

Appendix L

Tables of parameter values for the parametric transition intensities fitted to data grouped in 5-year age bands, calculated from the 1982, 1984, 1989 and 1994 NLTCS

The tables in this appendix give the parameter values for the parametric transition intensities ($\overset{\circ}{\mu}{}^{ij}_{x+t}$) fitted to the NLTCS in 5-year age bands. The method used is, for each transition intensity, fit both a staight line and a Makeham curve using weighted least squares, the weights (w^{ij}_{x+t}) being the inverse of the variance (i.e. $w^{ij}_{x+t} = 1/\text{Var}[\overline{\mu}^{ij}_{x+t}]$). Then chose the parametric form which provides the best fit, in terms of the smallest sum of weighted squared residuals (see Section 5.6 for more details). Tables L.161 and L.162 give the parameter values for the 1982–84 NLTCS for males and females—in aggregate the values are given in Table 5.67. The values for males, female and in aggregate for the 1984–89 and 1989–94 NLTCS are given in Tables L.163 to L.168.

Table L.161: Parameter values for the parametric transition intensities for males, calculated from the 1982 and 1984 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	1.06×10^{-2}	7.88×10^{-3}	1.18×10^{-1}	-
	1–2 ADLs	9.45×10^{-4}	8.20×10^{-3}	1.23×10^{-1}	-
	3–4 ADLs	-2.01×10^{-3}	6.66×10^{-3}	-4.27×10^{-2}	-
	5–6 ADLs	-2.66×10^{-2}	-	-	4.30×10^{-4}
	Inst'd	-4.63×10^{-3}	8.01×10^{-3}	5.54×10^{-2}	-
	Dead	-2.36×10^{-1}	-	-	3.83×10^{-3}
IADL only	Healthy	1.14×10^{0}	-	-	-1.24×10^{-2}
	1–2 ADLs	7.33×10^{-2}	9.38×10^{-2}	3.13×10^{-2}	-
	3–4 ADLs	-3.11×10^{-1}	-	-	4.42×10^{-3}
	5–6 ADLs	1.72×10^{-3}	-	-	5.71×10^{-4}
	Inst'd	-1.73×10^{-1}	-	-	2.67×10^{-3}
	Dead	5.90×10^{-3}	-	-	1.16×10^{-3}
1–2 ADLs	Healthy	5.39×10^{-2}	-	-	-4.02×10^{-5}
	IADL only	4.94×10^{-1}	-	-	-4.80×10^{-3}
	3–4 ADLs	2.24×10^{-1}	-	-	-4.91×10^{-4}
	5–6 ADLs	1.47×10^{-1}	-	-	-1.08×10^{-3}
	Inst'd	-9.68×10^{-2}	-	-	1.71×10^{-3}
	Dead	-2.96×10^{-1}	-	-	5.86×10^{-3}
3–4 ADLs	Healthy	-1.07×10^{-1}	2.00×10^{-1}	-2.91×10^{-2}	-
	IADL only	-7.60×10^{-2}	6.32×10^{-2}	3.76×10^{-2}	-
	1–2 ADLs	2.61×10^{-2}	3.93×10^{-1}	-1.25×10^{-1}	-
	5–6 ADLs	-5.53×10^{-1}	-	-	1.13×10^{-2}
	Inst'd	-1.73×10^{-1}	1.77×10^{-1}	2.77×10^{-2}	-
	Dead	1.35×10^{-1}	-	-	7.44×10^{-4}
5–6 ADLs	Healthy	1.44×10^{-1}	-	-	-1.56×10^{-3}
	IADL only	2.39×10^{-1}	-	-	-2.56×10^{-3}
	1–2 ADLs	1.50×10^{-1}	-	-	-1.16×10^{-3}
	3–4 ADLs	1.71×10^{-1}	-	-	-3.97×10^{-4}
	Inst'd	1.38×10^{-2}	-	-	6.90×10^{-4}
	Dead	1.86×10^{-1}	4.90×10^{-2}	7.21×10^{-2}	-
Inst'd	Healthy	-8.33×10^{-3}	3.35×10^{-2}	-5.30×10^{-2}	-
	IADL only	-8.54×10^{-2}	9.50×10^{-2}	-4.77×10^{-3}	-
	1–2 ADLs	2.05×10^{-2}	-	-	-2.17×10^{-4}
	3–4 ADLs	5.91×10^{-2}	-	-	-6.47×10^{-4}
	5–6 ADLs	1.39×10^{-2}	-	-	-1.52×10^{-4}
	Dead	-9.25×10^{-1}	-	-	1.56×10^{-2}

Table L.162: Parameter values for the parametric transition intensities for females, calculated from the 1982 and 1984 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-2.11×10^{-1}	-	-	3.38×10^{-3}
	1–2 ADLs	-5.58×10^{-2}	6.73×10^{-2}	1.74×10^{-2}	-
	3–4 ADLs	-5.27×10^{-2}	-	-	7.94×10^{-4}
	5–6 ADLs	1.26×10^{-3}	2.07×10^{-3}	8.71×10^{-2}	-
	Inst'd	-2.66×10^{-3}	4.02×10^{-3}	1.29×10^{-1}	-
	Dead	-7.96×10^{-3}	2.09×10^{-2}	5.86×10^{-2}	-
IADL only	Healthy	-1.87×10^{-1}	4.52×10^{-1}	-3.59×10^{-2}	-
	1–2 ADLs	-5.06×10^{-1}	-	-	1.11×10^{-2}
	3–4 ADLs	-2.01×10^{-3}	5.73×10^{-2}	-1.83×10^{-1}	-
	5–6 ADLs	7.89×10^{-3}	4.83×10^{-3}	1.28×10^{-1}	-
	Inst'd	-1.58×10^{-1}	-	-	2.63×10^{-3}
	Dead	-1.37×10^{-1}	-	-	2.51×10^{-3}
1–2 ADLs	Healthy	2.31×10^{-1}	-	-	-2.14×10^{-3}
	IADL only	5.75×10^{-1}	-	-	-4.86×10^{-3}
	3–4 ADLs	-3.29×10^{-1}	-	-	6.73×10^{-3}
	5–6 ADLs	-1.93×10^{-1}	2.22×10^{-1}	-4.99×10^{-3}	-
	Inst'd	-2.68×10^{-1}	-	-	4.21×10^{-3}
	Dead	-1.35×10^{-1}	2.03×10^{-1}	6.05×10^{-3}	-
3–4 ADLs	Healthy	2.40×10^{-2}	-	-	-2.07×10^{-4}
	IADL only	-3.50×10^{-2}	2.58×10^{-2}	5.06×10^{-2}	-
	1–2 ADLs	1.74×10^{0}	-	-	-1.83×10^{-2}
	5–6 ADLs	-1.07×10^{0}	-	-	1.82×10^{-2}
	Inst'd	-1.79×10^{-2}	4.31×10^{-2}	6.04×10^{-2}	-
	Dead	-1.25×10^{-2}	-	-	6.87×10^{-4}
5–6 ADLs	Healthy	7.75×10^{-2}	-	-	-6.38×10^{-4}
	IADL only	3.04×10^{-1}	-	-	-3.31×10^{-3}
	1–2 ADLs	-2.84×10^{-2}	6.49×10^{-2}	1.75×10^{-2}	-
	3–4 ADLs	-3.95×10^{-2}	-	-	2.40×10^{-3}
	Inst'd	-2.92×10^{-1}	-	-	5.06×10^{-3}
	Dead	1.12×10^{-1}	6.61×10^{-2}	4.35×10^{-2}	-
Inst'd	Healthy	2.66×10^{-3}	1.77×10^{-2}	-9.62×10^{-2}	-
	IADL only	1.63×10^{-2}	-	-	-1.43×10^{-4}
	1–2 ADLs	-1.22×10^{-1}	1.27×10^{-1}	-1.55×10^{-3}	-
	3–4 ADLs	6.59×10^{-3}	-	-	3.19×10^{-5}
	5–6 ADLs	5.13×10^{-3}	-	-	-1.95×10^{-5}
	Dead	-6.07×10^{-1}	-	-	1.01×10^{-2}

Table L.163: Parameter values for the parametric transition intensities for males, calculated from the 1984 and 1989 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	6.33×10^{-3}	6.30×10^{-3}	1.14×10^{-1}	-
	1–2 ADLs	-4.03×10^{-2}	4.90×10^{-2}	7.91×10^{-3}	-
	3–4 ADLs	-3.01×10^{-2}	-	-	4.38×10^{-4}
	5–6 ADLs	-1.22×10^{-3}	2.60×10^{-3}	-3.87×10^{-2}	-
	Inst'd	-1.18×10^{-2}	1.34×10^{-2}	5.36×10^{-2}	-
	Dead	1.30×10^{-3}	2.73×10^{-3}	8.11×10^{-2}	-
IADL only	Healthy	-4.32×10^{-1}	5.20×10^{-1}	-4.69×10^{-3}	-
	1–2 ADLs	5.33×10^{-2}	1.20×10^{-1}	-1.80×10^{-1}	-
	3–4 ADLs	-5.65×10^{-1}	-	-	8.16×10^{-3}
	5–6 ADLs	-1.90×10^{-1}	-	-	2.87×10^{-3}
	Inst'd	8.48×10^{-2}	-	-	-8.27×10^{-4}
	Dead	-4.88×10^{-1}	5.06×10^{-1}	3.97×10^{-4}	-
1–2 ADLs	Healthy	1.25×10^{-1}	-	-	-1.10×10^{-3}
	IADL only	4.22×10^{-2}	8.65×10^{-3}	1.20×10^{-1}	-
	3–4 ADLs	-1.17×10^{-1}	2.60×10^{-1}	-2.43×10^{-2}	-
	5–6 ADLs	-5.91×10^{-2}	-	-	1.29×10^{-3}
	Inst'd	-2.36×10^{-1}	2.41×10^{-1}	7.38×10^{-3}	-
	Dead	-6.51×10^{-2}	-	-	9.89×10^{-4}
3–4 ADLs	Healthy	-1.26×10^{-1}	1.36×10^{-1}	-4.54×10^{-3}	-
	IADL only	9.83×10^{-2}	-	-	-1.07×10^{-3}
	1–2 ADLs	9.77×10^{-1}	-	-	-1.06×10^{-2}
	5–6 ADLs	4.06×10^{-1}	-	-	-4.36×10^{-3}
	Inst'd	-5.37×10^{-1}	-	-	7.95×10^{-3}
	Dead	-1.20×10^{-1}	1.31×10^{-1}	1.44×10^{-2}	-
5–6 ADLs	Healthy	-2.00×10^{-2}	2.57×10^{-2}	-1.50×10^{-2}	-
	IADL only	9.69×10^{-2}	-	-	-1.05×10^{-3}
	1–2 ADLs	3.56×10^{-1}	-	-	-3.83×10^{-3}
	3–4 ADLs	4.87×10^{-2}	-	-	1.66×10^{-4}
	Inst'd	-4.85×10^{-2}	8.47×10^{-2}	1.93×10^{-2}	-
	Dead	-3.20×10^{-1}	3.32×10^{-1}	5.54×10^{-4}	-
Inst'd	Healthy	-1.20×10^{-1}	1.23×10^{-1}	4.86×10^{-3}	-
	IADL only	-1.90×10^{-2}	2.75×10^{-2}	-2.69×10^{-2}	-
	1–2 ADLs	-7.31×10^{-3}	-	-	1.11×10^{-4}
	3–4 ADLs	3.69×10^{-3}	-	-	-2.18×10^{-5}
	5–6 ADLs	-2.11×10^{-3}	2.34×10^{-3}	7.11×10^{-3}	-
	Dead	-8.51×10^{-2}	-	-	1.59×10^{-3}

Table L.164: Parameter values for the parametric transition intensities for females, calculated from the 1984 and 1989 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-4.52×10^{-2}	6.04×10^{-2}	1.77×10^{-2}	-
	1–2 ADLs	-1.64×10^{-1}	1.68×10^{-1}	1.22×10^{-2}	-
	3–4 ADLs	1.92×10^{-4}	2.15×10^{-3}	9.47×10^{-2}	-
	5–6 ADLs	-1.20×10^{-2}	1.56×10^{-2}	1.45×10^{-2}	-
	Inst'd	1.48×10^{-3}	1.98×10^{-3}	1.35×10^{-1}	-
	Dead	-1.20×10^{-2}	1.39×10^{-2}	1.64×10^{-2}	-
IADL only	Healthy	-8.35×10^{-2}	1.93×10^{-1}	-3.47×10^{-2}	-
	1–2 ADLs	7.88×10^{-2}	8.64×10^{-2}	2.13×10^{-2}	-
	3–4 ADLs	-5.76×10^{-2}	-	-	8.53×10^{-4}
	5–6 ADLs	-6.87×10^{-4}	1.62×10^{-2}	4.83×10^{-2}	-
	Inst'd	-8.76×10^{-2}	-	-	1.44×10^{-3}
	Dead	3.34×10^{-4}	4.79×10^{-3}	8.66×10^{-2}	-
1–2 ADLs	Healthy	-1.90×10^{-1}	2.17×10^{-1}	-3.24×10^{-3}	-
	IADL only	4.87×10^{-2}	-	-	-9.45×10^{-6}
	3–4 ADLs	-1.56×10^{-2}	8.68×10^{-2}	3.51×10^{-2}	-
	5–6 ADLs	-1.65×10^{-2}	-	-	5.16×10^{-4}
	Inst'd	-6.79×10^{-2}	8.32×10^{-2}	2.35×10^{-2}	-
	Dead	-1.58×10^{-3}	-	-	8.26×10^{-5}
3–4 ADLs	Healthy	6.89×10^{-3}	-	-	9.45×10^{-5}
	IADL only	2.46×10^{-2}	-	-	-2.07×10^{-4}
	1–2 ADLs	-1.38×10^{-1}	2.05×10^{-1}	2.37×10^{-3}	-
	5–6 ADLs	6.60×10^{-2}	-	-	2.95×10^{-4}
	Inst'd	-2.90×10^{-1}	-	-	4.30×10^{-3}
	Dead	9.01×10^{-3}	8.06×10^{-3}	5.11×10^{-2}	-
5–6 ADLs	Healthy	8.21×10^{-3}	-	-	-1.49×10^{-5}
	IADL only	8.97×10^{-2}	-	-	-9.84×10^{-4}
	1–2 ADLs	-1.68×10^{-1}	2.34×10^{-1}	-1.20×10^{-2}	-
	3–4 ADLs	-5.44×10^{-2}	1.51×10^{-1}	-2.57×10^{-2}	-
	Inst'd	3.82×10^{-2}	4.68×10^{-3}	1.22×10^{-1}	-
	Dead	-1.33×10^{-1}	-	-	1.95×10^{-3}
Inst'd	Healthy	-8.93×10^{-2}	9.63×10^{-2}	-5.60×10^{-4}	-
	IADL only	-4.12×10^{-3}	-	-	6.40×10^{-5}
	1–2 ADLs	-2.22×10^{-2}	3.13×10^{-2}	-1.41×10^{-2}	-
	3–4 ADLs	-2.01×10^{-1}	2.02×10^{-1}	-1.70×10^{-4}	-
	5–6 ADLs	-3.10×10^{-2}	3.12×10^{-2}	1.99×10^{-3}	-
	Dead	1.57×10^{-2}	2.08×10^{-3}	1.26×10^{-1}	-

