A TWO-FACTOR MODEL FOR STOCHASTIC MORTALITY WITH PARAMETER UNCERTAINTY

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Chicago, 24 April 2006

Plan

- Introduction + data
- A two-factor model for stochastic mortality
- Applications
  - Longevity bonds
  - Survivor caps and caplets
- Conclusions

The facts about mortality:

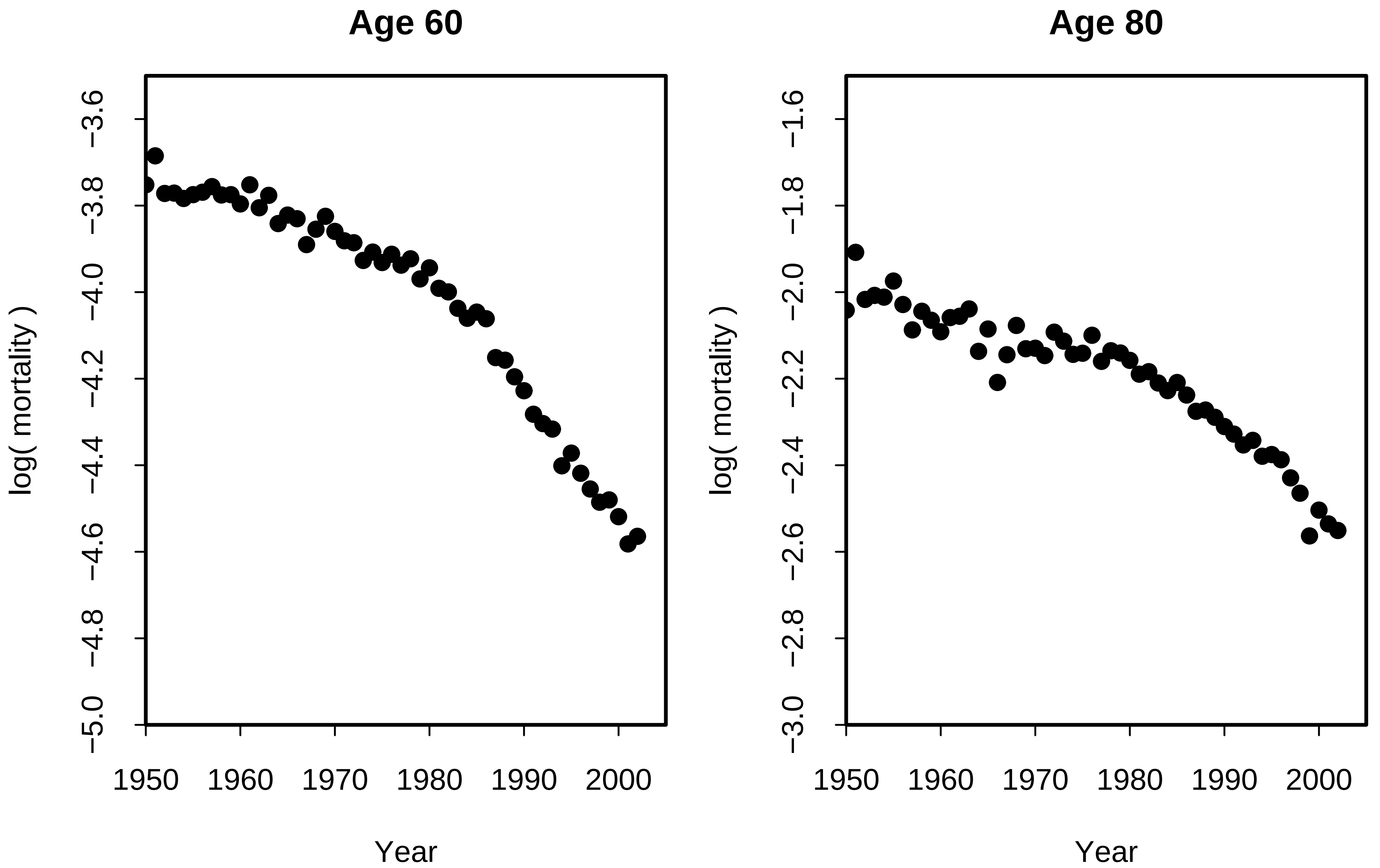
- Life expectancy is increasing.
- Future development of life expectancy is uncertain.

