HEDGING LONGEVITY RISK: A FORENSIC, MODEL-BASED ANALYSIS AND DECOMPOSITION OF BASIS RISK

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Longevity 6, Sydney, 9-10 September 2010

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Aim

- Two populations
  - PENSION PLAN's own population $(k = 2)$
  - INDEX population $(k = 1)$
- $L(T) =$ PENSION PLAN liability value at time T
- Aim: to reduce the risk associated with $L(T)$ using hedge instruments linked to INDEX population

  $+$ understand the contributors to risk reduction

Plan

- Aim
- Process for estimating hedge effectiveness
  - Simulation model
  - Valuation model
- Case Study: model $+$ data
- Forensic analysis of basis risk and correlation

Longevity risk hedging

- Cashflow hedging *versus* Value hedging
- INDEX-based hedge ($k = 1$) *versus* CUSTOMISED hedge ($k = 2$)

Key quantities

- $T =$ future liability valuation date
- $a_k(T, x) =$
  - value at T
  - life annuity of 1 per annum
  - for an individual aged x at T, in population k
- $a_k(T, x)$ depends upon:
  - experience up to $T \Rightarrow$ time T base mortality table
  - mortality projection model at $T \Rightarrow$ time T 2-D mortality table
- $a_k(T, x) = \sum_{s=1}^{\infty} (1 + r)^{-s} \ {}_s p_x(t)$

Simple example

- $T = 10$
- Liability value $L(T) = a_2(T, 65)$
- Hedging instrument: deferred annuity swap

$$H(T) = a_k(T, x) - \hat{a}_k^{\text{fxd}}(0, T, x)$$

$$\hat{a}_k^{\text{fxd}}(0, T, x) = \text{value at } T \text{ of swap fixed leg}$$

- $k = 2 \Rightarrow$ CUSTOMISED hedge
- $k = 1 \Rightarrow$ INDEX hedge

Steps in constructing and evaluating a hedge (*)

1. Objectives

2. Hedging instrument

3. Method for hedge effectiveness assessment

4. Calculate hedge effectiveness

5. Forensic analysis and interpretation of results

(*) Coughlan et al. (2010) *Longevity hedging: A framework for longevity basis risk analysis and hedge effectiveness* To appear in NAAJ

Steps in constructing and evaluating a hedge

Step 1: Objectives	
Risk to be hedged	Liability value, $L(T)$
Horizon	$T = 10$
Amount of risk to be hedged	Partial risk reduction

Steps in constructing and evaluating a hedge

Step 2: Hedging instrument	
Choice of instrument	Deferred annuity swap, value at T: $$H(T) = a_k(T,x) - \hat{a}_k^{\text{fxd}}(0,T,x)$$ (no collateral or margin calls)
Structure hedge	Static: $L_H(T) = L(T) + h \times H(T)$
Calibrate hedge ratio	$h^* = -\rho_{LH} \times SD(L(T))/SD(H(T))$

h^* minimises $Var(L_H(T))$.

Steps in constructing and evaluating a hedge

Step 3: Method for assessment of hedge effectiveness	
Risk metric	$Var\left(L_H(T)\right)$
Basis for comparison	$1 - Var\left(L_H(T)\right)/Var\left(L(T)\right)$
Retrospective vs. Prosp.	Prospective
Simulation model	two-population Age-Period-Cohort
Valuation model	$2\times$ one-population APC models with consistent projections

Steps in constructing and evaluating a hedge

Step 4: Hedge effectiveness calculation

Simulate future mortality rates up to T

Evaluate assets and liabilities at T

Evaluate hedge effectiveness

Step 5: Forensic analysis and interpretation of results

Simulation

1	Past mortality rates for INDEX population (up to time "$t = 0$")	Past mortality rates for PENSION PLAN (up to time "$t = 0$")
2	Fit two-population model	
3	Simulation of two-population underlying mortality rates for $t = 1, \ldots, T$	
4	INDEX population: Add Poisson risk to death counts	PENSION PLAN: Add Poisson risk to death counts
5	Future scenarios for INDEX mortality experience $t = 1, \ldots, T$	Future scenarios for PENSION PLAN mortality experience $t = 1, \ldots, T$

Evaluation

Simulation

1A	Past mortality rates for INDEX	Past mortality rates for PENSION PLAN
1B	$+$ Future mortality scenarios for INDEX	$+$ Future mortality scenarios for PENSION PLAN

Valuation model

2	Scenario $+$ Model $\Rightarrow$ calibration for hedging instrument valuation	Scenario $+$ Model $\Rightarrow$ calibration for portfolio liability valuation
3	Consistent valuation model mortality projections	
4	For each scenario: INDEX hedge instrument valuation	For each scenario: PENSION PLAN liability valuation
5	Calculate hedge effectiveness	

Hedge Effectiveness: basic idea

- $L =$ liability value
- $H =$ value of hedging instrument
- $\rho = cor(L, H)$
- $h =$ units of H
- Hedged portfolio value$= P(h) = L + h \times H$
- $h^* = -\rho \times SD(L)/SD(H)$
- Optimal Hedge Effectiveness
  $R^2(h^*) = 1 - Var(P(h^*))/Var(L) = \rho^2$

