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Longevity hedging: A framework for longevity basis risk analysis and hedge effectiveness

Guy D. Coughlan, Marwa Khalaf-Allah, Yijing Ye, Sumit Kumar, Andrew J.G. Cairns, David Blake and Kevin Dowd

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The Pensions Institute
Cass Business School
City University
106 Bunhill Row London
EC1Y 8TZ
UNITED KINGDOM

<http://www.pensions-institute.org/>

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Guy D. Coughlan,* Marwa Khalaf-Allah,* Yijing Ye,* Sumit Kumar,*
Andrew J.G. Cairns,# David Blake@ and Kevin Dowd@

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Abstract

Basis risk is an important consideration when hedging longevity risk with instruments based on longevity indices, since the longevity experience of the hedged exposure may differ from that of the index. As a result, any decision to execute an index-based hedge requires a framework for (i) developing an informed understanding of the basis risk, (ii) appropriately calibrating the hedging instrument and (iii) evaluating hedge effectiveness. We describe such a framework and apply it to two case studies: one for the UK (which compares the population of assured lives from the Continuous Mortality Investigation with the England & Wales national population) and one for the US (which compares the population of California with the US national population). The framework is founded on an analysis of historical experience data, together with an appreciation of the contextual relationship between the two related populations in social, economic and demographic terms. Despite the different demographic profiles, each case study provides evidence of stable long-term relationships between the mortality experiences of the two populations. This suggests the important result that high levels of hedge effectiveness should be achievable with appropriately-calibrated, static, index-based longevity hedges. Indeed, this is borne out in detailed calculations of hedge effectiveness for hypothetical pension portfolios where the basis risk is based on these case studies.

Keywords: Longevity hedging, longevity index, longevity basis risk, stochastic mortality, two populations, hedge effectiveness.

* Pension Advisory Group, JP Morgan Chase Bank, 125 London Wall, London EC2Y 5AJ, United Kingdom.

Maxwell Institute for Mathematical Sciences, and Actuarial Mathematics and Statistics, Heriot-Watt University, Edinburgh, EH14 4AS, United Kingdom.

@ Pensions Institute, Cass Business School, 106 Bunhill Row, London, EC1Y 8TZ, United Kingdom.

Corresponding author: guy.coughlan@jpmorgan.com

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1. Introduction

Longevity risk – the risk that life spans exceed expectation – has become the focus of much attention for defined benefit pension plans and life insurers with large annuity portfolios. Until recently, the only way to mitigate longevity risk was via an insurance solution: pension plans bought annuities from, or sold their liabilities to, insurers and insurers bought reinsurance. Then 2008 saw the first capital markets solutions for longevity risk management executed by Lucida plc and Canada Life in the UK.¹ Both these transactions were significant catalysts for the development of the longevity risk transfer market, bringing additional capacity, flexibility and transparency to complement existing insurance solutions.

While customized (i.e., indemnity-based) capital markets longevity transactions have received more publicity following Canada Life's pioneering longevity swap, it is significant that the very first capital markets transaction to transfer the longevity risk associated with pension payments was a standardized, index-based hedge. In this transaction, Lucida plc executed a mortality forward rate contract called a "q-forward",² the payoff of which was linked to the LifeMetrics longevity index³ for England & Wales. Hedging longevity risk with index-based hedging instruments can be beneficial for several reasons. First, by standardizing the longevity exposure to reflect an index, there is the potential to create greater liquidity and lower the cost of hedging. Second, some pension plans are just too large to hedge the full extent of their exposure to longevity risk in other ways. Third, it is currently the most practical solution for hedging the longevity risk associated with deferred pensions and deferred annuities (see Coughlan (2009a)).⁴

Against the benefits of index-based longevity hedges, one must weigh the disadvantages, the primary one being basis risk. Because the mortality experience of the index will differ from that of the pension plan or annuity portfolio, the hedge will be imperfect, leaving a residual amount of risk, known as basis risk. When contemplating whether and how to hedge, it is clearly essential to evaluate the size of this risk and weigh the degree of risk reduction against the cost of the hedge.

Unfortunately, until now, there has been little work published by academics or practitioners on longevity basis risk and its impact on the effectiveness of longevity

¹In January 2008, Lucida, a UK pension buyout insurer, executed a standardized index-based longevity hedge designed to hedge liability value (see Lucida (2008) and Symmons (2008)). Then in July 2008, Canada Life executed a customized hedge of its UK liability cash flows (Trading Risk (2008) and Life and Pensions (2008)). See also Coughlan (2009b) for more details.

² The q-forward instrument is described in detail in Coughlan *et al.* (2007b).

³ The LifeMetrics longevity index is documented in Coughlan *et al.* (2007a, 2007c, 2008). See also www.lifemetrics.com.

⁴ For deferred longevity risk, index-based hedges are a viable solution, whereas customized hedges are generally either not available or else very costly. Furthermore, the risk prior to retirement is a pure valuation risk, since no payments are made before retirement. This valuation risk is driven by uncertainty in mortality improvements which are generally calibrated from improvement forecasts for large populations (e.g. national population indices). In addition, the underlying longevity exposure of deferred members of pension plans is not well defined owing to various member options, such as early retirement, lump sums, spouse transfers, etc, so exact, customized hedging is neither practicable nor desirable.

hedges. Nor has hedge effectiveness as it relates to longevity risk been well understood by practitioners and consultants in the pension and insurance industries. Lacking a proper framework, it has been common practice in some quarters to “assess” hedge effectiveness by making qualitative value judgements based on the differences in observed mortality rates. This has led to a widely held misconception that index-based longevity hedges are ineffective.

This paper addresses these issues, first, by proposing a framework for assessing longevity basis risk and hedge effectiveness and, second, by presenting two practical examples – one for the UK and one for the US – that illustrate this framework and demonstrate that index-based hedges can indeed be highly effective.

The framework we propose sets out the key principles and steps involved in a structured approach to determining the effectiveness of longevity hedges. The key initial step is a careful analysis of the basis risk between the population associated with the pension plan or annuity portfolio (the “exposed population”) and the population associated with the hedging instrument (the “hedging population”). This is illustrated by the two examples mentioned above. Each example compares the experience of the national population with a particular affluent sub-population, which, historically, has enjoyed, on average, lower rates of mortality and higher mortality improvements over time than the national population. Using this basis risk analysis, we then conduct an evaluation of hedge effectiveness for a hypothetical pension plan with the same mortality characteristics as the affluent subpopulation for a static (i.e., not dynamically rebalanced) hedge, based on a longevity index linked to the national population.

This framework provides a practical approach that hedgers can use to calibrate index-based hedges, develop an informed understanding of basis risk and evaluate hedge effectiveness.

These examples demonstrate that, despite the different demographic profiles of the population pairs, there is evidence of stable long-term relationships between their mortality experiences. This has favourable implications for the effectiveness of appropriately-calibrated, index-based longevity hedges. From this, we conclude that longevity basis risk between a pension plan, or annuity book, and a hedging instrument linked to a broad population-based longevity index can, in principle, be reduced very considerably.

The paper is organized as follows. In the next section, we describe the relationship between longevity basis risk and hedge effectiveness, and review the existing literature on the subject. Section 3 then presents the framework for analysing basis risk and hedge effectiveness. In Sections 4 and 5, we apply the framework to the two case studies mentioned above. Finally, Section 6 is devoted to conclusions.

2. Basis risk and hedge effectiveness

2.1 What is basis risk?

Basis risk arises whenever there are differences, or mismatches, between the underlying hedged item and the hedging instrument. These differences can take many forms, ranging from differences in the timing of cash flows to differences in the underlying variables that determine the cash flows. The presence of basis risk means that hedge effectiveness will not be perfect and that, post implementation, the hedged position will still have some residual risk.

It is important to note that basis risk is present to some degree in most financial hedges and it does not automatically invalidate the case for hedging. For example, the interest rate and inflation hedges used by pension plans and insurance companies almost always have some basis risk. Contrary to common practice, basis risk should always be quantified because, in many cases, it can be minimized through careful structuring and calibration of the hedging instrument to ensure high hedge effectiveness. A ‘good’ hedge is therefore one in which the basis risk is small, relative to the risk of the initial unhedged position.

2.2 Longevity basis risk

In the context of longevity and mortality, basis risk often relates to mismatches in demographics between the “exposed population” (e.g., the population of members of a pension plan or the beneficiaries of an annuity portfolio) and the “hedging population” associated with the hedging instrument (i.e., the population that determines the payoff on the hedge).⁵ These demographic mismatches can arise because the two populations are completely different, or because one population is a subpopulation of the other, or because just a few individuals are different. But regardless of how they arise, such mismatches can be classified according to a small number of demographic characteristics (Richards and Jones (2004)), such as gender, age, socioeconomic class, geographical location, etc.

If two populations have similar profiles for these characteristics, then one would generally expect basis risk to be small. On the other hand, if the two populations have vastly different profiles, then basis risk could be large, but this is not necessarily the case as we shall see below.

Examples of basis risk between populations include that originating from the mismatch in mortality rates between males and females (the “gender basis”), the mismatch between mortality at different ages (the “age basis”), the mismatch between national mortality and the mortality of a particular subpopulation (the “subpopulation basis”) and the mismatch between mortality in different countries (the “country basis”).

⁵ In the context of longevity hedging, basis risk can also arise from the structure of the hedging instrument independently of any demographics. In this paper we ignore this kind of basis risk.

The basis risk associated with gender or age is generally not an issue for index-based hedges, as it can be minimized by appropriate structuring of the hedging instrument. Indeed, most broad-based population longevity indices are broken down into sub-indices by gender and age, thereby permitting the hedging instrument to be matched to the gender and age profile of the underlying pension plan or annuity portfolio through appropriate combinations of sub-indices. As a result, the most important determinant of basis risk in most index-based hedging situations is that associated with socioeconomic class.⁶

For the purposes of this paper, we distinguish between basis risk and sampling risk, the latter being the risk associated with small populations.⁷ Our goal is to explore the elements of basis risk that cannot be explained by the size of the population, although in our examples, variability due to sampling risk will be a factor for higher ages, since the population at higher ages is relatively small and the numbers of deaths – and hence mortality rates – will be highly variable from one year to the next.

2.3 Hedge effectiveness

While hedge effectiveness is an intuitive concept, it is not yet widely understood or applied in the context of longevity hedging.⁸ Certainly, the presence of longevity basis risk reduces the effectiveness of longevity hedges, but the relationship between the two is not as straightforward as one might suppose. Whereas basis risk is typically measured in demographic terms, hedge effectiveness should be measured in economic terms and demographic mismatches do not necessarily result in significant economic costs, as we shall illustrate. The key to designing an appropriate method for assessing hedge effectiveness is to start from hedging objectives that reflect the nature of the risk being hedged and focus on the degree of risk reduction in economic terms. One way to do this is by using a Monte Carlo simulation to generate forward-looking scenarios for the evolution of mortality rates for both the exposed and hedging populations. Ideally, a two-population stochastic mortality model, calibrated from a suitable basis risk analysis, should be used to do this in a consistent fashion. However, we wish, in this paper, to avoid the additional complexity of dealing with a formal model in order to concentrate on producing a straightforward exposition of longevity hedge effectiveness. Accordingly, we adopt a model-free (or nonparametric) approach to measuring hedge effectiveness, based largely on historical data on mortality rates and mortality rate improvements.

⁶ Variations in mortality associated with different regions within a country can largely be explained by socioeconomic or lifestyle differences. See, for example, Richards and Jones (2004).

⁷ Even if we had a sample from a larger population without basis risk, because it was free from gender, age and sub-population biases, the small population would still experience sampling – random variation – risk relative to the large population which would cause its mortality experience to diverge from that of the large population over time.

⁸ It is worth drawing the parallel that when the requirement to assess hedge effectiveness was first introduced in an accounting context under US GAAP (SFAS 133) and IFRS (IAS 39), the subject was at that time poorly understood by both practitioners and accountants.

2.4 Existing literature

Several authors have explored the basis risk between populations associated with annuity portfolios and life insurance portfolios. Cox and Lin (2007) found empirical evidence of a (partial) natural hedge operating between such portfolios, implying that the basis risk between them is relatively small.⁹ Coughlan *et al.* (2007a: pp 85-87) provided a calculation of the risk reduction between hypothetical annuity and life insurance portfolios using historical mortality experience data: the results suggest significant benefits in terms of reduction in risk and economic capital. Sweeting (2007) explored the basis risk associated with longevity swaps in a more qualitative fashion but draws similar conclusions.

Recently, a number of researchers have developed mortality models for two or more related populations (see Li and Lee (2005), Jarner (2008), Jarner and Kryger (2009), Plat (2009), Li and Hardy (2009), Cairns *et al.* (2010) and Dowd *et al.* (2010)). These models are all based on the principle that, on the grounds of biological reasonableness,¹⁰ the mortality rates of related populations should not diverge over the long term. Whereas all these papers are motivated by a desire to develop coherent and consistent forecasts, only the latter four express an explicit goal related to the measurement of basis risk and hedge effectiveness. Ideally, a coherent two-population model is needed to evaluate basis risk prospectively, but such a model also needs to be both intuitively appealing and appropriately calibrated, and these, in turn, depend on a sound understanding of historical mortality experience.

Hedge effectiveness testing in a general context has been addressed by Coughlan *et al.* (2004) and its application to longevity hedging has been briefly discussed in Coughlan *et al.* (2007a) and Coughlan (2009b). More recently, Ngai and Sherris (2010) have evaluated some hedge effectiveness metrics for hedges of various annuity products in the Australian market.

⁹ Similar results have been found by Dahl and Møller (2006), Friedberg and Webb (2007), and Wang *et al.* (2010).