"Longevity risk"

Longevity Risk $=$ the risk that future mortality rates are lower than anticipated

Focus here: Mortality rates above age 60

England and Wales log mortality rates 1950-2002

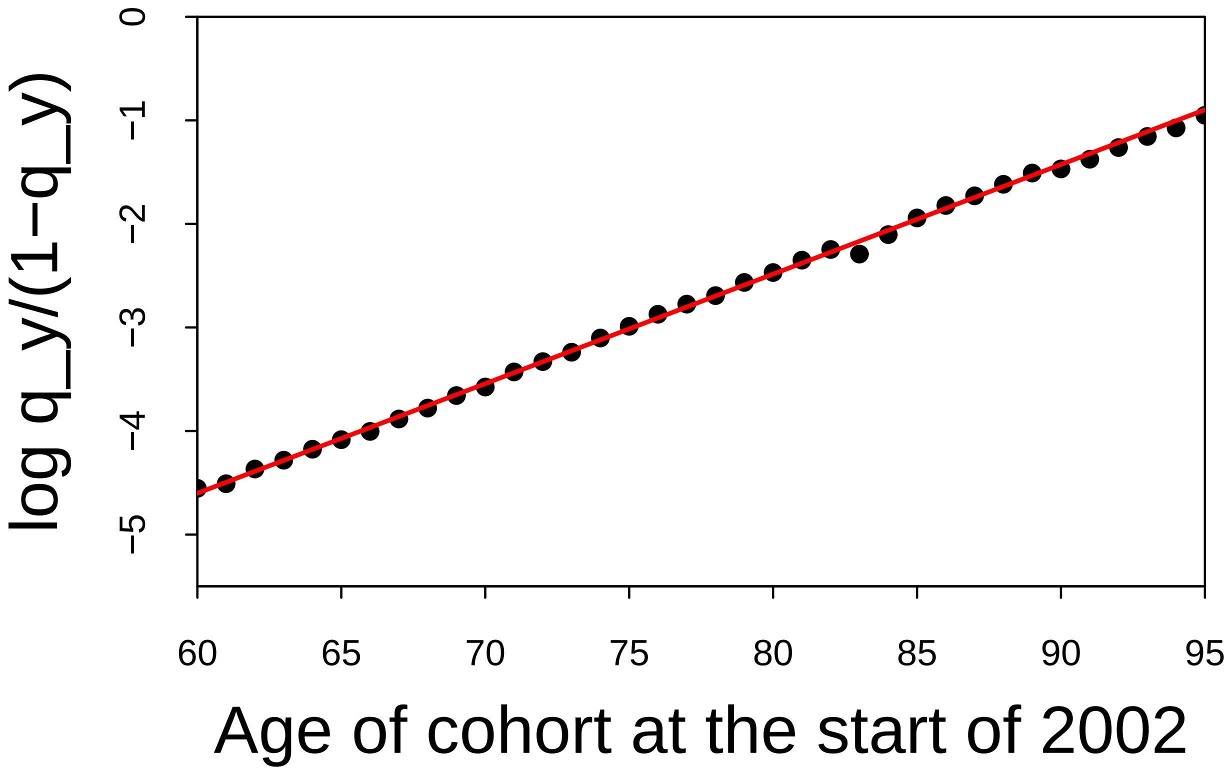


Stochastic Models

Limited historical data $\Rightarrow$

- No single model is ‘the right one’
- Parameter risk
- Model risk

Case study: England and Wales males, age 60-95



$q_y =$ mortality rate at age y in 2002

Data suggests $\log q_y/(1 - q_y)$ is linear

A TWO-FACTOR 'SHORT-RATE' MODEL

Cohort: Age x at time $t = 0$

Mortality rates for the year t to $t + 1$:

$$q(t, x) = \frac{e^{A_1(t) + A_2(t)(x+t)}}{1 + e^{A_1(t) + A_2(t)(x+t)}}$$

$(x + t)$= age at time t

We model $A(t) = (A_1(t), A_2(t))'$ as a random-walk with drift

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Model: Random walk with drift

$$A(t+1) - A(t) = \mu + CZ(t+1)$$

- $V = CC' =$ variance-covariance matrix
- Estimate μ and V
- Quantify parameter uncertainty in μ and V

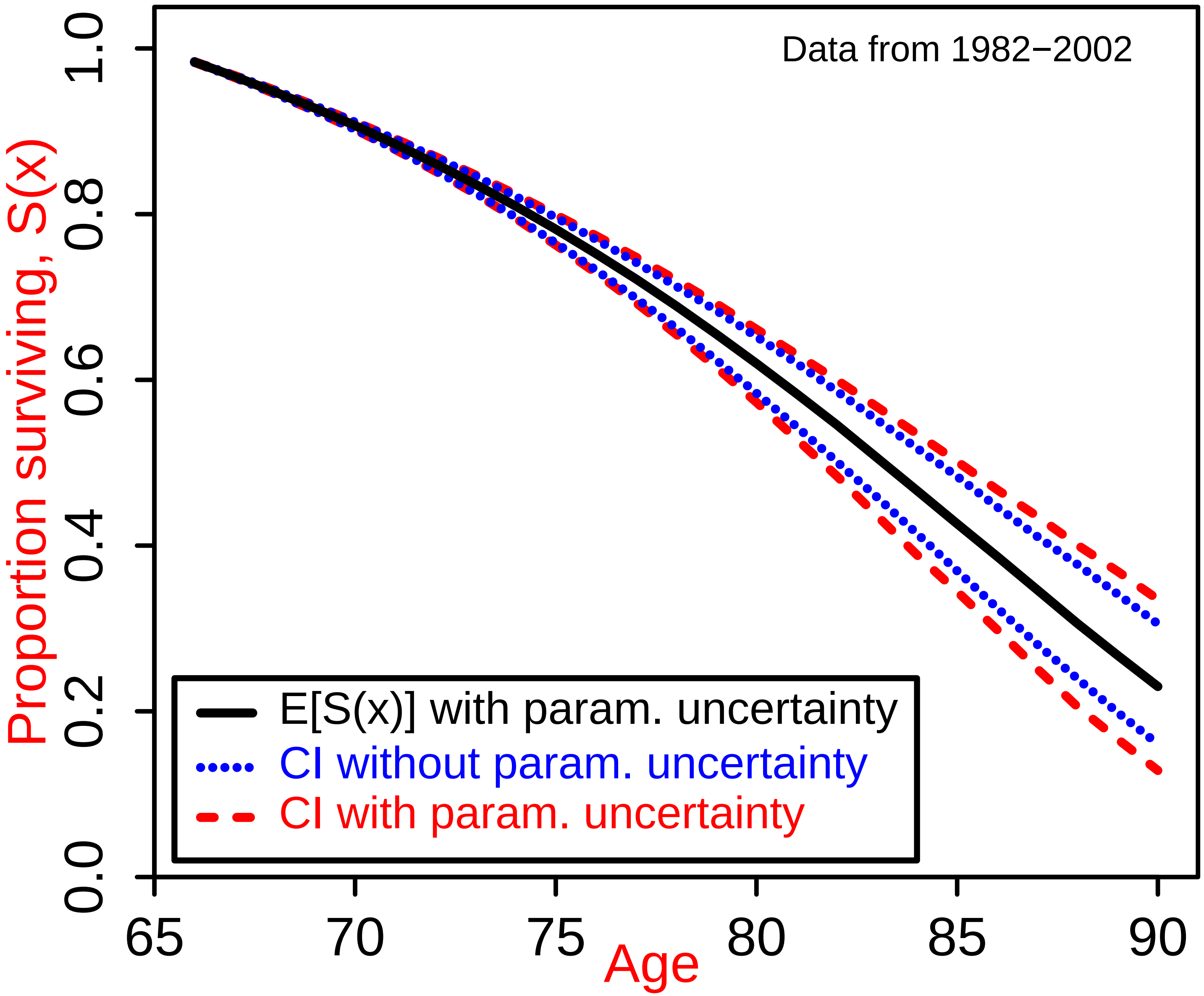
Application: longevity bonds

- Cohort: Age x at time $t = 0$
- $S(t, x) =$ survivor index at t

  proportion surviving from time 0 to time t

$$S(t,x) \;=\; (1 - q(0,x)) \times \ldots \times (1 - q(t-1,x))$$

90% Confidence Interval (CI) for Cohort Survivorship



Stochastic Mortality: General Conclusions

- Less than 10 years:
  - Systematic risk not significant
- Over 10 years
  - Systematic risk becomes more and more significant over time
- Over 20 years
  - Model and parameter risk begin to dominate

Risk-neutral pricing

$$\begin{pmatrix} A_1(t+1) \\ A_2(t+1) \end{pmatrix} = \begin{pmatrix} A_1(t) \\ A_2(t) \end{pmatrix} + \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{pmatrix} \tilde{Z}_1(t+1) + \lambda_1 \\ \tilde{Z}_2(t+1) + \lambda_2 \end{pmatrix}$$

where $\tilde{Z}_1(t+1)$ and $\tilde{Z}_2(t+1)$ are i.i.d. $\sim N(0,1)$

<u>under a risk-neutral pricing measure $Q(\lambda)$</u>

λ_1 and λ_2 are market prices of risk

Comments

- The market is highly incomplete
- The switch from P to Q is a modelling assumption
- (Simple) Key assumption:

  market prices of risk λ_1 and λ_2 are constant.
- As a market develops this assumption becomes a testable hypothesis

One data point: the EIB-BNP longevity bond

- Offer price (ultimately unsuccessful) $\Rightarrow$ risk premium of 20 basis points if held to maturity
- What values of λ_1, λ_2 are consistent with the 20b.p.'s risk premium?
- One price, two parameters $\Rightarrow$ many solutions

