STOCHASTIC MORTALITY MODELS:

Criteria for Assessing and Comparing Models

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PLAN FOR TALK

- Background remarks
- Two families of models
- Criteria for evaluating different models
  - quantitative
  - qualitative
- Closing remarks

The Problem

2007: What we know as the facts:

- Life expectancy is increasing.
- Future development of life expectancy is uncertain.

"Longevity risk"

$\Rightarrow$ Systematic risk for pension plans and annuity providers

The Problem – UK Defined-Benefit Pension Plans:

- Before 2000:
  - High equity returns masked impact of longevity improvements
- After 2000:
  - Poor equity returns, low interest rates
  - Decades of longevity improvements now a problem

Why do we need stochastic mortality models?

Data $\Rightarrow$ future mortality is uncertain

- Good risk management
- Setting risk reserves
- Life insurance contracts with embedded options
- Pricing and hedging mortality-linked securities

Stochastic mortality

- Many models to choose from

- Limited data $\Rightarrow$ model and parameter risk

(*) Cairns et al. (2007) A quantitative comparison of stochastic mortality models.... Online: www.lifemetrics.com

Measures of mortality

- $q(t,x)$ = underlying mortality rate in year t at age x
- $m(t,x)$ = underlying death rate
- Assume $q(t,x) = 1 - \exp[-m(t,x)]$

Poisson model:

Actual deaths $D(t,x) \sim$ Poisson $(m(t,x)E(t,x))$

Two general families of models

$$\log m(t,x) = \beta_x^{(1)}\kappa_t^{(1)}\gamma_{t-x}^{(1)} + \ldots + \beta_x^{(N)}\kappa_t^{(N)}\gamma_{t-x}^{(N)}$$

OR

$$\text{logit}q(t,x) = \beta_x^{(1)}\kappa_t^{(1)}\gamma_{t-x}^{(1)} + \ldots + \beta_x^{(N)}\kappa_t^{(N)}\gamma_{t-x}^{(N)}$$

- $\beta_x^{(k)}$ = age effect for component k
- $\kappa_t^{(k)}$ = period effect for component k
- $\gamma_{t-x}^{(k)}$ = cohort effect for component k

e.g. Lee-Carter (1992) model

$$\log m(t,x) \;=\; \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} = \sum_{i=1}^{2} \beta_x^{(i)}\kappa_t^{(i)}\gamma_{t-x}^{(i)}$$

- $N = 2$ components
- $\beta_x^{(1)}$, $\beta_x^{(2)}$ age effects
- $\kappa_t^{(2)}$ single random period effect
- $\kappa_t^{(1)} \equiv 1$
- $\gamma_{t-x}^{(1)} = \gamma_{t-x}^{(2)} \equiv 1$ (model has no cohort effect)

Six Models

Lee-Carter (1992) LC
$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)}$

Renshaw-Haberman (2006) RH
$\log m(t,x) = \beta_x^{(1)} + \beta_x^{(2)} \kappa_t^{(2)} + \beta_x^{(3)} \gamma_{t-x}^{(3)}$

Age-Period-Cohort APC
$\log m(t,x) = \beta_x^{(1)} + 1 \times \kappa_t^{(2)} + 1 \times \gamma_{t-x}^{(3)}$

Cairns-Blake-Dowd (2006) CBD-1

$$\text{logit}\,q(t,x) = \kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)}$$

Cairns et al. (2007) CBD-2

$$\text{logit}\,q(t,x) =$$

$$\kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)} + \left((x-\bar{x})^2 - \sigma_x^2\right)\kappa_t^{(3)} + \gamma_{t-x}^{(4)}$$

Cairns et al. (2007) CBD-3

$$\text{logit}\,q(t,x) = \kappa_t^{(1)} + (x-\bar{x})\kappa_t^{(2)} + (x_c - x)\gamma_{t-x}^{(3)}$$

How to compare stochastic models (*)

- Quantitative criteria
- Qualitative criteria
  - parsimony and transparency
  - robust relative to age and period range
  - biologically reasonable
  - forecasts are reasonable

Quantitative Criteria

Bayes Information Criterion (BIC)

- Model k: $\hat{l}_k =$ model maximum likelihood
- BIC penalises over-parametrised models
- $BIC_k = \hat{l}_k - \frac{1}{2} n_k \log N$
  - $n_k =$ number of parameters
  - $N =$ number of observations