¹⁰ A method of reasoning used to establish a causal association (or relationship) between two factors that is consistent with existing medical knowledge.

3. A framework for analysing basis risk and hedge effectiveness

Any decision to execute an index-based longevity hedge requires a framework for (i) developing a deep understanding of the basis risk involved, (ii) calibrating the hedging instrument and (iii) evaluating hedge effectiveness.

In most situations involving real pension plans and annuity portfolios, the amount of historical data available will be too short to draw rigorous statistical conclusions about basis risk. Nevertheless, by examining available data carefully and trying to identify key demographic – especially socioeconomic – characteristics, one can usually develop an informed assessment of the nature and magnitude of this risk. As fits with risk management best practice in other areas, hedging decisions are ultimately based on professional judgment supported data analysis and experience.

3.1 Basis risk analysis

Basis risk analysis should be appropriately aligned with the hedging objectives in terms of the metric, the time horizon and the analytical method.

3.1.1 Metric

There are many different metrics that can be used to gain a perspective on the basis risk associated with longevity hedges. Because of the complex relationships between mortality experience across age, time (or period) and year of birth (or cohort), it is necessary to examine the historical performance of all key metrics. These are:

- mortality rates (either crude rates or graduated rates)
- mortality improvements (i.e., percentage changes in mortality rates)
- survival rates
- life expectancies
- liability cash flows
- liability values.

Since mortality rates constitute the basic raw data associated with longevity, they have been the most commonly used metric for assessing basis risk. Unfortunately, a direct comparison between the mortality rates of two populations provides a naïve and often misleading perspective on basis risk and the effectiveness of longevity hedges. This is for several reasons. First, mortality rates as metrics are not directly related to the effectiveness of longevity hedges. Therefore, one needs to be careful in drawing any conclusions regarding the impact of basis risk, as observed in mortality rate comparisons, on longevity hedge effectiveness.

Second, the data corresponding to annual mortality rates at particular ages contain a lot of noise or sampling variability. This noise is present both though time and across ages, and is evidenced in the observed year-on-year fluctuations of mortality rates around their long

term trends.¹¹ The noise can be reduced by (i) graduating mortality rates across ages using a smoothing routine; (ii) bucketing adjacent ages together when calculating mortality rates; and (iii) evaluating the changes in mortality rates over the longer time horizons that are more typical of the timescales associated with longevity trends emerging. So, in using mortality-rate comparisons to evaluate the basis risk of longevity hedges, it is important to incorporate these three elements – graduation, age bucketing and longer horizons – into the analysis.

Survival rates and life expectancy both address the above shortcomings as metrics for basis risk analysis. Because survival rates in a pensioner population correspond to the number of members who are still alive to receive a pension and life expectancy corresponds to the expected period over which a pension needs to be paid, these metrics are more closely related to the hedge effectiveness objective than mortality rates. Moreover, both survival rates and life expectancy are calculated from many different mortality rates for different ages and at different times, so there is natural smoothing out of the noise that is associated with individual mortality rates in individual years. For example, the 10-year survival rate for 65-year-old males depends on the mortality rates for males aged 65 through to 74. Similarly, life expectancy for 65-year-olds depends on the mortality rates for every age above 65.

Although useful in developing an understanding of basis risk, none of the above metrics is ideal for quantifying hedge effectiveness. Since most hedging exercises are focused on mitigating the variability in the liability cash flows or the variability in the value of these cash flows, basis risk studies should ideally focus on the impact on cash flow and/or value. These metrics directly reflect the monetary impact of basis risk and are the appropriate metrics for evaluating the effectiveness of longevity hedges.

While cash flow and value are more useful for quantifying basis risk than the other metrics discussed above, they suffer from one main disadvantage in that they depend on the specific details of the benefit structure of the particular pension plan or annuity portfolio and, as such, involve complex calculations, including discounting of future cash flows, that must be repeated in full for each situation. By contrast, mortality rates, survival rates and life expectancy are independent of the details of the specific benefit structure and can, with appropriate interpretation, give useful insights into basis risk.

3.1.2 Time Horizon

The choice of time horizon is important in assessing basis risk. Longevity risk, as it applies to large populations, is a slowly-building, cumulative trend risk that should be evaluated over long time horizons. To be consistent with this, metrics should be evaluated over horizons of at least several years. For example, in comparing the evolution of mortality rates for two populations, it is desirable to evaluate changes in their mortality rates over multi-year horizons, rather than year on year.

¹¹ Indeed, mortality rates typically exhibit a negative autocorrelation, so that a high mortality rate in one year is followed by a low mortality rate the next year (Coughlan et al. (2007a)).

Unfortunately, using long horizons means that there are fewer independent observations available from a given historical data set. So selecting the time horizon for analyzing basis risk involves making a trade-off between a horizon long enough to identify trends and short enough to provide enough independent data points to give a robust analysis.

3.1.3 Analytical Method

The analytical method for evaluating basis risk should, as for the metric and time horizon, also be appropriately aligned with the hedging objective. This means deciding on various details of the analysis, such as whether to compare the levels of a particular metric or changes in the metric for each population. If comparing changes, we need to specify how these changes should be defined. For example: What time period is optimal? Should we use overlapping periods or non-overlapping periods? Should we use one-period changes or cumulative multi-period changes?

Other methodological choices include, for example, whether and how to bucket age groups and whether and how to graduate mortality rates.¹² The use of both age-group bucketing and graduation are generally desirable to reduce noise and can be justified because mortality rates for adjacent ages are similar and highly correlated.¹³ As a result, bucketing and graduation do not destroy the integrity of the data, rather they bring the twin benefits of simplification and noise reduction, thereby rendering a clearer perspective on basis risk and hedge effectiveness. In practice, mortality curves – as represented in mortality tables – are graduated as a key part of the valuation process for liabilities whenever pension or annuity portfolios are transferred between different counterparties. Furthermore, hedges constructed using bucketed age groups have already been transacted in the capital markets and their effectiveness in reducing risk has not been compromised by the age bucketing.

Once these aspects of the analysis have been decided, the next decision is how to compare the results across the two populations. This can be done qualitatively in a graphical format, or quantitatively using statistical analyses, such as correlation.

Correlation is a common way of evaluating basis risk in a general setting, but care should be taken in the context of longevity. Correlation in the annual improvements in mortality rates between two populations reflects the short-term relationship between these populations and, by virtue of the noise inherent in mortality data, can give a very misleading indication of the strength of the relationship between their long-term trends. By contrast, correlations between long-term mortality improvements are more relevant indicators, but, as mentioned, there will be far fewer independent data points for long-term improvements.

The essence of longevity basis risk analysis is the search for a stable long-term relationship between the two populations. If such a relationship can be identified, then an

¹² The graduation and bucketing methods used in this paper are explained in Appendices A1 and A2.

¹³ The “age basis” is typically small for adjacent ages in a large population (see Coughlan et al. (2007a)).

appropriate index-based longevity hedge can be calibrated by determining the optimal hedge ratios for the hedging instrument.

3.2 Hedge calibration

Hedge calibration refers to the process of designing the hedging instrument to maximize its effectiveness in reducing risk, relative to the hedging objectives. It involves two elements. The first is the determination of the appropriate structure and characteristics of the hedging instrument (e.g., type of instrument, maturity, index to be used, etc.). The second is the determination of the optimal amount of the hedge required to maximize hedge effectiveness. This involves determining optimal “hedge ratios” for each of the sub-components of the hedging instrument.

As a simple example, consider a hedging instrument with just one component designed to hedge the value of a pension liability at a future time, that we call the “hedge horizon”. Suppose we have bought h units of the hedge for each unit of the liability: h is the hedge ratio. Then the total (net) value of the combined exposure is:

$$V_{\text{Total}} = V_{\text{Liability}} + h \cdot V_{\text{Hedge}} \quad (1)$$

The optimization element referred to above involves selecting h to maximize hedge effectiveness by minimizing the uncertainty in V_{Total} . It can be shown that, assuming the values are normally distributed and risk is measured by standard deviation, then the optimal hedge ratio is given by (Coughlan *et al.* (2004)):

$$h_{\text{Optimal}} = -\mathbf{r} \cdot (\mathbf{s}_{\text{Liability}} / \mathbf{s}_{\text{Hedge}}) \quad (2)$$

where $\mathbf{s}_{\text{Liability}}$ and $\mathbf{s}_{\text{Hedge}}$ are the standard deviations of the values of the liability and hedging instrument, respectively, at the hedge horizon and \mathbf{r} is the correlation between them.

It is evident from this simple example that basis risk analysis is an essential prerequisite for optimal hedge calibration.

3.3 Hedge effectiveness methodology

Assessing hedge effectiveness requires taking account of the hedging objectives and the nature of the risk that is being hedged to develop a methodology that is appropriate. Table 1 summarizes the key steps involved in this process (derived from Coughlan *et al.* (2004)). A key part of the methodological choice is whether hedge effectiveness is to be assessed *retrospectively* or *prospectively*.

Retrospective hedge effectiveness analysis involves using actual historical data to assess how well a hedging instrument would have performed in the past. In this kind of

effectiveness test, basis risk is taken account of by virtue of the historical relationships between the observed mortality outcomes for both the hedging and exposed populations.

By contrast, prospective hedge effectiveness analysis involves developing forward-looking scenarios to anticipate how well a hedging instrument might perform in the future. This involves a Monte Carlo simulation of potential future paths for mortality rates from which the performance of the hedging instrument can be assessed relative to the underlying longevity exposure. In this case, basis risk must be explicitly taken into account, with the simulation of scenarios for future mortality rates reflecting, in a consistent way, the observed relationship between the hedging and exposed populations. As mentioned above, this ideally requires a two-population stochastic mortality model to be used.

Let us discuss the hedge effectiveness framework presented in Table 1 in greater detail. Step 1 involves defining hedging objectives, in particular designating the precise risk being hedged. This includes the risk class (i.e., longevity risk), as well as the precise nature of what is being hedged (e.g., longevity trend improvements above 2% per year over the next 10 years, or the total uncertainty in survivorship over 40 years, etc.). An essential part of this is defining the hedge horizon of the hedging relationship as well the performance metric (e.g., hedging liability cash flows or liability value).

Step 2 in the process is to select the hedging instrument and calibrate the optimal hedge ratio. The latter should be chosen to maximize the degree of risk reduction and should be determined from an appropriate basis risk analysis, as suggested by the simple example in the previous section (see, for example, equation (2)).

Step 3 in the process – which defines the hedge effectiveness methodology – is important, because an inappropriate choice can lead to spurious and misleading results with effective hedges being deemed ineffective, or vice versa. Defining the methodology involves several choices. The first choice involves selecting between a retrospective and prospective effectiveness test which we have discussed above. The second choice to be made is the “basis for comparison” which involves specifying how the performance of the unhedged and hedged exposures are to be compared. A simple choice is in terms of the degree of risk reduction:

$$\text{Relative Risk Reduction} = RRR = 1 - \text{Risk}_{(\text{Liability} + \text{Hedge})} / \text{Risk}_{\text{Liability}} \quad (3)$$

Clearly, a perfect hedge reduces the risk to zero, corresponding to 100% risk reduction.

If the hedging objectives are framed in terms of hedging the liability value, another key element of the basis for comparison choice is the pricing (i.e., valuation) model used to determine the values of the liability and the hedging instrument under different scenarios at the hedge horizon.

The next choice to be made is the selection of the risk metric. If the hedging objectives are couched in terms of hedging liability value, then an example of an appropriate risk

metric might be the value-at-risk (VaR) of the liabilities at the hedge horizon relative to an expected, or a median, outcome and calculated at a particular confidence level.

The final methodological choice relates to selecting the type of simulation model used to generate the scenarios needed for the test.

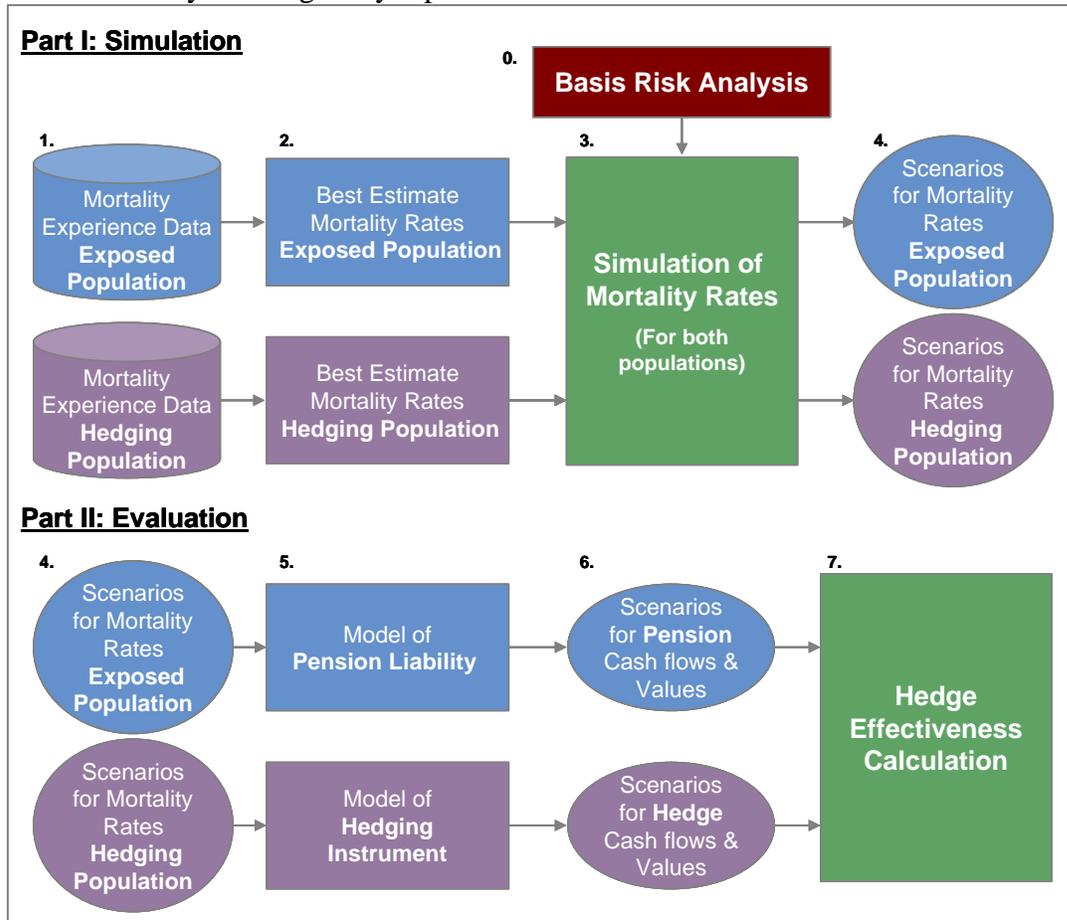
Step 4 addresses the actual calculation of hedge effectiveness. This involves an implementation of the method defined in the previous step as a two-stage process: (i) simulation and (ii) evaluation. Note that the simulation of mortality risk is a separate process from the evaluation of the impact of the scenarios on the liability and the hedging instrument, as illustrated in Figure 1. In particular, the evaluation process is the same for any set of mortality scenarios, regardless of how the set of scenarios is generated. Figure 1 also shows the important role of basis risk analysis in the calculation of hedge effectiveness.