Table L.165: Parameter values for the parametric transition intensities for males and females, calculated from the 1984 and 1989 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	4.34×10^{-3}	1.06×10^{-2}	7.28×10^{-2}	-
	1–2 ADLs	-1.38×10^{-2}	2.02×10^{-2}	5.07×10^{-2}	-
	3–4 ADLs	-1.05×10^{-4}	2.08×10^{-3}	1.08×10^{-1}	-
	5–6 ADLs	-1.55×10^{-2}	1.80×10^{-2}	9.15×10^{-3}	-
	Inst'd	-1.46×10^{-3}	4.00×10^{-3}	1.04×10^{-1}	-
	Dead	-3.91×10^{-3}	6.69×10^{-3}	3.94×10^{-2}	-
IADL only	Healthy	-6.90×10^{-3}	1.27×10^{-1}	-7.71×10^{-2}	-
	1–2 ADLs	1.30×10^{-1}	1.46×10^{-2}	7.45×10^{-2}	-
	3–4 ADLs	-1.52×10^{-1}	-	-	2.24×10^{-3}
	5–6 ADLs	-2.33×10^{-2}	3.66×10^{-2}	4.25×10^{-2}	-
	Inst'd	-5.52×10^{-2}	-	-	1.05×10^{-3}
	Dead	-3.27×10^{-2}	4.23×10^{-2}	1.21×10^{-2}	-
1–2 ADLs	Healthy	1.08×10^{-1}	-	-	-1.02×10^{-3}
	IADL only	1.29×10^{-2}	-	-	5.82×10^{-4}
	3–4 ADLs	6.14×10^{-2}	2.28×10^{-2}	7.14×10^{-2}	-
	5–6 ADLs	1.39×10^{-2}	-	-	1.81×10^{-4}
	Inst'd	-6.77×10^{-2}	8.05×10^{-2}	2.28×10^{-2}	-
	Dead	-2.89×10^{-3}	-	-	1.12×10^{-4}
3–4 ADLs	Healthy	-5.47×10^{-4}	-	-	1.55×10^{-4}
	IADL only	-1.71×10^{-2}	2.78×10^{-2}	-2.15×10^{-2}	-
	1–2 ADLs	7.92×10^{-2}	4.22×10^{-2}	-1.10×10^{-1}	-
	5–6 ADLs	6.15×10^{-2}	-	-	4.11×10^{-4}
	Inst'd	-3.42×10^{-1}	-	-	5.09×10^{-3}
	Dead	1.28×10^{-2}	4.62×10^{-3}	1.00×10^{-1}	-
5–6 ADLs	Healthy	1.39×10^{-2}	-	-	-1.17×10^{-4}
	IADL only	1.08×10^{-1}	-	-	-1.17×10^{-3}
	1–2 ADLs	-1.06×10^{-1}	1.81×10^{-1}	-1.76×10^{-2}	-
	3–4 ADLs	-3.48×10^{-2}	1.34×10^{-1}	-2.44×10^{-2}	-
	Inst'd	4.28×10^{-2}	3.80×10^{-3}	1.31×10^{-1}	-
	Dead	-3.52×10^{-2}	3.96×10^{-2}	2.76×10^{-2}	-
Inst'd	Healthy	6.82×10^{-4}	5.50×10^{-3}	1.01×10^{-2}	-
	IADL only	-3.12×10^{-2}	3.63×10^{-2}	-5.42×10^{-3}	-
	1–2 ADLs	2.07×10^{-2}	-	-	-2.12×10^{-4}
	3–4 ADLs	-3.01×10^{-3}	7.26×10^{-3}	-3.56×10^{-2}	-
	5–6 ADLs	-6.11×10^{-2}	6.21×10^{-2}	6.57×10^{-4}	-
	Dead	-5.45×10^{-2}	7.38×10^{-2}	1.67×10^{-2}	-

Table L.166: Parameter values for the parametric transition intensities for males, calculated from the 1989 and 1994 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-6.39×10^{-2}	-	-	1.11×10^{-3}
	1–2 ADLs	2.55×10^{-3}	6.92×10^{-3}	1.12×10^{-1}	-
	3–4 ADLs	-2.37×10^{-2}	-	-	3.81×10^{-4}
	5–6 ADLs	-6.26×10^{-3}	9.59×10^{-3}	1.25×10^{-2}	-
	Inst'd	-6.57×10^{-2}	6.86×10^{-2}	4.87×10^{-3}	-
	Dead	-4.56×10^{-2}	-	-	6.92×10^{-4}
IADL only	Healthy	9.98×10^{-1}	-	-	-1.15×10^{-2}
	1–2 ADLs	-4.21×10^{-1}	-	-	7.05×10^{-3}
	3–4 ADLs	1.12×10^{-2}	4.43×10^{-2}	-1.45×10^{-1}	-
	5–6 ADLs	-8.01×10^{-3}	8.38×10^{-3}	6.95×10^{-2}	-
	Inst'd	-7.84×10^{-3}	-	-	3.51×10^{-4}
	Dead	8.94×10^{-2}	-	-	-9.11×10^{-4}
1–2 ADLs	Healthy	2.06×10^{-2}	3.97×10^{-2}	-1.44×10^{-1}	-
	IADL only	-1.30×10^{-1}	-	-	2.17×10^{-3}
	3–4 ADLs	5.79×10^{-2}	2.36×10^{-2}	8.29×10^{-2}	-
	5–6 ADLs	-2.43×10^{-1}	-	-	3.44×10^{-3}
	Inst'd	-4.66×10^{-1}	4.95×10^{-1}	5.18×10^{-3}	-
	Dead	-8.01×10^{-3}	1.67×10^{-2}	-6.88×10^{-2}	-
3–4 ADLs	Healthy	-2.68×10^{-1}	3.02×10^{-1}	-4.11×10^{-3}	-
	IADL only	2.61×10^{-1}	-	-	-3.05×10^{-3}
	1–2 ADLs	-2.01×10^{-3}	6.93×10^{-2}	-2.77×10^{-1}	-
	5–6 ADLs	4.38×10^{-1}	-	-	-3.79×10^{-3}
	Inst'd	-4.59×10^{-1}	-	-	6.60×10^{-3}
	Dead	-4.02×10^{-1}	4.04×10^{-1}	1.00×10^{-2}	-
5–6 ADLs	Healthy	4.00×10^{-3}	-	-	1.45×10^{-4}
	IADL only	3.93×10^{-2}	-	-	-4.54×10^{-4}
	1–2 ADLs	-2.64×10^{-1}	3.09×10^{-1}	-7.59×10^{-3}	-
	3–4 ADLs	-1.86×10^{-1}	-	-	2.56×10^{-3}
	Inst'd	2.03×10^{-1}	-	-	-1.72×10^{-3}
	Dead	7.31×10^{-2}	-	-	-6.86×10^{-4}
Inst'd	Healthy	-4.99×10^{-1}	5.05×10^{-1}	1.11×10^{-3}	-
	IADL only	-6.12×10^{-2}	6.97×10^{-2}	-5.71×10^{-3}	-
	1–2 ADLs	-3.14×10^{-2}	-	-	4.85×10^{-4}
	3–4 ADLs	-1.10×10^{-1}	1.10×10^{-1}	5.33×10^{-3}	-
	5–6 ADLs	-5.10×10^{-2}	5.12×10^{-2}	1.31×10^{-3}	-
	Dead	-9.11×10^{-2}	1.06×10^{-1}	9.35×10^{-3}	-

Table L.167: Parameter values for the parametric transition intensities for females, calculated from the 1989 and 1994 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-1.08×10^{-2}	2.33×10^{-2}	4.58×10^{-2}	-
	1–2 ADLs	-6.11×10^{-3}	2.51×10^{-2}	4.19×10^{-2}	-
	3–4 ADLs	-9.01×10^{-2}	8.94×10^{-2}	4.95×10^{-3}	-
	5–6 ADLs	7.73×10^{-4}	1.84×10^{-3}	8.18×10^{-2}	-
	Inst'd	-6.16×10^{-3}	1.10×10^{-2}	7.45×10^{-2}	-
	Dead	1.43×10^{-3}	-	-	1.34×10^{-5}
IADL only	Healthy	-6.86×10^{-3}	1.46×10^{-1}	-6.44×10^{-2}	-
	1–2 ADLs	-8.04×10^{-1}	-	-	1.25×10^{-2}
	3–4 ADLs	-1.02×10^{-2}	4.46×10^{-2}	-1.31×10^{-1}	-
	5–6 ADLs	8.92×10^{-2}	-	-	-8.66×10^{-4}
	Inst'd	-1.03×10^{-1}	-	-	1.54×10^{-3}
	Dead	-8.64×10^{-2}	-	-	1.27×10^{-3}
1–2 ADLs	Healthy	2.67×10^{-1}	-	-	-2.82×10^{-3}
	IADL only	-4.96×10^{-2}	-	-	1.46×10^{-3}
	3–4 ADLs	-1.22×10^{-1}	-	-	3.47×10^{-3}
	5–6 ADLs	1.15×10^{-2}	-	-	2.88×10^{-4}
	Inst'd	-9.01×10^{-3}	7.19×10^{-3}	1.22×10^{-1}	-
	Dead	-5.56×10^{-2}	-	-	8.34×10^{-4}
3–4 ADLs	Healthy	7.05×10^{-3}	-	-	1.02×10^{-4}
	IADL only	1.88×10^{-1}	-	-	-2.09×10^{-3}
	1–2 ADLs	-5.03×10^{-1}	-	-	7.52×10^{-3}
	5–6 ADLs	-3.63×10^{-1}	-	-	5.81×10^{-3}
	Inst'd	-9.55×10^{-2}	1.60×10^{-1}	1.20×10^{-2}	-
	Dead	1.94×10^{-2}	-	-	-8.64×10^{-5}
5–6 ADLs	Healthy	-9.46×10^{-2}	1.07×10^{-1}	5.51×10^{-3}	-
	IADL only	2.42×10^{-3}	-	-	8.68×10^{-6}
	1–2 ADLs	-1.20×10^{-1}	-	-	1.78×10^{-3}
	3–4 ADLs	7.84×10^{-3}	-	-	8.10×10^{-4}
	Inst'd	-1.95×10^{-1}	2.04×10^{-1}	1.68×10^{-2}	-
	Dead	-4.04×10^{-2}	-	-	9.13×10^{-4}
Inst'd	Healthy	-1.22×10^{-2}	2.98×10^{-2}	-1.33×10^{-2}	-
	IADL only	-2.67×10^{-3}	-	-	5.17×10^{-5}
	1–2 ADLs	2.51×10^{-4}	2.31×10^{-2}	-1.08×10^{-1}	-
	3–4 ADLs	-1.95×10^{-4}	-	-	2.69×10^{-5}
	5–6 ADLs	5.08×10^{-3}	-	-	-4.82×10^{-5}
	Dead	-4.52×10^{-3}	-	-	3.80×10^{-4}

Table L.168: Parameter values for the parametric transition intensities for males and females, calculated from the 1989 and 1994 NLTCS, grouped in 5-year age bands.

From State	To State	Parameter A	B	C	D
Healthy	IADL only	-1.04×10^{-1}	-	-	1.67×10^{-3}
	1–2 ADLs	1.07×10^{-4}	1.51×10^{-2}	6.17×10^{-2}	-
	3–4 ADLs	-2.10×10^{-2}	2.08×10^{-2}	2.05×10^{-2}	-
	5–6 ADLs	1.46×10^{-3}	1.85×10^{-3}	7.01×10^{-2}	-
	Inst'd	3.29×10^{-4}	3.98×10^{-3}	1.15×10^{-1}	-
	Dead	1.03×10^{-3}	2.05×10^{-3}	7.19×10^{-2}	-
IADL only	Healthy	6.24×10^{-1}	-	-	-6.70×10^{-3}
	1–2 ADLs	-8.99×10^{-1}	-	-	1.38×10^{-2}
	3–4 ADLs	-4.31×10^{-2}	9.06×10^{-2}	-6.31×10^{-2}	-
	5–6 ADLs	1.08×10^{-2}	-	-	7.14×10^{-5}
	Inst'd	-8.76×10^{-2}	-	-	1.36×10^{-3}
	Dead	-4.53×10^{-2}	-	-	7.58×10^{-4}
1–2 ADLs	Healthy	2.74×10^{-1}	-	-	-2.91×10^{-3}
	IADL only	-9.52×10^{-2}	-	-	1.95×10^{-3}
	3–4 ADLs	-6.42×10^{-2}	-	-	2.92×10^{-3}
	5–6 ADLs	-1.45×10^{-1}	1.60×10^{-1}	1.01×10^{-2}	-
	Inst'd	-2.30×10^{-2}	2.28×10^{-2}	8.01×10^{-2}	-
	Dead	-5.40×10^{-2}	5.75×10^{-2}	1.47×10^{-2}	-
3–4 ADLs	Healthy	-6.40×10^{-2}	8.68×10^{-2}	-8.23×10^{-3}	-
	IADL only	2.09×10^{-1}	-	-	-2.35×10^{-3}
	1–2 ADLs	6.81×10^{-2}	3.63×10^{-3}	1.72×10^{-1}	-
	5–6 ADLs	-2.30×10^{-2}	-	-	1.81×10^{-3}
	Inst'd	3.61×10^{-2}	2.44×10^{-2}	6.58×10^{-2}	-
	Dead	-1.95×10^{-1}	2.07×10^{-1}	3.16×10^{-3}	-
5–6 ADLs	Healthy	-2.22×10^{-2}	-	-	5.21×10^{-4}
	IADL only	-5.96×10^{-3}	-	-	1.44×10^{-4}
	1–2 ADLs	-2.94×10^{-2}	7.07×10^{-2}	-2.76×10^{-2}	-
	3–4 ADLs	-1.30×10^{-1}	-	-	2.43×10^{-3}
	Inst'd	-4.54×10^{-2}	8.60×10^{-2}	2.50×10^{-2}	-
	Dead	-2.93×10^{-2}	-	-	8.14×10^{-4}
Inst'd	Healthy	-3.60×10^{-2}	5.31×10^{-2}	-5.38×10^{-3}	-
	IADL only	-1.89×10^{-4}	-	-	3.71×10^{-5}
	1–2 ADLs	6.36×10^{-2}	-	-	-6.58×10^{-4}
	3–4 ADLs	-8.25×10^{-3}	-	-	1.38×10^{-4}
	5–6 ADLs	1.60×10^{-3}	-	-	-3.20×10^{-6}
	Dead	-2.55×10^{-2}	-	-	6.49×10^{-4}

Appendix M

Graphs of the constrained (positive) MLEs of the transition intensities, 95% confidence intervals and parametric fits, using data in 5-year age bands from the 1982 and 1984 NLTCS

Figures M.89 to M.94 and M.95 to M.100 give graphs of the transition intensities, for males and females respectively, out of states 1–6 in turn, for the 1982–84 NLTCS. The graphs for males and females together are given in Figures 5.49 to 5.54. They show the point estimates (constrained (positive) MLEs) for the data grouped in 5-year age bands, the confidence intervals calculated from the variance estimates (in Section 5.4) and the parametric form of the transition intensities (calculated in Section 5.6).