Quantitative Criteria – BIC

England & Wales males, 1961-2004, ages 60-89

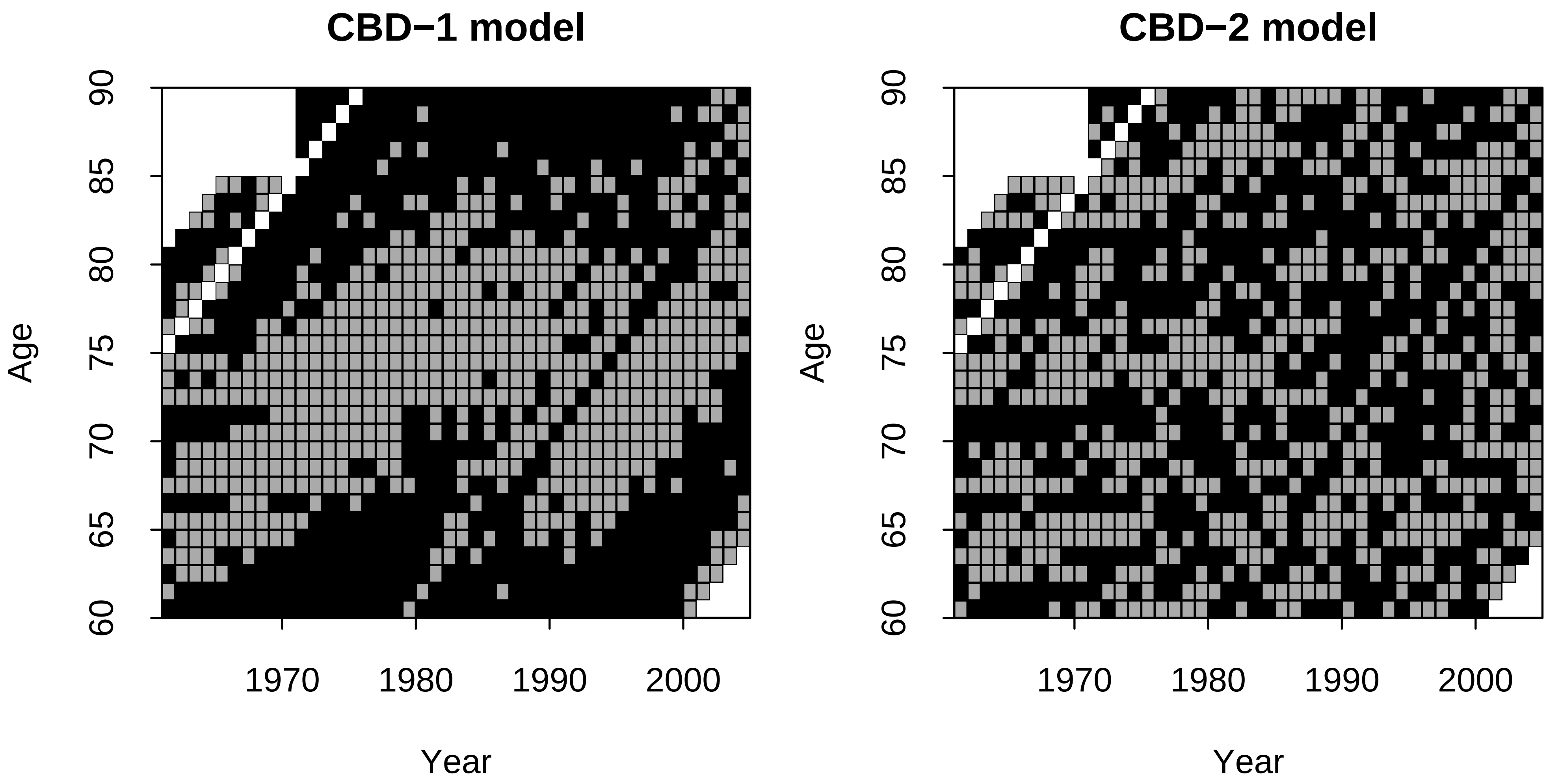
Model	Max log-lik.	# parameters	BIC (rank)
LC	-8912.7	102	-9275.8 (5)
RH	-7735.6	203	-8458.1 (3)
APC	-8608.1	144	-9120.6 (4)
CBD-1	-10035.5	88	-10348.8 (6)
CBD-2	-7702.1	202	-8421.1 (2)
CBD-3	-7823.7	161	**-8396.8 (1)**

The BIC doesn't tell us the whole story ...