Step 5 in the process – the final step in the framework – involves interpreting the hedge effectiveness results.

Table 1: Framework for assessing hedge effectiveness

Step 1	Define hedging objectives <ul style="list-style-type: none"> • Metric • Hedge horizon • Risk to be hedged (full or partial)
Step 2	Select hedging instrument <ul style="list-style-type: none"> • Structure hedge • Calibrate hedge ratio
Step 3	Select method for hedge effectiveness assessment <ul style="list-style-type: none"> • Retrospective vs. prospective test • Basis for comparison (comparing hedged and unhedged performance, valuation model, etc.) • Risk metric • Simulation model to be used
Step 4	Calculate the effectiveness of hedge <ul style="list-style-type: none"> • Simulation of mortality rates for both populations • Evaluation of effectiveness based on the simulations
Step 5	Interpret the effectiveness results

Figure 1: The process of hedge effectiveness assessment (step 4 in the hedge effectiveness framework) involves two distinct parts: simulation and evaluation, with basis risk analysis being a key input into the former.



4. UK basis risk case study

In this section, we present the results of an empirical analysis of the basis risk between the national population of England & Wales males (based on data from the Office for National Statistics, ONS) and the population of UK males who own life assurance policies (based on data from the Continuous Mortality Investigation, CMI). The ONS is the UK government agency that compiles official national mortality statistics. The CMI is a body, funded by the UK life insurance industry and run by the UK Actuarial Profession, which publishes mortality rates for assured lives, derived from data submitted by UK life insurers. The data used in this analysis cover the 45-year period 1961-2005.

The CMI data come from an affluent subset of the UK population, whose mortality rates have consistently been lower and mortality improvements higher than those of the national population.¹⁴ It is important to note that the population of assured lives behind the CMI data is a subset of the national population that changes from year to year, depending on which insurers choose to submit their data: the CMI data are therefore not only a subset of the national population, but also a (changing) subset of the population of assured lives. Furthermore, the number of lives in the CMI assured lives data set has fallen significantly over the past 20 years. Currie (2009) has a chart showing how the exposure by age has changed through time, from a peak of over 200,000 lives at an age of around 40 in 1985 to a peak of fewer than 50,000 lives for males in their late 50s in 2005. For higher ages, the exposures are even lower. Such changes have inevitably introduced additional noise into the CMI data and are likely to lead to a higher measured basis risk than genuinely exists between the two populations. As a consequence, the results of estimating basis risk that we present in this paper are likely to be conservative.

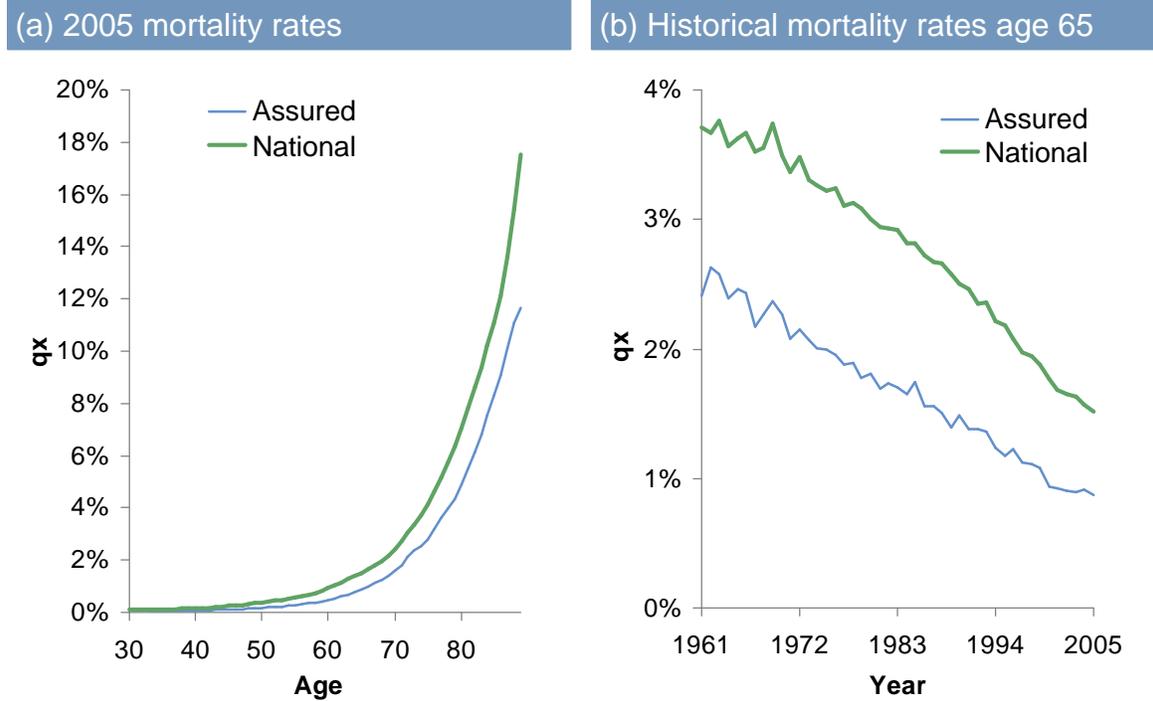
In this analysis, we used graduated initial mortality rates, whose calculation is described in Appendix A.2. Moreover, we only consider data up to age 89, as beyond this age the ONS only publishes aggregated mortality statistics and the results may be affected by the modelling choice for higher ages used in the graduation of mortality rates. We begin by looking at mortality rates, before moving on to examine other metrics.

4.1 Mortality rates and mortality improvements

Figure 2 shows a graphical comparison of graduated mortality rates for the assured population and the national population. The most obvious feature of the data, which is common to all ages over 35, is the significant difference in the level of mortality rates for the two populations: assured mortality is much lower than national mortality. But what is also evident is that the long-term downward trends are quite similar, suggesting that there might be a long-term relationship between the mortality rates of the two populations. Certainly from year to year, there is volatility around each trend and the assured data set appears to be noisier (particularly at higher ages), but broadly speaking the observed improvements in mortality are moving together and not diverging.

¹⁴ The CMI data are a subset of the UK population, so are not strictly a subset of the population of England & Wales. However, in the context of this analysis the difference is small.

Figure 2: A comparison of male mortality rates for the UK assured and England & Wales national populations: (a) Spot mortality curves for 2005, (b) Historical evolution of graduated mortality rates for 65-year-old males, 1961-2005.



Comparing the average levels of mortality rates (Table 2) for the two populations, we see that assured mortality in 2005 was on average 57% of national mortality, having fallen from 68% in 1961. So there has been a pronounced decrease in relative assured mortality rates since 1961. Moreover, the relative rates vary by age, with assured mortality for the younger, pre-retirement ages of 40-64 currently averaging just 46% of national mortality and for older post-retirement ages of 65-89 averaging 68%.

Over the period 1961-2005, observed mortality improvements (Table 3) have averaged 2.04% p.a. for the assured population, compared with 1.62% p.a. for the national population. Furthermore, the younger pre-retirement ages have experienced much higher improvements of 2.32% p.a. and 1.67% p.a., respectively, for assured males and national males, while for the older post-retirement ages improvements have been lower, at 1.75% and 1.57%, respectively.

Table 2: UK assured male mortality rates as a percentage of England & Wales national male mortality rates, averaged over age.

Ratio of mortality rates (Assured/National)	1961	2005
Overall: 40-89	68%	57%
Younger: 40-64	62%	46%
Older: 65-89	74%	68%

Table 3: Annualized male mortality improvements for the England & Wales national and UK assured populations, averaged over age groups, 1961-2005

Mortality improvements 1961-2005 (annualized)	National (% p.a.)	Assured (% p.a.)	Difference (percentage points)
Overall: 40-89	1.62	2.04	0.42
Younger: 40-64	1.67	2.32	0.65
Older: 65-89	1.57	1.75	0.18

As we have already mentioned, the differences in both the levels of and the improvements in mortality rates between the two populations do *not* necessarily mean that the effectiveness of longevity hedges will be poor. Indeed, there appears to be a relatively stable long-term relationship between them that can be exploited to construct hedges that are highly effective. That this is the case is evidenced by evaluating aggregate correlations in the observed changes in mortality rates for the two populations (Tables 4 and 5).

Table 4 lists the “aggregate correlations” for changes in mortality rates over different horizons calculated from individual ages. The calculation of these correlations is described in Appendix A.3, and involves evaluating the correlation for changes in mortality rates over non-overlapping periods jointly for each individual age. The correlations are calculated for both absolute changes in mortality rates and relative, percentage changes, i.e., mortality improvements. Note that the aggregate correlations in year-on-year changes based on individual ages are quite small, just 36%, but they increase with the length of the time horizon. Correlations are around 97% or more for a 20-year horizon and around 80% for a 10-year horizon.

Table 4: Aggregate correlations of changes in male mortality rates for individual ages between the UK assured and England & Wales national populations, 1961-2005.

Individual ages	Correlation between absolute changes in mortality rates		Correlation between improvement rates (relative changes)	
	Individual ages: 40-89	Individual ages: 50-89	Individual ages: 40-89	Individual ages: 50-89
20-year horizon	97%	97%	93%	96%
10-year horizon	80%	77%	81%	85%
5-year horizon	69%	66%	65%	70%
1-year horizon	36%	36%	29%	36%

Note: Correlations are calculated across time (using non-overlapping periods) and across individual ages (without any age bucketing), using graduated mortality rates. See Appendix A.3.

Using age buckets helps remove noise from the mortality data and leads to much higher correlations as shown in Table 5. With 10-year age buckets (50-59, 60-69, 70-79 and 80-89), the aggregate correlation for year-on-year changes is 54%, rising to 94% for a 10-year horizon and to 99% for a 20-year horizon.

But it should be noted that for long horizons, there is a limited number of data points. Small numbers of data points lead to an upward bias in the correlation results and increased sampling noise, so the results should be considered as indicative only. Despite this lack of formal statistical robustness, we should take comfort from the fact that the aggregate correlation results are collectively consistent and intuitive. Furthermore, the results of the other analyses in this section provide additional support for the existence a long-term relationship between the two populations.

Table 5: Aggregate correlations of changes in male mortality rates for 10-year age buckets between the UK assured and England & Wales national populations, 1961-2005.

Age buckets: 50-59, 60-69, 70-79, 80-89	Correlation between absolute changes in mortality rates	Correlation between improvement rates (relative changes)
20-year horizon	99%	98%
10-year horizon	94%	93%
5-year horizon	91%	85%
1-year horizon	54%	51%
Note: See note to Table 4.		

4.2. Survival rates

Comparing long-term survival rates provides a different perspective on basis risk and the relationship between the longevity experiences of the two populations. This is because long-term (multi-year) survival rates involve mortality rates for different ages across different years.¹⁵ Figure 3(a) shows the evolution of 10-year survival rates for the two populations for 65-year-old males over the period 1970-2005. The 10-year survival rate at age 65 therefore shows the proportion of 65-year-olds surviving to age 75. Both survival rates have been increasing over time but, more importantly, the ratio between them has been more or less constant over time, as shown in Figure 3(b). The latter chart suggests a relatively stable long-term relationship between the survival rates of the two populations.

Note that the survival ratio of assured to national survival rates is greater than one for all ages and increases with age. The average survival ratios over the period are listed in Table 6 along with some summary statistics on the variation in the survival ratio through time. For example, 45-year-old males have an average survival ratio of 1.03 compared

¹⁵ Short-term survival rates, by contrast, provide the same perspective as mortality rates since a 1-year survival rate is just one minus the corresponding mortality rate.

with an average of 1.55 for 80-year-olds. This means that the assured population has 3% more 45-year-old males surviving to be aged 55 than the national population. Similarly the assured population has 55% more 80-year-olds surviving to age 90.

Figure 3: Historical 10-year male survival rates for the UK assured and England & Wales national populations based on data over the period 1961-2005: (a) Historical evolution of 10-year survival rates for males reaching 65 in different years between 1970 and 2005, (b) Ratio of the 10-year survival rate for the assured population to the 10-year survival rate for the national population for males reaching various ages in different years between 1970 and 2005.