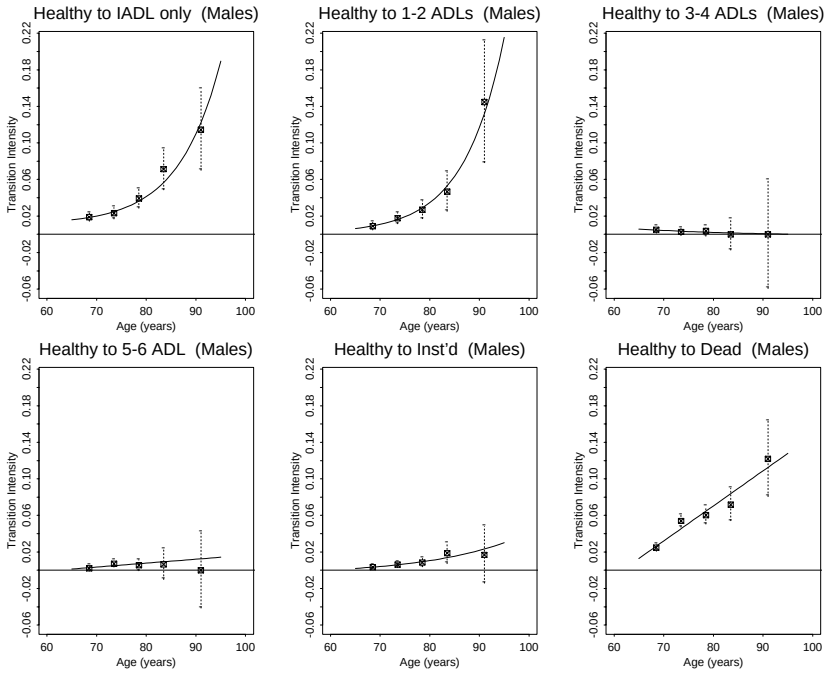

Figure M.89: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males grouped in 5-year age bands in the 1982–84 NLTCS.

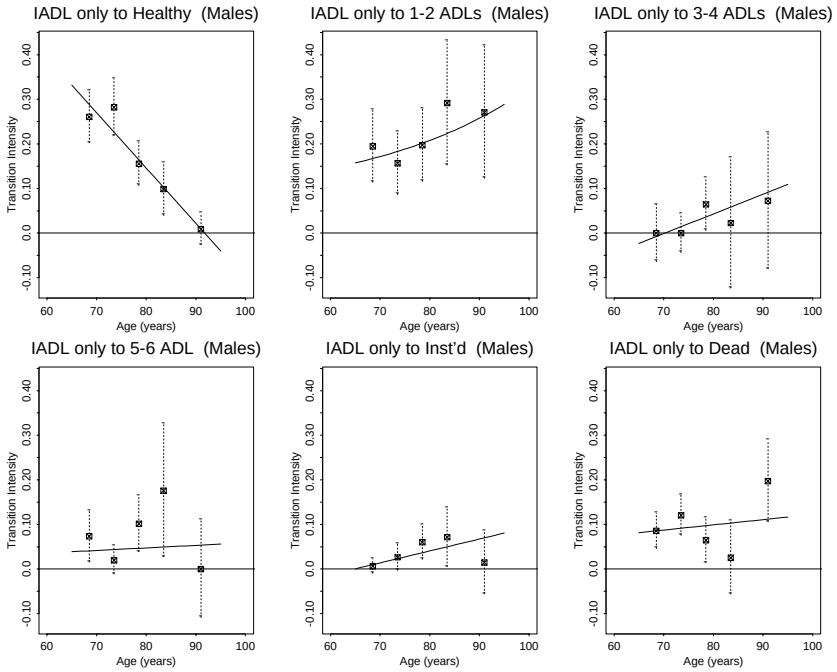

Figure M.90: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males grouped in 5-year age bands in the 1982–84 NLTCS.

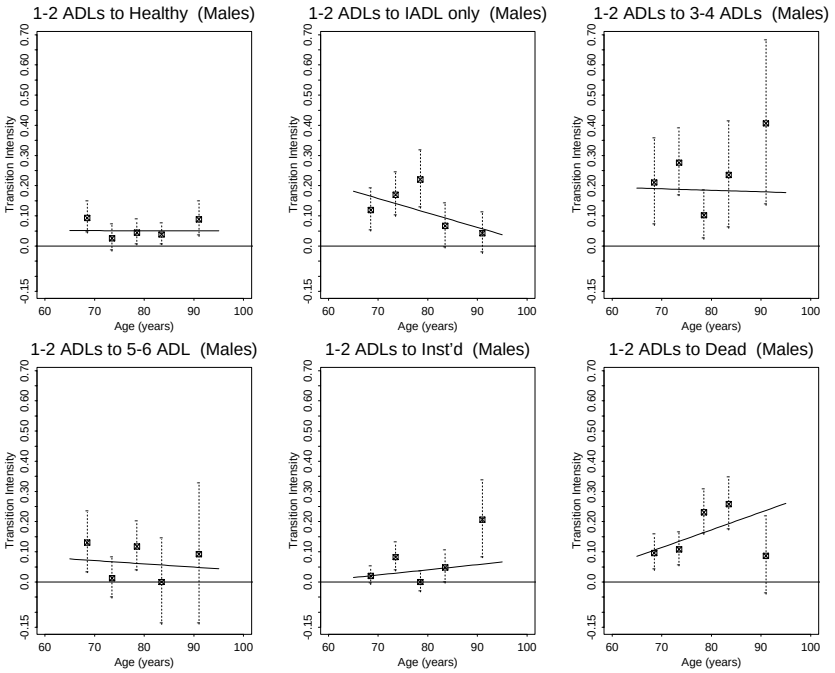

Figure M.91: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males grouped in 5-year age bands in the 1982–84 NLTCS.

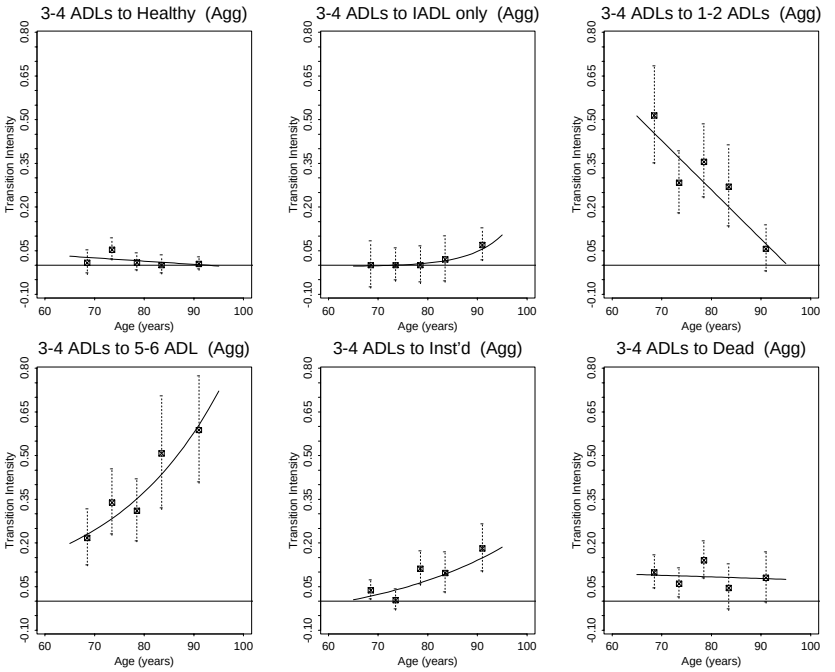

Figure M.92: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males grouped in 5-year age bands in the 1982–84 NLTCS.

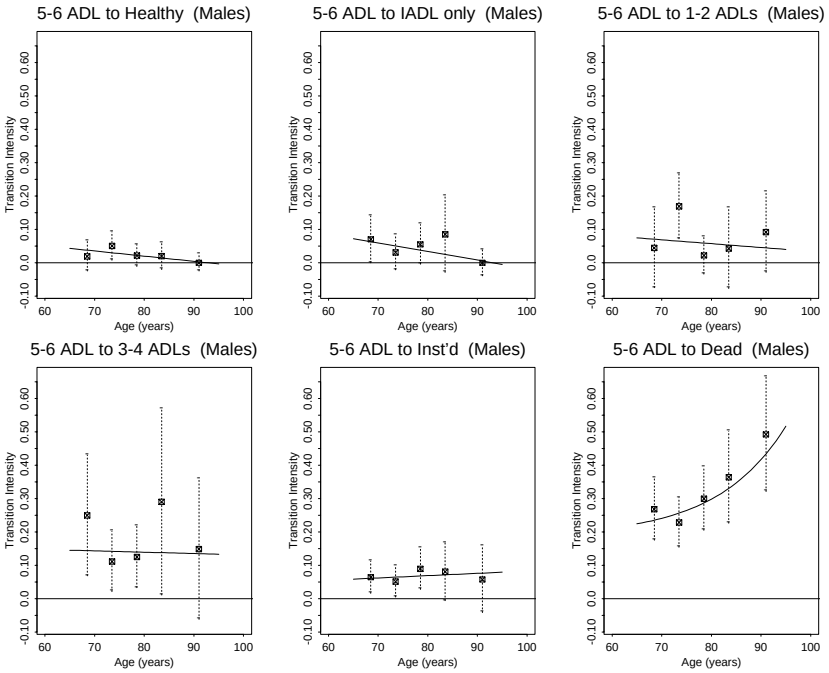

Figure M.93: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males grouped in 5-year age bands in the 1982–84 NLTCS.

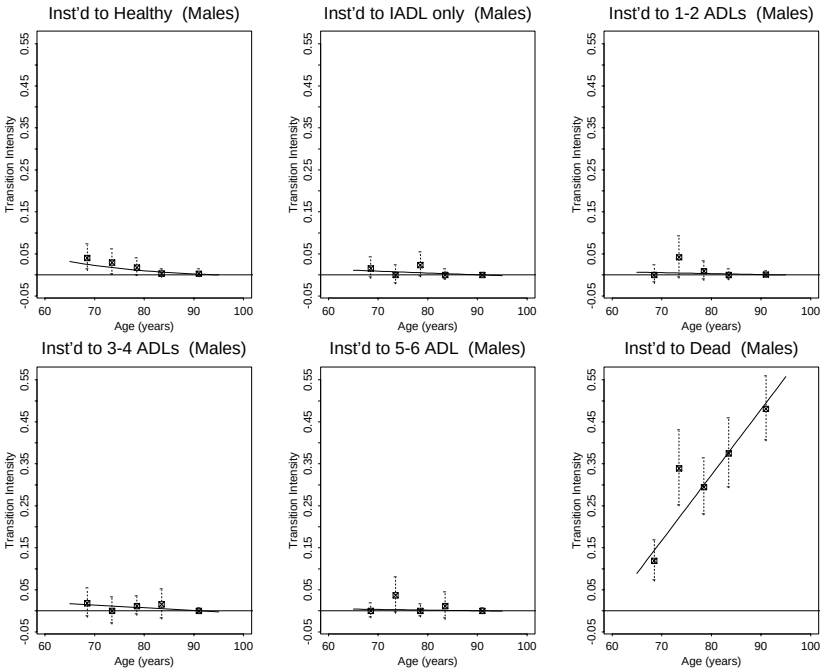

Figure M.94: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males grouped in 5-year age bands in the 1982–84 NLTCS.

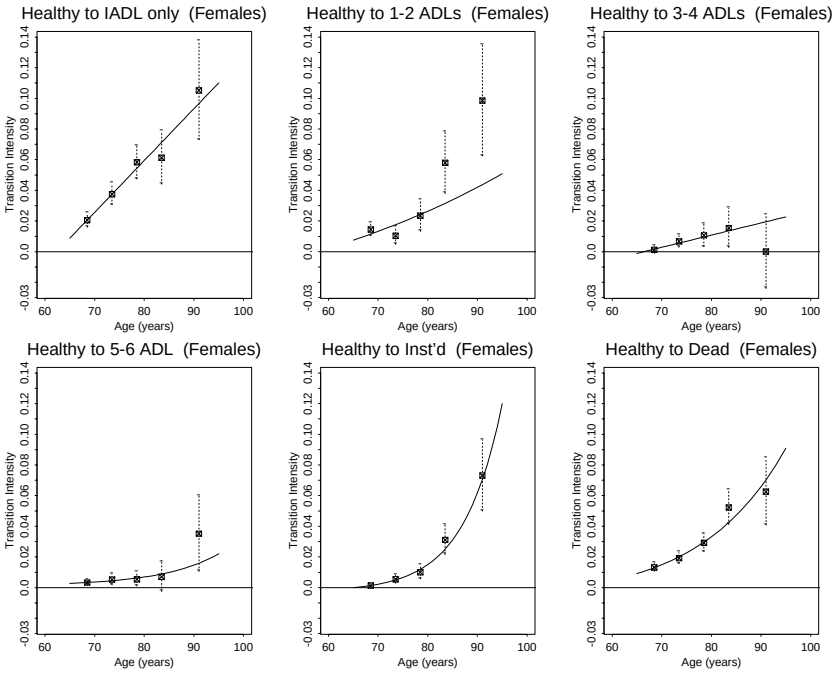

Figure M.95: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for females grouped in 5-year age bands in the 1982–84 NLTCS.

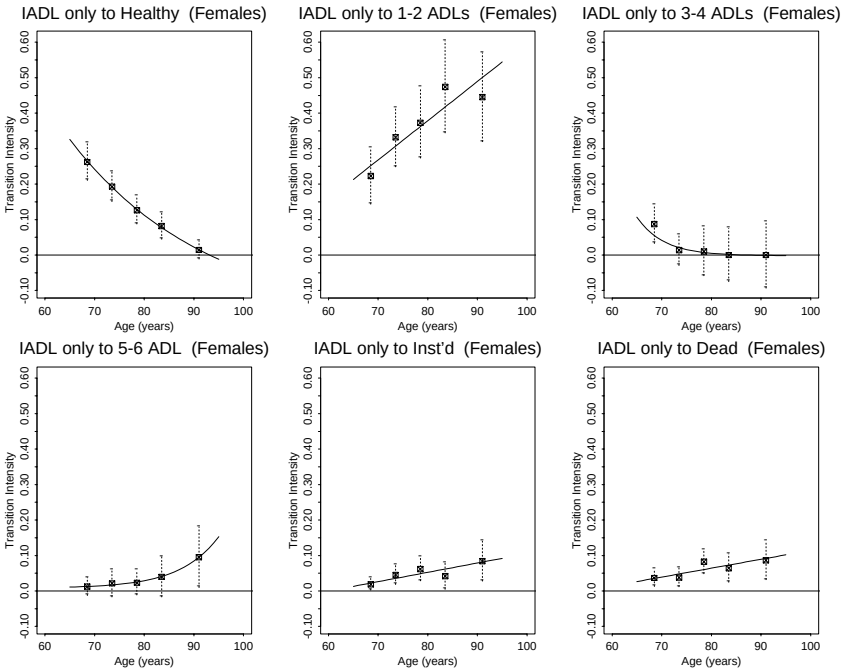

Figure M.96: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for females grouped in 5-year age bands in the 1982–84 NLTCS.

