The International Actuarial Association

MODELS FOR STOCHASTIC MORTALITY

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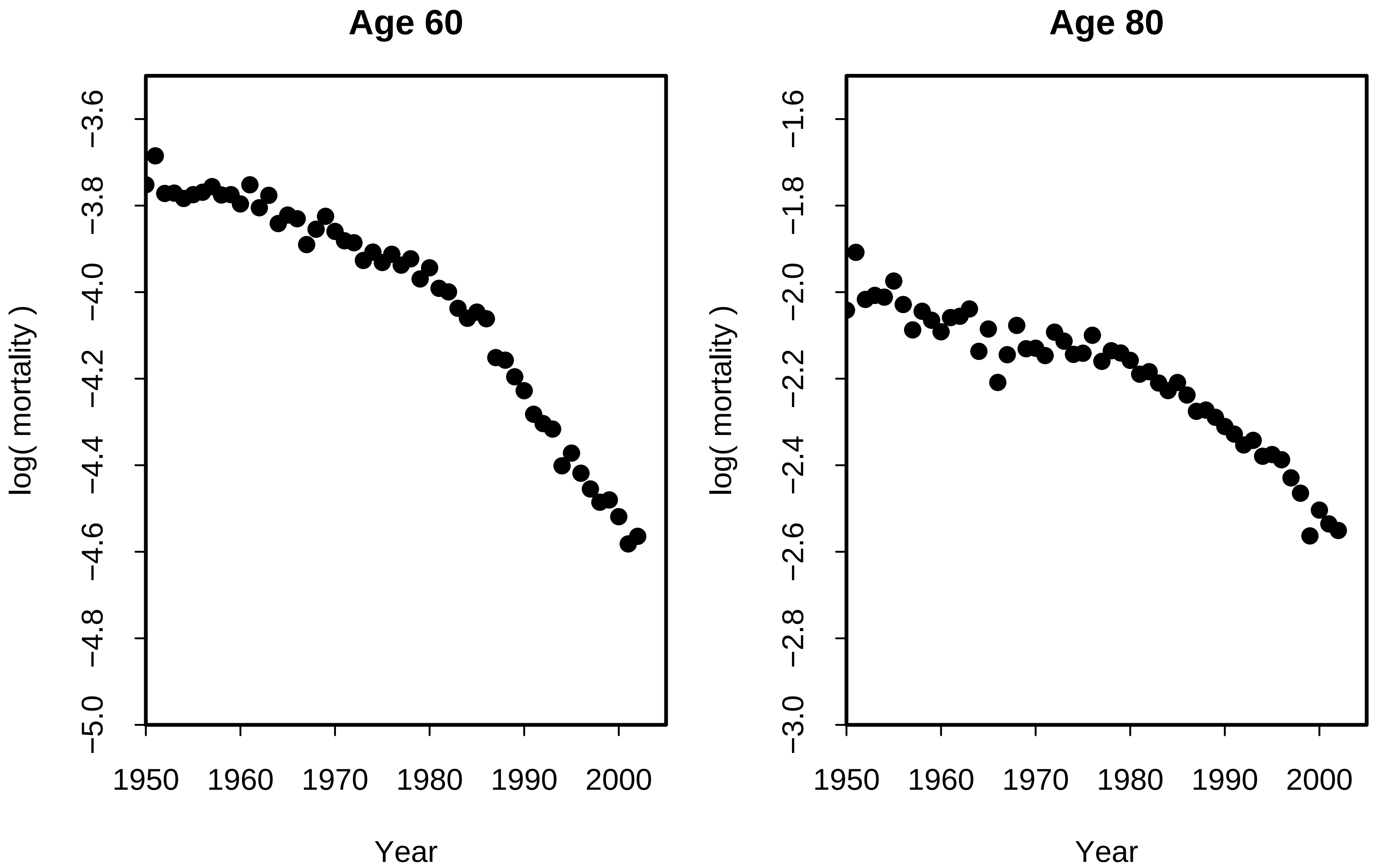
Plan

- Introduction + data
- A simple model for stochastic mortality
  - Parameter uncertainty
- Comparing different models
  - Model risk
  - Model selection criteria

England and Wales log mortality rates 1950-2002



The facts about mortality:

- Mortality rates are falling
- Systematic randomness from year to year
- Uncertain improvement rate
- Different ages $\Rightarrow$ Different improvement rates

Stochastic Models for Future Mortality

Limited historical data $\Rightarrow$

- No single model is 'the right one'

  limited data $\Rightarrow$ Model risk

- Even with the right model

  limited data $\Rightarrow$ Parameter risk

Why is it important to model stochastic mortality?