Hedge Effectiveness

- Hedge Effectiveness

  $$R^2(h) = 1 - Var(P(h))/Var(L) \leq \rho^2$$

- Hedge Effectiveness depends on
  - Correlation, $\rho = cor(L, H)$
  - Choice of h versus h^*

Coming up

- Forensic analysis of

$$cor(L, H) = cor\Big(a_2(T, 65),\ a_k(T, x)\Big)$$

- Hedge effectiveness example

Case Study

- Population 1: England and Wales males
- Population 2: UK CMI assured lives, males
- 1961–2005; ages 50-89
- Here: 2-population model (Cairns et al., 2010)
- Model here: just one example

  (simple model: but both period and cohort effects)

Age-Period-Cohort model (APC) (M3-2 pops)

$m_k(t,x) =$ population k death rate

$$\log m_k(t,x) = \beta^{(k)}(x) + \kappa^{(k)}(t) + \gamma^{(k)}(t-x)$$

$\beta^{(1)}(x),\ \ \beta^{(2)}(x)$ population 1 and 2 age effects

$\kappa^{(1)}(t),\ \ \kappa^{(2)}(t)$ period effects

$\gamma^{(1)}(c),\ \ \gamma^{(2)}(c)$ cohort effects

A 2-population model (one large, one small)

- Large population 1
  - $\kappa^{(1)}(t)$: random walk with drift, μ_1
  - $\gamma^{(1)}(c)$: AR(2) around linear drift ($\longrightarrow$ ARIMA(1,1,0))
- Spreads:
  - $S_2(t) = \kappa^{(1)}(t) - \kappa^{(2)}(t)$: AR(1)
  - $S_3(c) = \gamma^{(1)}(c) - \gamma^{(2)}(c)$: AR(2)

Why mean reversion in spreads

- Hypothesis (e.g. Li and Lee, 2005):

  For each age x, $\dfrac{m_1(t,x)}{m_2(t,x)}$ does not diverge over time

Bayesian statistical approach

Prior judgement

$\times$ model likelihood of data (Poisson $+$ ARIMA)

$=$ posterior distribution for parameters

Bayesian output

Bayesian *posterior* distribution for

- Process parameters (e.g. $\kappa^{(1)}(t)$ random-walk drift, μ_1)
- Underlying latent state variables
  - age, period and cohort effects
  - especially important for small populations
- Full parameter uncertainty

Implementation

- Simulation – Stage 1
  - EW, CMI males data for 1961-2005, ages 50-89
  - Fit the 2-population model using MCMC
- Simulation – Stage 2
  - Full PU simulation of 2-pop model

    $\Rightarrow$ underlying $m_1(t,x), m_2(t,x)$ for

    $t = 2006, \ldots, 2015$

- Simulation – Stage 3 – future Poisson deaths

  – Specify exposures, $E_1(t,x)$, $E_2(t,x)$ for $t = 2006, \ldots$

Case 1:	$E_1(t,x) = E_1(2005,x)$,	$E_2(t,x) = E_2(2005,x)$
Case 2:	$E_1(t,x) = 100 \times E_1(2005,x)$,	$E_2(t,x) = 100 \times E_2(2005,x)$

  – Simulate independent Poisson death counts $D_k(t,x) \sim \text{Poisson}\Big(m_k(t,x)E_k(t,x)\Big)$ for $t = 2006, \ldots, 2015$