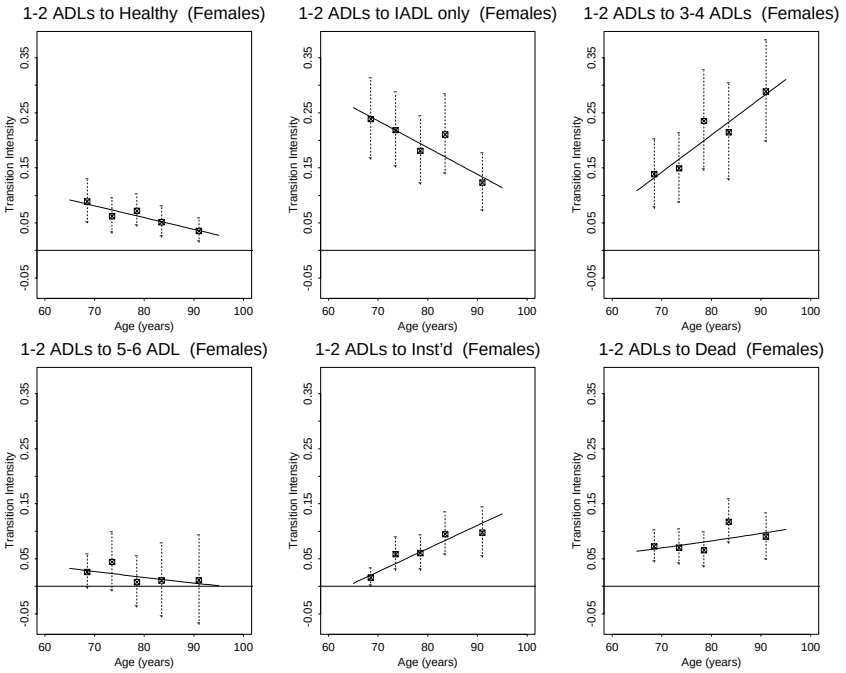

Figure M.97: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for females grouped in 5-year age bands in the 1982–84 NLTCS.

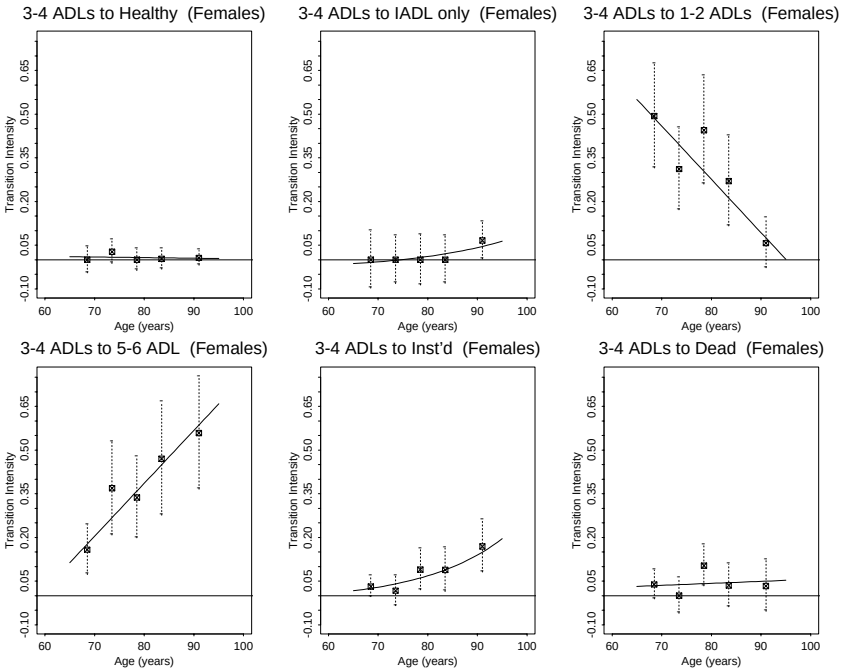

Figure M.98: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for females grouped in 5-year age bands in the 1982–84 NLTCS.

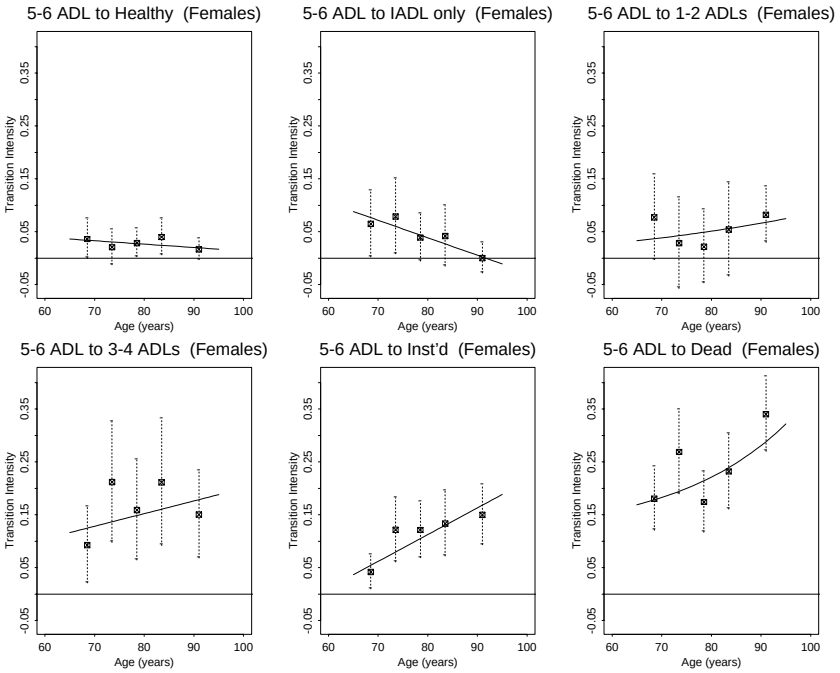

Figure M.99: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for females grouped in 5-year age bands in the 1982–84 NLTCS.

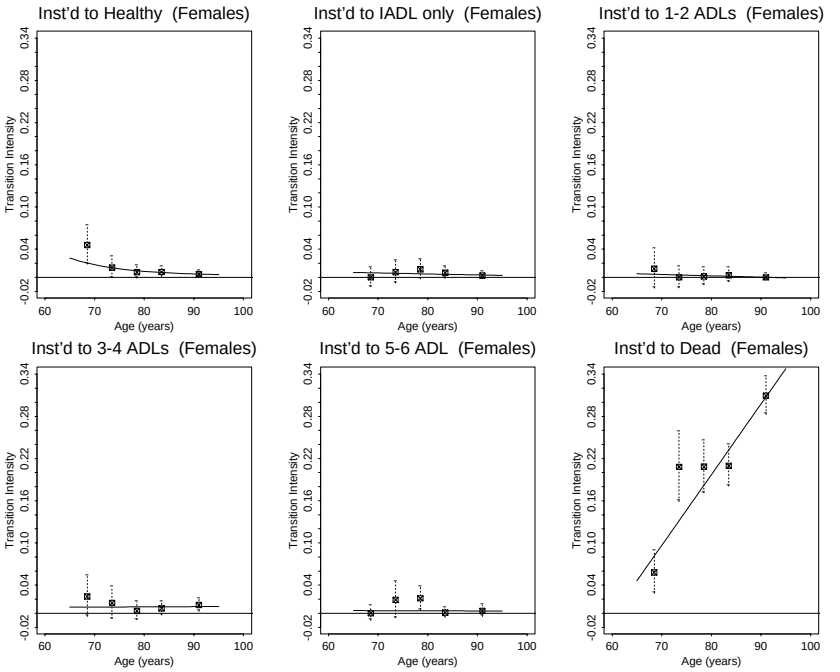

Figure M.100: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for females grouped in 5-year age bands in the 1982–84 NLTCS.

Appendix N

Graphs of the constrained (positive) MLEs of the transition intensities, 95% confidence intervals and parametric fits, using data in 5-year age bands from the 1984 and 1989 NLTCS

Figures N.101 to N.106, N.107 to N.112 and N.113 to N.118 give graphs of the transition intensities, for males, females and in aggregate respectively, out of states 1–6 in turn, for the 1984–89 NLTCS. They show the point estimates (constrained (positive) MLEs) for the data grouped in 5-year age bands, the confidence intervals calculated from the variance estimates (in Section 5.4) and the parametric form of the transition intensities (calculated in Section 5.6).

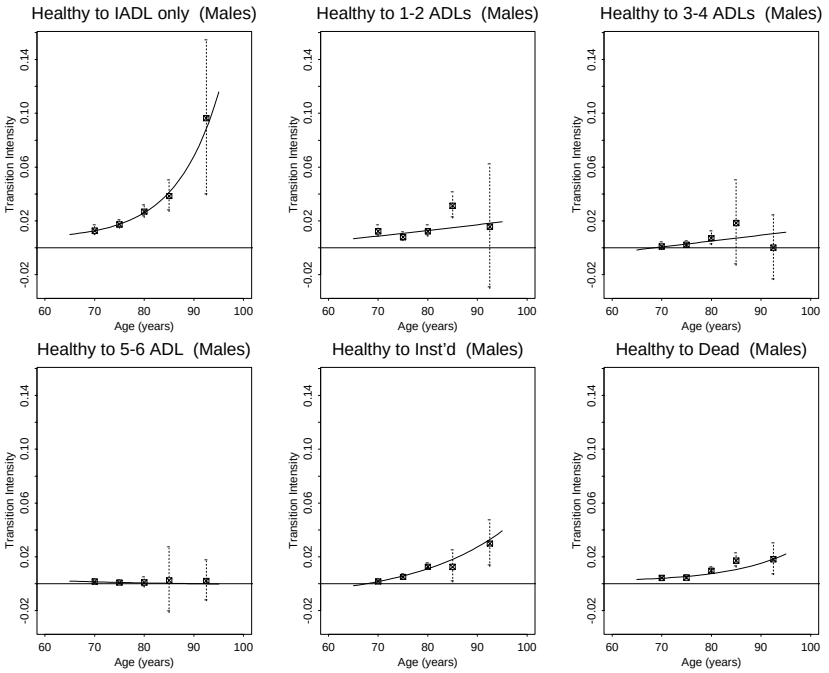

Figure N.101: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males grouped in 5-year age bands in the 1984–89 NLTCS.

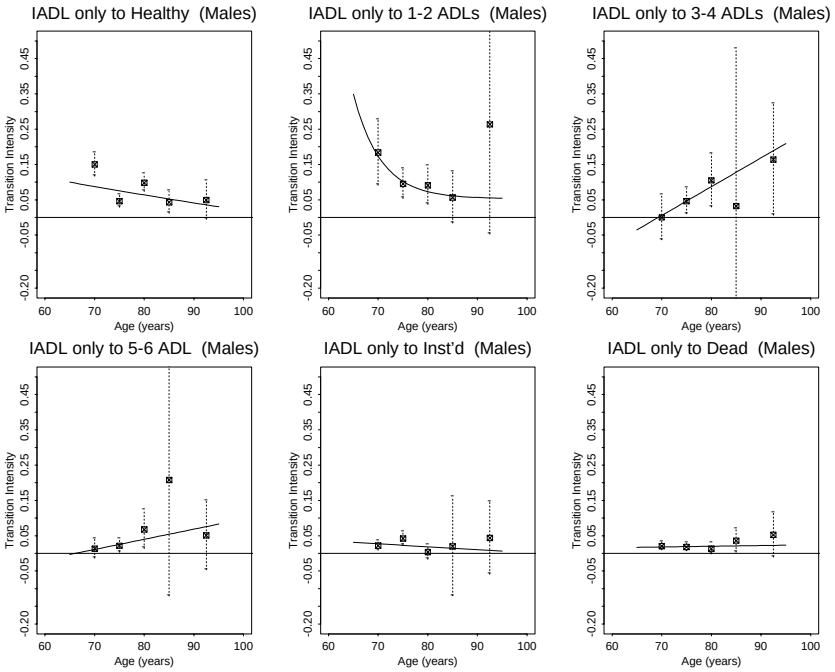

Figure N.102: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males grouped in 5-year age bands in the 1984–89 NLTCS.

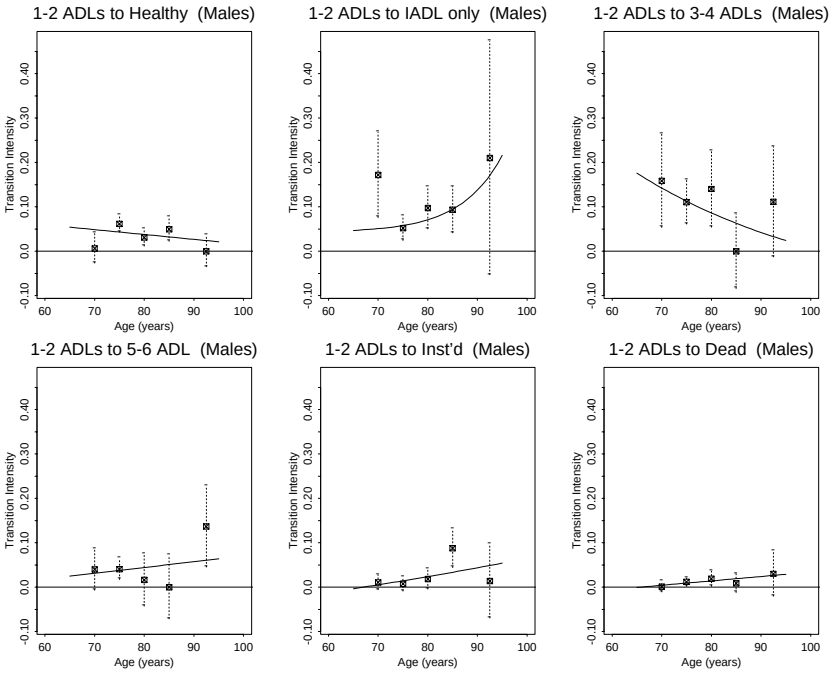

Figure N.103: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males grouped in 5-year age bands in the 1984–89 NLTCS.

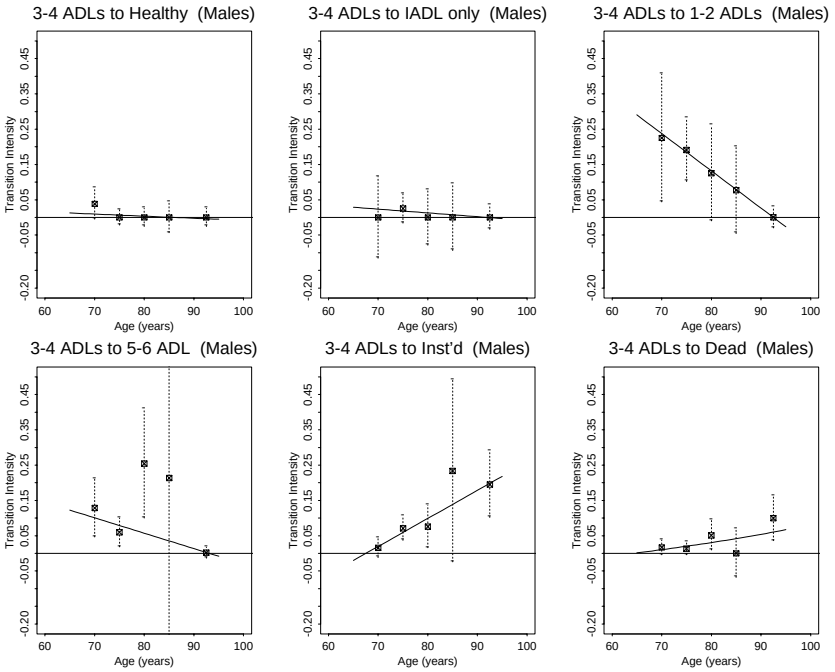

Figure N.104: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males grouped in 5-year age bands in the 1984–89 NLTCS.