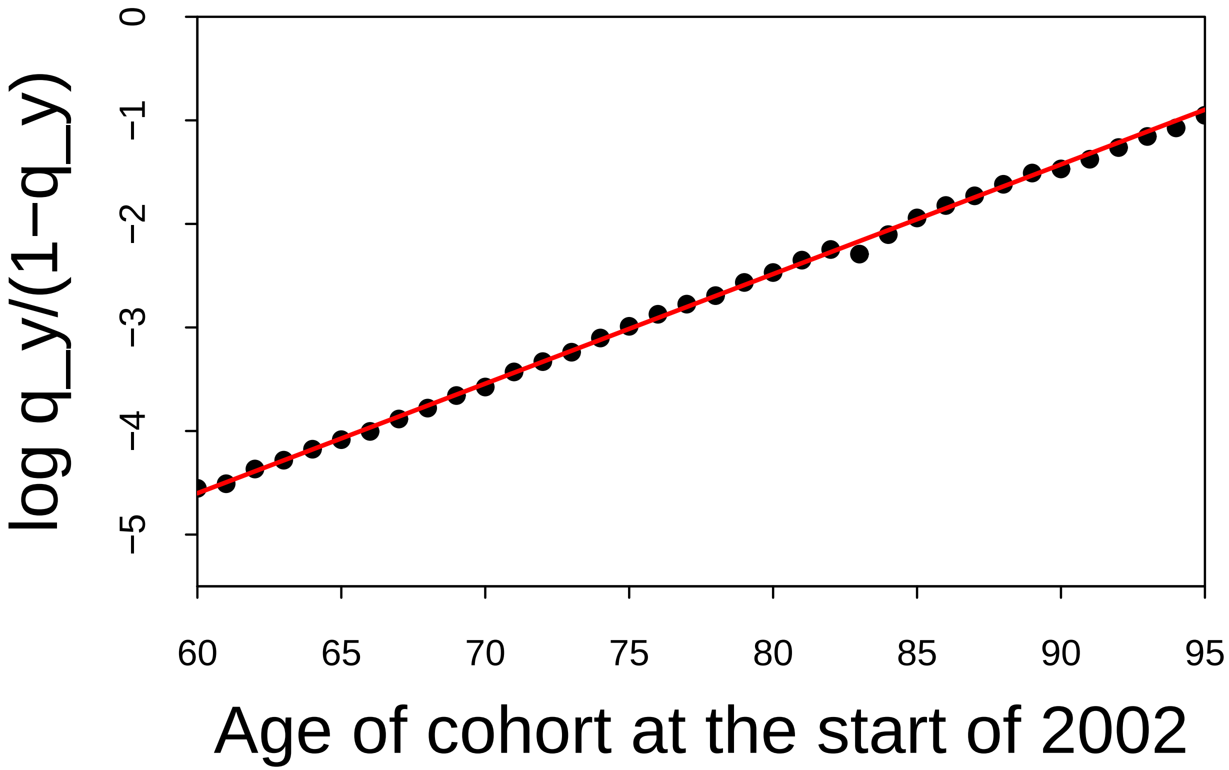
- Good risk management

  [Process risk, parameter uncertainty, model risk]

- Calculating *quantile* reserves

- Valuing and hedging complex life insurance contracts with guarantees

Case study: England and Wales males, age 60-90



$q_y =$ mortality rate at age y in 2002

Data suggests $\log q_y/(1-q_y)$ is linear

A TWO-FACTOR PARAMETRIC TIME-SERIES MODEL

x= age at time t

Mortality rates for the year t to $t+1$:

$$\text{logit}\, q(t,x) = \kappa_1(t) + \kappa_2(t)\,(x - \bar{x})$$

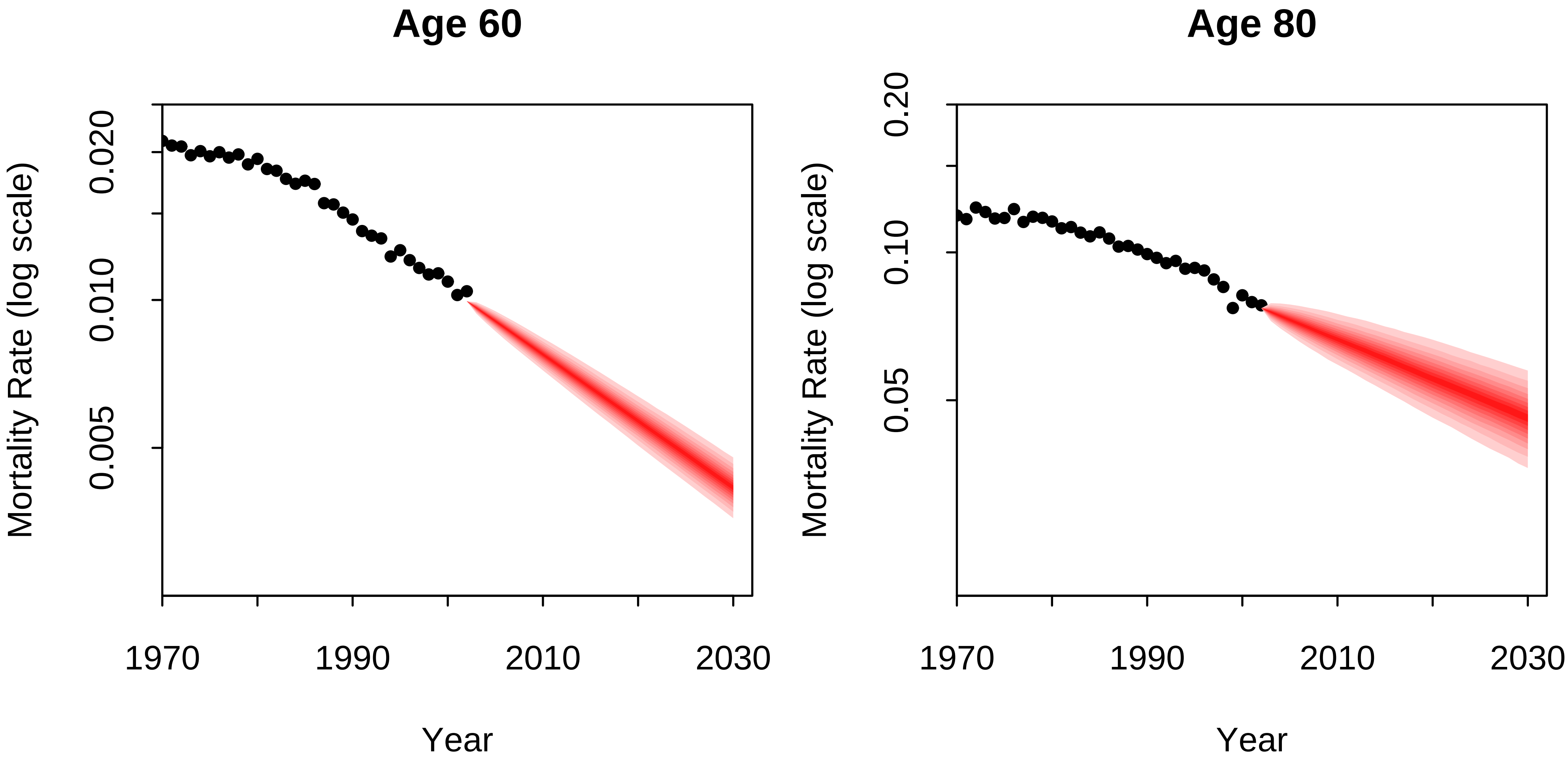
We model $\kappa(t) = (\kappa_1(t), \kappa_2(t))'$ as a random-walk

