Robust Modelling and Management of Longevity Risk

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Acknowledgements: David Blake, Kevin Dowd, Guy Coughlan

Plan

- Robustness
- Genealogy
- New directions in modelling
- Hedging pension plan longevity risk

  Reasons for (maybe) going beyond customised longevity swaps

JFK: ”*We choose to go...not because [it is] easy, but because [it is] hard. .....*”

- Developing new models is easy.
- Recommending a customised longevity swap is ”easy”.
- Developing new models that are robust is hard!
- Setting ERM strategy that we have confidence in, that is optimal and that we know is robust is harder still!

Robustness

- Issue of robustness $\Rightarrow$ key question for business
- Many forms:
  - Model fit
  - Model forecasts
  - Business decisions and ERM strategy
  - Details of how model implemented
- Interplay with business objectives

Robustness: model fit

- Sensitivity of age, period + cohort effects to
- Changes in range of years or ages
- Method of calibration

See: Cairns et al. (2009); Richards and Currie (2009)

Robustness: model forecasts + decisions

- Sensitivity of key outputs
  - forecasts of future mortality rates
  - financial variables e.g. MCV liabilities; q-fwd price
  - ERM decisions and strategy
- Relative to
  - change in range of years or ages
  - change in calibration methodology

See: Cairns et al. (2011,2012); Cairns (2012)

Robustness: Details of Implementation

- Statistical issues
  - conditional independent Poisson *vs* regression
  - amount of smoothing
  - treatment of parameter uncertainty
  - treatment of model risk
  - Bayesian versus frequentist
- Forecasting
  - choice of time series model
  - recalibration risk at future valuation dates

Robustness: Interplay with Business Objectives

- Risk appetite, tolerances and limits
- Time horizon
- Cashflow or value hedge
- What range of ERM choices are being considered?

  Attitude to ambiguity / Knightian uncertainty etc.
- What metrics are being considered?
  - optimal hedge ratios
  - hedge effectiveness or other metric e.g. change in $E[\text{utility}]$
  - price for hedge

Development of New Models

- Many new stochastic mortality models since Lee-Carter
- Are they fit for purpose?
- Are they robust?

GENEALOGY: 1st GENERATION MODELS

Currie/Richards (M4)
2-D P-splines
2002, ...