- Valuation Model: Stage 1 – Calibration

  Choose calibration window

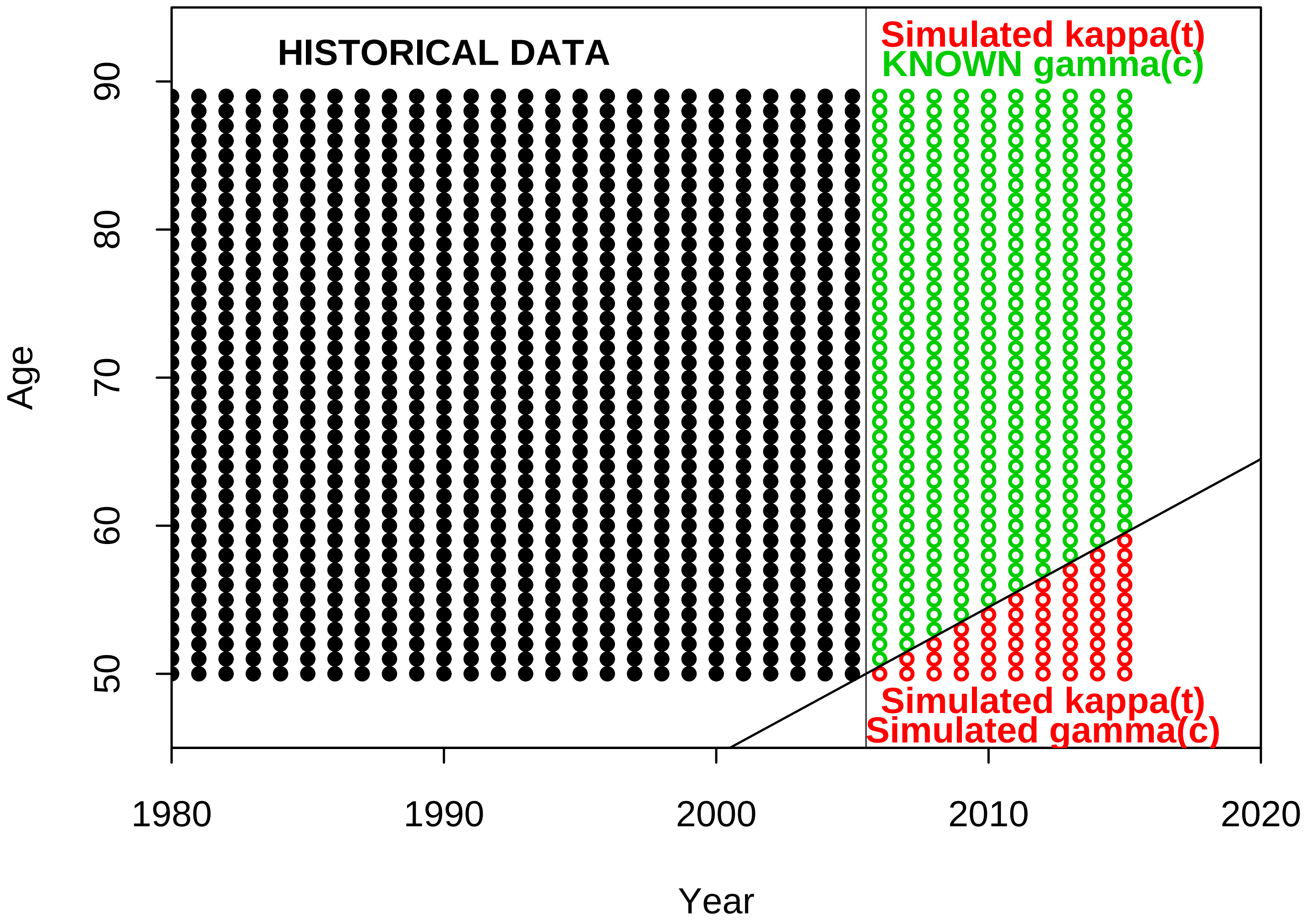
  Each stochastic scenario:

  – Full re-calibration of single-pop APC model to 2015 EW data

  – Full re-calibration of single-pop APC model to 2015 CMI data

  – Calibrate $\kappa^{(1)}(t)$ trend: μ_1

Treatment of the cohort effect



- Stage 2 – Valuation

For each stochastic scenario at $T = 2015$

– Calculate $a_1(T, x)$

– Calculate $a_2(T, x)$

"Ideal": calculate $a_k(T, x)$ using expectations under full 2-pop stochastic model

BUT: impractical (and unrealistic in practice??)

- Stage 2 – Valuation: how to calculate $a_1(T,x)$

$\beta^{(1)}(y)$, $\gamma^{(1)}(T-x-1)$ are known

$$+$$

$\kappa^{(1)}(t)$ projected beyond $T = 2015$

$$\Downarrow$$

$$m_1(T+1,x),\ m_1(T+2,x+1),\ m_1(T+3,x+2),\ldots$$

$$+$$

Discount Factors

$$\Downarrow$$

$$a_1(T,x)$$

- Stage 2 – Valuation: $a_1(T, x)$

  Key assumption

  Deterministic approximation to stochastic $\kappa^{(1)}(t)$:

$$\hat{\kappa}^{(1)}(T+s) = \kappa^{(1)}(T) + s \times \mu_1$$

  Similarly: Calculate $a_2(T, x)$

  $\kappa^{(2)}(t)$ needs projection beyond $T = 2015$

$$\hat{\kappa}^{(2)}(T+s) = \kappa^{(2)}(T) + s \times \mu_2$$

- Stage 2 – Valuation
  - μ_1 based on 2015 full recalibration of $\kappa^{(1)}(t)$

    Data from T_0 to $T = 10$ (2015)

    Random walk model

    $$\Rightarrow \mu_1 = \left(\kappa^{(1)}(T) - \kappa^{(1)}(T_0)\right) / (T - T_0)$$

  - Important assumption

    $$\mu_2 = \mu_1$$

- Stage 2 – Annuity price summary

  – Deterministic projection approx: Nielsen (2010) (Solvency II)

  – Other approximations ...