$$\kappa(t+1) = \kappa(t) + \mu + CZ(t+1)$$

- Estimate μ and $V = CC'$
- Quantify parameter uncertainty in μ and V

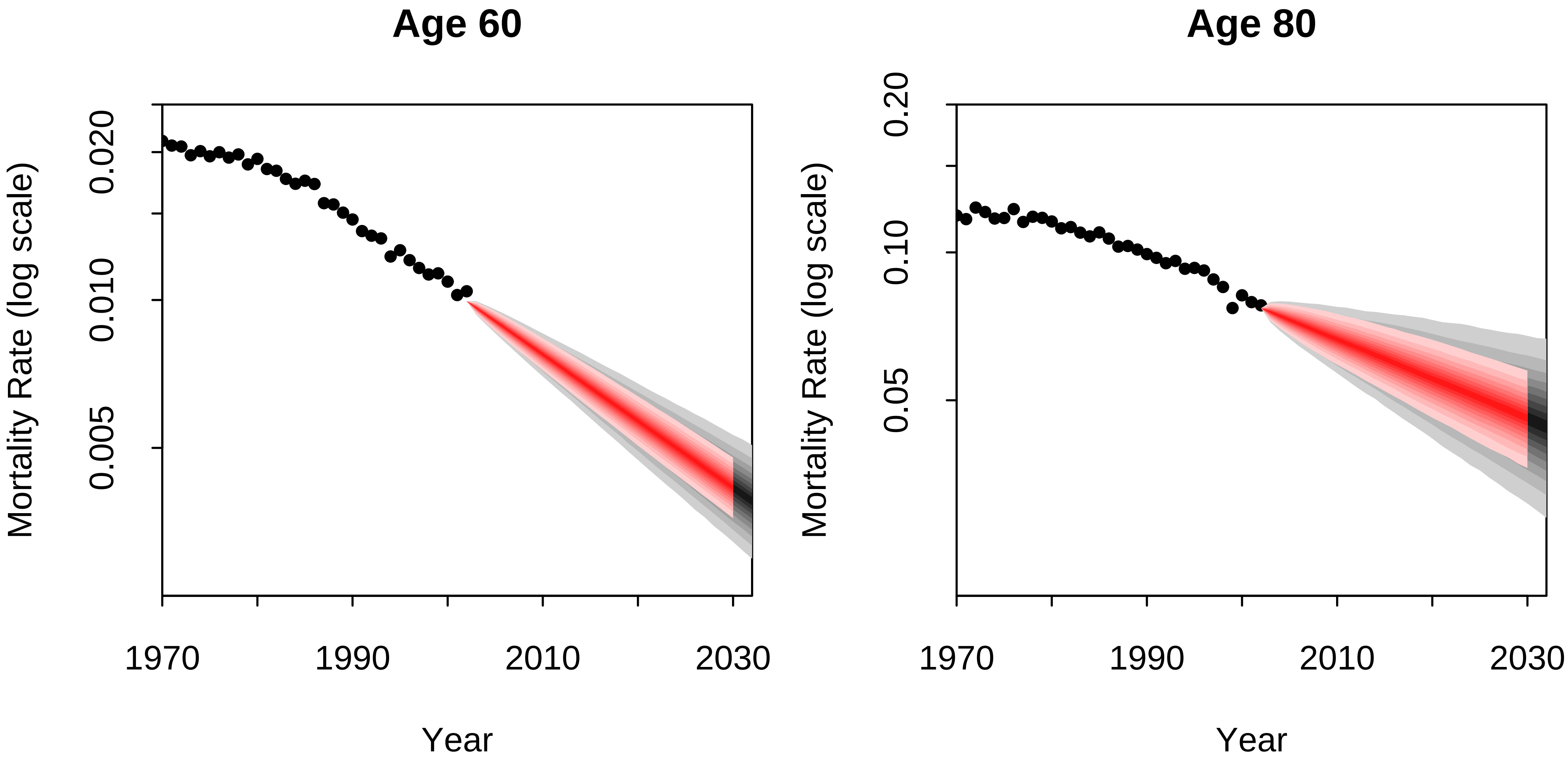
Simulated mortality rates: 90% confidence intervals