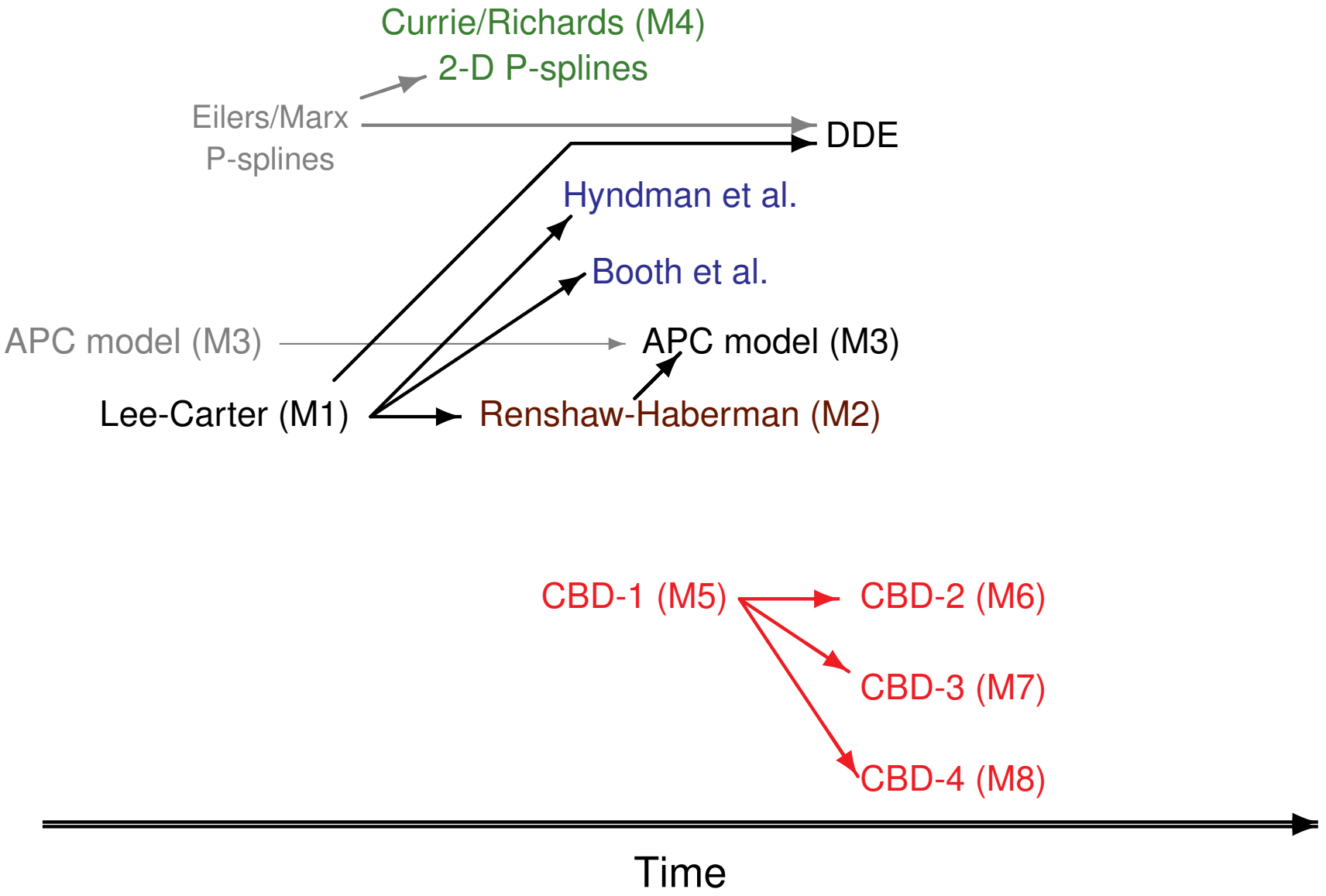
Eilers/Marx
P-splines

Lee-Carter (M1)
1992

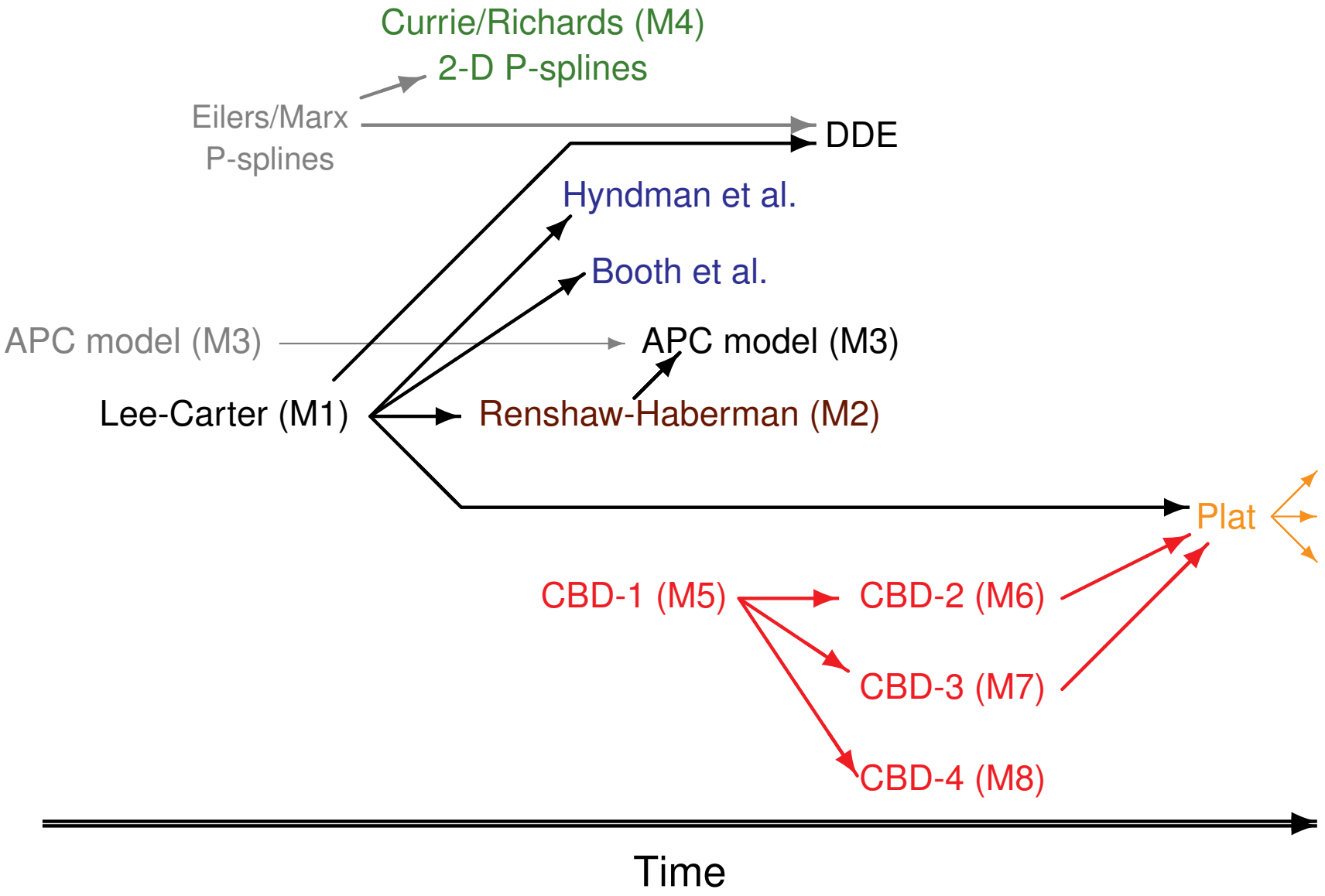