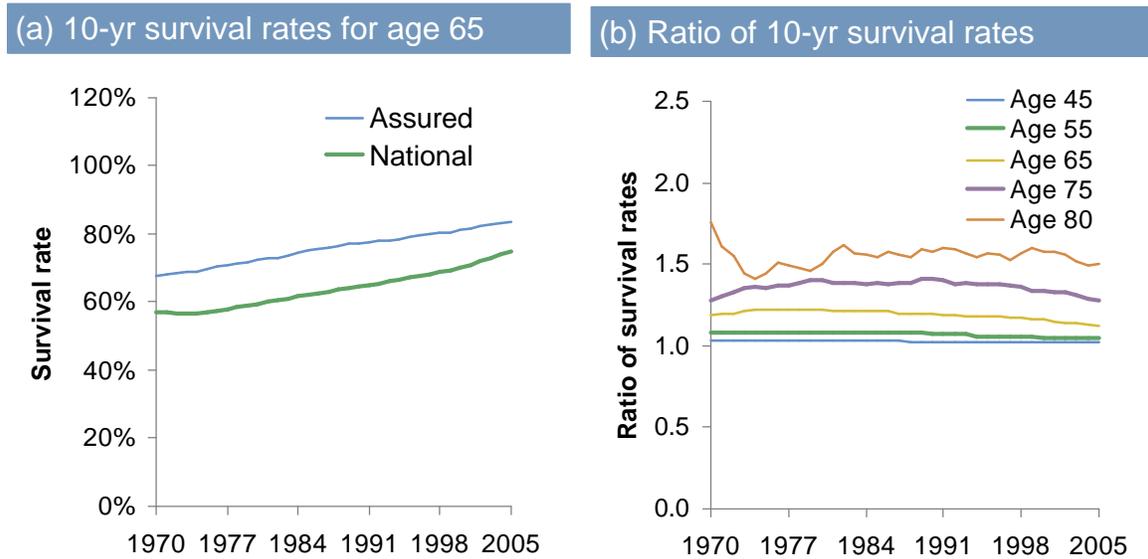


Table 6: Key statistics on the male survival ratio between the UK assured and England & Wales national populations. The survival ratio is defined as the 10-year survival rate for the assured population to the 10-year survival rate for the national population over the period 1970-2005.

10-year survival ratio (Assured / National)	Age 45	Age 55	Age 65	Age 75	Age 80
Average survival ratio	1.03	1.07	1.19	1.36	1.55
Standard deviation	0.004	0.011	0.029	0.037	0.063
Coeff of variation (std dev / average)	0.4%	1.1%	2.4%	2.7%	4.1%
Worst case (max / average)	0.6%	2.0%	6.2%	6.1%	13.8%

Notes: Survival rates are calculated for each age cohort using graduated mortality rates. The quoted age represents the age at the start of the 10-year period.

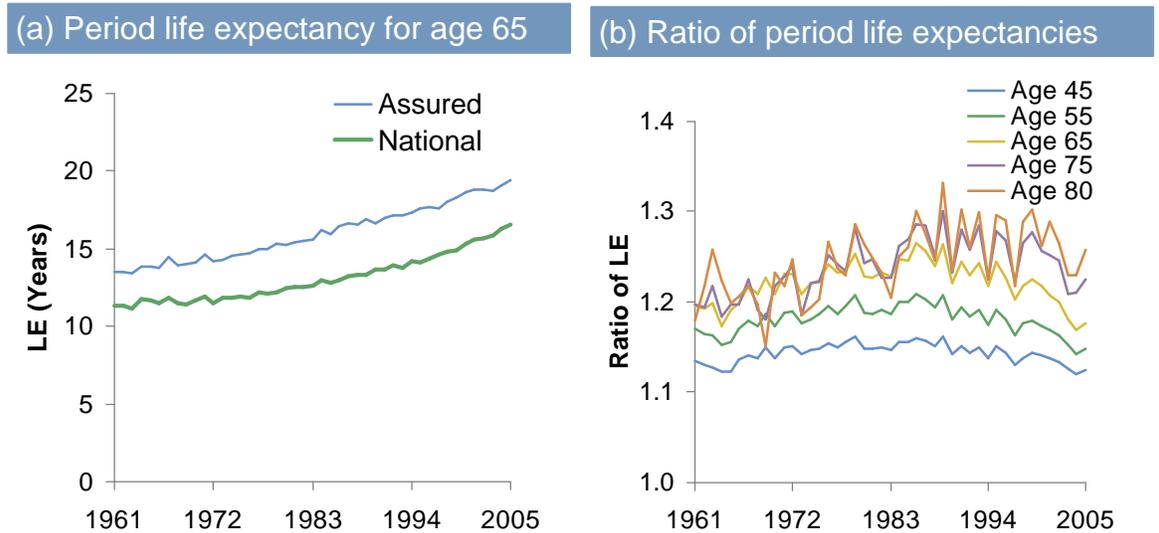
4.3 Life expectancy

Another perspective on basis risk comes from *period* life expectancy.¹⁶ This is a measure of how much longer on average individuals would be expected to live and, in a pension context, of how much longer one would expect that retirement income must be paid. Note that these results are dependent on the method used to estimate mortality rates at very high ages for which only limited mortality data are available.

Figure 4 shows the evolution of (curtate) period life expectancy for selected ages over 1961-2005. Note that despite the different levels of life expectancy, the ratio between the life expectancies of the two populations is relatively constant through time and increases with age. The ratio averages 1.14 at age 45, 1.22 at age 65 and 1.24 at age 80.

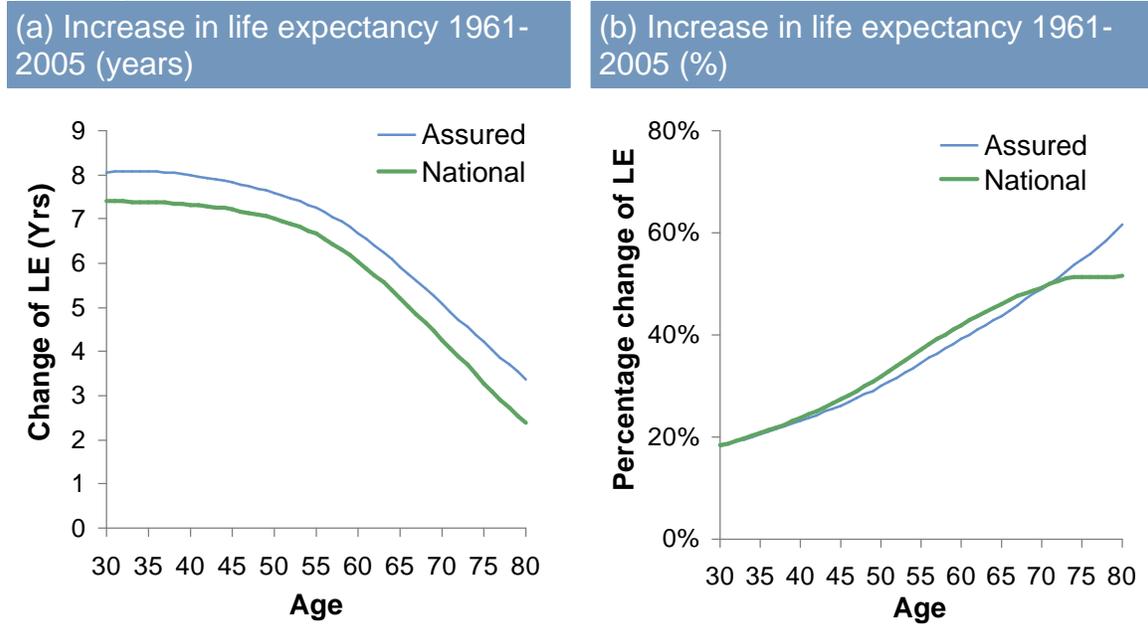
Over the entire 45-year period 1961-2005, period life expectancy has increased significantly for both populations at all ages. The assured data show greater increases in life expectancy than the national data. In particular, the highest increases have occurred for assured males in their 30s, with life expectancy for 33-year-olds increasing by 8.09 years, compared with 7.39 years for national population males of the same age. Figure 5 compares the change in life expectancy by age for the two populations. We stop calculating period life expectancy at age 80 to avoid the results being overly impacted by the method of graduating mortality rates higher ages, i.e., age 90 and above.

Figure 4: Evolution of male period life expectancy for the UK assured and England & Wales national populations, 1961-2005: (a) Life expectancy for 65-year-old males measured in years, (b) Ratio of life expectancy for the assured population to the life expectancy for the national population for various ages.



¹⁶ Period life expectancy is calculated from the spot (i.e., current) mortality curve, assuming no further mortality improvements. It is generally acknowledged that period life expectancy underestimates actual (i.e., cohort) life expectancy because mortality rates are widely expected to continue to fall through time. However, period life expectancy has the advantage of being an objective metric.

Figure 5: Increase in male period life expectancy for the UK assured and England & Wales national populations, 1961-2005: (a) Increase measured in years, (b) Increase in percentage terms.



Between ages 30 and 80, Figure 5(a) shows that the difference in life expectancy between the two populations varies from 0.58 years for 50-year-olds to 0.98 years for 80-year-olds. It is evident that there are clear differences between the populations in terms of the increase in period life expectancy measured in years. However, when we consider the relative percentage increase in life expectancy over the period, we find that the two populations have behaved in a very similar way, as illustrated in Figure 5(b). Both populations exhibit virtually the same percentage increases for ages 30 to 75. There is a divergence above 75, which may well be caused by differences in the graduation methodology used for higher ages.

The conclusion we draw from the analysis above is that the data for period life expectancy, like the other metrics we have examined, are indicative of a stable long-term relationship between the two populations which is likely to have a favourable impact on hedge effectiveness.

4.4 Liability (annuity) cash flows

Comparing the historical cash flows paid by annuities for different cohorts in each population provides yet another perspective on basis risk. To minimize the noise in comparing the two populations we focus on cumulative cash flows over periods of 10 years.

This particular metric is closely related to survivorship; in fact, it essentially corresponds to the average survival rate over the 10-year period. This should not be surprising. The annuity cash flow in any given year is proportional to the survival rate to the end of that year. So the 10-year cumulative cash flow is proportional to the sum of survival rates over periods ranging from one year to 10 years (Figure 6(a)). The calculation assumes that the annuity pays £1 each year to each surviving member of each population.

Figure 6: Cumulative cash flows over 10-year horizons for liabilities (annuities) based on the UK assured and England & Wales national male populations over the period 1970-2005: (a) Cumulative cash flow for 65-year-old males measured in GBP, (b) Ratio of cumulative cash flows for the assured population to that for the national population.

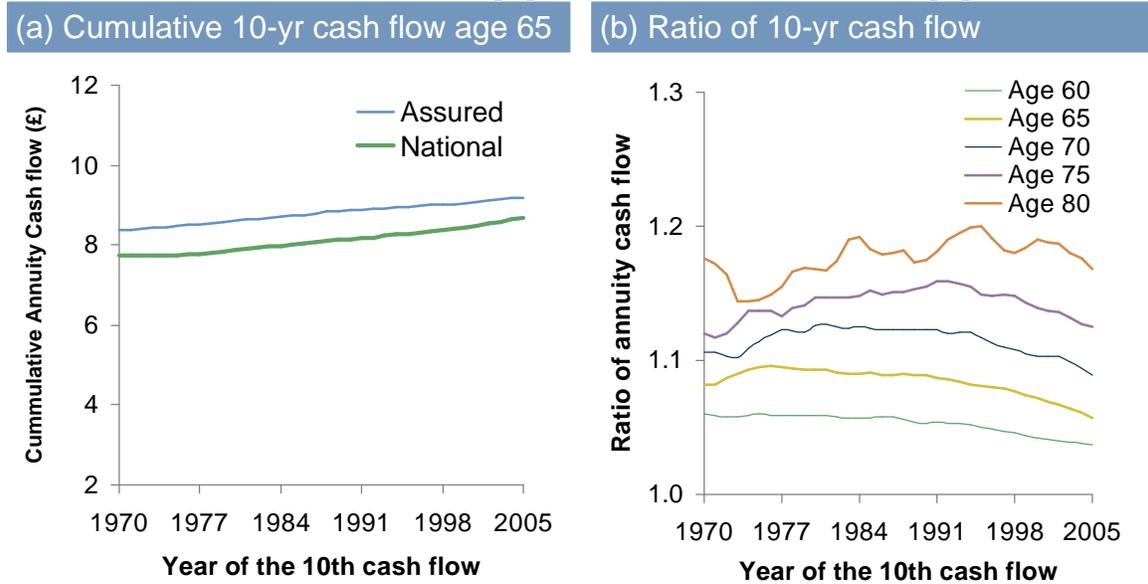


Figure 6(b) shows the ratio of 10-year cumulative annuity cash flows for the assured population to those of the national population. Each line represents the ratio over time for the same initial age. The chart shows the evolution of this ratio between 1970 and 2005, where the age at the start of the 10-year period runs from 60 to 80 years.¹⁷ Several points are worthy of note:

- The ratios are all greater than one, again reflecting higher survival rates for the assured population. They vary between approximately 1.04 to 1.20, depending on the cohort and the year.
- The ratios are reasonably stable, and trend downwards very slightly through time for cohorts with an initial age below 70. In particular, the ratio for the cohort with

¹⁷ Note that this metric cannot start before 1970, since it requires 10 years of historical cash-flow data to calculate the first cumulative cash-flow value.

an initial age of 60 varies between 1.04 and 1.06 over the period and for an initial age of 70 between 1.09 and 1.13.