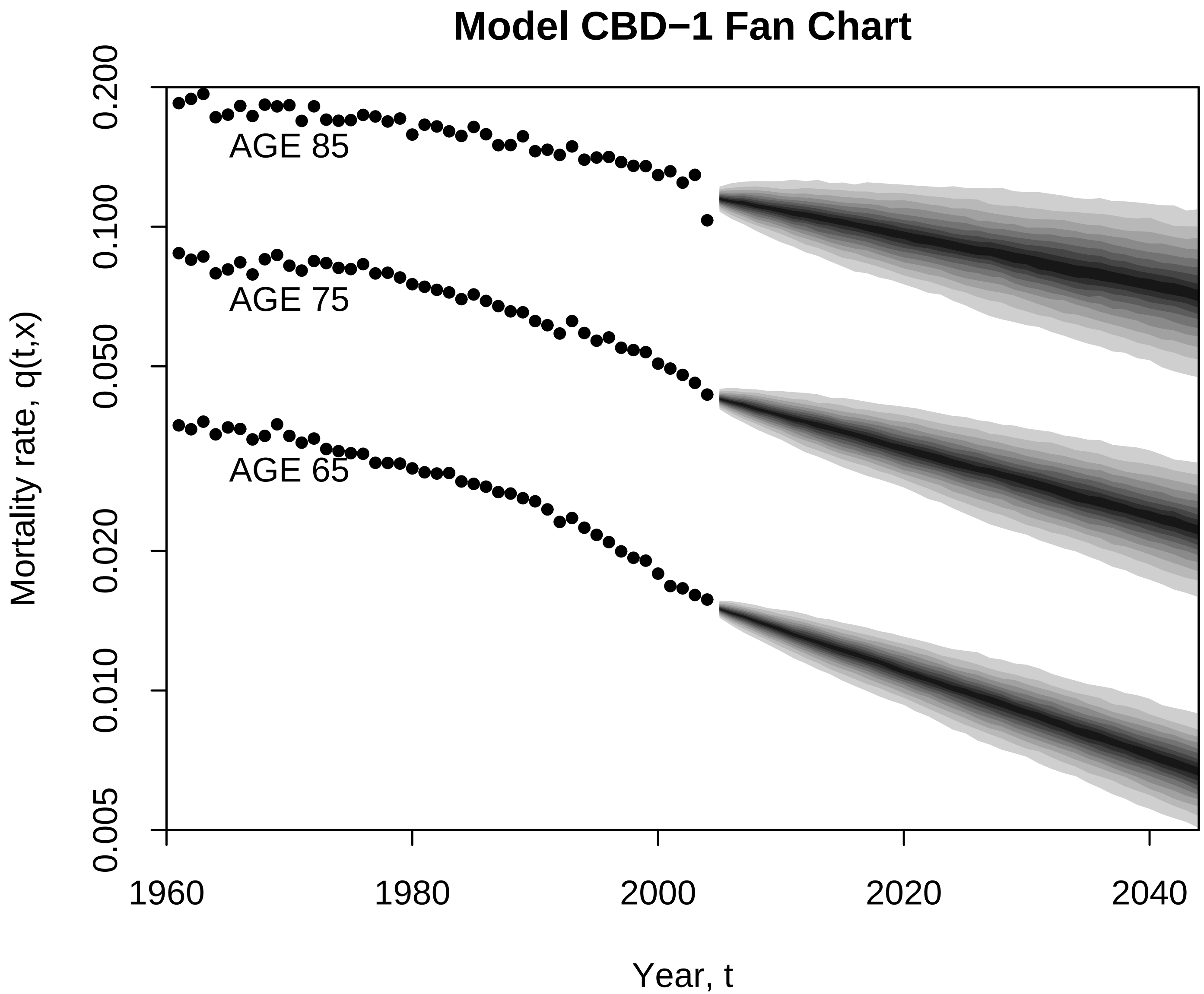
Qualitative Criteria – Graphical diagnostics

- Poisson model $\Rightarrow (t, x)$ cells are all independent.