without parameter uncertainty



Simulated mortality rates: 90% confidence intervals

with and without parameter uncertainty



Other models

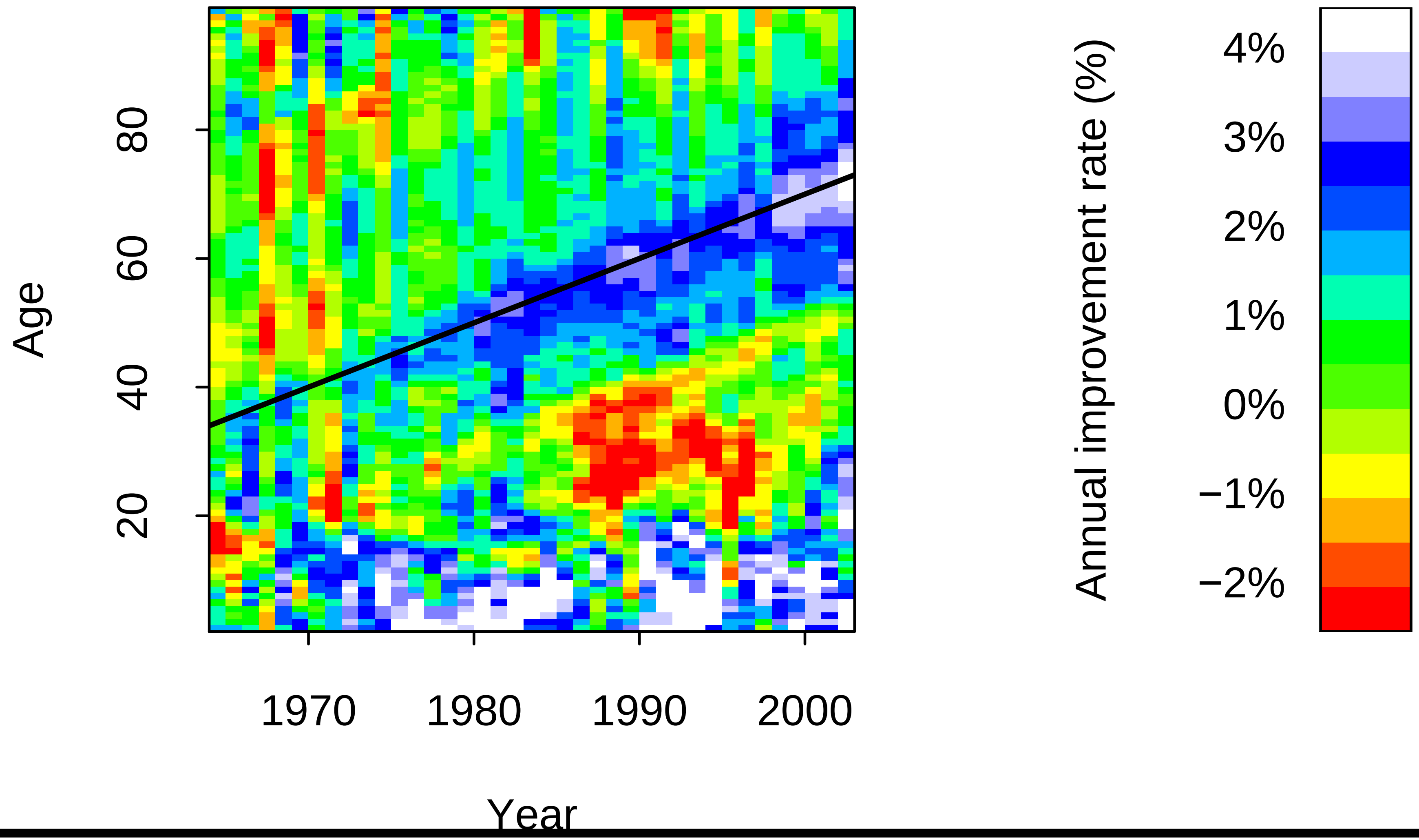
General family:

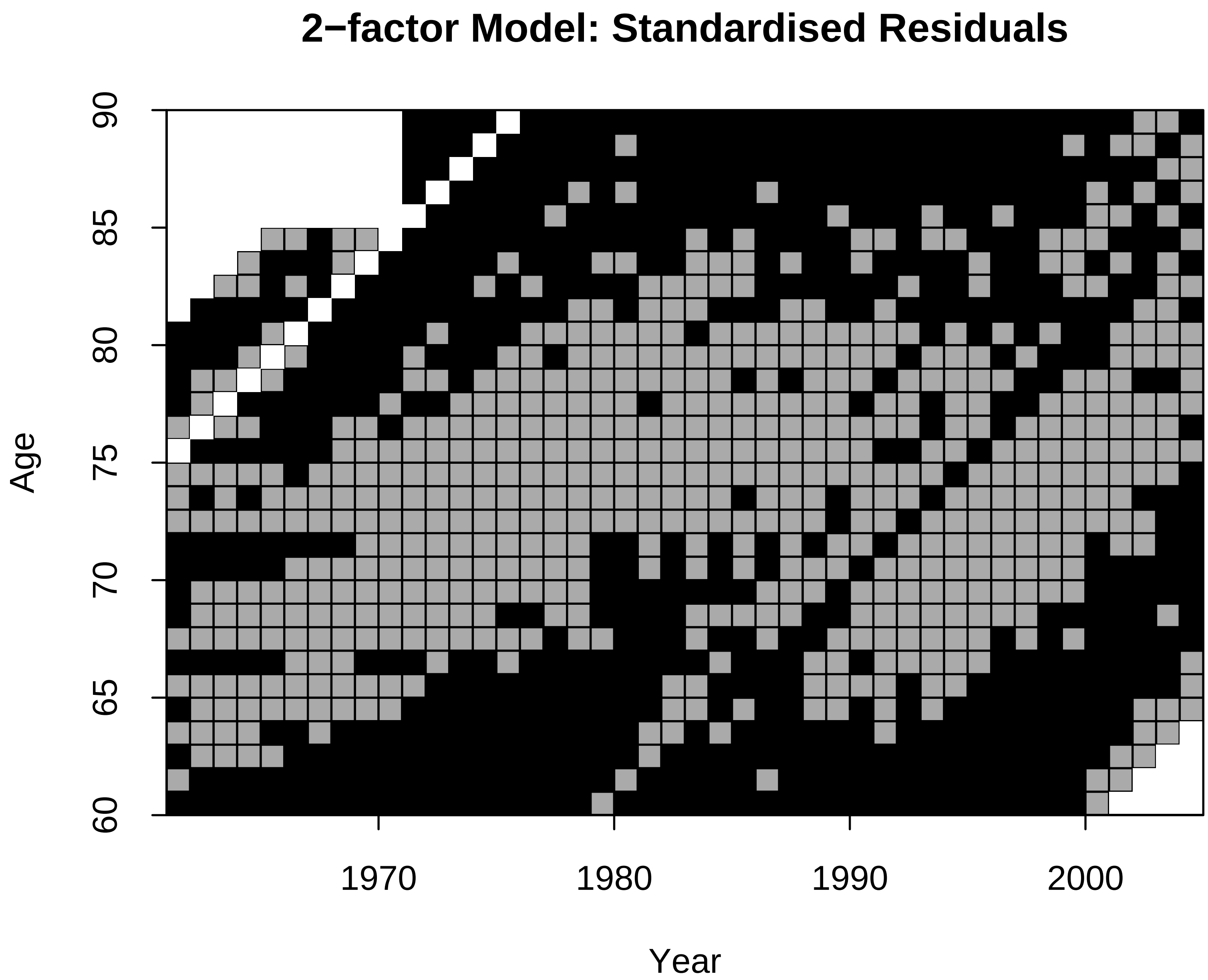
$$\text{logit}\, q(t,x) = \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} \gamma_{t-x}^{(i)}$$