- Cohorts born in earlier years (and therefore of higher initial age) have consistently higher ratios than those born in later years.
- Volatility increases for older birth cohorts (i.e., as the initial age increases). This is consistent with higher levels of noise associated with the higher age data.

4.5 Liability (annuity) values

The prices of lifetime annuities give an alternative perspective on basis risk that relates directly to hedge effectiveness in monetary terms. A life annuity purchased from a life office by a pension plan for each of its members on their retirement provides a perfect hedge for the member's longevity risk. Since adequate data on actual annuity prices for the assured and national populations are not publicly available, we compare theoretical annuity prices based on the kind of pricing (or valuation) model that the life office might itself use to price annuities. The results are therefore more dependent on modelling considerations than the other analyses in the paper: in particular, a projection model needs to be used to forecast future mortality rates and there also needs to be a model to calibrate mortality rates at ages higher than those available in published data (see Appendix A.2).¹⁸

The annuity pricing model takes the graduated mortality rates from the previous year as the mortality base table for each population in a particular year. Then future mortality rate forecasts relative to that base table are made by applying a projection model to historical data. We will suppose the life office uses a (single-population) Lee-Carter (1992) model with 30 years of historical data to generate these projections for each population for every year from 1991 to 2006.¹⁹ ²⁰ The resulting set of mortality rates constitutes a complete cohort mortality table for each base year. From this cohort mortality table, the life office will calculate the stream of expected cash flows paid out for each annuity. Finally, the annuity price is just the present value of the expected cash flows and we assume the life office uses a constant discount rate of 5%. One benefit of this assumption is that it allows us to focus on the change in annuity prices that are caused purely by changes in longevity.²¹

Figure 7 shows the results of comparing annuity prices based on assured and national mortality data for the same age. Figure 7(a) shows the stable, slightly upward-sloping relationship over time between the annuity prices for the assured and national populations for age 65. Figure 7(b) shows the ratio of the prices of assured population annuities to national population annuities through time for ages 60 to 80, with each line showing how

¹⁸ The price of a lifetime annuity depends on mortality rates for all ages above the current age, and in particular, on the mortality rates at very high ages for which there are many fewer data points available and consequently higher levels of noise.

¹⁹ Note that we cannot start before 1991, in order that we have 30 years of historical data to calibrate the projection model.

²⁰ Note that this projection model is used only for pricing (valuation) of the annuities, not for simulating scenarios.

²¹ In practice, interest-rate risk does matter, but this could be hedged using an interest-rate swap.

the ratio evolves through time for a given age. The annuities are assumed to pay equal amounts as long as an individual is alive at the end of the relevant year. The annuities are first priced on 1 January 1991, using the 1990 graduated mortality rates as the initial mortality base table and mortality rate projections generated from this date using historical rates between 1961 and 1990. In the following year, 1991 graduated mortality rates are used as the base table and mortality rate projections are generated using historical rates between 1962 and 1991, and so on.

The main observations from Figure 7(b) are as follows:

- The ratios are all greater than one, again confirming that the average cost of providing pensions for the assured population is higher than that for the national population. The ratio varies between approximately 1.09 to 1.27, depending on the age and the year.
- Higher ages demonstrate consistently higher ratios.
- The ratios are reasonably stable, but trend downwards slightly through time. In particular the ratio for age 60 varies between 1.09 and 1.15 over the period and for age 70 between 1.13 and 1.21.
- Volatility around the trend increases as the age increases.

Figure 7: Liability values for selected ages through time for the UK assured and England & Wales national male populations over the period 1991-2006: (a) Liability values for 65-year-old males measured in GBP, (b) Ratio of liability values for the assured population to that for the national population.

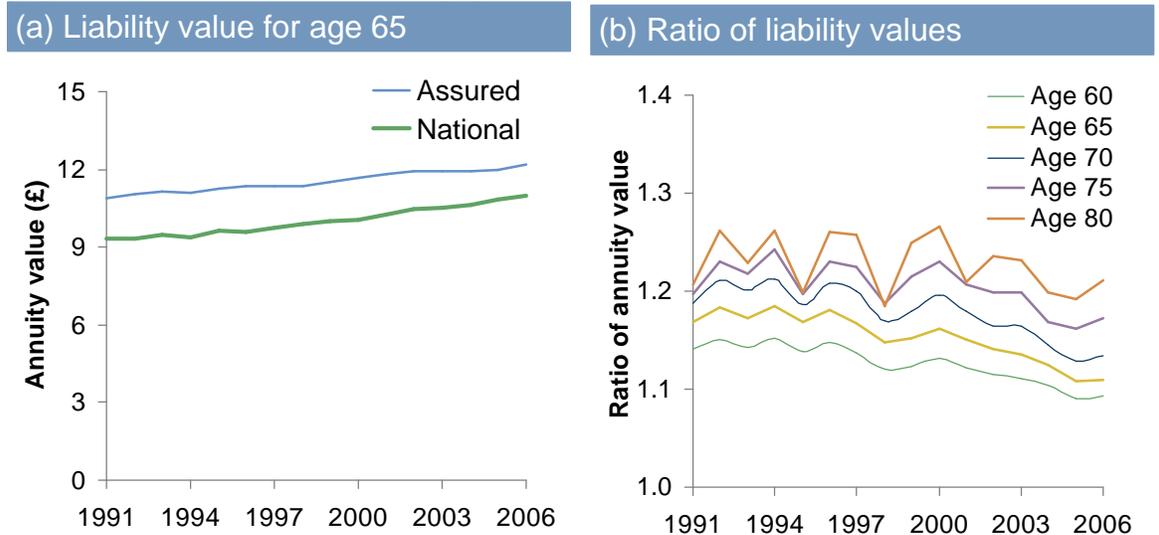
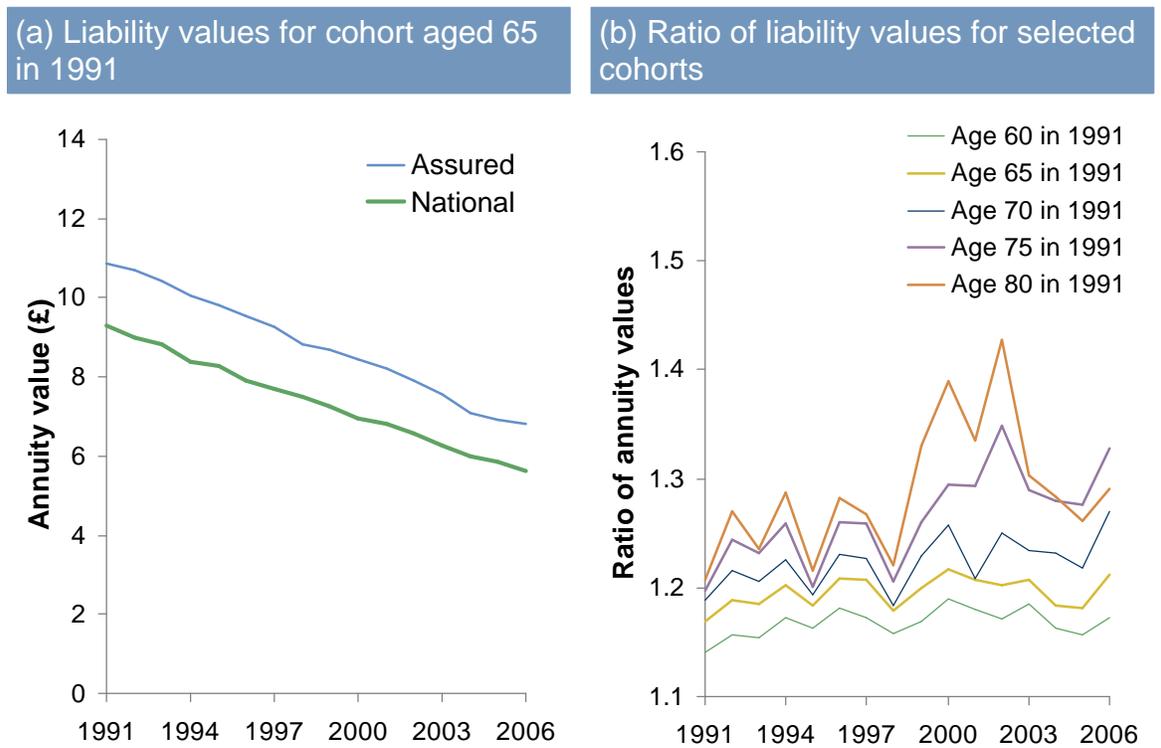


Figure 8 provides a different presentation of the data in which the lines follow the same cohort through time (rather than the same age but at different times). Figure 8(a) shows that, as the cohort aged 65 in 1991 gets older and its remaining life expectancy falls, the liability value also falls in a stable way for both assured and national populations. Figure 8(b) shows the ratio of the prices of assured population annuities to national population

annuities through time for cohorts aged between 60 and 80 years at the start of the analysis period. Each line shows how the ratio of annuity prices evolves through time for a single cohort. This chart exhibits features which are consistent with the previous chart, namely:

- The ratios are all greater than one.
- Older birth cohorts demonstrate consistently higher ratios than those born in later years.
- The ratios are reasonably stable, but trend upwards slightly through time. In particular, the ratio for the cohort with an initial age of 60 varies between 1.14 and 1.19 over the period and for an initial age of 70 between 1.18 and 1.27.
- Volatility increases for older birth cohorts and as a cohort ages.

Figure 8: Liability values for selected cohorts through time for the UK assured and England & Wales national male populations over the period 1991-2006: (a) Liability values for male cohorts aged 65 in 1991 measured in GBP, (b) Ratio of liability values for selected cohorts for the assured population to that for the national population.



These results constitute a retrospective hedge effectiveness test and are consistent with those of Coughlan *et al.* (2007a: pp 80-81).

Despite the modelling assumptions that are an inevitable part of the annuity pricing calculation, we again see evidence of a relatively stable relationship between the two populations from both a period and a cohort perspective.

4.6 Hedge effectiveness example

Considering the above analyses along with the context of the CMI assured population as an affluent, but more volatile, subset of the national population, the evidence suggests that there has been a stable long-term relationship between their mortality experiences. This is particularly significant given their different mortality levels, their different mortality improvements and the amount of noise in the CMI assured population data.

The implications of this for longevity hedges indexed to national population mortality data are clear. The long-term effectiveness of such hedges – providing they are appropriately calibrated – should be relatively high, leading to a significant reduction in longevity risk.

To illustrate this, we have evaluated the effectiveness of a static longevity hedge linked to the LifeMetrics Index for England & Wales in reducing the longevity risk of a hypothetical pension plan with the same mortality behaviour as the CMI assured population. We focus on a particular retrospective hedge effectiveness test based on historical data.²² The approach to calculating hedge effectiveness is the one outlined in Section 3.3 and summarized in Table 1 and Figure 1.

4.6.1 Step 1: Hedging objectives

The pension plan is assumed to consist entirely of deferred male members currently aged 55, whose mortality characteristics are the same as the CMI assured population and who will receive a fixed pension of GBP 1 for life, beginning at retirement in 10 years' time (the hedge horizon) at age 65. The hedging objective is to remove the uncertainty in the *value* of the pension at retirement due to longevity risk.

4.6.2 Step 2: Hedging instrument

The hedging instrument is a 10-year deferred annuity swap that pays out on the basis of a survival index for the national population of England & Wales for 55-year-old males. As we are considering a hedge of value, we can assume (without impacting the economics) that the hedging instrument is cash-settled at the hedge horizon at the market value prevailing at that time. In other words, in 10 years' time, the pension plan receives a payment reflecting the market value of the hedging annuity at that time in return for making a fixed payment at that time. So the hedging instrument involves a net settlement that is the difference between the fixed payment and the market value of the hedging annuity in 10 years' time.

²² This avoids the requirement, essential for a prospective test, of having to choose a two-population stochastic mortality model.

Because the hedging population is different from the exposed population, the hedge ratio needs to be calibrated to reflect the relationship between their mortality improvements. Applying equation (2) results in a hedge ratio of 0.987, meaning that to hedge a GBP 1 liability requires GBP 0.987 of the hedging instrument.

4.6.3 Step 3: Method for hedge effectiveness assessment

As mentioned above, we are performing a retrospective effectiveness test, based on historical data. The basis for comparison that we use is two-fold involving evaluation of (i) the correlations between the unhedged and hedged liability and (ii) the degree of risk reduction. Since the objective is to hedge the value of the pension liability, we focus on a risk metric corresponding to the value-at-risk (VaR) in 10 years' time, where the VaR is measured at a 95% confidence level relative to the median. We measure hedge effectiveness by comparing the VaR of the pension before and after hedging. We use historical mortality data to directly evaluate historical scenarios for the evolution of mortality rates over a 10-year horizon, from which the VaR of the pension can be calculated. For asymmetric, non-normal distributions a risk metric such as VaR is preferred over standard deviation, as it more accurately captures the downside risk and the associated implications for economic capital. However, other risk metrics generally give similar results.

The hedge effectiveness is calculated in terms of the relative risk reduction in equation (3) and implemented as follows:

$$RRR = 1 - \text{VaR}_{(\text{Liability} + \text{Hedge})} / \text{VaR}_{\text{Liability}} \quad (4)$$

We construct scenarios for the hedge effectiveness analysis in a model-independent way directly from the historical mortality data. Because the amount of available historical data is limited to 45 years, we form historical scenarios using the approach summarized in Table 7. This approach involves combining the set of historically realized mortality improvements with the set of realized mortality base tables. In particular, we construct 1575 scenarios for each population by applying realized mortality improvements coming from the full historical set of 35 overlapping 10-year periods or *windows* (1961-1971, 1962-1972, ..., 1995-2005) to each of the realized mortality *base tables* defined by the observed mortality rates in each of the 45 years (1961, 1962, ..., 2005).