  – r = risk-free interest rate (fixed)

  – $a_1(T,x) = f\left(r, \beta^{(1)}(x), \kappa^{(1)}(T), \gamma^{(1)}(T-x-1), \mu_1\right)$

  – $a_2(T,x) = f\left(r, \beta^{(2)}(x), \kappa^{(2)}(T), \gamma^{(2)}(T-x-1), \mu_1\right)$

Variants

- Full parameter uncertainty (PU)
- Full parameter certainty (PC):
  - PC age, period and cohort effects (up to 2005)
  - μ_1 fixed in 2005
- Partial PC:
  - PC age, period and cohort effects (up to 2005)
  - μ_1 recalibrated in 2015 using latest $\kappa^{(1)}(t)$
- With and without Poisson Risk

Role of parameter uncertainty

- $L = L_{Base} + L_{PU}$
- $H = H_{Base} + H_{PU}$
- Base case: process risk only $\Rightarrow$ correlation ρ_{Base}
- Additional parameter uncertainty $\Rightarrow$ $\rho_{Base} \longrightarrow \rho_{PU}$
- Correlation can go up or down

Value hedging

- Cash value of a hedging instrument at time T

  versus

- Cash value of liability: $a_2(T, 65)$ (CMI)

- e.g.

  - $a_2(T, 65)$ versus $a_2(T, x)$ (CUSTOMISED hedge)
  - $a_2(T, 65)$ versus $a_1(T, x)$ (INDEX hedge)

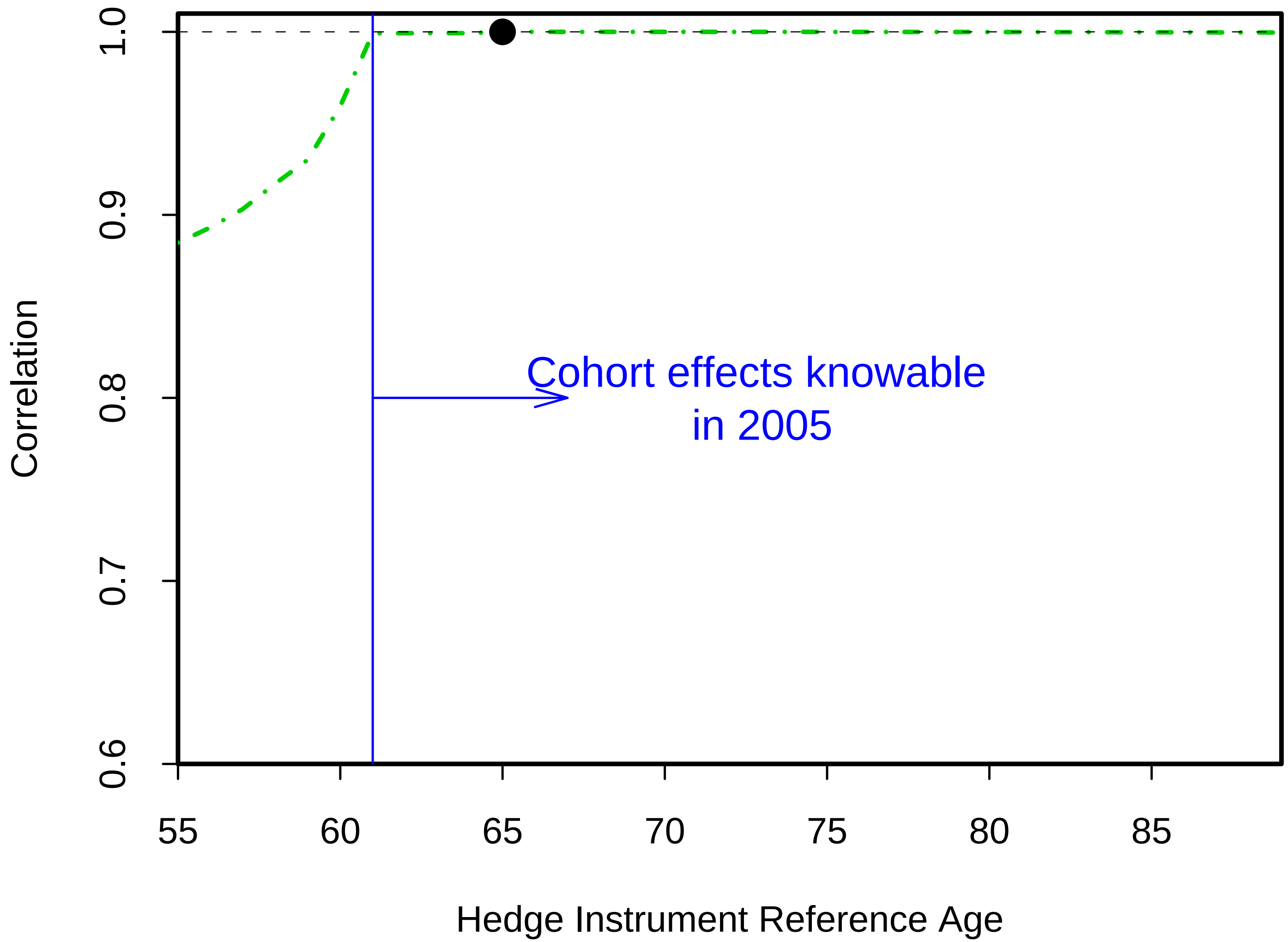
Value hedging

Recap: $a_k(T, x)$ depends on:

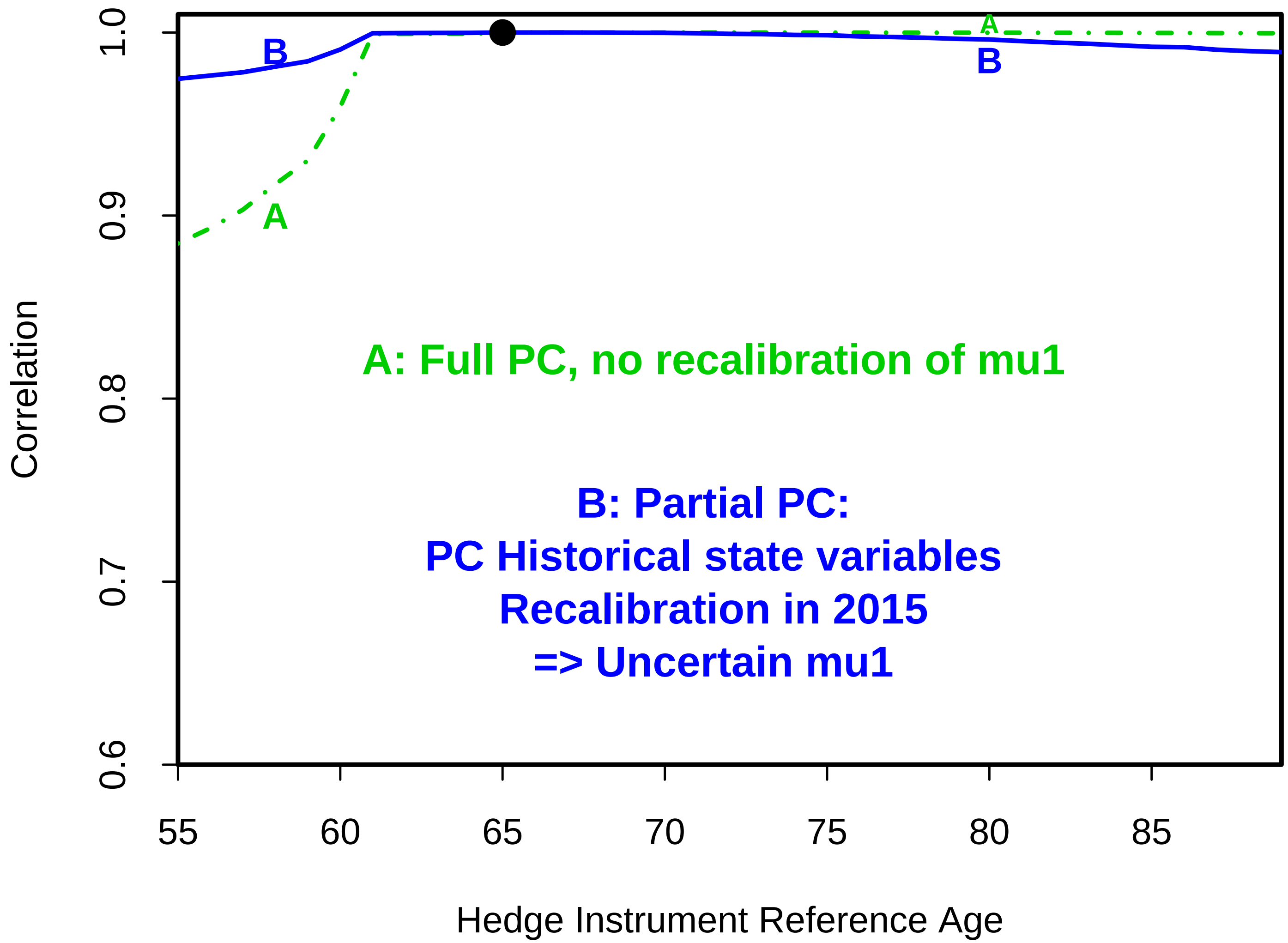
- State variables up to time T
  ($\kappa^{(k)}(t)$ and $\gamma^{(k)}(c)$)
- Estimate of $\kappa_t^{(1)}$ drift, μ_1, beyond T
  - PC case: μ_1 known at time 0
  - PU case: μ_1 not known until time T

CUSTOMISED hedge; full parameter certainty (PC)