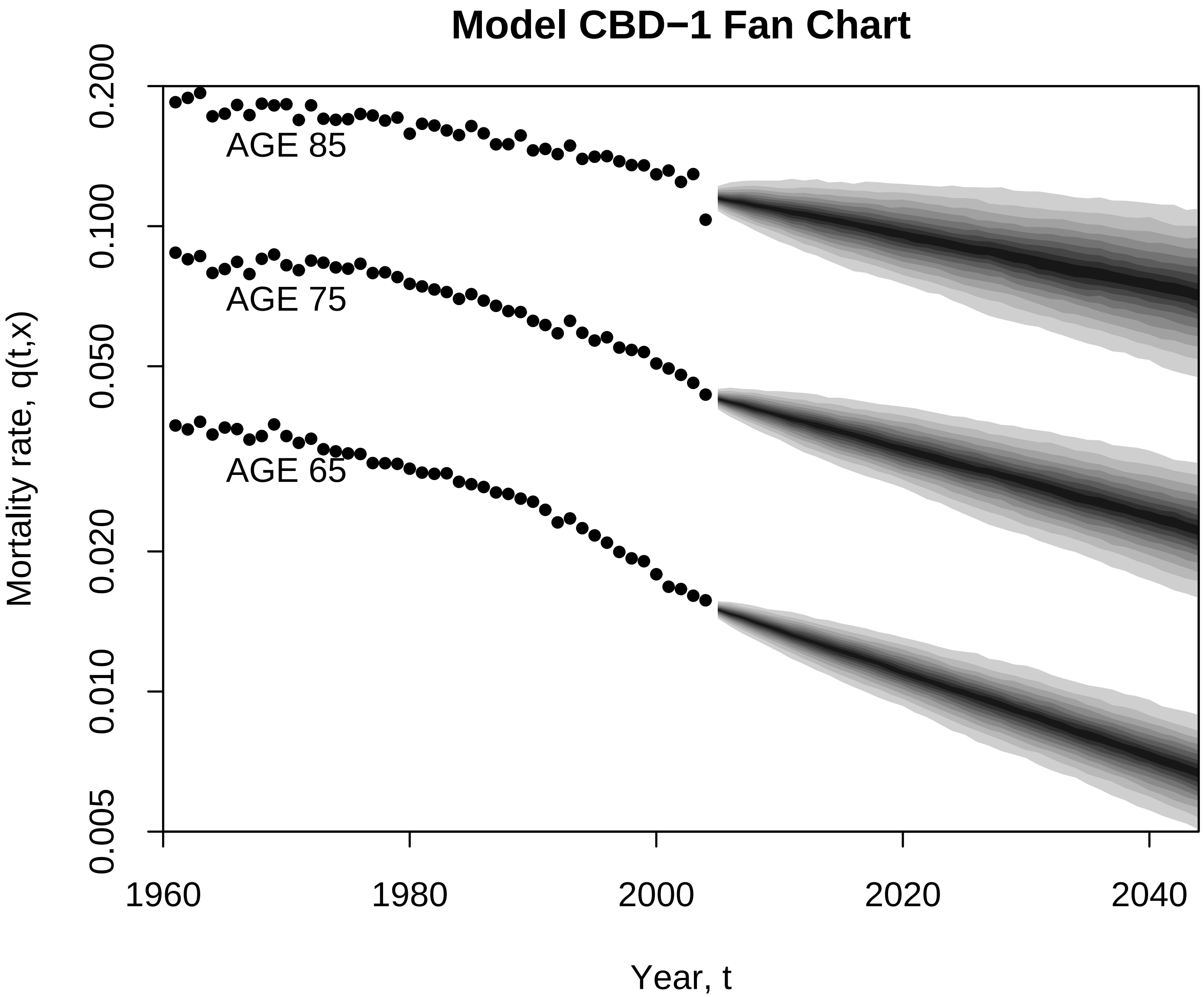
Are standardised residuals i.i.d.?