To make this more explicit, let us denote the inception date of the hedge $t = 0$ and the hedge horizon $t = 10$. Let q_x^b denote the realized historical mortality rate in year b for age x . For a given value of b , we will use q_x^b to provide us with a base table at time 0 in our generation of a set of scenarios. For each of the 45 base tables, we have $w = 1, \dots, 35$ ten-year historical windows that we use to generate 35 scenarios. For each pair (b, w) , we generate one mortality-rate scenario $q_x^{(b,w)}(t)$ for each time $t = 0, 1, \dots, 10$. Specifically, the mortality rate scenarios for each population are given by:

$$q_x^{(b,w)}(t) = q_x^b \cdot \frac{q_x^{w+t}}{q_x^w} \quad (5)$$

where $b = 1961, 1962, \dots, 2005$; $w = 1961, 1962, \dots, 1995$; $t = 0, 1, \dots, 10$. Equation (5) consists of the base table giving us the scenario's mortality rates for $t = 0$, and q_x^{w+t}/q_x^w is the t -year improvement factor at age x in window w .

This approach provides us with 45 sets of scenarios with each set comprising 35 different 10-year scenarios. We evaluate hedge effectiveness by examining (i) the 45 sets of scenarios separately and (ii) all the sets in aggregate (1575 scenarios in total).

The valuation, or pricing model, that we use to value both the liability and the hedging instrument at time $t = 10$ involves projecting the expected (i.e., best estimate) future stream of cash flows beyond time 10 for the liability and the hedging instrument, and then discounting those cash flows back to time 10, the time at which the valuation is to be made. In particular, the valuation under the different scenarios at time 10 requires us to come up with a best estimate projection of mortality rates for that scenario beginning at time $t = 10$.

In order to value the pension liability and the hedging instrument at time 10, we use a very simple approach to project best estimate mortality rates into the future beginning from $t = 10$ in each scenario. This simple projection method averages the one-year percentage mortality improvements from $t = 0$ to $t = 10$ and applies this average improvement rate to the scenario mortality rate at $t = 10$ for all future years. In other words, in each scenario, starting at year 10, we develop a forecast for the future path of best estimate mortality based on the observed history up until that point in time.²³ We will assume for the purpose of valuation that the mortality improvement projections for the members of the pension plan are the same as those for national population. We justify this on the grounds that common industry practice is to use mortality projections derived from a large, broad based population (e.g., the national population) to calibrate the mortality projections for a specific pension plan for the purposes of valuation. This is because, in practice, the vast majority of pension plans do not have sufficient data to develop their own mortality projections from their historical experience alone. It can also be justified intuitively on the grounds that the long-term relationship between the two populations that we have demonstrated above implies that the long-run projections for the two populations cannot diverge in a systematic way. This is the principle that lies behind the two-population mortality models discussed in Section 2.4. We have explored the implications of relaxing this assumption and find a modest fall in hedge effectiveness if separate mortality improvement projections are used.

To value the liability in a particular scenario, we start from the best estimate mortality projection in year 10, calculate the stream of expected cash flows that will be paid out in

²³ So the value in each scenario at the horizon in 10 years' time depends on a projection based on the history of that particular scenario.

that scenario and then calculate the present value of the expected cash flows at $t = 10$. The value of the pension liability in that scenario is therefore given by

$$V_{\text{Liability}}^{(b,w)}(10) = {}_{10}P_{55}^{(b,w)}(0) \sum_{t=1}^{\infty} {}_t\hat{p}_{65}^{(b,w)}(10).DF(t). \quad (6)$$

Here ${}_{10}P_{55}^{(b,w)}$ is the (b,w) scenario's historical 10-year survival rate for 55-year-olds in scenario (b,w) from $t = 0$ until $t = 10$ and ${}_t\hat{p}_{65}^{(b,w)}(10)$ is the projected t -year survival rate at the hedge horizon for the same cohort (now aged 65) in that scenario, with the hat over the p to indicate its dependence on the choice of valuation model. The discount factor for calculating the present value is denoted by DF and our calculations assume a flat discount rate of 5% for simplicity. The hedging instrument is valued in a similar manner.

Table 7: The approach used to generate 1575 ten-year scenarios for mortality rates. This combines the 45 historical base tables with the 35 (overlapping) 10-year historical mortality improvements for each base table.

Component of the approach:	Applies at:	Calculated from:	Data used for pension liability:	Data used for hedging instrument:
1. Base mortality table	$t = 0$	Historical period mortality rates for each year 1961-2005	Member-specific mortality data	National mortality index data
2. Realized mortality rates for each scenario	$t = 1,2,3,\dots,10$	Historical 10-year mortality improvements applied to each base table in panel (1)	Member-specific mortality data	National mortality index data

Note: Each component of the approach builds on the previous one. The inception of the hedge corresponds to time $t = 0$ and the hedge horizon to time $t = 10$.

4.6.4 Step 4: Calculation of hedge effectiveness

The first part of the calculation of hedge effectiveness requires the simulation of the 1575 historical scenarios for the evolution of mortality rates using the method described above.

The second part of the calculation of hedge effectiveness involves determining the values of the pension and the hedging instrument in each scenario at the end of the 10-year hedge horizon. This is done by applying the valuation model described above in equation (6) to each of the scenarios at the hedge horizon.

This provides us with a distribution of 1575 possible values for the pension and a distribution for 1575 possible values for the hedging instrument in 10 years' time. From these, we calculate the correlation between the liability values and the hedging instrument values, as well as the relative risk reduction. The results of these calculations are

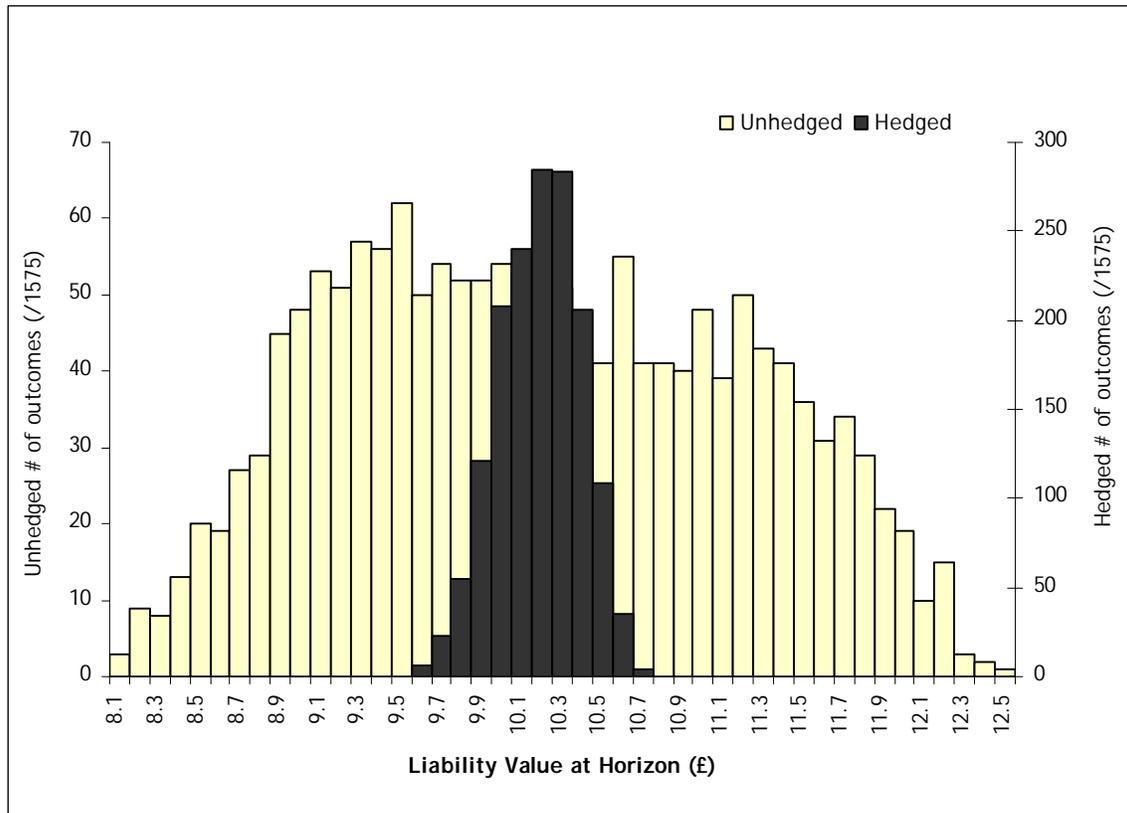
summarized in Table 8. Implementing the scenarios as 45 different sets, each with a different base table, gives high correlations between the values in all scenario sets. However, the relative risk reduction is modest. One factor contributing to this modest risk reduction is the fact that we are dealing with a small number of scenarios in each set (i.e., only 35 scenarios) and these involve overlapping periods. As a result, there is a relatively small amount of volatility in each scenario set, leading to a modest risk reduction.

Table 8: Results of the hedge effectiveness analysis for the hypothetical UK pension plan

Method of using the scenarios	Correlation between value of liability and hedging instrument	Hedge effectiveness	Comments
45 sets of 10-year scenarios, each set with a different mortality base table and 35 scenarios	Range: 0.93 – 0.97 Average: 0.96	Range: 67% – 79% Average: 73%	<ul style="list-style-type: none"> • Scenarios in each set involve overlapping 10-year improvements and are not fully independent. • 35 scenarios is a small number for a fully comprehensive analysis
Aggregate set of 1575 scenarios	0.98	82.4%	<ul style="list-style-type: none"> • Adequately large number of scenarios • Scenarios include different base tables, which increases the range of scenario outcomes
Note: Hedge effectiveness is measured in terms of relative risk reduction with VaR as the risk metric, as in equation (4).			

By contrast, aggregating the 1575 scenarios introduces considerably more volatility in the scenarios set and leads to much higher hedge effectiveness results. For the aggregate scenario set, the hedge reduces the impact of longevity risk on the value of the pension liability by 82.4%, demonstrating a high level of hedge effectiveness. This is shown in the histogram in Figure 9. The light coloured bars represent the range of possible outcomes for the unhedged value of the pension liability in 10 years’ time coming from the 1575 scenarios. The dark shaded bars reflect a much narrower range of possible outcomes for the value of the pension liability after hedging. The fact that the histogram for the hedged case is much narrower than that for the unhedged case provides a visual illustration of the high degree of hedge effectiveness.

Figure 9: Hedge effectiveness analysis using an index-based hedging instrument linked to England & Wales national male mortality to hedge the longevity risk of a pension plan with the same mortality characteristics as the UK assured male population.



5. US basis risk case study

The analysis of the previous section has been repeated in a US context as a robustness check of the results. The populations used were the US national population and the population of the state of California (both based on data sourced from the Centers for Disease Control and Prevention, CDC, and the National Census Bureau). The data in this analysis covered the 25-year period 1980-2004. California has a higher level of affluence than the nation as a whole with per capita GDP 11% above the national average,²⁴. This greater affluence is reflected in mortality rates that have been consistently lower and mortality improvements higher than those of the national population. The results are very similar to those obtained in the UK example above, leading to a similarly high degree of hedge effectiveness associated with an appropriately-calibrated index-based hedge linked to US national population longevity. Details can be found in Appendix B.

²⁴ US Census Bureau and US Bureau of Economic Analysis, 2008 figures.

6. Conclusions

In this paper, we have developed a framework for analysing longevity basis risk and its implications for the effectiveness of longevity hedges. Such a framework is essential for:

- Understanding basis risk
- Calibrating two-population stochastic mortality models
- Calibrating index-based longevity hedges
- Measuring hedge effectiveness

The framework is built on a quantitative analysis of data, together with a qualitative understanding of the contextual relationship between the populations involved. The quantitative analysis involves examining the historical experience of the populations in terms of different metrics. The nature of longevity risk dictates that the time horizon for the analysis should be long, but the framework acknowledges that there will not, in general, be enough data for a robust analysis over such horizons. As a result, an assessment of the nature and magnitude of basis risk must rest on professional judgment, informed by evidence coming from analysis, experience and context.

A key element of this framework is a structured approach to hedge effectiveness assessment, based on a proven approach developed for derivatives accounting. To obtain a valid and meaningful hedge effectiveness test requires careful attention to (i) the design of the test, (ii) the generation (simulation) of appropriate scenarios and (iii) the valuation methodology for the underlying exposure and the hedging instrument in each scenario.

We have applied the framework to two case studies involving empirical analyses in a UK and US context of the relationship between the mortality experience of the national population and that of a more affluent subpopulation. Despite the different demographic profiles between the pairs of populations, we demonstrate evidence of high correlations in mortality improvements between the two related populations and stable long-term relationships across different metrics. These results have very favourable implications for the effectiveness of appropriately-calibrated, index-based longevity hedges. From this, we conclude that longevity basis risk between a pension plan, or annuity portfolio, and a hedging instrument linked to a broad national population-based longevity index can in principle be reduced very considerably.

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Appendix A: Technical issues

A.1 Age buckets

As discussed in the text, bucketing ages together is frequently beneficial in removing noise from mortality rates and is often implemented in actual transactions to hedge both mortality risk and longevity risk. Where age buckets are used in this paper, the buckets all involve simple averages over each age in the bucket. So, for example, in computing the mortality rate for the age bucket 60-69 years old, we take a simple unweighted average of the mortality rates for each age $q_{60}, q_{61}, q_{62}, \dots, q_{69}$. Similarly, the survival rate for the age bucket 60-69 is an average of the survival rates for each individual age $p_{60}, p_{61}, p_{62}, \dots, p_{69}$.