Answer: 20 b.p. spread equates to

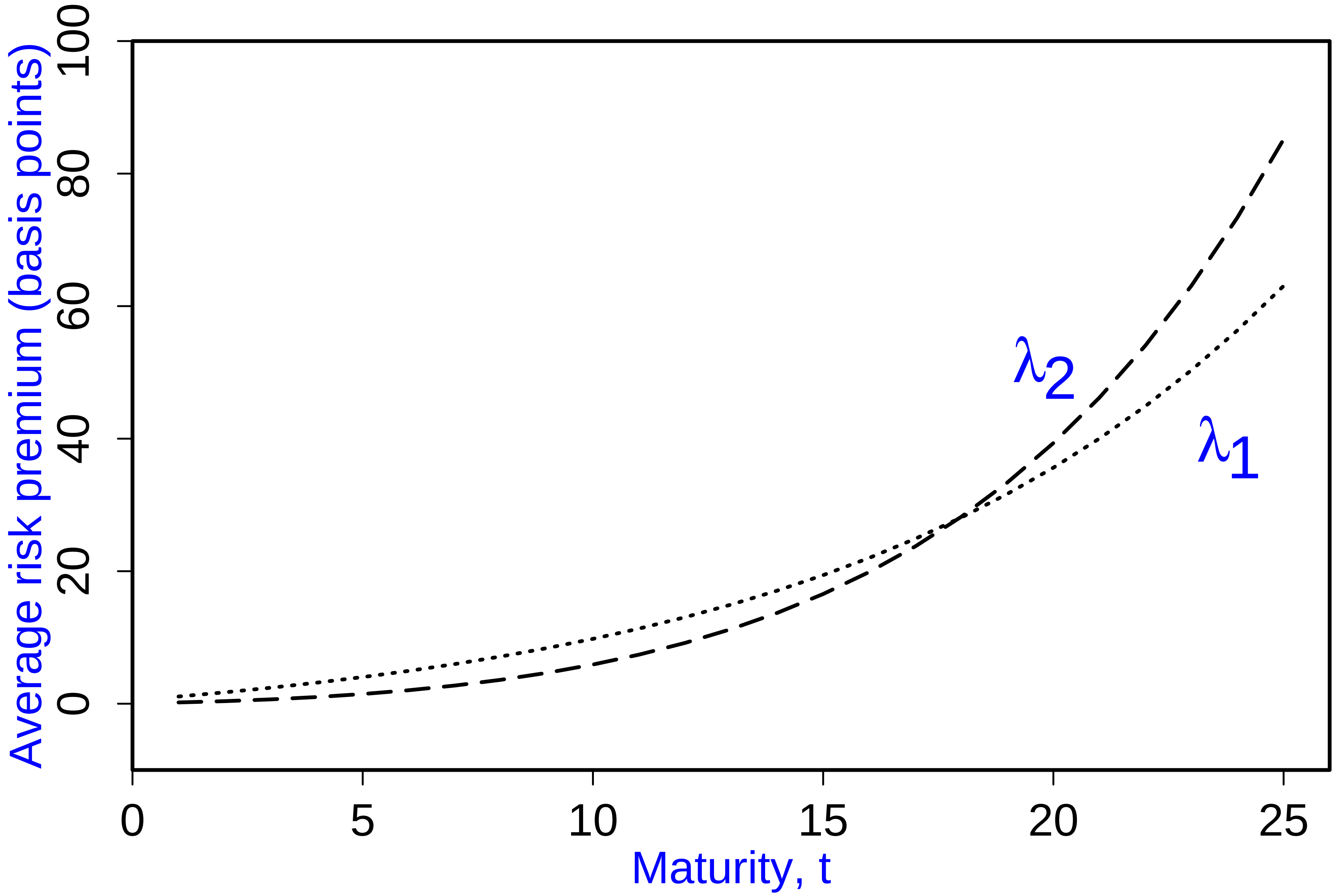
$$\begin{array}{llll} \lambda_1 = & 0.375, & \lambda_2 = & 0 \\ & \downarrow & & \downarrow \\ \lambda_1 = & 0, & \lambda_2 = & 0.315 \end{array}$$

Do these values represent a *good deal*?

Why do we need to know λ_1, λ_2?

$\Rightarrow$ info. on how to price new issues in the future.

Zero-coupon Longevity bonds: avg. risk premium p.a.