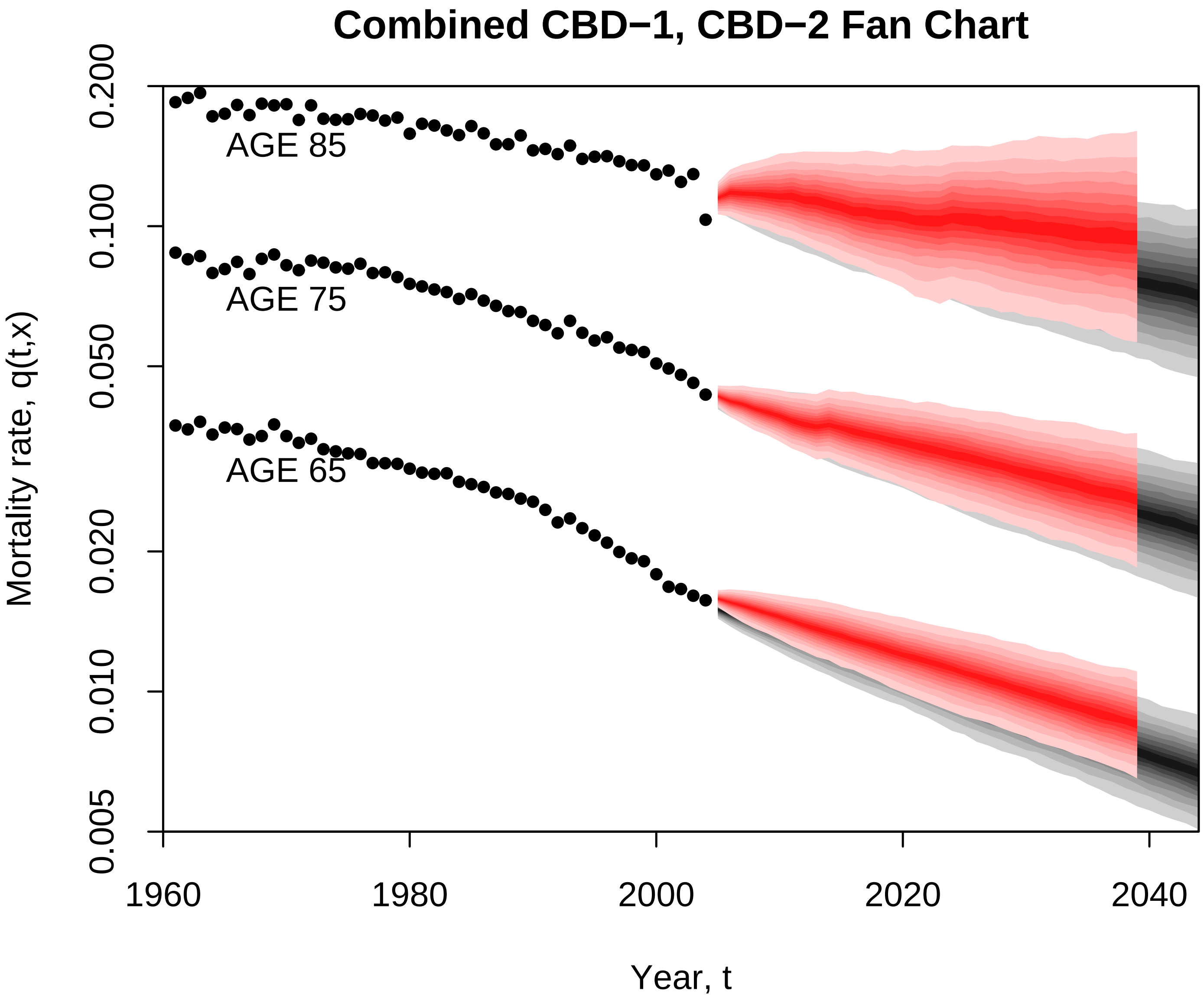
Mortality Fan Charts + A plausible set of forecasts