CBD-1 (M5)
2006

Time

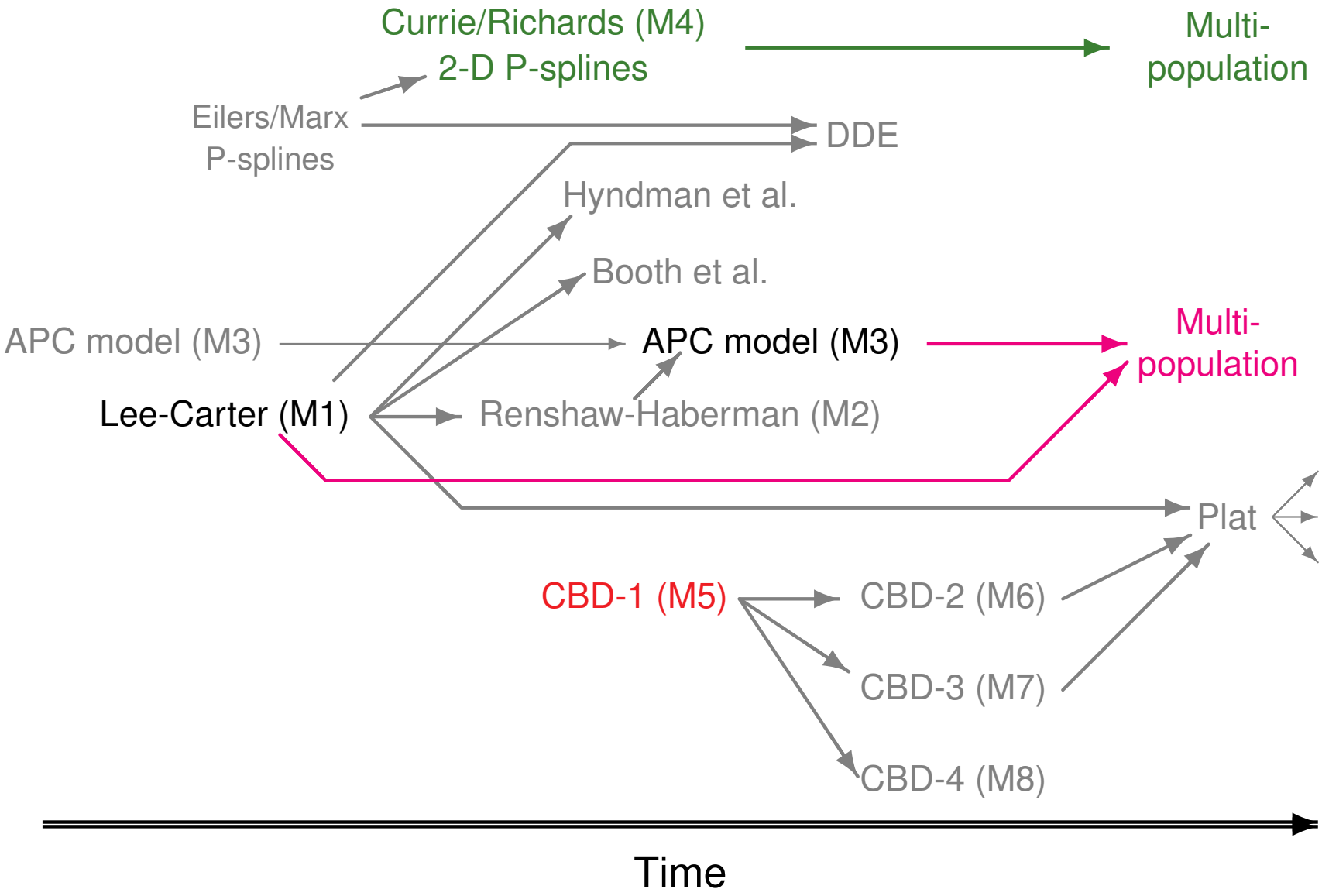
Improvements + more complexity



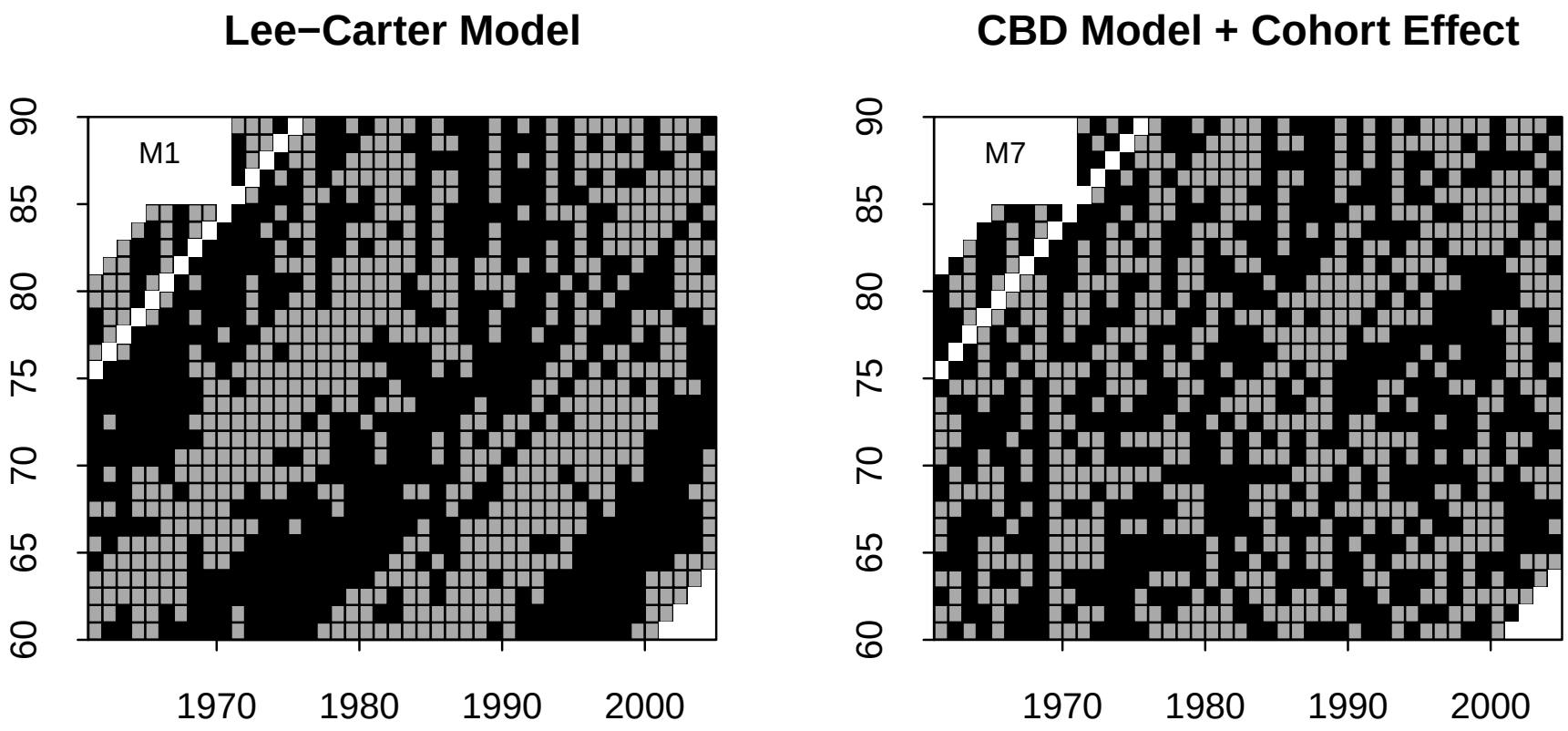
More improvements + even more complexity



Multiple population modelling



Why do we need complexity?