- $\beta_x^{(i)}$ = age effects
- $\kappa_t^{(i)}$ = period effects
- $\gamma_{t-x}^{(i)}$ = cohort effects
- N = number of components

Cohort Effects (e.g. Willetts, 2004)

Annual mortality improvement rates (Engl. & Wales, males)




2−factor Model: Standardised Residuals
Age
90
85
80
75
70
65
60
1970
1980
1990
2000
Year

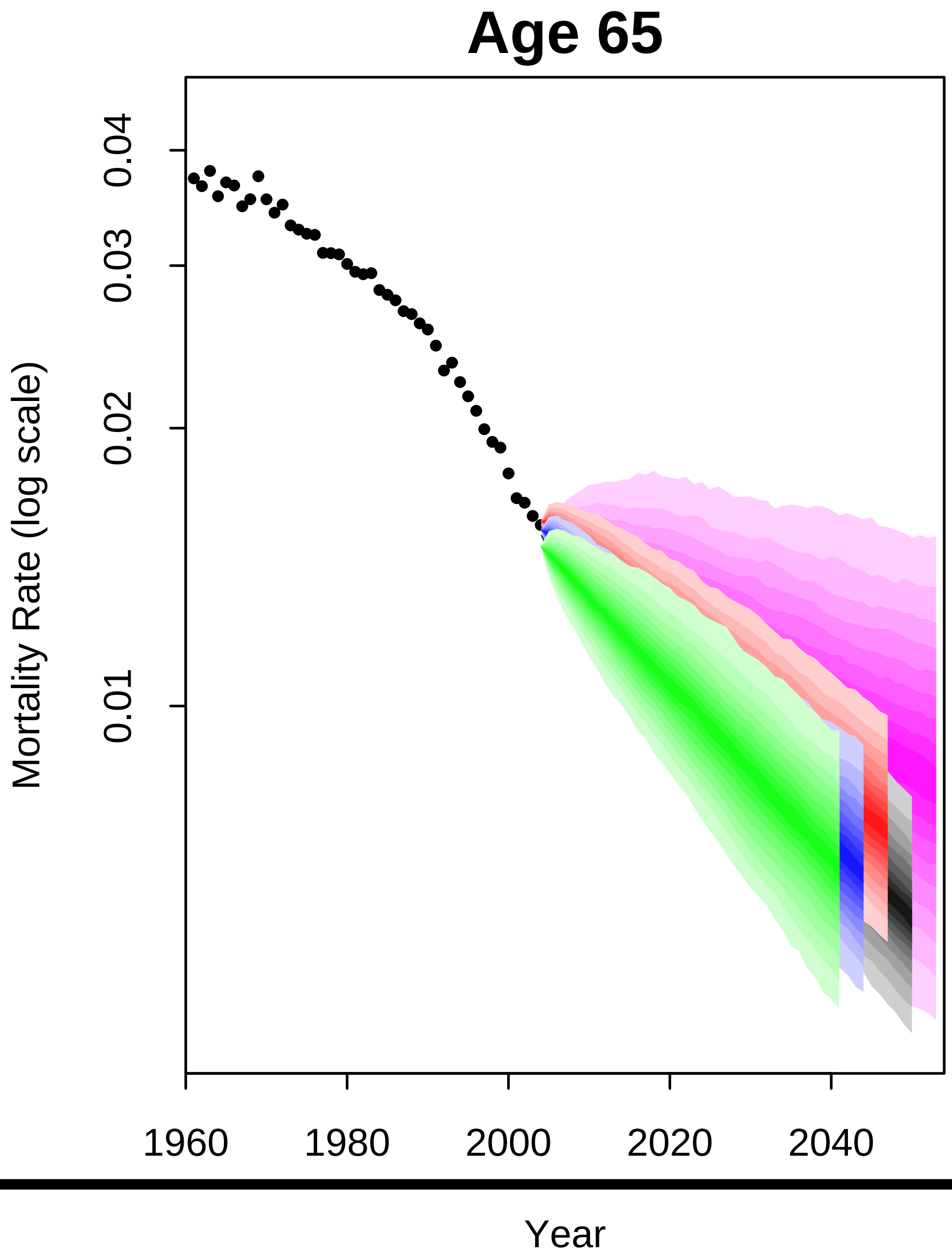
Eight models compared (5 here; England & Wales data)