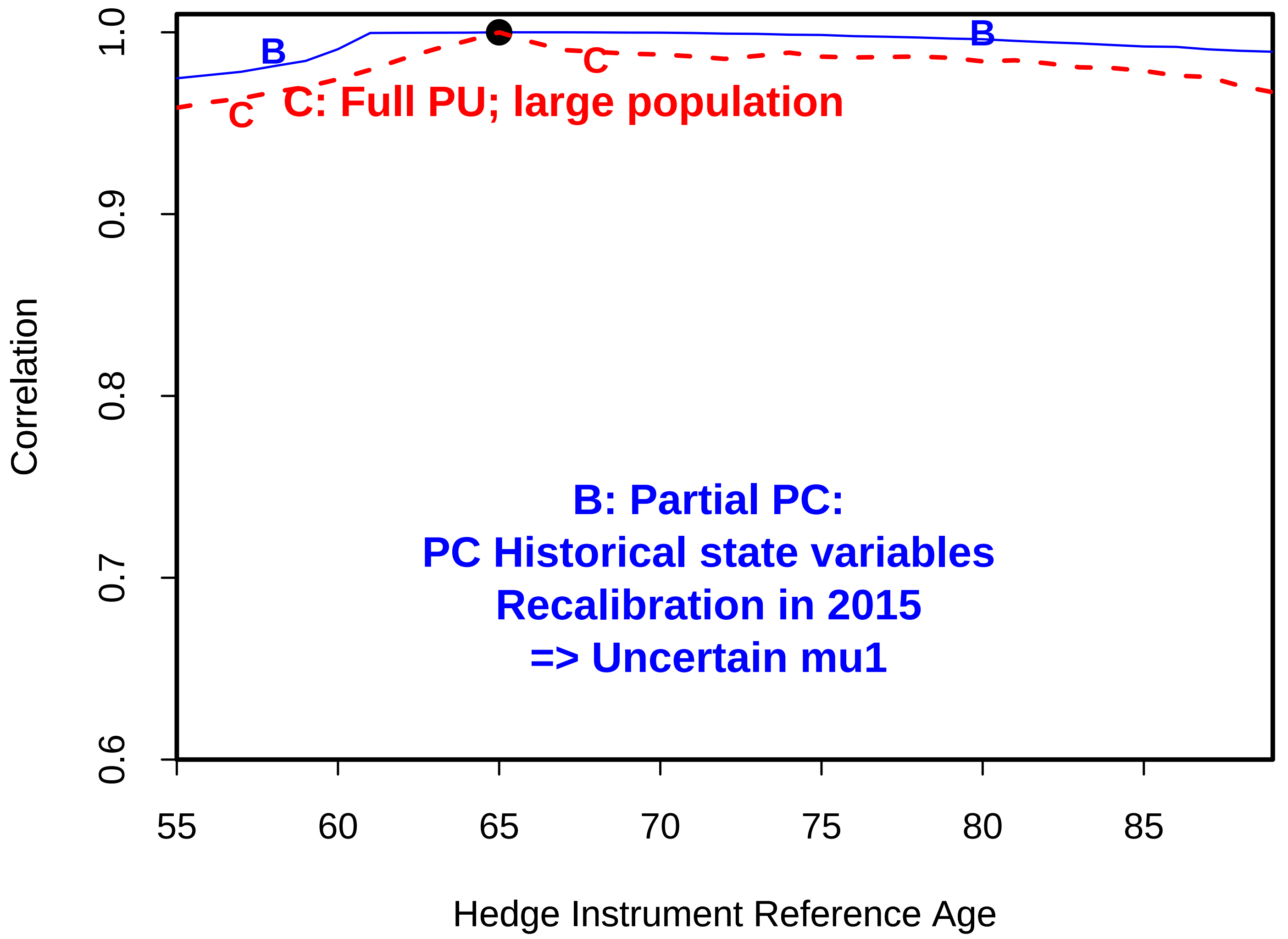
Hedging $a_2(T, 65)$ using $a_2(T, x)$: Correlation plot