Black $\Rightarrow$ model *over*-estimates $m(x,t)$ death rate

Gray $\Rightarrow$ model *under*-estimates $m(x,t)$ death rate

LC: non-random clusters $+$ errors are too big

Issues on complexity

- Lee-Carter, CBD-1: simple and robust

  BUT underlying assumptions are violated:

  A: Deaths, $D(x,t)$ are cond. Poisson$\Big(m(x,t)E(x,t)\Big)$

  B: Death counts in neighbouring (x,t) cells are independent

- More complexity e.g. CBD-1 $\rightarrow$ CBD-3 $\rightarrow$ Plat ...
  - Underlying assumptions now okay
  - But excessive complexity $\Rightarrow$ less robust forecasts???

- Dowd et al. (2010a,b): out-of-sample backtesting

  Models that fit *much better* in sample

  are *not obviously better* at out-of-sample forecasting

Issues on complexity

- More complex $\Rightarrow$ More random processes
- More random processes $\Rightarrow$

  MUCH more difficult to model multiple populations

A Possible Way Forward

Single-population models

- Paradigm shift away from *independent* Poisson model
- Focus on small number of key drivers

  $\Rightarrow$ much easier to extend to multi-populations
- Focus on greater robustness of forecasts

Key Idea: CBD/Plat Revisited

Underlying $\log\ m(x,t) =$

- $\beta_1(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$: two key drivers

  PLUS

  $R(x,t)$ *Residuals*

- Assume: vector $R(t) \rightarrow R(t+1)$ mean reverting process

  $\Rightarrow$ long term risk depends on two key drivers

Possible models for $R(x,t)$

1. $R(x,t) = \phi R(x-1,t-1) + Z_R(x,t)$

2. $R(x,t) = \phi R(x-1,t-1) +$ diffusion $+ Z_R(x,t)$

3. Smooth underlying period effects, $\kappa_1(t), \kappa_2(t)$

   plus annual shocks

   e.g. $R(1), R(2), \ldots$ are i.i.d. vectors, correlated

   across ages

Multi-population and (???) robust modelling

$$\log m(x,t) = \text{simple age/period} + R(x,t)$$

- Focus effort on modelling $\kappa_1(t), \kappa_2(t)$

  Fewer core processes $\Rightarrow$ more robust (?)