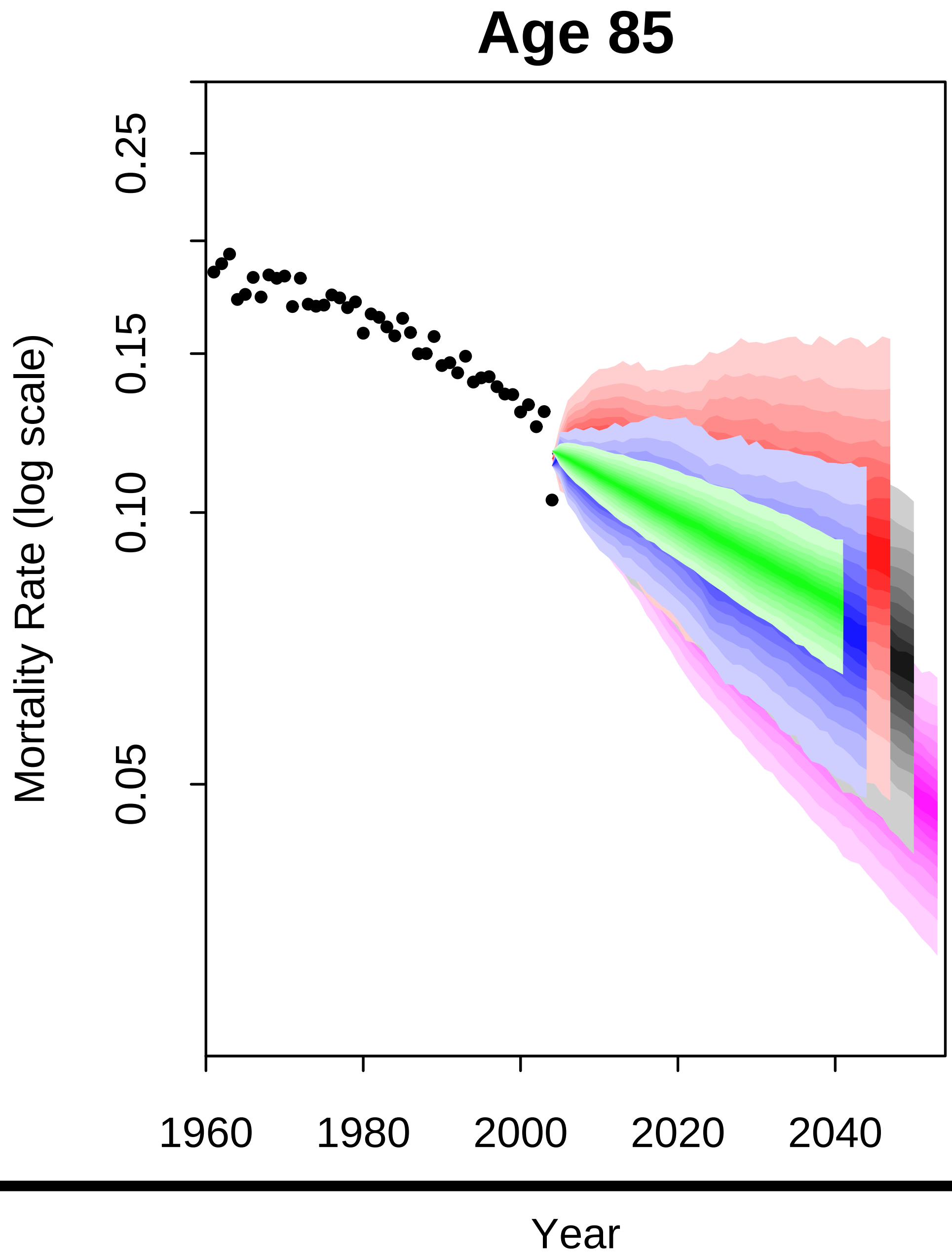
Model	Maximum log-likelihood (*)	Effective number of parameters	BIC (rank)
M1 (Lee-Carter)	-8912.7	102	-9275.8 (6)
M2 (Renshaw-Haberman)	-7735.6	203	-8458.1 (3)
M5 (Cairns-Blake-Dowd)	-10035.5	88	-10348.8 (8)
M7 (new)	-7702.1	202	-8421.1 (2)
M8 (new)	-7823.7	161	**-8396.8 (1)**

(*) Poisson model for deaths; E & W males, ages 60-89

US data: top three ranks are reversed

Model risk is important





Model evaluation criteria

- Quality of fit (BIC)
- Robustness of parameter estimates
- Plausibility of forecasts
- Biological reasonableness
- ......
- ......