$a_2(T,65)$ vs $a_2(T,x)$: Impact of Recalibration Risk

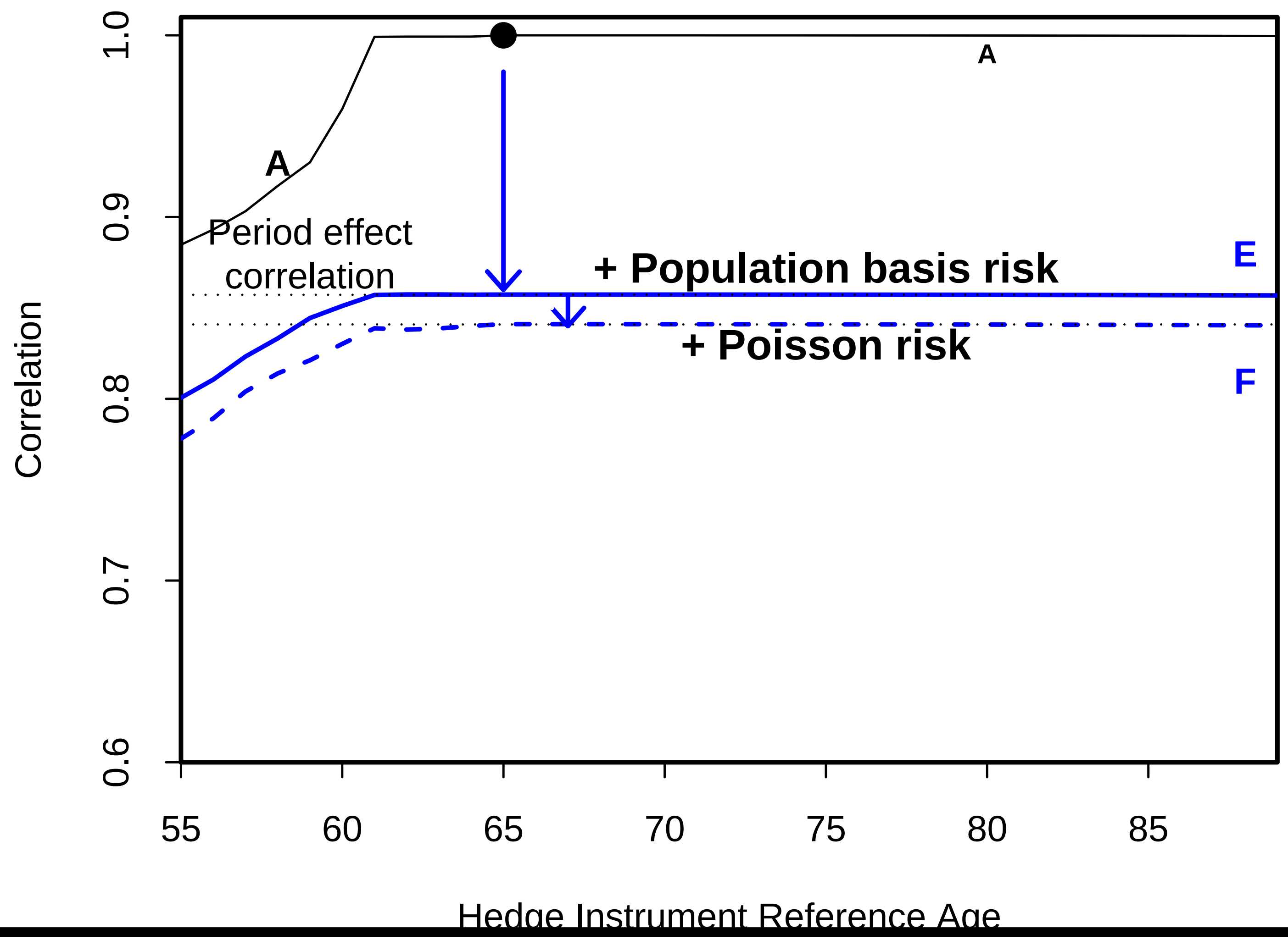


$a_2(T,65)$ vs $a_2(T,x)$: Impact of full PU

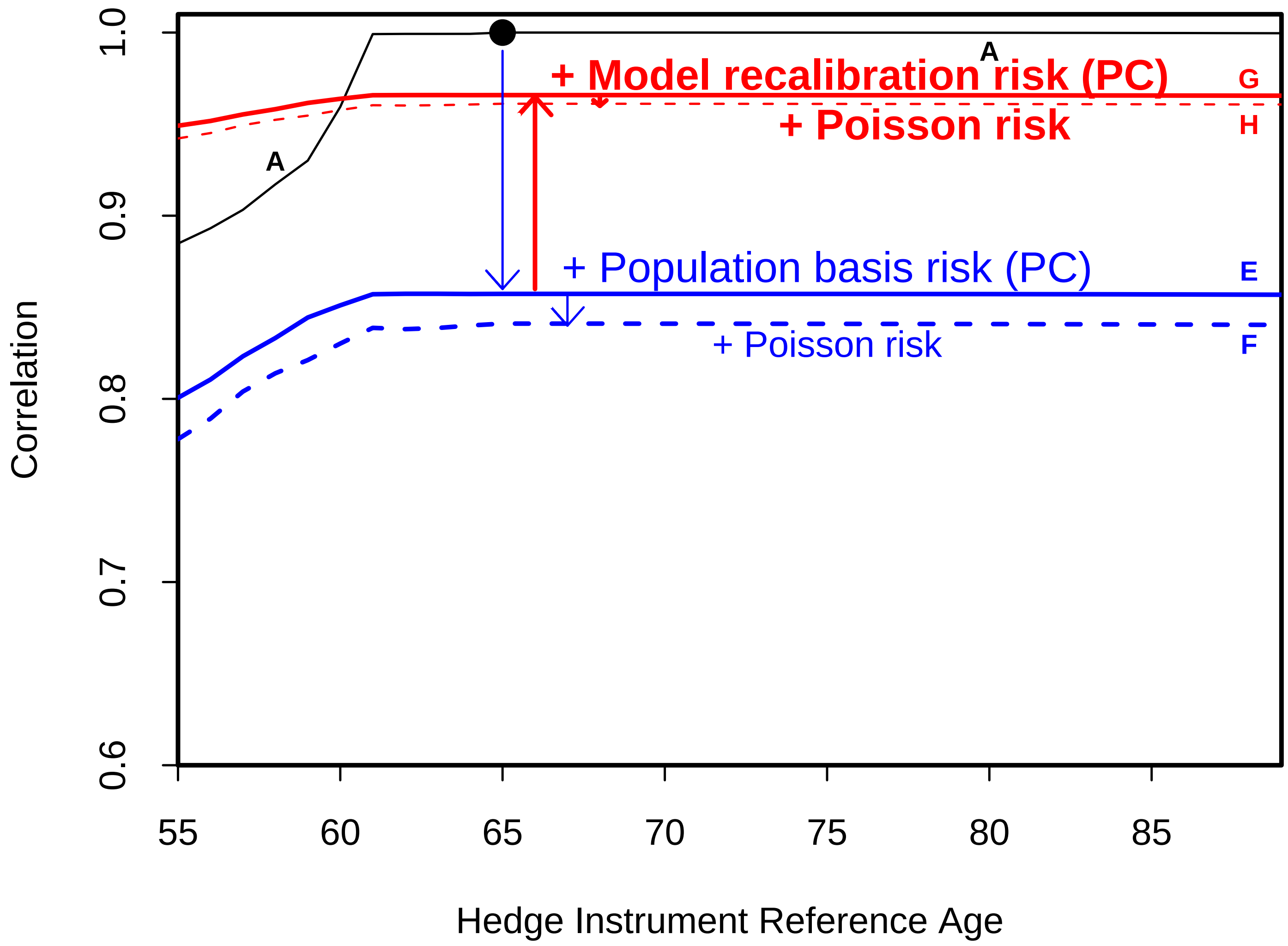