  But *work in progress*

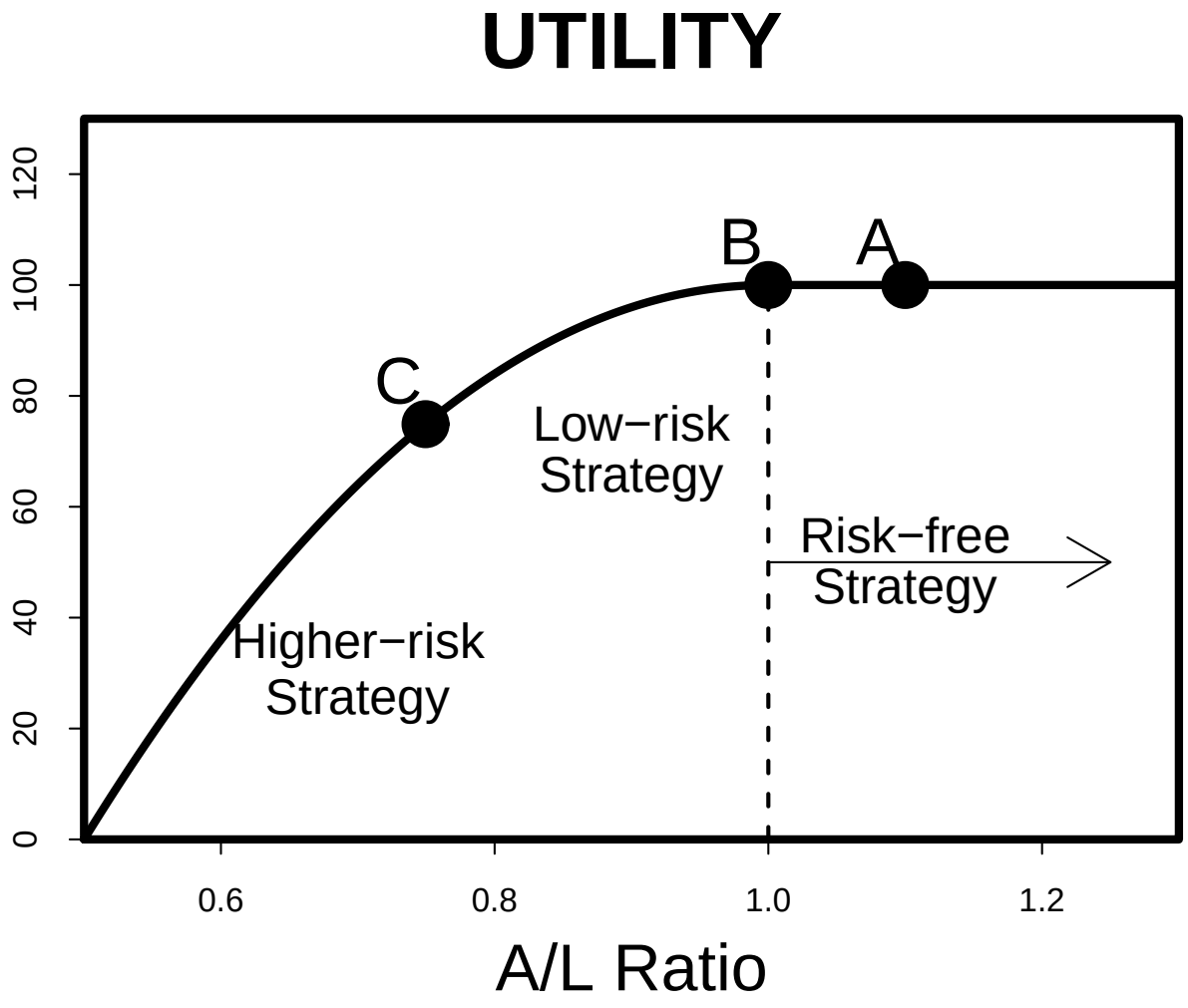
- Focus effort on multi-population model for $\kappa_1^{(i)}(t), \kappa_2^{(i)}(t)$ for $i = 1, 2, ...$

Risk Management Decisions

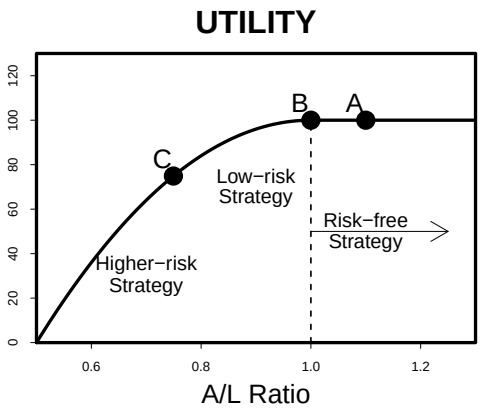
Are pension plans getting the right advice?

Why have there been so few index-linked longevity transactions?

Pension Plan Derisking – Simplified?



Utility consistent (?) with derisking strategies



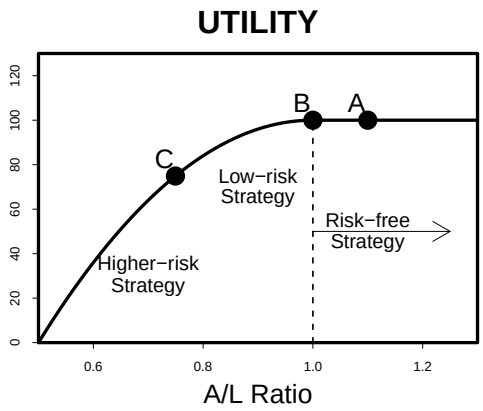
Plan closed to future accrual + salary inflation

A $\Rightarrow$ Customised Longevity Swap optimal + full risk reduction

C $\Rightarrow$??? Customised Longevity Swap + equities: BUT

- timing
- equities $\Rightarrow$ reaching bliss, B, not certain

$\Rightarrow$ Is a Customised Longevity Swap really optimal?