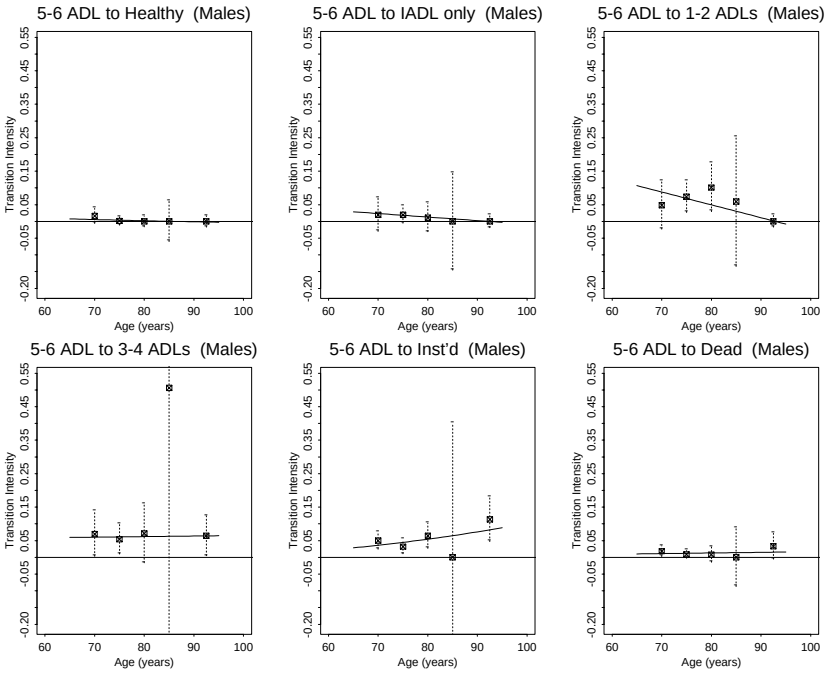

Figure N.105: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males grouped in 5-year age bands in the 1984–89 NLTCS.

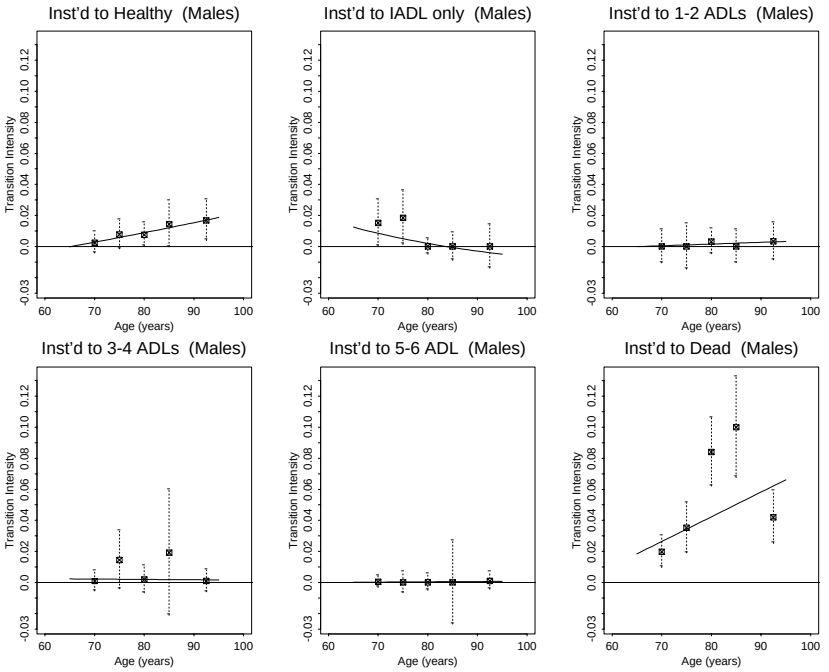

Figure N.106: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males grouped in 10-year age bands in the 1984–89 NLTCS.

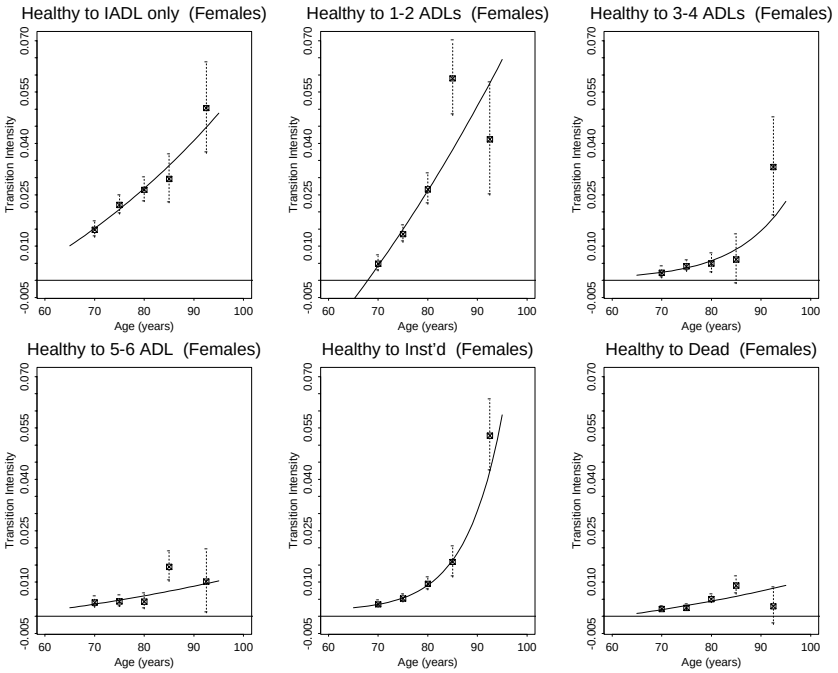

Figure N.107: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for females grouped in 5-year age bands in the 1984–89 NLTCS.

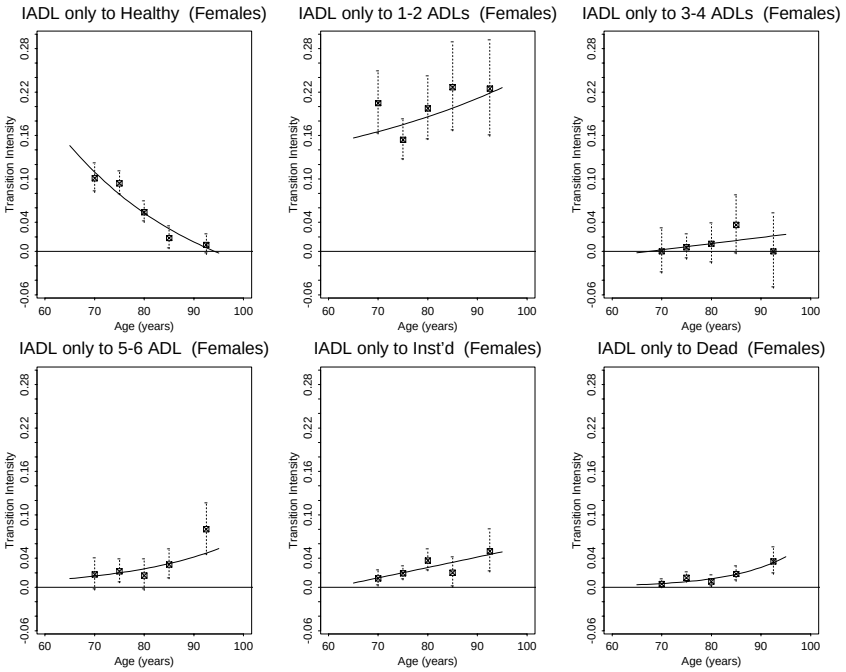

Figure N.108: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for females grouped in 5-year age bands in the 1984–89 NLTCS.

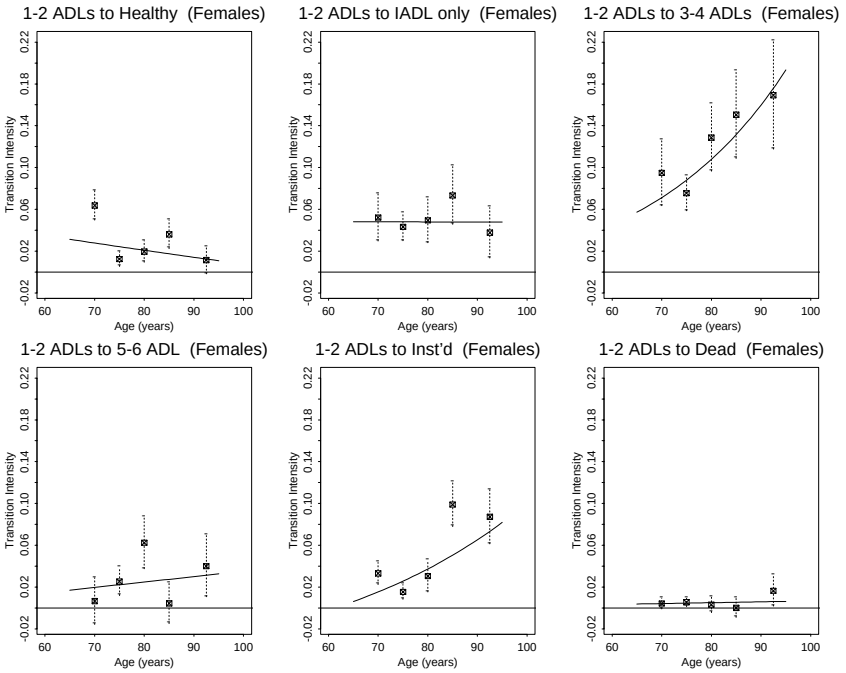

Figure N.109: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for females grouped in 5-year age bands in the 1984–89 NLTCS.

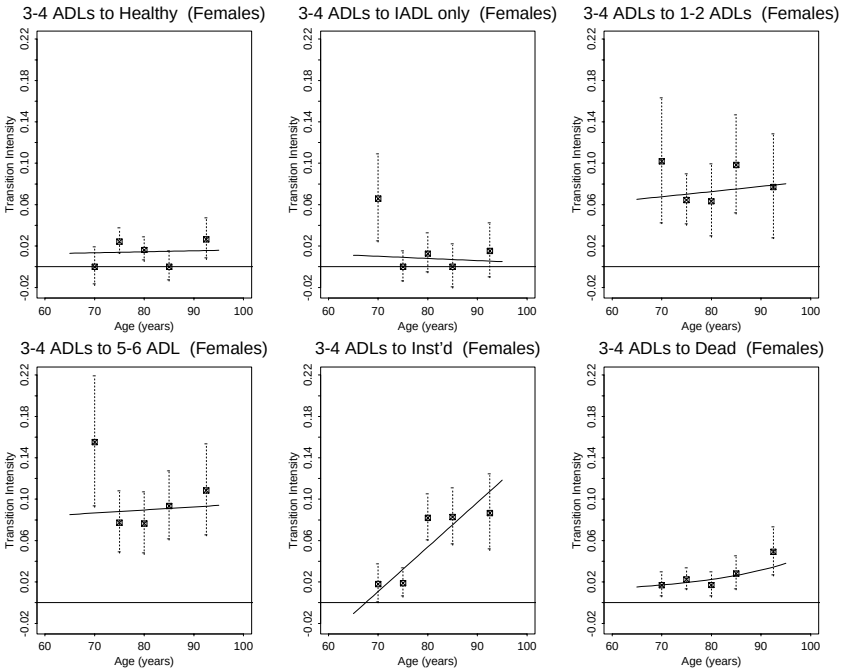

Figure N.110: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for females grouped in 5-year age bands in the 1984–89 NLTCS.

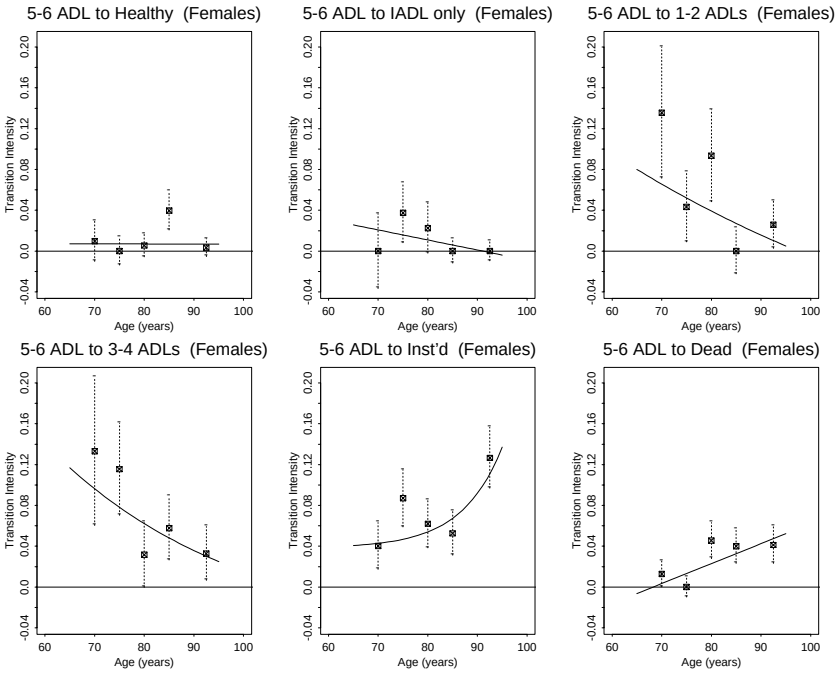

Figure N.111: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for females grouped in 5-year age bands in the 1984–89 NLTCS.

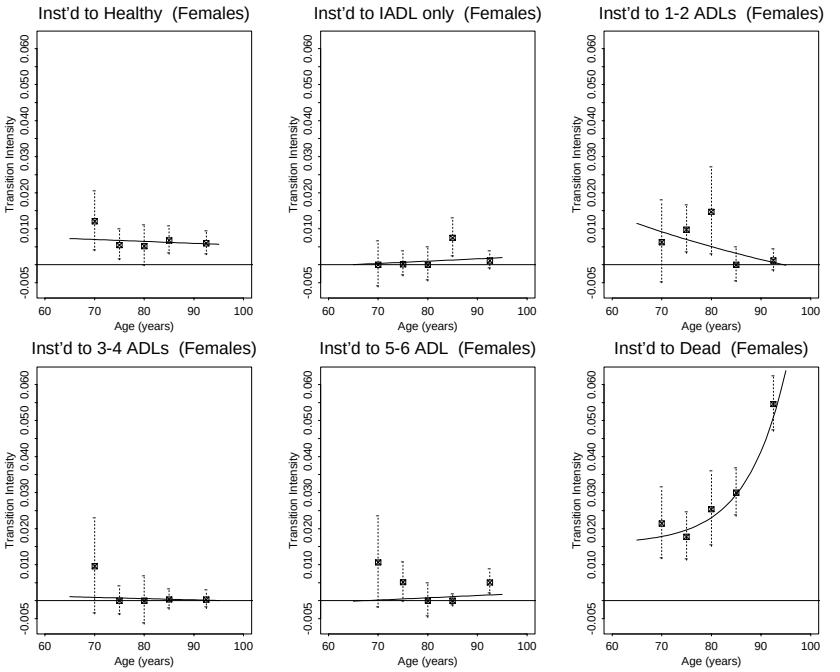

Figure N.112: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for females grouped in 5-year age bands in the 1984–89 NLTCS.