Model risk

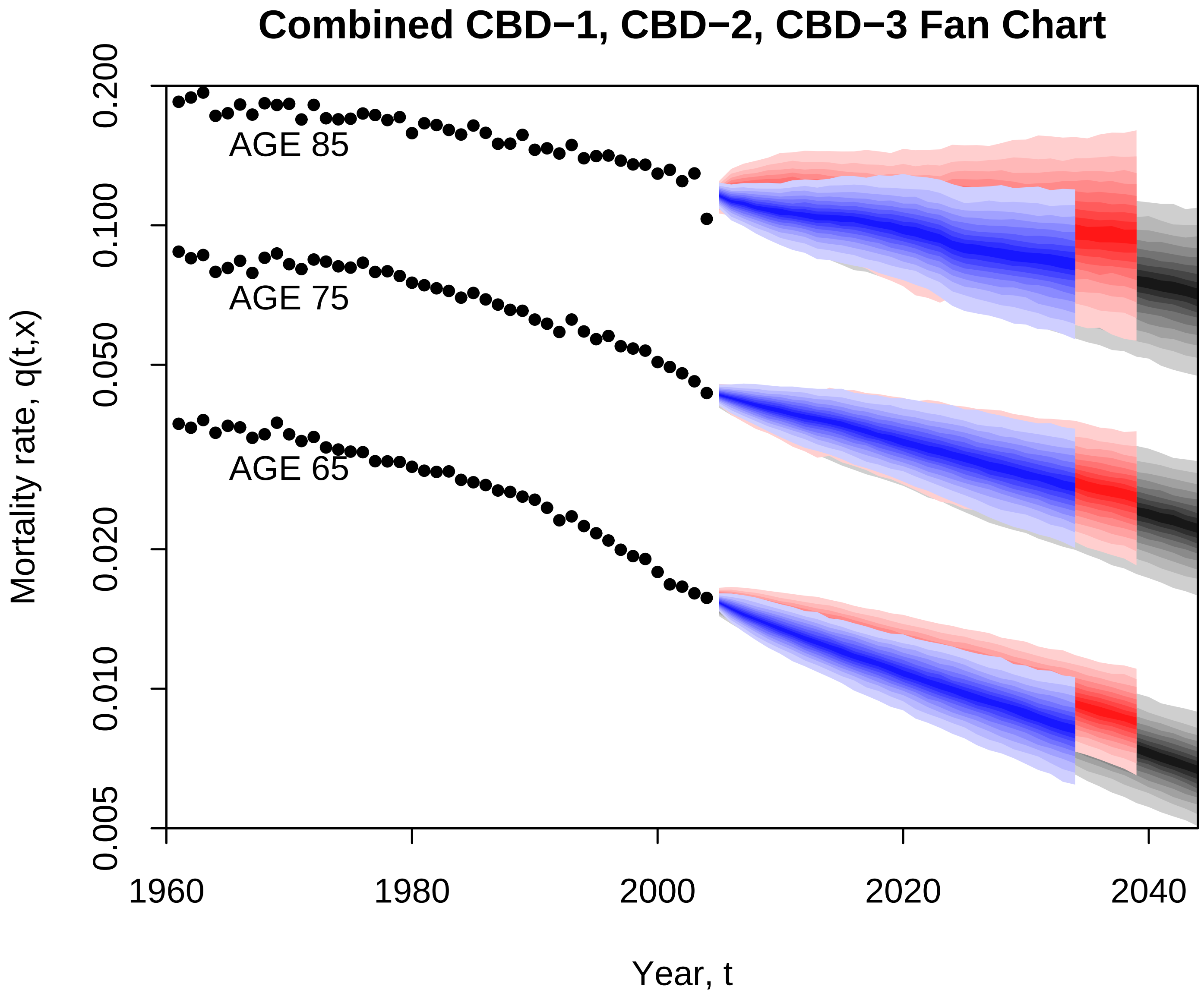

Model CBD−1 Fan Chart
AGE 85
AGE 75
AGE 65
Mortality rate, q(t,x)
0.200
0.100
0.050
0.020
0.010
0.005
1960
1980
2000
2020
2040
Year, t