Plan exposed to future accrual + salary inflation

A $\Rightarrow$ Customised longevity swap for pensioners + equities

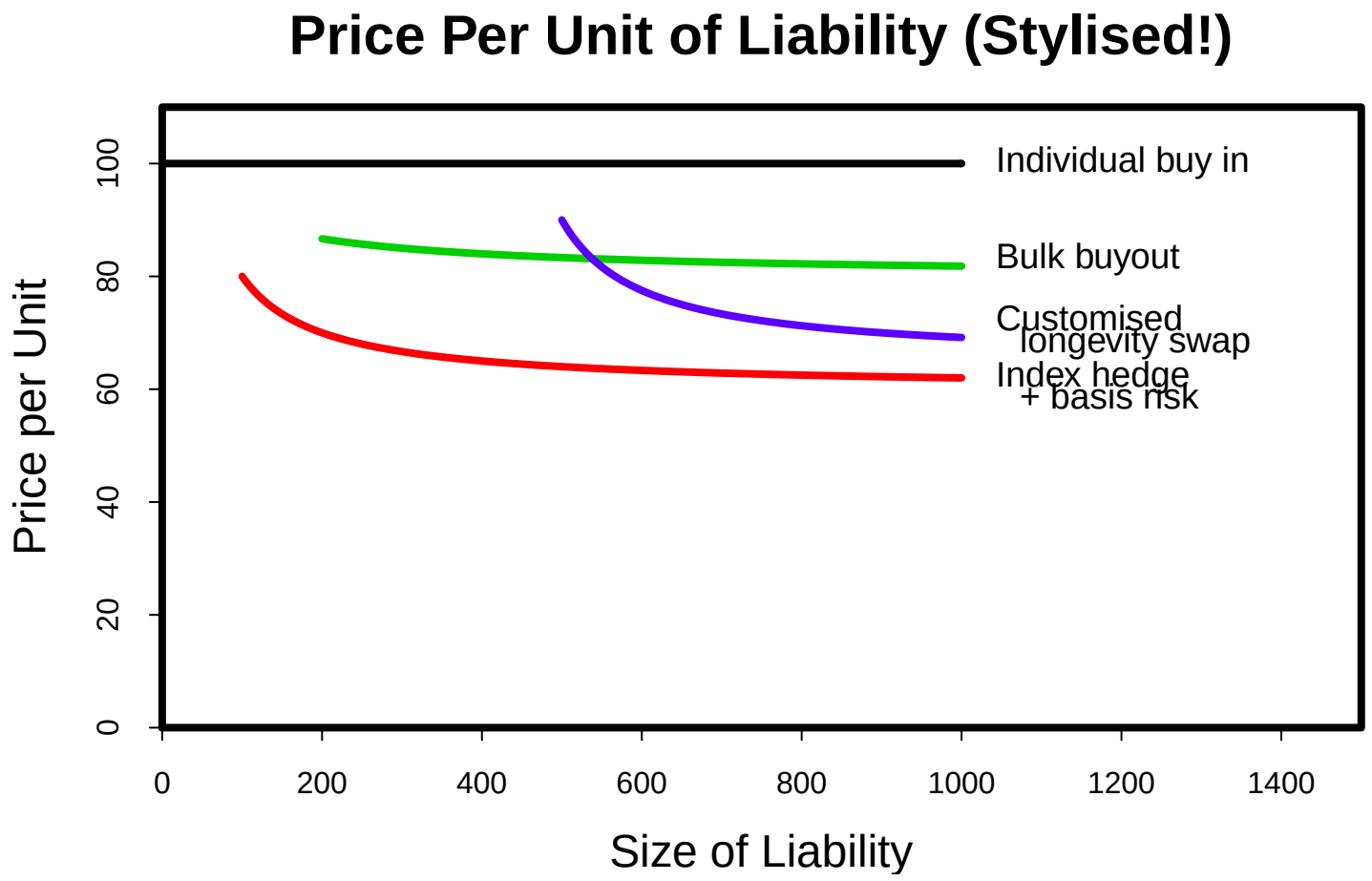
+ further phased derisking

- BUT non-hedgeable liabilities + equities

  $\Rightarrow$ Falling below B is possible

  $\Rightarrow$ Is Customised Longevity Swap really optimal?