Robustness of parameter estimates and forecasts

- Fit model to 1961-2004

  Forecasts based on 1961-2004 parameter estimates

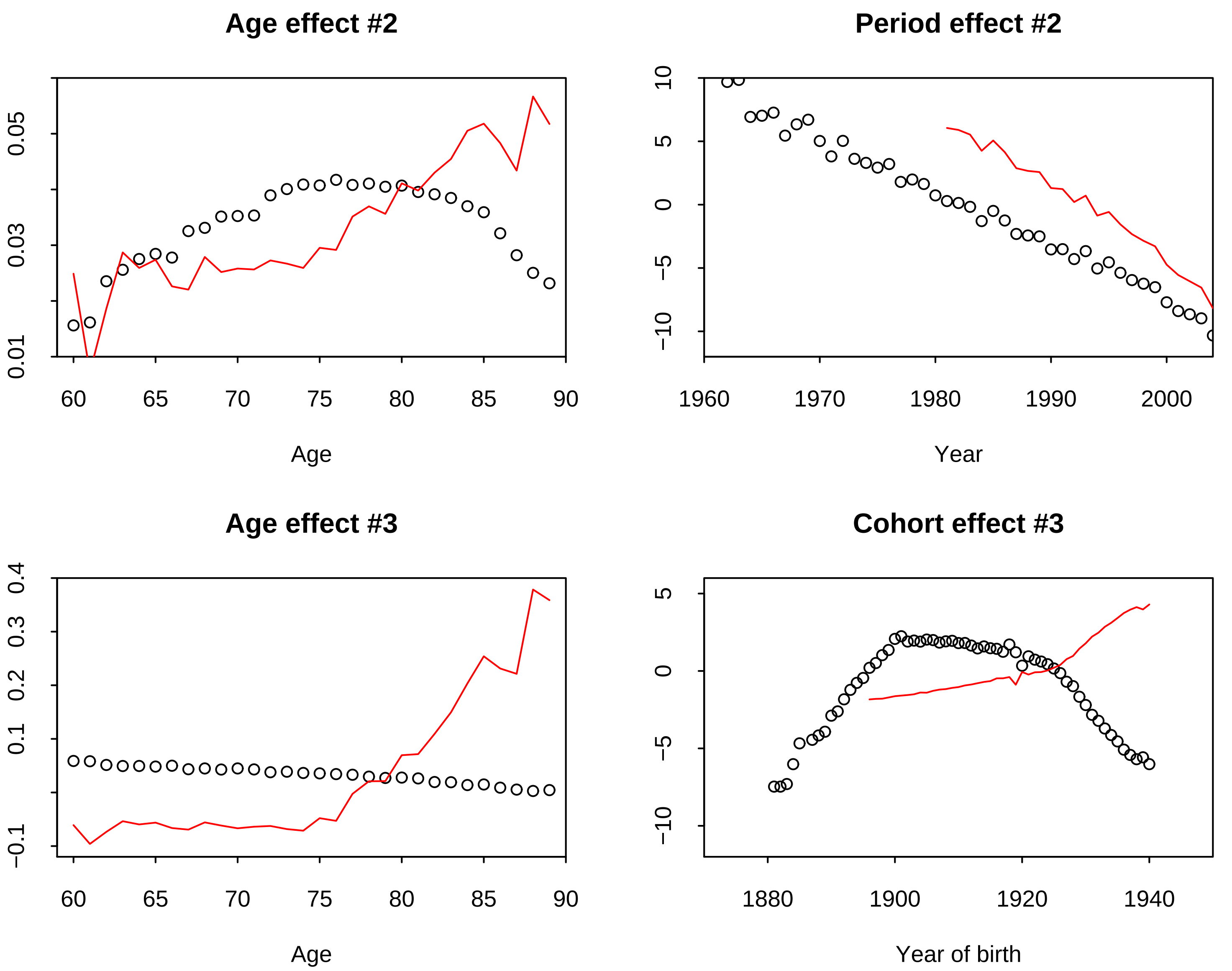
- Fit model to 1981-2004

  Forecasts based on 1981-2004 parameter estimates

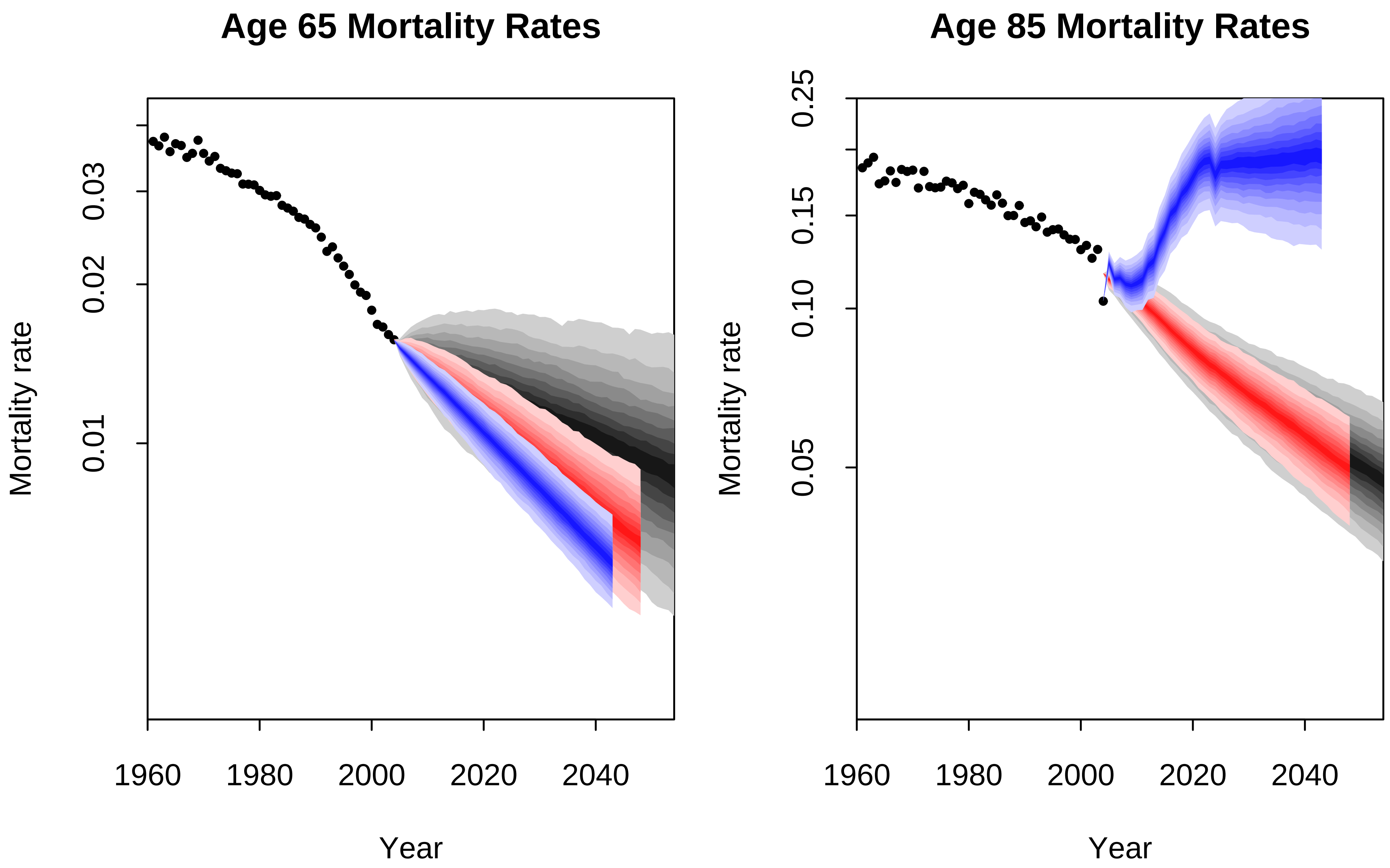
- Fit model to 1961-2004

  Forecasts based on 1981-2004 parameter estimates

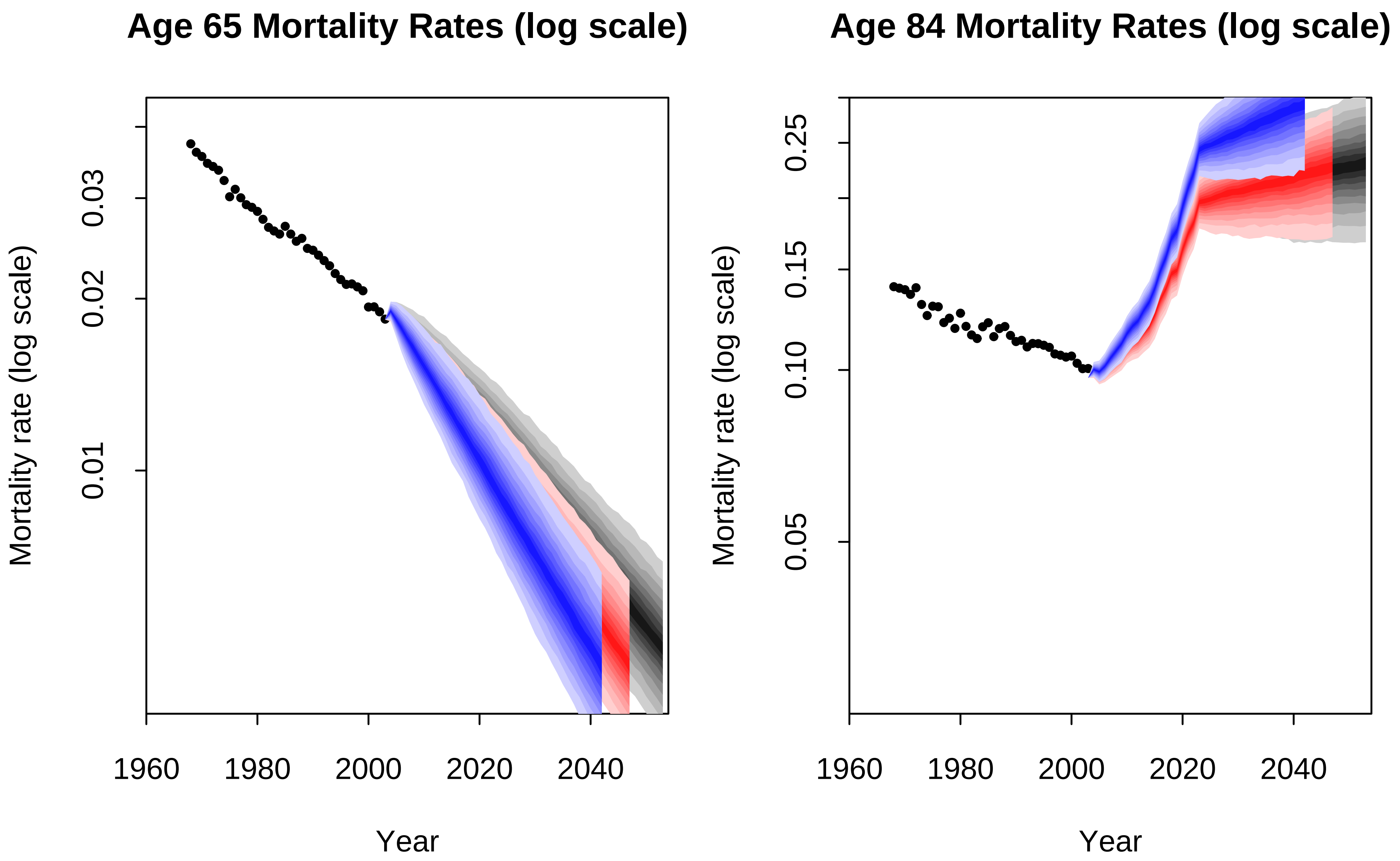
Model M2 parameter estimates not robust



Model M2 forecasts not robust



Model M8 US males forecasts are *not plausible*



Possible problems with M2 and M8

- M2 only ???:
  - Likelihood has many local maxima
  - Lack of smoothness in age effects
- M2 and M8:
  - Age-Cohort effects being used to compensate for too few Age-Period effects

Conclusions

- Model and parameter risk is important
- Need to use several model selection criteria
  - Quantitative: e.g. BIC
  - Qualitative