

MATHEMATICAL MODELS  
AND  
THE CREDIT CRUNCH

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## Plan

- Philosophy
- Background (brief!)
- Questions:
  - assumptions and limitations
  - model types
  - concealment of complexity and model risk
- What to do in the future

## Background to this presentation

### Acknowledgements to

- Numerous friends & colleagues
- Osmosis (i.e. accumulation of uncorroborated evidence over many months)
- Several speakers this week

## Credit Crunch

- Turner Review (UK regulator):
  - [apparent] *misplaced reliance on sophisticated maths*
  - complexity  $\Rightarrow$   
*difficult for top management and boards to assess and exercise judgement over risks being taken*
  - complexity of market not matched by improvements in modelling
  - VaR partly to blame

## Financial mathematicians must take some blame

Different individuals: some or all of

- Allowing models to be used inappropriately
- Not carrying out due diligence
- Not warning senior management about risks
- Allowing bonus culture to over-rule common sense

→ operational risks

## Question 1: assumptions and limitations

Did users of models understand **assumptions** and **limitations** of models?

- Hypothesis:

*nothing wrong with the underlying maths*

**BUT require full specification + testing**

- some models are better than others
- models must be fully scrutinised and tested
- **underlying assumptions and limitations must be communicated upwards**

## Assumptions

- Is a specific assumption: (A) true, (B) approximately correct, (C) laughably wrong?
- What will happen if the assumption is incorrect?
- What can be done to mitigate incorrect assumptions?
- e.g. Black-Scholes model + delta hedging
  - **Gamma hedging**: rebalancing at discrete times, jumps in prices
  - **Vega hedging**: volatility changes from time to time

## Limitations

- Model designed for a specific contract  
then applied to other contracts
- What about less complex contracts?
- What about more complex contracts?
- Model  $\Rightarrow$  price + risk management strategy
- Model might fail if market gets too big

## Question 2: pricing versus risk-management models

Did users understand the difference between

- pricing models
- risk-management models
- risk-measurement models?

## Pricing models

- Also known as *market models*
- e.g. Black-Scholes model
- Model a subset of all risks
- No-arbitrage assumption + dynamic hedging
- Risk-neutral pricing measure
- Simple enough to allow quick calculation of prices
- Calibration of parameters using today's market prices

## Pricing models

### Pros:

- Model is *consistent* with what we observe *today* in the market
- Avoids mispricing of very similar contracts

### Cons:

- Model might not be consistent with historical dynamics and data
- Approach to calibration might not be consistent with model assumptions
- e.g. recalibration of  $\sigma$  in B-S model

## Pricing models

### Dangers:

- avoids mispricing of **very similar** contracts BUT
- extension of pricing to new, **less similar** contracts creates a market based on the *assumed* truth of the model
- e.g. (???) **Gaussian copula model + credit market**
- Reality: **embryo market:**  
pricing models A and B both consistent with limited data  
BUT A and B  $\Rightarrow$  different prices in expanded market

## Risk MANAGEMENT models

- Also known as *real-world models*
  - Wider range of risks
  - Calibrated to historical data
  - Regular recalibration
  - Rigorous statistical testing; model + parameter risk
  - Economic reasonableness
  - Rational economic dynamics
- ⇒ okay for risk control and optimisation

## Risk management models

### Pros:

- Consistent with the past
- Realistic
- Proper assessment of risk

### Cons:

- Difficult to calibrate in real time
- Difficult to price derivatives
- Theoretical prices not exactly equal to market prices

## Risk MEASUREMENT models

- Real-world models
- Incorporate market irrationality; inefficiency
  - information asymmetry
  - negative risk premiums
  - pro/counter cyclical dynamics
  - behavioural finance
    - e.g. overconfidence; understatement of risks
- **DO NOT attempt to optimise!** ( $\Rightarrow$  excessive leverage)
- Okay for: **robustness of strategy**  $\Rightarrow$  ??? risk mitigation

## Question 3: complexity and model risk

Do quants and/or traders have the incentive to

- conceal the extent of contract and model complexity from investors?
- downplay model risk?