Model risk



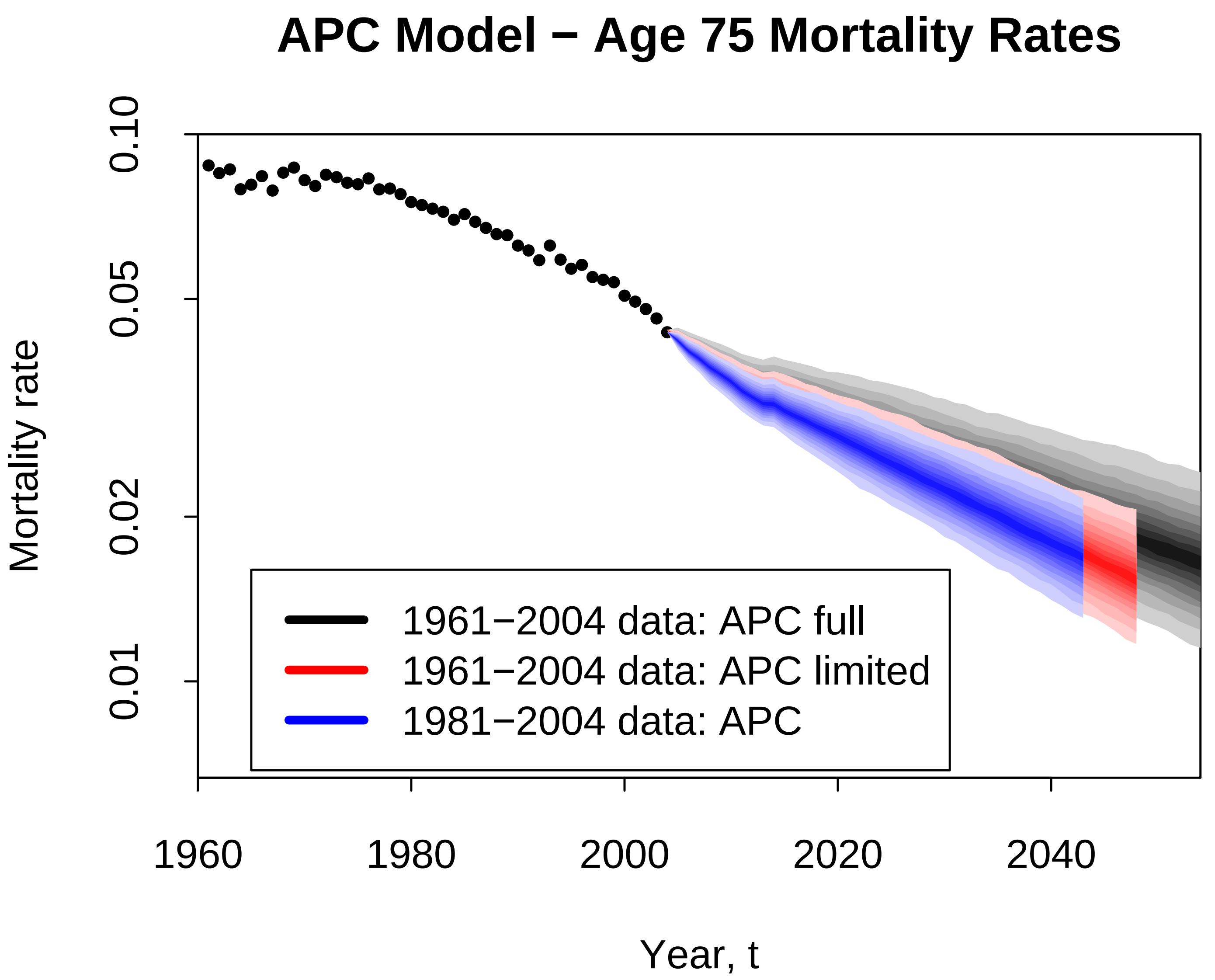
Combined CBD−1, CBD−2 Fan Chart
AGE 85
AGE 75
AGE 65
Mortality rate, q(t,x)
0.005
0.010
0.020
0.050
0.100
0.200
1960
1980
2000
2020
2040
Year, t