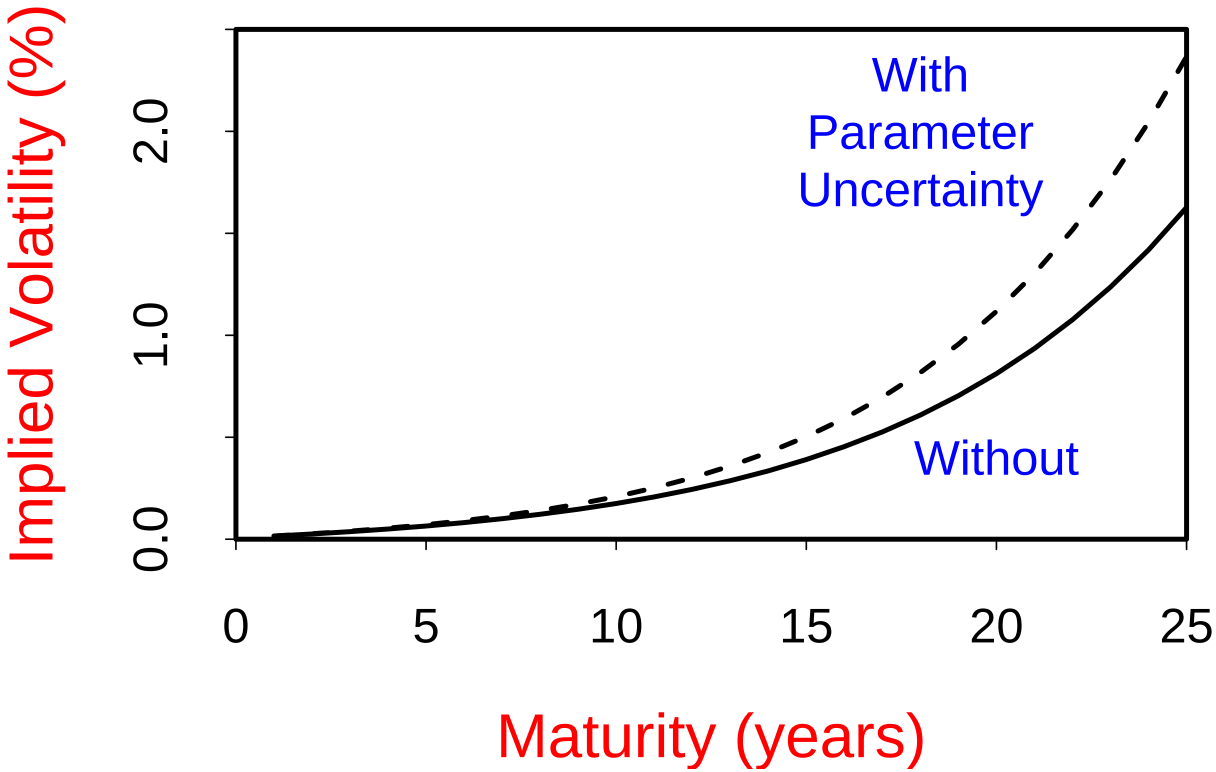
Longevity Bond Risk Premiums: $\lambda = (0.375, 0)$

Dependency on term and initial age:

		Initial age of cohort, x		
		60	65	70
Bond	20	8.9	14.7	23.1
Maturity	25	12.7	20.0	28.7
T	30	16.9	24.3	31.5

At-the-Money Call Options on $S(t)$

Payoff: $\max\{S(T) - K,\ 0\}$ where $K = E_{Q(\lambda)}[S(T)]$



25-year survivor cap: parameter uncertainty adds 33%

Conclusions

- Stochastic models important for
  - risk measurement $\longrightarrow$ assessment of risk premium
  - pricing contracts with option characteristics
- One model out of many possibilities
- Significant longevity risk in the medium/long term
- Model and parameter risk is important

Selected References

Cairns, A.J.G., Blake, D., and Dowd, K. (2006) "A Two-Factor Model for Stochastic Mortality with Parameter Uncertainty," Preprint.

Blake, D., Cairns, A.J.G., and Dowd, K. (2006) "Living with mortality: longevity bonds and other mortality-linked securities," Faculty of Actuaries Sessional Meeting, 16 January, 2006.

Cairns, A.J.G., Blake, D., and Dowd, K. (2005) " Pricing death: Frameworks for the valuation and securitization of mortality risk," Preprint, Heriot-Watt University.

Cairns, A.J.G., Blake, D., Dawson, P., and Dowd, K. (2005) "Pricing the risk on longevity bonds," *Life and Pensions,* October: 41-44.

Lee, R.D., and Carter, L.R. (1992) "Modeling and forecasting U.S. mortality," *Journal of the American Statistical Association,* 87: 659-675.

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