↔ Creating complexity in order to profit from the ignorance of others

## A scenario – the illusion of understanding

- Marketing team: good idea for a new product
- Investors will only buy when
  - they **think** they understand the risk profile
  - they **believe** the product will help hedge risk or make money (alpha)
- Quants enlisted to help “educate” investors

## A scenario – the illusion of understanding

- Quants enlisted to help “educate” investors
- One model, one calibration:

Result: “enlightenment” and SALES

- Many models + parameter uncertainty

Result: **confusion** and NO sales and NO bonuses  
(even when the product is good for risk reduction)

## Variation – The Marketing team

- incentive to conceal complexity and model & parameter risk from senior managers and directors

Regulators: need to enforce **Prudent Person Principle**

- Would you sell this product to your grandmother?
- Do **you** understand the risks fully?
- Does **your customer** understand the risks fully?

## What do we need to be doing in the future?

- Improved stochastic modelling
- Better availability and use of historical data
- A stronger voice for quants
- Alternatives to short-horizon quantile risk measures
- Stronger dialogue between academics, regulators + banks
- ...

## Future Model Types

- Solvency II  $\Rightarrow$

Need combined Pricing + risk management models

- Why?

S-II  $\Rightarrow$  need market-consistent values in 1 year

**BUT: is S-II too focused on short-term balance-sheet volatility?**

## Combined pricing + RM models

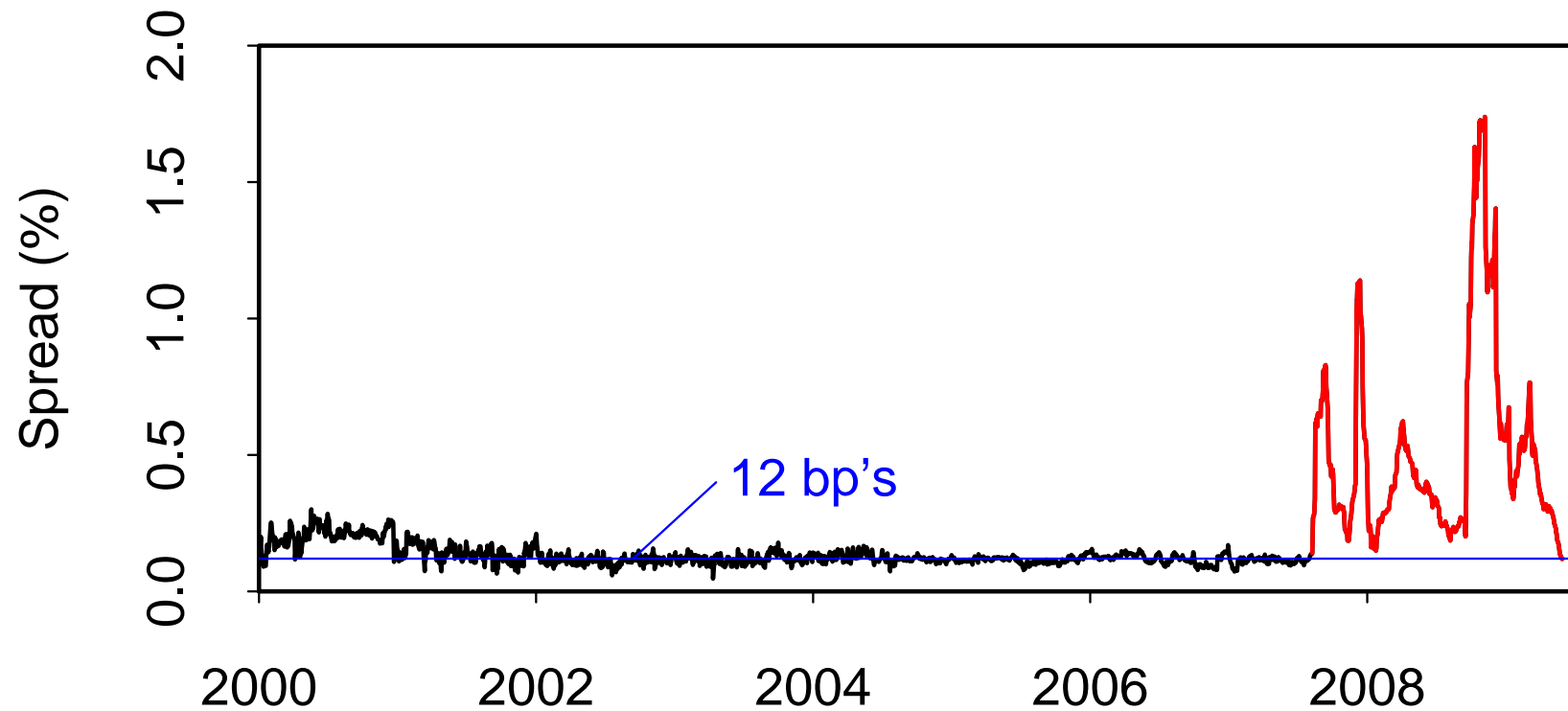
### Requirements:

- Realistic, multi-factor
- Process parameters ( $\mu$ ,  $\sigma$ ,  $\rho$ , . . .) calibrated using historical data
- State variables ( $S(t)$ ,  $r(t)$ ,  $\sigma(t)$ , . . .) *calibrated* using market prices
- Dynamics of state variables consistent with model assumptions (c.f. pricing models)

## Regime shifts

Unsecured versus collateralised short-term loans

1-month LIBOR minus 1-month REPO



## Improved risk-management/measurement models

- Liquidity; buying/selling spreads; asymmetric info.
- Extreme regime shifts
  - Liquidity, volatility, (perceived) information asymmetry, ...?
- Other latent variables
- Large-scale, destabilising feedback, hysteresis
- Fat tails, stochastic volatility
- Market irrationality, behavioural finance etc.

## FTSE-100, 13-17 October 2008; log returns

+7.94%, +3.17%, -7.43%, -5.50%, +5.09%

- A: i.i.d. normal model
- B: non-central-t distribution + stochastic volatility

*p*-values (should be i.i.d.  $\sim U[0, 1]$ ):

Day	1	2	3	4	5
A	0.9999999999999994	0.998	0.0000000000000001	0.00000001	0.999999
B	0.9792	0.7447	0.0610	0.1293	0.8476

## Improved modelling: Augmented by

- thorough analysis of **model and parameter risk**  
⇒ discourages excessive leverage!
- scenario analysis, stress tests and black swans

## A stronger voice for quants

- Quants; risk management team; external experts
- Walker Review (UK):

Senior management and non-executive directors

(NEDs<sup>\*</sup>)  $\Leftarrow$  external advice

(\*) NED: *Scots slang*: Non-Educated Delinquent

## A stronger voice for quants

- a greater number of NEDs should have a strong knowledge of QRM & ERM:  
*enough to be able to ask the right questions*
- Regulator  $\Rightarrow$  “fit and proper” test
- Risk Committee NEDs should have access to middle and junior staff

## The role of Value-at-Risk

- In theory: VaR  $\Rightarrow$  quantile
- In practice ?????
  - “VaR”  $\Rightarrow$  quantile + i.i.d. multivariate normality
- *VaR is not a coherent risk measure*
  - “non-coherence” was not a cause of the crisis
  - Expected-shortfall + stochastic volatility + fat tails  $\Rightarrow$  bigger crisis ???
  - BUT optimise VaR  $\Rightarrow$  small probability, high-severity risks

## Improving on traditional Value-at-Risk

- Use better models!
- How to avoid pro-cyclicality?
  - ?????
  - Take the long term view
    - e.g. run-off of life insurance liabilities
    - ( $\Rightarrow$  greater emphasis on cashflow matching)
  - Does Solvency II go far enough?

# Securitisation

## Reasons:

- Risk transfer and risk reduction
- Reduced capital requirements
- Convert illiquid into liquid assets
- Capitalise future illiquid cashflows
- Trading **new** risks  $\Rightarrow E_{\text{subj}}[\text{utility}] \nearrow$  for all

## Issues:

- **Complex repackaging** of existing liquid traded risks  $\Rightarrow$  **DANGER**  
???
- Securitisation of OWN risks  $\Rightarrow$  moral hazard
- Insurable interest

- Risk-reduction or gambling?

## Better availability of data

- Longer runs of data
- Buying and selling prices (liquidity)
- Available for free (with a time lag) for non-commercial research