Model risk

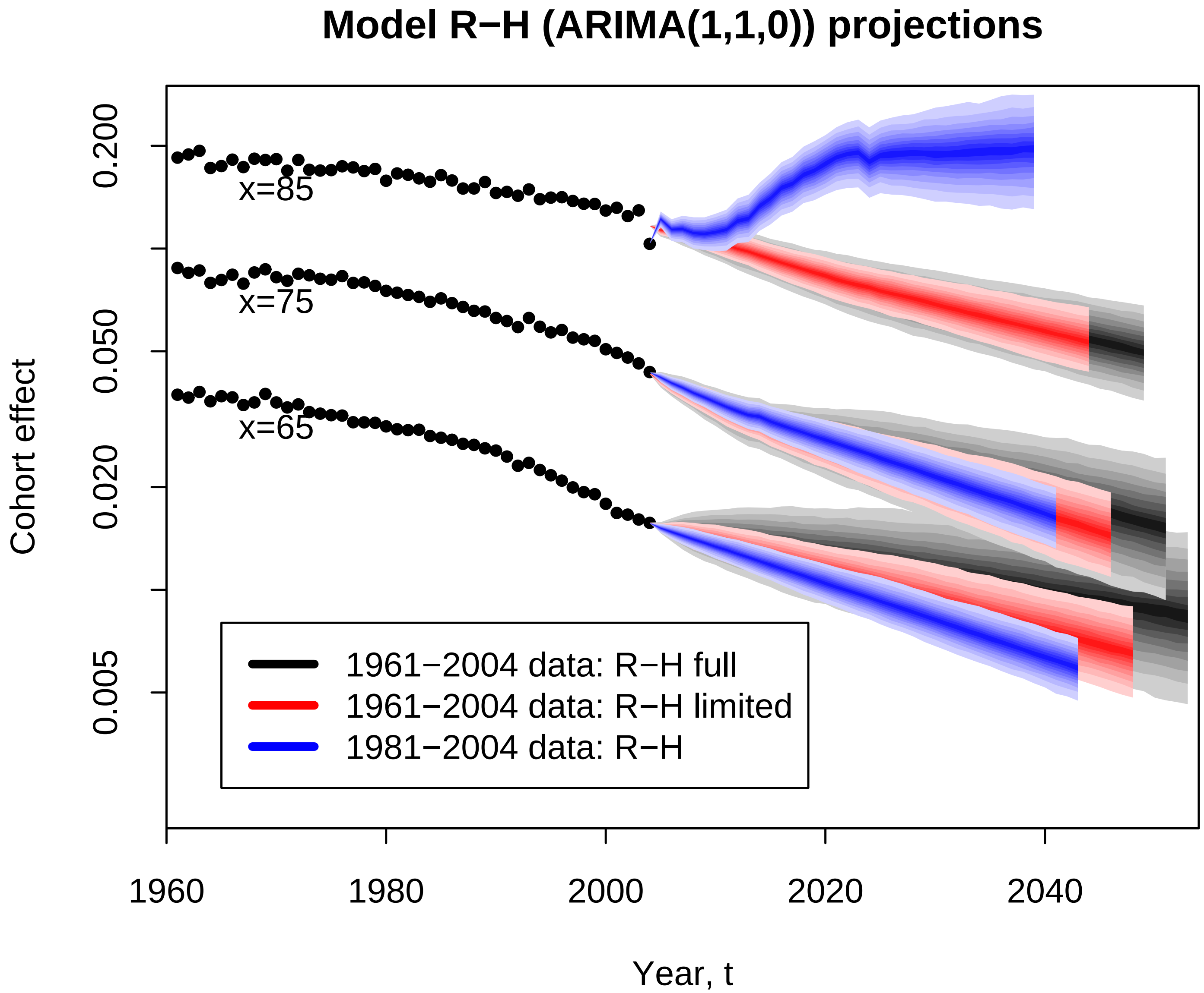


Combined CBD−1, CBD−2, CBD−3 Fan Chart
AGE 85
AGE 75
AGE 65
Mortality rate, q(t,x)
0.005
0.010
0.020
0.050
0.100
0.200
1960
1980
2000
2020
2040
Year, t

Robustness: e.g. Age-Period-Cohort model



Not all models are robust: Renshaw-Haberman model



Robustness Problem

- Likely reason: Likelihood function has multiple maxima
- Consequences:
  - Lack of robustness within sample
  - Lack of robustness in forecasts
    - central trajectory
    - prediction intervals
  - Some sample periods $\Rightarrow$ implausible forecasts

Concluding remarks

- Range of models to choose from
- Quantitative criteria is only the starting point
- Additional criteria $\Rightarrow$
  - Some models pass
  - Some models fail

References

- Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Ong, A., and Belevich, I. (2007) A quantitative comparison of stochastic mortality models using data from England and Wales and the United States. Preprint.
- Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G., Epstein, D., and Khallaf-Allah M. (2008) The Plausibility of Mortality Density Forecasts: An Analysis of Six Stochastic Mortality Models. Preprint - forthcoming.