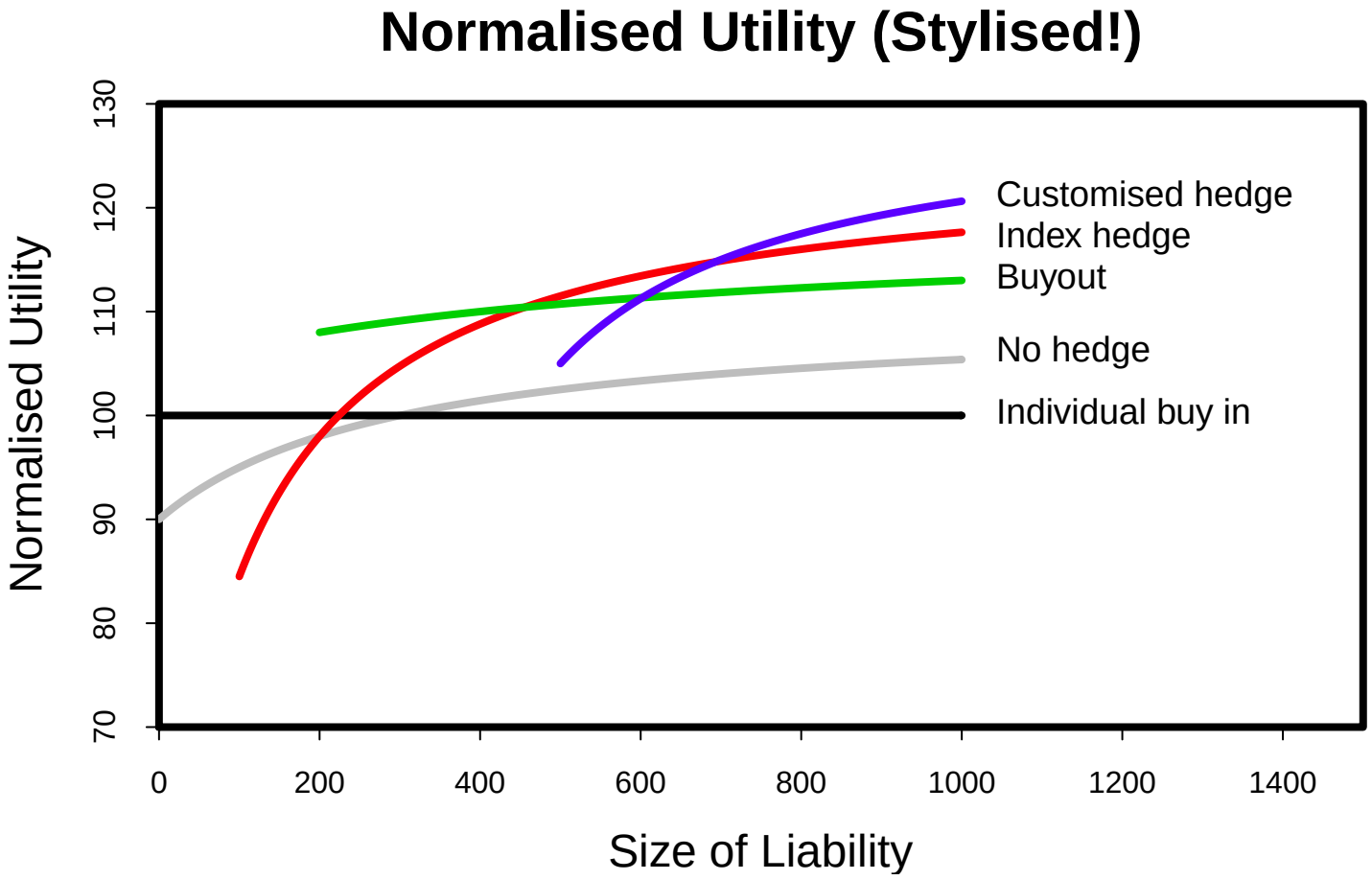
Longevity risk management options



Issues: size thresholds; fixed costs; basis risk; Poisson risk

WARNING: this figure has no scientific basis!!!!

Choosing between the options



Issues: Varying unit price; Poisson risk; basis risk; risk aversion

WARNING: this figure has no scientific basis!!!!

Discussion

- Index-linked hedges have great potential
- Index-linked hedges have greater potential for robustness problems
- But these can be overcome:
  - More robust multi-population models
  - Careful choice of hedging instrument and maturity
  - Robust hedging strategies (e.g. Nuga hedging)

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