A.2 Graduation method for mortality rates

The two case studies presented in this paper are based on mortality rates that have been calculated and graduated using a method very similar to that used in the LifeMetrics Longevity Index (see Coughlan *et al.* (2007a) for details). The graduation method is an objective one which is applied consistently in each year to each data set. In particular, we have used the same cubic spline approach as used for the LifeMetrics Index to smooth the crude central mortality rates, but have used a variant of the higher age methodology to come up with graduated mortality rates for ages above 80.

Graduated mortality rates for higher ages, which are needed to calculate period life expectancies and annuity prices, are obtained by the method described below:

1. Calculate crude central mortality rates m_x for each age x for the year in question.
2. Graduate the crude central mortality rates for all ages up to age 89 using the cubic splines approach described in Coughlan *et al.* (2007a). Denote these first-stage graduated rates by m'_x .
3. For ages below 80, set the graduated central mortality rate m^s_x to be m'_x .
4. For ages 80-89:
 - a. Perform a regression of $\log m'_x$ against age, and back out the fitted central mortality rates m''_x for each age.
 - b. Set the graduated central mortality rate m^s_x to be a blend of m'_x and m''_x , such that at age 80 the blend is 100% m'_x and at age 89 it is 100% m''_x . The following monotonic blending function is used:

$$f(z) = \begin{cases} \exp(1) \cdot \exp(-1/(1-z^2)) & \text{for } 0 \leq z < 1 \\ 0 & \text{for } z = 1 \end{cases} \quad (\text{A1})$$

where $z = (x - 80)/9$ for ages x between 80 and 89.

5. For ages 90 and over, calculate the graduated central mortality rate m_x^g using the algorithm outlined in Coughlan *et al.* (2007a) which involves fitting a cubic polynomial.
6. Calculate graduated initial mortality rates q_x for each age using the transformation:

$$q_x = m_x^g / (1 + m_x^g / 2) \tag{A2}$$

A.3 Computational method for “aggregate correlations”

In calculating correlations in mortality rates between two populations, the correlations relate to both changes and relative changes in mortality rates dq for each population over a particular time horizon, H , that varies from one year to 20 years. Non-overlapping periods are used, which means that, for long horizons, there are fewer historical data points available. Note that because of this lack of data when considering long horizons, we calculate what we call “aggregate correlations”, which jointly compare changes for different periods and for different ages. In other words, we correlate matrices $[dq^1_{x,t}]$ and $[dq^2_{x,t}]$, where x labels the age and t labels the time period t to $t + H$. Consider the example of calculating aggregate correlations for the mortality rate changes for individual ages from 50 to 89 over a five-year horizon ($H = 5$ years). The calculation involves computing the correlation between two 40×9 matrices of the following form:

$$\left[\begin{array}{cccc} dq(x=50, & dq(x=50, & dq(x=50, & \dots & dq(x=50, \\ t=1961-1966) & t=1966-1971) & t=1971-1976) & & t=2001-2005) \\ dq(x=51, & dq(x=51, & dq(x=51, & \dots & dq(x=51, \\ t=1961-1966) & t=1966-1971) & t=1971-1976) & & t=2001-2005) \\ dq(x=52, & dq(x=52, & dq(x=52, & \dots & dq(x=52, \\ t=1961-1966) & t=1966-1971) & t=1971-1976) & & t=2001-2005) \\ \dots & \dots & \dots & \dots & \dots \\ dq(x=89, & dq(x=89, & dq(x=89, & \dots & dq(x=89, \\ t=1961-1966) & t=1966-1971) & t=1971-1976) & & t=2001-2005) \end{array} \right]$$

When aggregate correlations are calculated from age buckets rather than individual ages, the calculation is identical except that the rows in the $[dq^1_{x,t}]$ and $[dq^2_{x,t}]$ matrices are indexed by the age bucket rather than the individual single year of age.

Note that we use the same symbol above for absolute and relative changes in mortality rates, although they are defined differently. Absolute changes in mortality rates are defined by $q(t+1) - q(t)$ and relative changes are defined by $[q(t+1) - q(t)] / q(t)$.

Appendix B: US basis risk case study

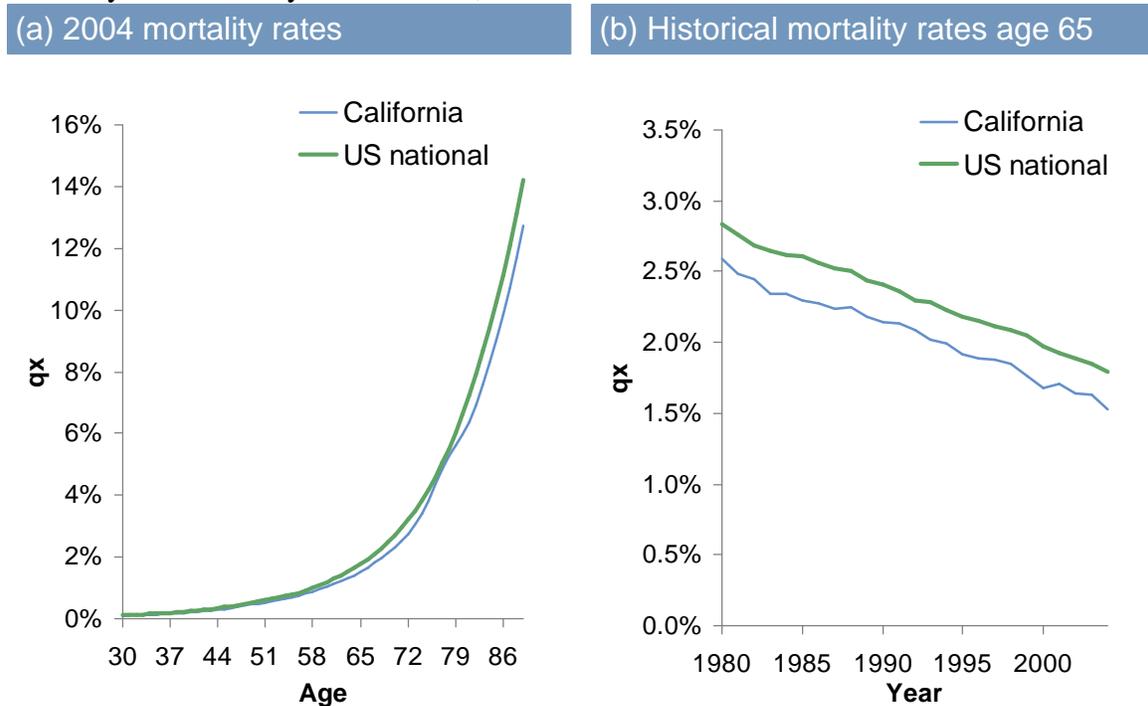
In this appendix, we present the results of the empirical analysis of the basis risk between the US national population and the population of the state of California (both based on data sourced from the Centers for Disease Control and Prevention, CDC, and the National Census Bureau). The data used cover the 25-year period 1980-2004. In 2008, California boasted a population of 36.8 million, representing 12% of the US national population and a GDP per capita of 11% above the national average,²⁵ reflecting a higher level of affluence than the nation as a whole. This greater affluence is reflected in historically observed lower mortality rates and higher mortality improvements.

We follow the same analytical steps as for the UK case study.

B.1 Mortality rates and mortality improvements

Figure 10 shows a graphical comparison of male graduated mortality rates for California and the US national population. A common feature of the data is the difference in the level of mortality rates: California mortality is much lower than national mortality. What is also evident is that the long-term downward trends are similar, suggesting that there might be a long-term relationship between the mortality rates of the two populations. The observed improvements in mortality are evolving similarly and not diverging.

Figure 10: A comparison of male mortality rates for California and the US national populations, (a) Spot mortality curves for 2004, (b) Historical evolution of graduated mortality rates for 65-year-old males, 1980-2004.



²⁵ US Census Bureau and US Bureau of Economic Analysis, 2008 figures.

Table 9 compares the average levels of mortality rates for the two populations, showing that California mortality in 2004 averaged 88% of national mortality, having fallen from 92% in 1980. Over the period 1980-2004, observed mortality improvements (Table 10) have averaged 1.65% p.a. for California males, compared with 1.48% p.a. for the national population. Furthermore, it is interesting to note that the younger pre-retirement ages have experienced lower improvements than the older post retirement ages in contrast with the experience of English & Welsh males.

Table 9: California male mortality rates as a percentage of US national male mortality rates, averaged over age.

Ratio of mortality rates (California/US national)	1980	2004
Overall: 40-89	92%	88%
Younger: 40-64	90%	87%
Older: 65-89	94%	89%

Table 10: Annualized male mortality improvements for the US national and California populations, averaged over age groups, 1980-2004.

Mortality improvements 1980-2004	National (% p.a.)	California (% p.a.)	Difference (percentage points)
Overall: 40-89	1.48	1.65	0.17
Younger: 40-64	1.44	1.57	0.13
Older: 65-89	1.51	1.73	0.22

Table 11 lists the aggregate correlations for changes in mortality rates over different horizons calculated from individual ages. Note that these aggregate correlations in year-on-year changes based on individual ages are quite small, just 40-50%, but they increase with the length of the time horizon. Correlations are around 65-85% for a 5-year horizon and up to 97% for a 10-year horizon.²⁶

Using 10-year age buckets rather than individual ages leads to higher correlations for one-year and 10-year horizons of 51-54% and 94-99%, respectively, as shown in Table 12. Again, we should note that for long horizons and bucketed ages, there is a limited number of data points, and the correlation results should be considered as indicative only. However, taken together with the results of the other analyses below, they support the existence of a strong long-term relationship between the mortality experience of the two populations.

²⁶ Note the lowish result of just 66% for the correlation of relative changes for ages 40-89 over a 10-year horizon. This seems to be the result of noise, as the correlation rises to 87% over the age range 35-89.

Table 11: Aggregate correlations of changes in male mortality rates for individual ages between the California and US national populations, 1980-2004.

Individual ages	Correlation between absolute changes in mortality rates		Correlation between improvement rates (relative changes)	
	Individual ages: 40-89	Individual ages: 50-89	Individual ages: 40-89	Individual ages: 50-89
10-year horizon	97%	97%	66%	91%
5-year horizon	68%	65%	83%	77%
1-year horizon	41%	41%	52%	41%
Note: See note to Table 4.				

Table 12: Aggregate correlations of changes in male mortality rates for 10-year age buckets between the California and US national populations, 1980-2004.

Age buckets: 50-59, 60-69, 70-79, 80-89	Correlation between absolute changes in mortality rates	Correlation between improvement rates (relative changes)
10-year horizon	99%	94%
5-year horizon	60%	77%
1-year horizon	54%	51%
Note: See note to Table 4.		

B.2. Survival rates

Figure 11(a) shows the evolution of 10-year survival rates for the two populations for 65-year-old males over the period 1989-2004. Survival rates for both populations have been increasing over time but, more importantly, the ratio between survival rates has been more or less constant over time, except at very high ages, as shown in Figure 11(b).

The latter chart suggests a relatively stable long-term relationship between the survival rates of the two populations, with the ratio of California to national survival rates greater than one for all ages and increasing with age. The average survival ratios over the period are listed in Table 13 and are much closer to one than in the previous case study, with Californian males aged 80 experiencing only a 10% higher survival rate to age 90 than the nation overall.

Figure 11: Historical 10-year male survival rates for California and the US national populations based on data over the period 1980-2004: (a) Historical evolution of 10-year survival rates for males reaching 65 in different years between 1989 and 2004, (b) Ratio of the 10-year survival rates for California to the 10-year survival rate for the US national population for males reaching various ages in different years between 1989 and 2004.

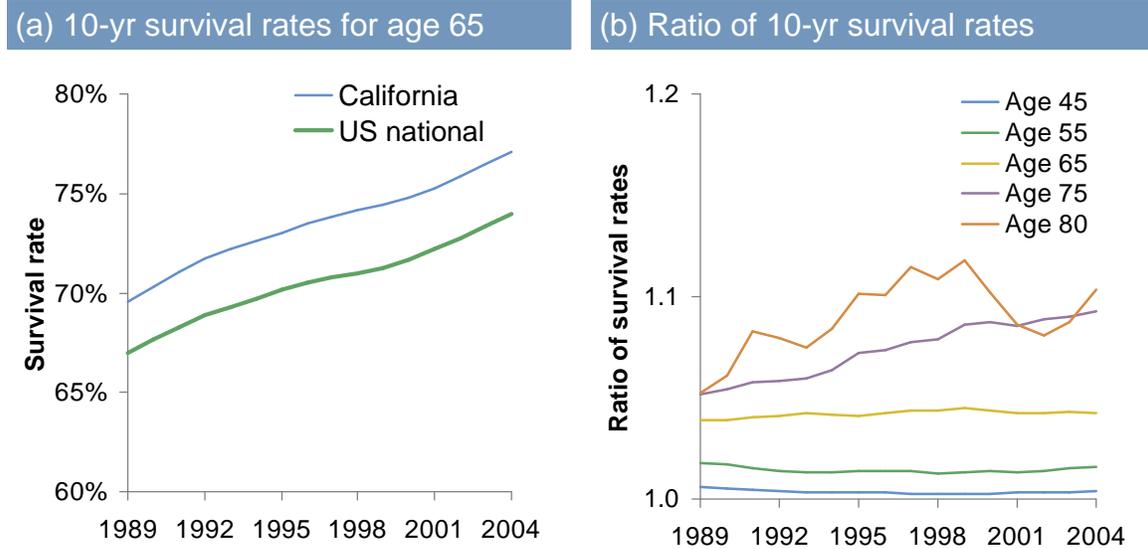


Table 13: Key statistics on the male survival ratio between California and US national male populations. The survival ratio is defined as the 10-year survival rate for California to the 10-year survival rate for the national population over the period 1989-2004.