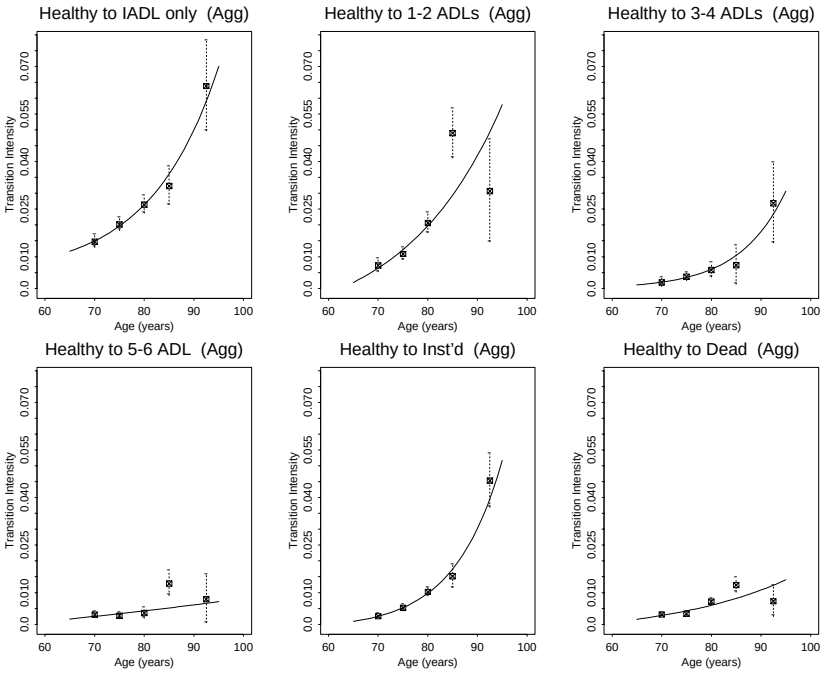

Figure N.113: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males and females grouped in 5-year age bands in the 1984–89 NLTCS.

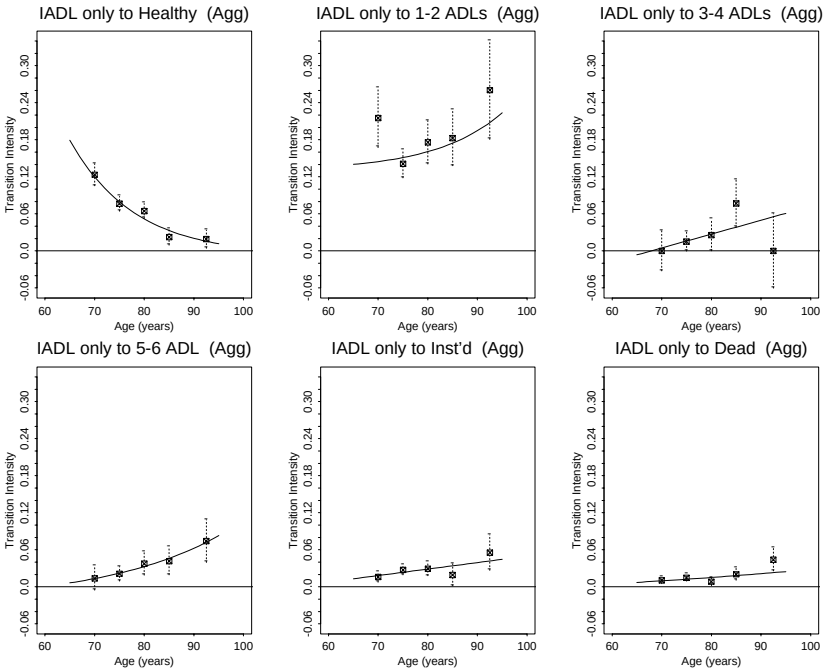

Figure N.114: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males and females grouped in 5-year age bands in the 1984–89 NLTCS.

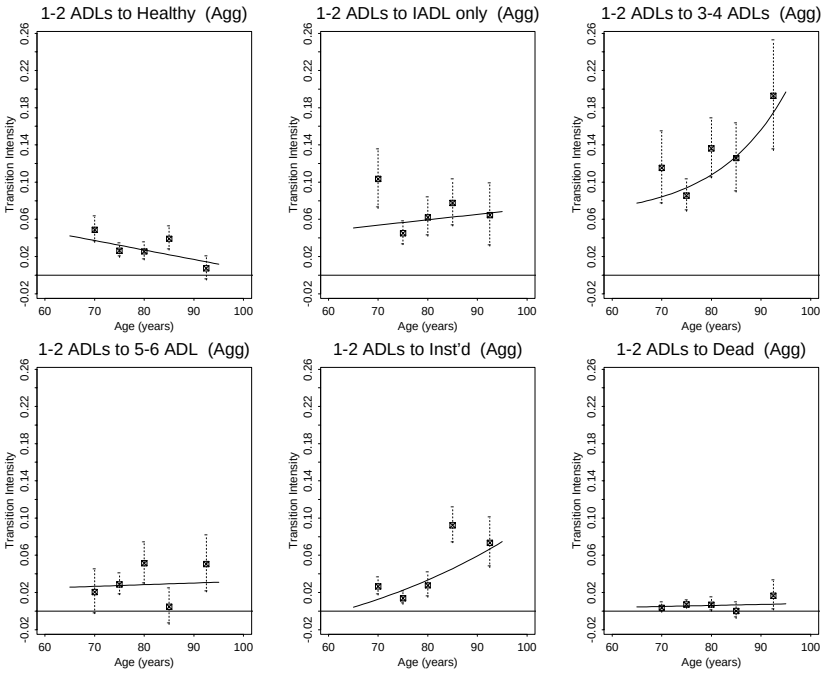

Figure N.115: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males and females grouped in 5-year age bands in the 1984–89 NLTCS.

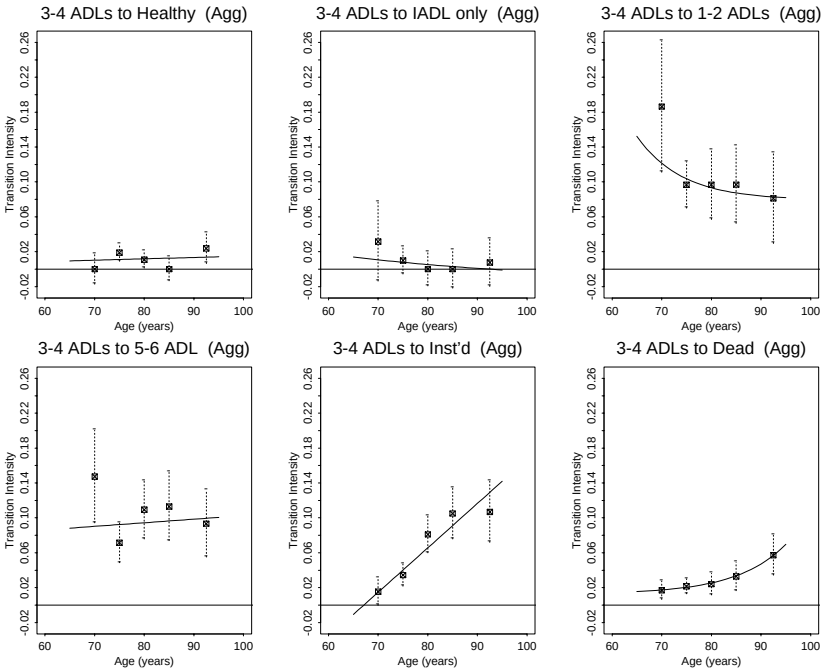

Figure N.116: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males and females grouped in 5-year age bands in the 1984–89 NLTCS.

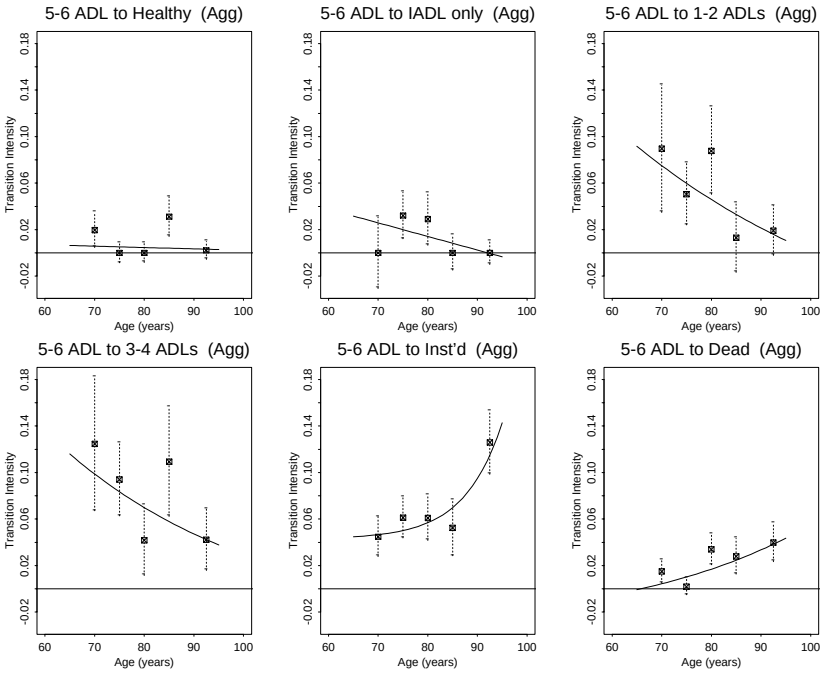

Figure N.117: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males and females grouped in 5-year age bands in the 1984–89 NLTCS.

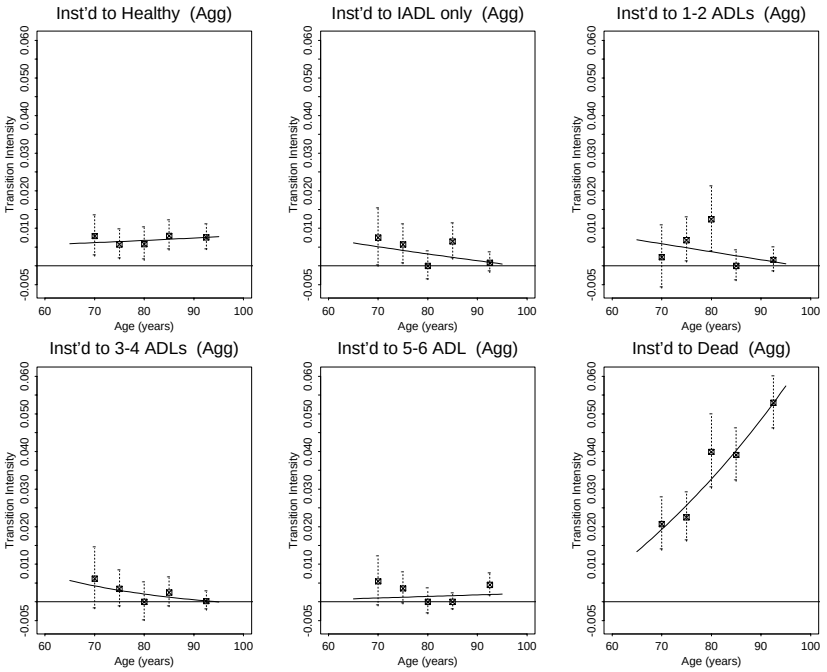

Figure N.118: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males and females grouped in 5-year age bands in the 1984–89 NLTCS.

Appendix O

Graphs of the constrained (positive) MLEs of the transition intensities, 95% confidence intervals and parametric fits, using data in 5-year age bands from the 1989 and 1994 NLTCS

Figures O.119 to O.124, O.125 to O.130 and O.131 to O.136 give graphs of the transition intensities, for males, females and in aggregate respectively, out of states 1–6 in turn, for the 1989–94 NLTCS. They show the point estimates (constrained (positive) MLEs) for the data grouped in 5-year age bands, the confidence intervals calculated from the variance estimates (in Section 5.4) and the parametric form of the transition intensities (calculated in Section 5.6).

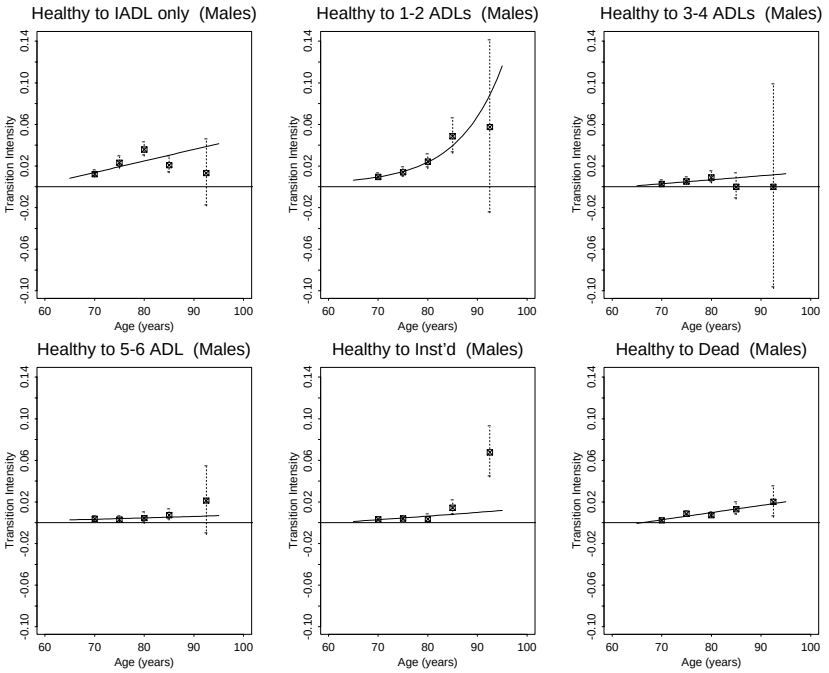

Figure O.119: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males grouped in 5-year age bands in the 1989–94 NLTCS.

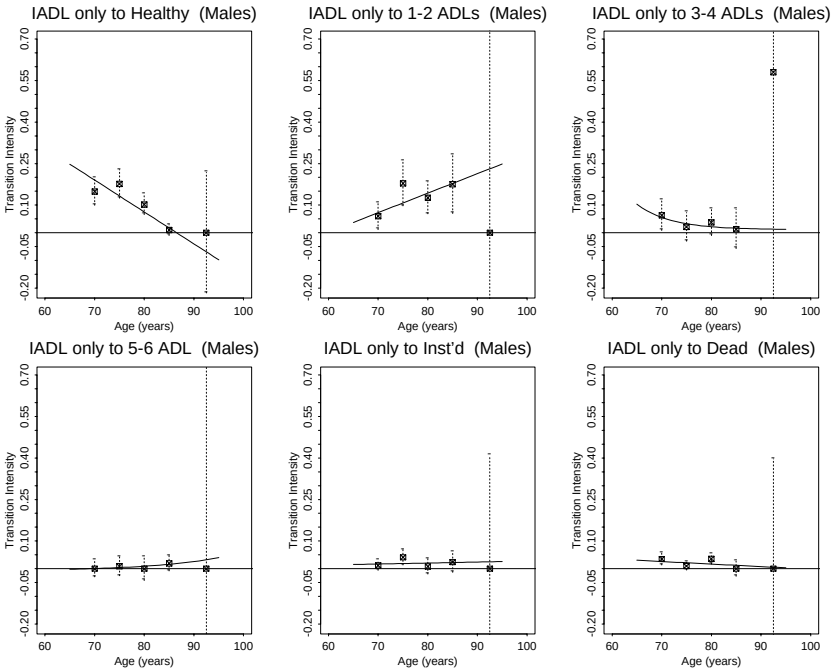

Figure O.120: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males grouped in 5-year age bands in the 1989–94 NLTCS.