10-year Survival Ratio (California / National)	Age 45	Age 55	Age 65	Age 75	Age 80
Average survival ratio	1.00	1.01	1.04	1.07	1.09
Standard deviation	0.001	0.002	0.002	0.014	0.018
Coeff of variation (std dev / average)	0.1%	0.2%	0.2%	1.3%	1.7%
Worst case (max / average)	0.2%	0.4%	0.3%	2.0%	3.5%
Note: See notes to Table 6.					

B.3 Life expectancy

From 1980 to 2004, period life expectancy increased significantly for both populations at all ages. Figures 12 and 13 show that the California data exhibit both higher and greater increases in period life expectancy than the national data. Despite this, the ratio of life expectancies has been relatively constant as shown in Figure 12(b). In particular, the ratio of California to US national life expectancies have, over the 25-year period, averaged 1.03 at age 45, 1.05 at age 65, 1.05 at age 75 and 1.05 at age 80. Moreover, as Figure 13(b) shows, the percentage increases in life expectancies have been very similar, only beginning to diverge above age 75, which again might be due to assumptions about the mortality rates at higher ages.

Figure 12: Evolution of male period life expectancy for the California and US national populations, 1980-2004: (a) Life expectancy for 65-year-old males measured in years, (b) Ratio of life expectancy for the California population to the life expectancy for the national population for various ages.

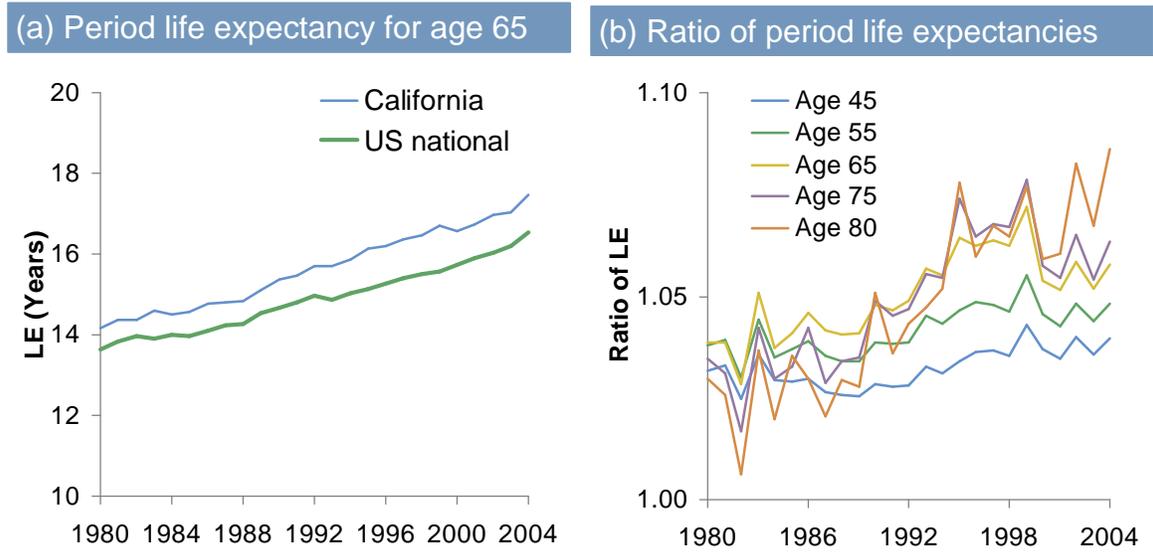
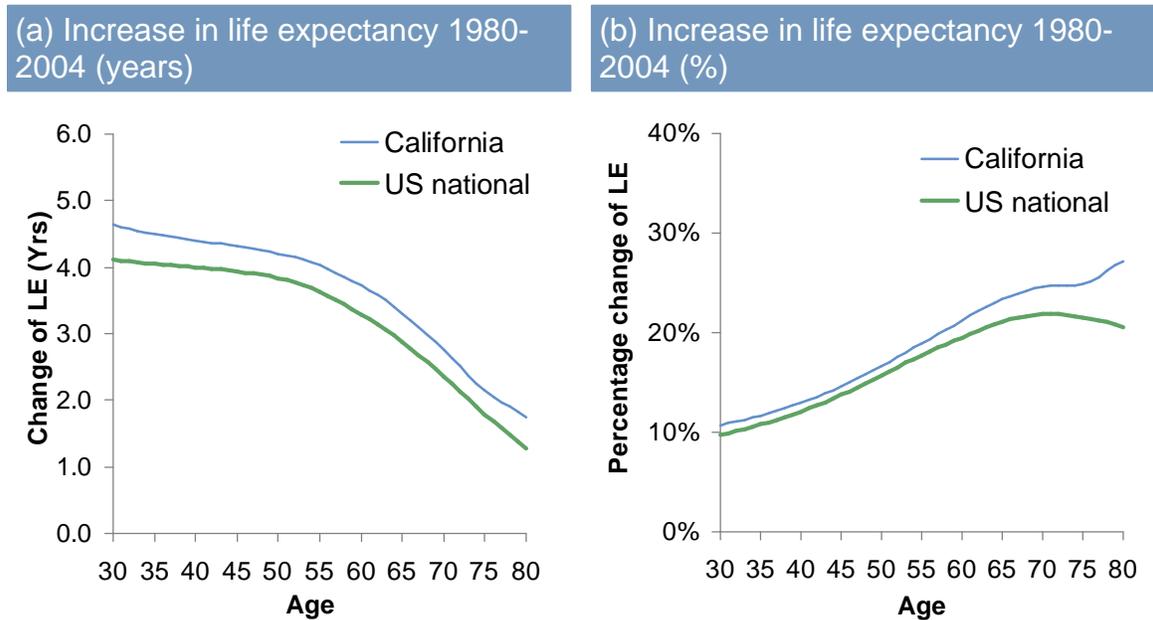


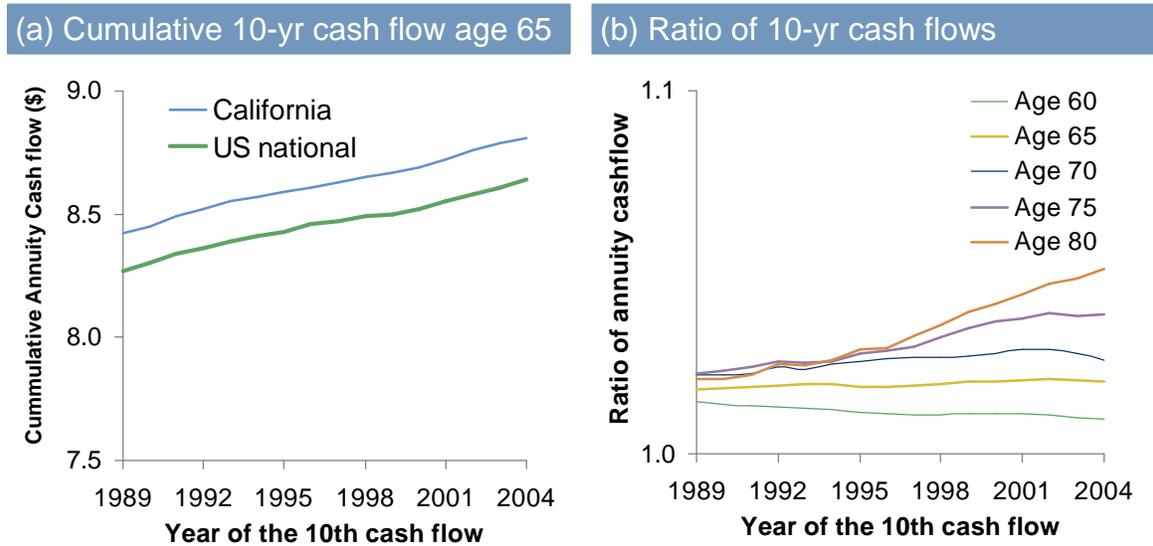
Figure 13: Increase in male period life expectancy for the California and US national populations, 1980-2004: (a) Increase measured in years, (b) Increase in percentage terms.



B.4 Liability (annuity) cash flows

We now compare the historical cash flows from annuities for different cohorts in each population (Figure 14). As before, we minimize the noise in comparing the two populations by calculating cumulative cash flows over periods of 10 years (Figure 14(a)). The calculation assumes that the annuity pays \$1 each year to each surviving member of each population. Figure 14(b) shows the ratio of 10-year cumulative annuity cash flows for the Californian male population to those of the national population. Each line represents the ratio over time for the same initial age.

Figure 14: Cumulative cash flows over 10-year horizons for liabilities (annuities) based on the California and US national male populations over the period 1989-2004: (a) Cumulative cash flows for 65-year-old males measured in USD, (b) Ratio of cumulative cash flow for the California population to that for the national population.



We note the following features, which are similar to the UK case study:

- The ratios are all greater than one, reflecting higher survival rates for the Californian population. The ratio varies from approximately 1.01 to 1.05, depending on the cohort and the year.
- The ratios are reasonably stable. In particular, the ratio for the cohort with an initial age of 60 varies between 1.01 and 1.014 over the period, while that for an initial age of 70 varies between 1.02 and 1.03.

B.5 Liability (annuity) values

The more limited historical data for this example (just 25 years compared with 45 years for the UK case study) means that calculating historical annuity prices is challenging, as there are not really enough data to calibrate the mortality forecasts used in the valuation model. As a result, we omit this analysis from this case study.

B.6 Hedge effectiveness example

Given the long-term, stable relationship between the US national and California populations in terms of their mortality experiences, the long-term effectiveness of an appropriately calibrated hedge of the latter using a hedging instrument based on the former should be high.

We have evaluated an example of a static hedge of longevity risk in a hypothetical pension plan with the same mortality behaviour as males in the state of California and a hedging instrument linked to the LifeMetrics Index of male mortality for the US national population. We perform the same kind of retrospective hedge effectiveness test as we did in the UK example in Section 4.6, using the method defined in Table 7.

B.6.1 Step 1: Hedging objectives

The pension plan consists entirely of deferred male members currently aged 55, whose mortality characteristics are the same as Californian males and who will receive a fixed pension of USD 1 for life beginning at retirement in 10 years' time (the hedge horizon) at age 65. The hedging objective is to remove the uncertainty in the *value* of the pension at retirement due to longevity risk.

B.6.2 Step 2: Hedging instrument

As in the UK case, the hedging instrument is a 10-year deferred annuity swap that pays out on the basis of a survival index for the US national population for 55-year-old males. As we are considering a hedge of value, we can again assume that the hedging instrument is cash-settled at the hedge horizon at the market value prevailing at that time. The hedge ratio was calibrated using equation (2) resulting in a hedge ratio of 1.07, implying that to hedge a USD 1 liability requires a USD 1.07 hedging instrument.

B.6.3 Step 3: Method for hedge effectiveness assessment

We measure hedge effectiveness using the same method as for the UK case study. The only difference is that there is a more limited historical dataset available.

With available historical data limited to 25 years, we use the same method we used in Section 4.6 to form historical scenarios by combining the set of realized mortality

improvements with the set of realized mortality base tables. In particular, we construct scenarios for each population by applying realized mortality improvements coming from the full historical set of 15 overlapping 10-year periods (1980-1990, 1981-1991, ..., 1994-2004) to each of the realized mortality base tables defined by the observed mortality rates in each of the 25 years (1980, 1981, ..., 2004). The mortality rate scenarios for each population are given by equation (5), but now with $b = 1980, 1981, \dots, 2004$; $w = 1980, 1981, \dots, 1994$; $t = 1, 2, \dots, 10$. The more limited history leads to just 25 sets of scenarios with only 15 scenarios per set. In aggregate we now have only 375 scenarios.

B.6.4 Step 4: Calculation of hedge effectiveness

The results of the analysis are shown in Table 14 and are qualitatively similar to the UK case study. Implementing the scenarios as 25 different sets, each with a different base table, gives high correlations between the values in all scenario sets. However, the relative risk reduction is modest. This is because we are dealing with a small number of scenarios in each set (i.e., only 15 scenarios) and these involve overlapping periods. As a result, there is a relatively small amount of volatility in each scenario set, leading to a modest risk reduction.

Again, we see that aggregating the 375 scenarios introduces considerably more volatility in the scenarios set and leads to much higher hedge effectiveness results. For the aggregate scenario set, the hedge reduces the impact of longevity risk on the value of the pension liability by 86.5%. The increase in hedge effectiveness is illustrated in histogram shown in Figure 15.

Table 14: Results of the hedge effectiveness analysis for the hypothetical US pension plan

Method of using the scenarios	Correlation between value of liability and hedging instrument	Hedge effectiveness	Comments
25 sets of 10-year scenarios, each set with a different mortality base table and 15 scenarios	Range: 0.94 – 0.95 Average: 0.95	Range: 71% – 82% Average: 77%	<ul style="list-style-type: none"> Scenarios in each set involve overlapping 10-year improvements and are not fully independent. 15 scenarios is a small number for a fully comprehensive analysis
Aggregate set of 375 scenarios	0.99	86.5%	<ul style="list-style-type: none"> Adequately large number of scenarios Scenarios include different base tables, which increases the range of scenario outcomes

Note: Hedge effectiveness is measured in terms of relative risk reduction with VaR as the risk metric, as in equation (4).

Figure 15: Hedge effectiveness analysis using an index-based hedging instrument linked to US national male mortality to hedge the longevity risk of a pension plan with the same mortality characteristics as the California male population.