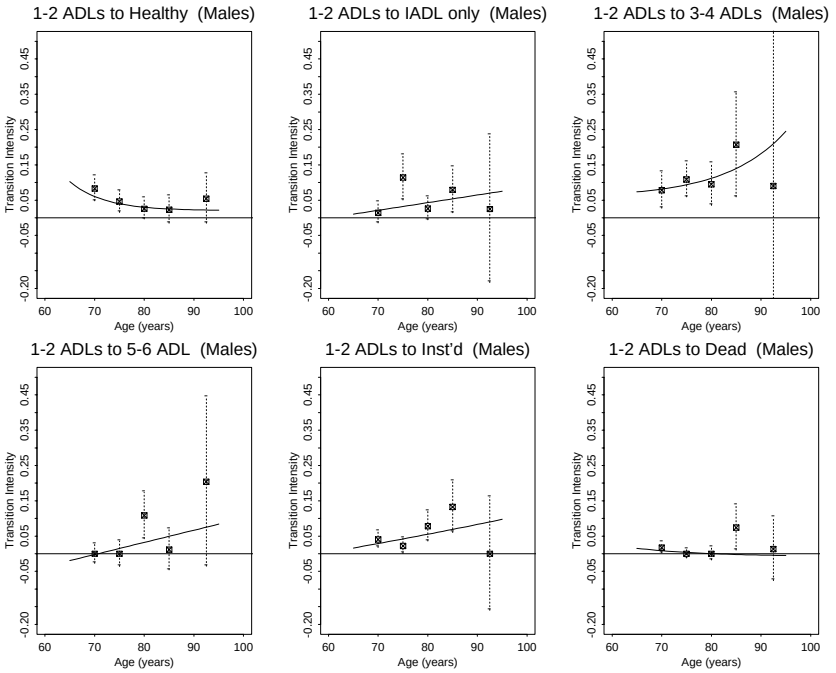

Figure O.121: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males grouped in 5-year age bands in the 1989–94 NLTCS.

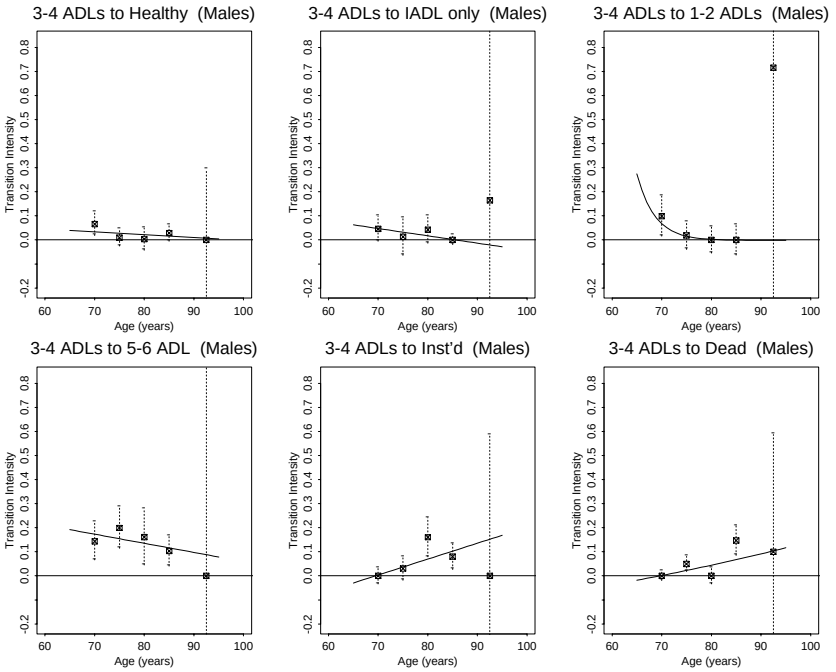

Figure O.122: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males grouped in 5-year age bands in the 1989–94 NLTCS.

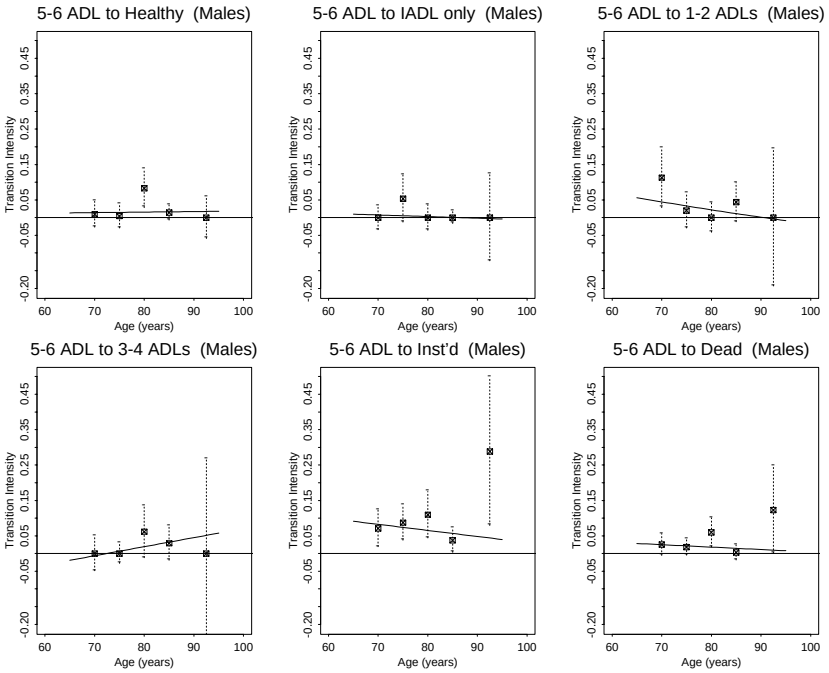

Figure O.123: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males grouped in 5-year age bands in the 1989–94 NLTCS.

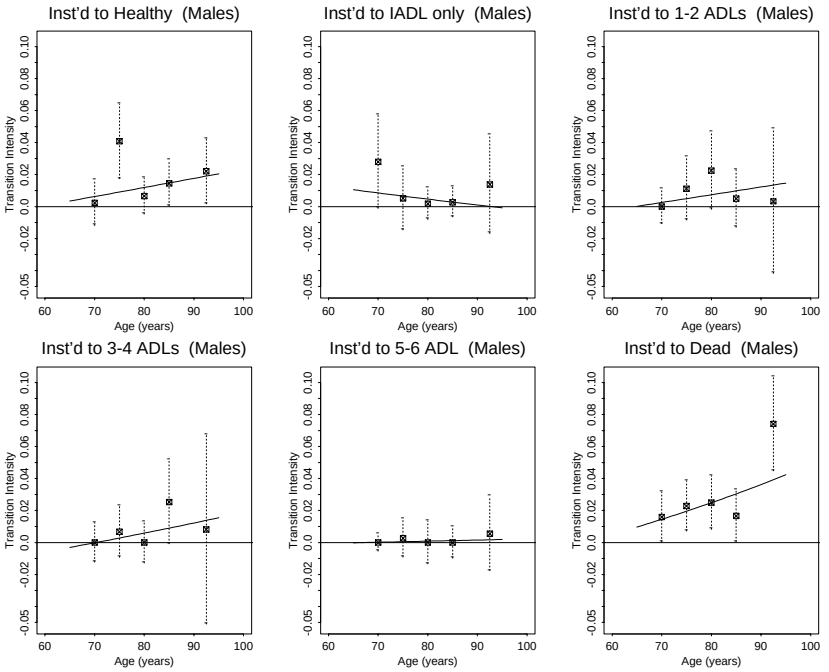

Figure O.124: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males grouped in 5-year age bands in the 1989–94 NLTCS.

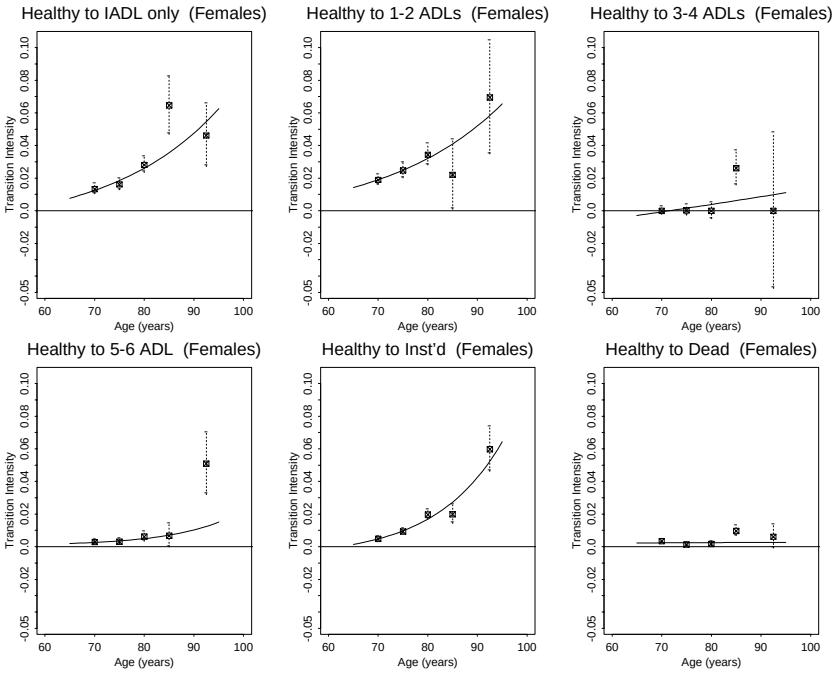

Figure O.125: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for females grouped in 5-year age bands in the 1989–94 NLTCS.

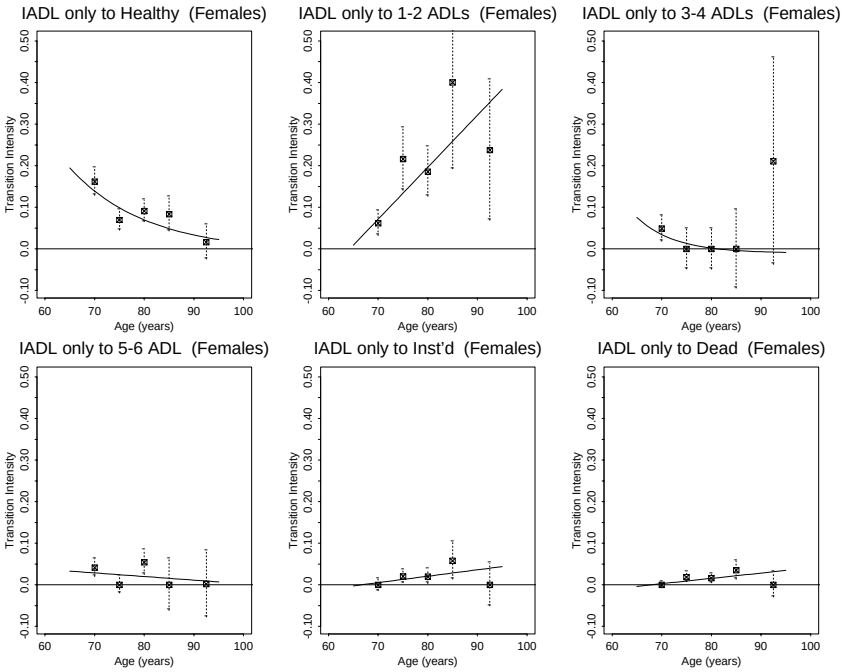

Figure O.126: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for females grouped in 5-year age bands in the 1989–94 NLTCS.

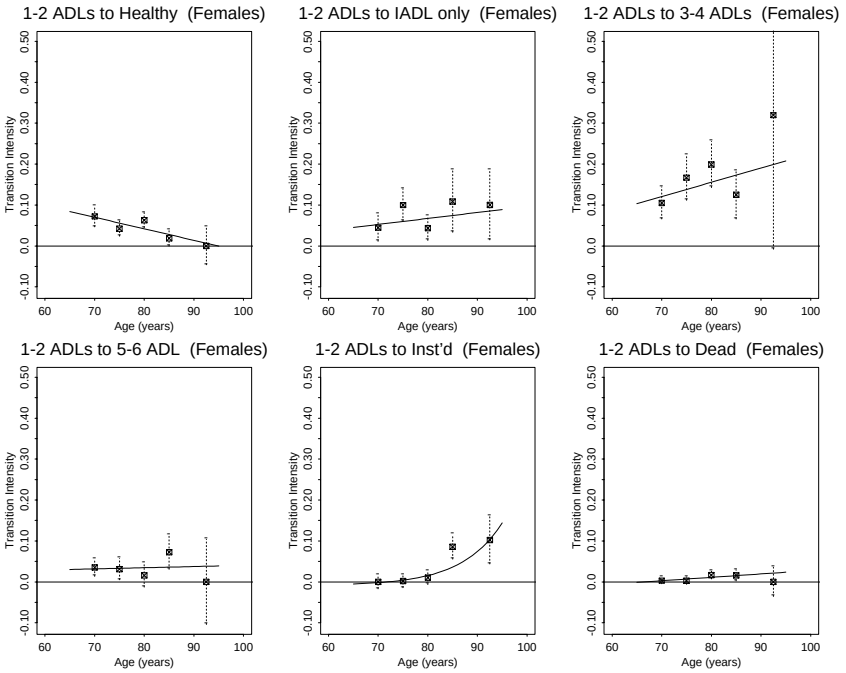

Figure O.127: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for females grouped in 5-year age bands in the 1989–94 NLTCS.

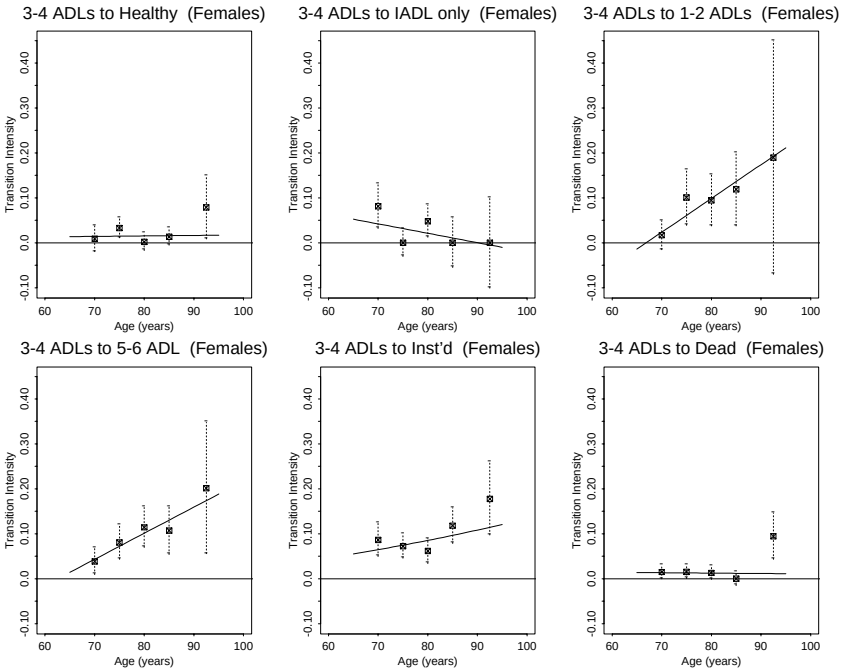

Figure O.128: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for females grouped in 5-year age bands in the 1989–94 NLTCS.

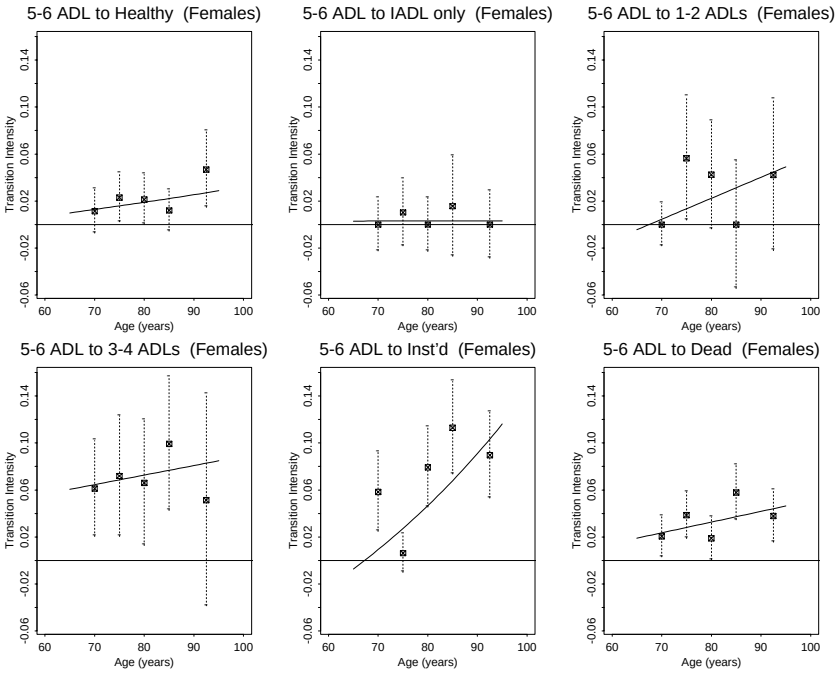

Figure O.129: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for females grouped in 5-year age bands in the 1989–94 NLTCS.

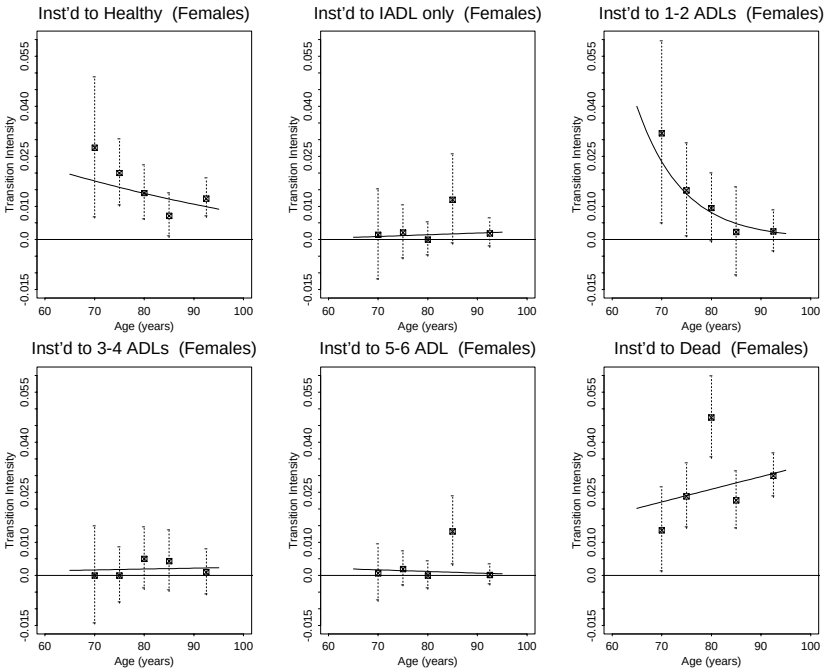

Figure O.130: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for females grouped in 5-year age bands in the 1989–94 NLTCS.

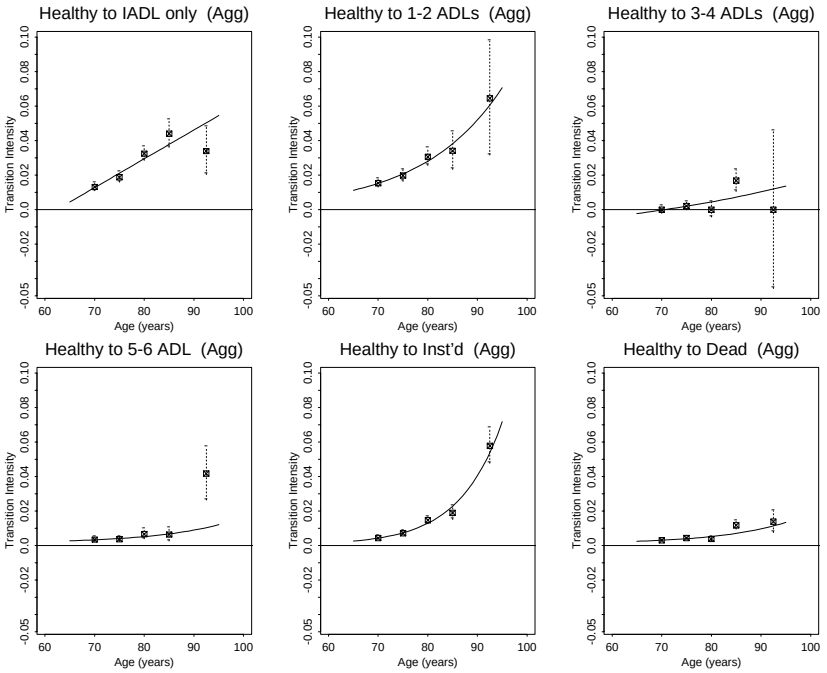

Figure O.131: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Healthy' state for males and females grouped in 10-year age bands in the 1989–94 NLTCS.

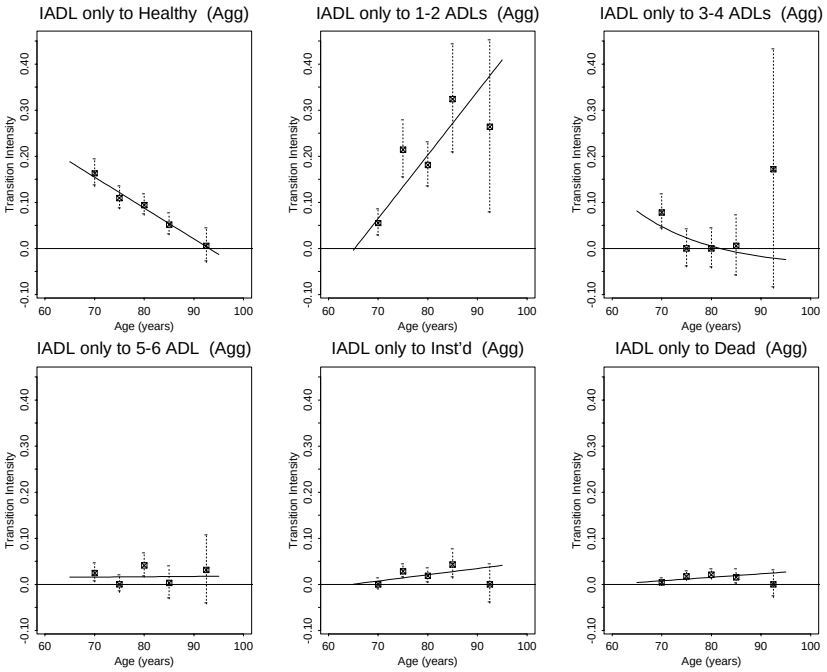

Figure O.132: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'IADL only' state for males and females grouped in 5-year age bands in the 1989–94 NLTCS.

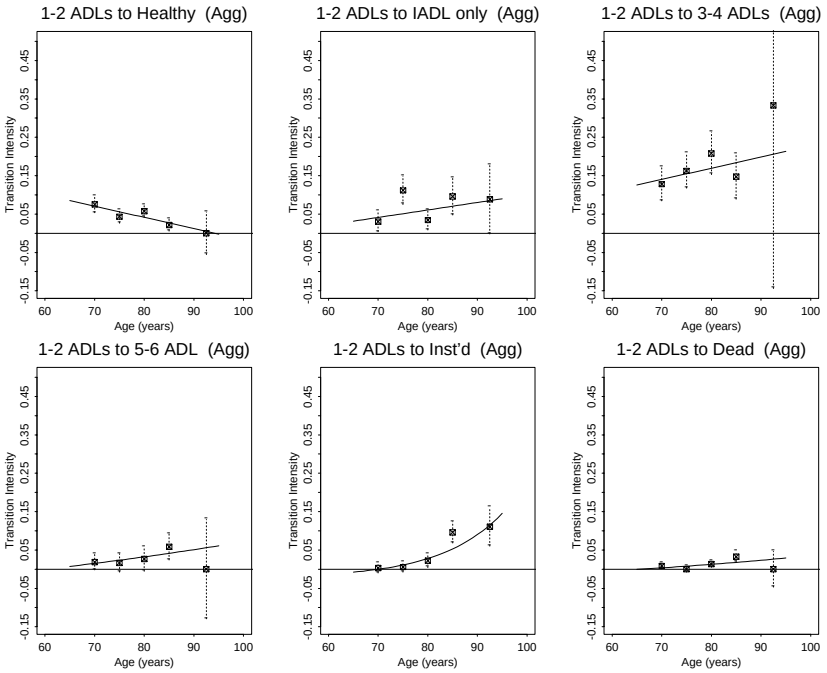

Figure O.133: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '1–2 ADLs' state for males and females grouped in 5-year age bands in the 1989–94 NLTCS.

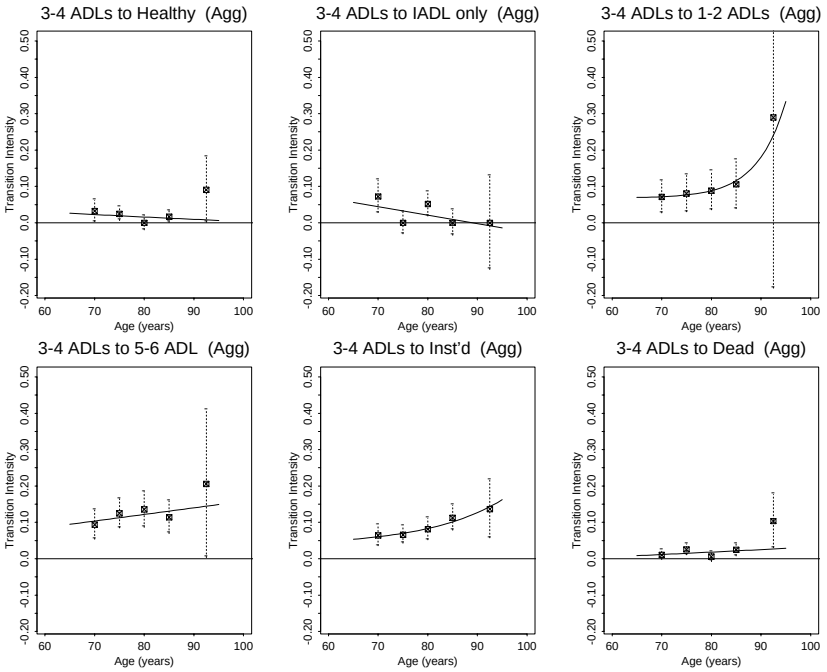

Figure O.134: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '3–4 ADLs' state for males and females grouped in 5-year age bands in the 1989–94 NLTCS.

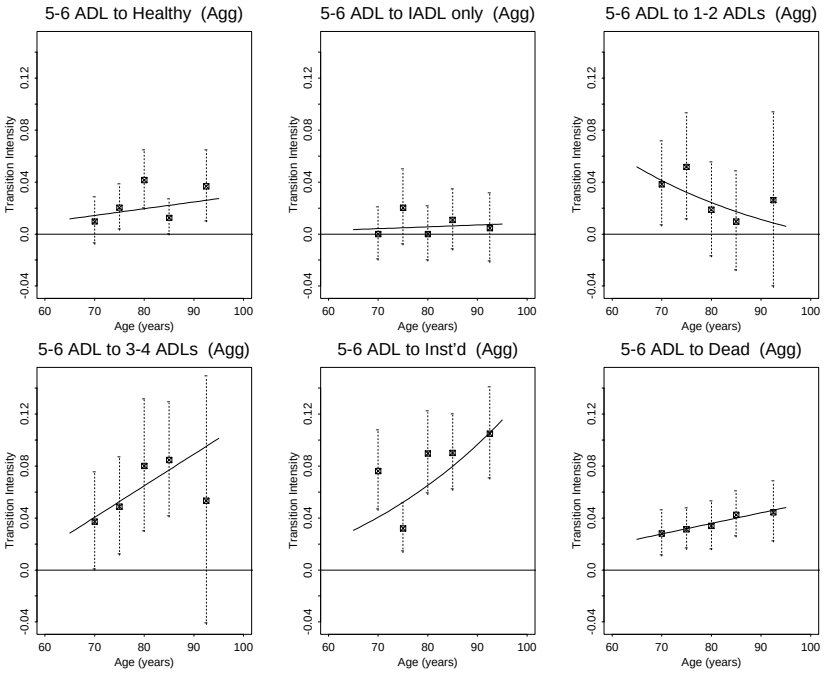

Figure O.135: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the '5–6 ADLs' state for males and females grouped in 5-year age bands in the 1989–94 NLTCS.

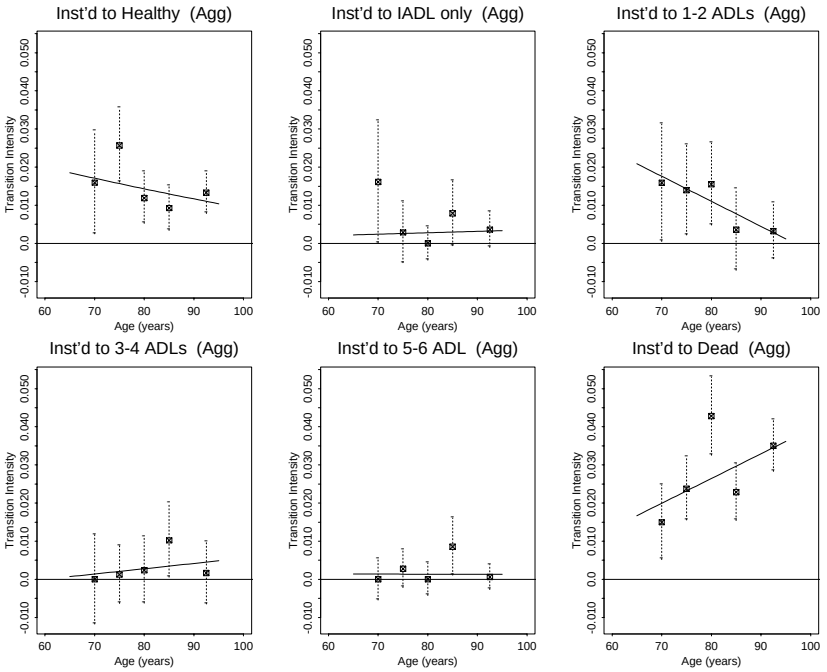

Figure O.136: Graphs of point estimates (CMLE), approximate 95% confidence intervals and parametric fits of the transition intensities out of the 'Institutionalized' state for males and females grouped in 5-year age bands in the 1989–94 NLTCS.