INDEX hedge; full parameter certainty (PC)

$a_2(T, 65)$ vs $a_1(T, x)$: PC $+$ Population basis risk



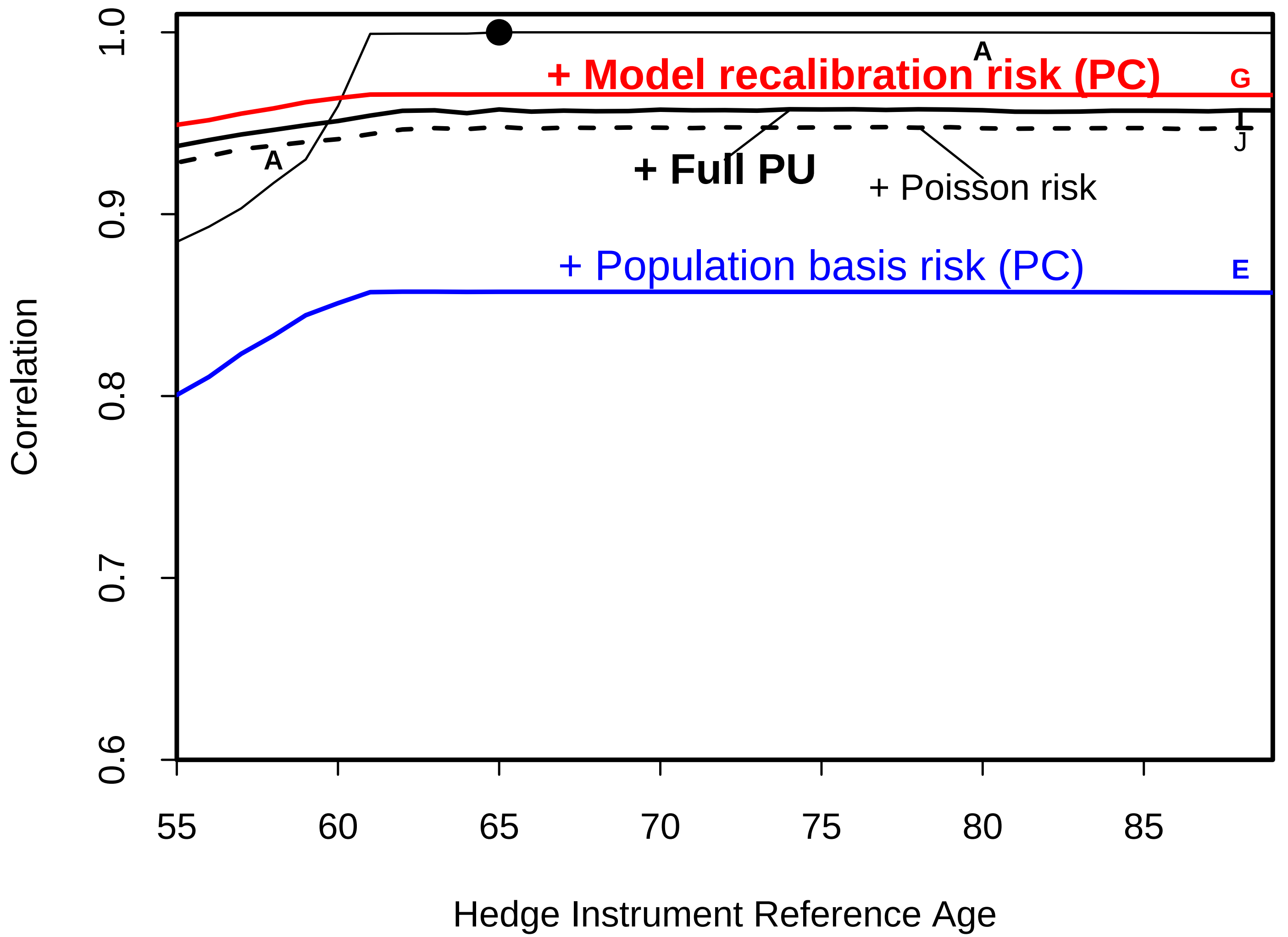
$a_2(T, 65)$ vs $a_1(T, x)$: Impact of recalibration risk



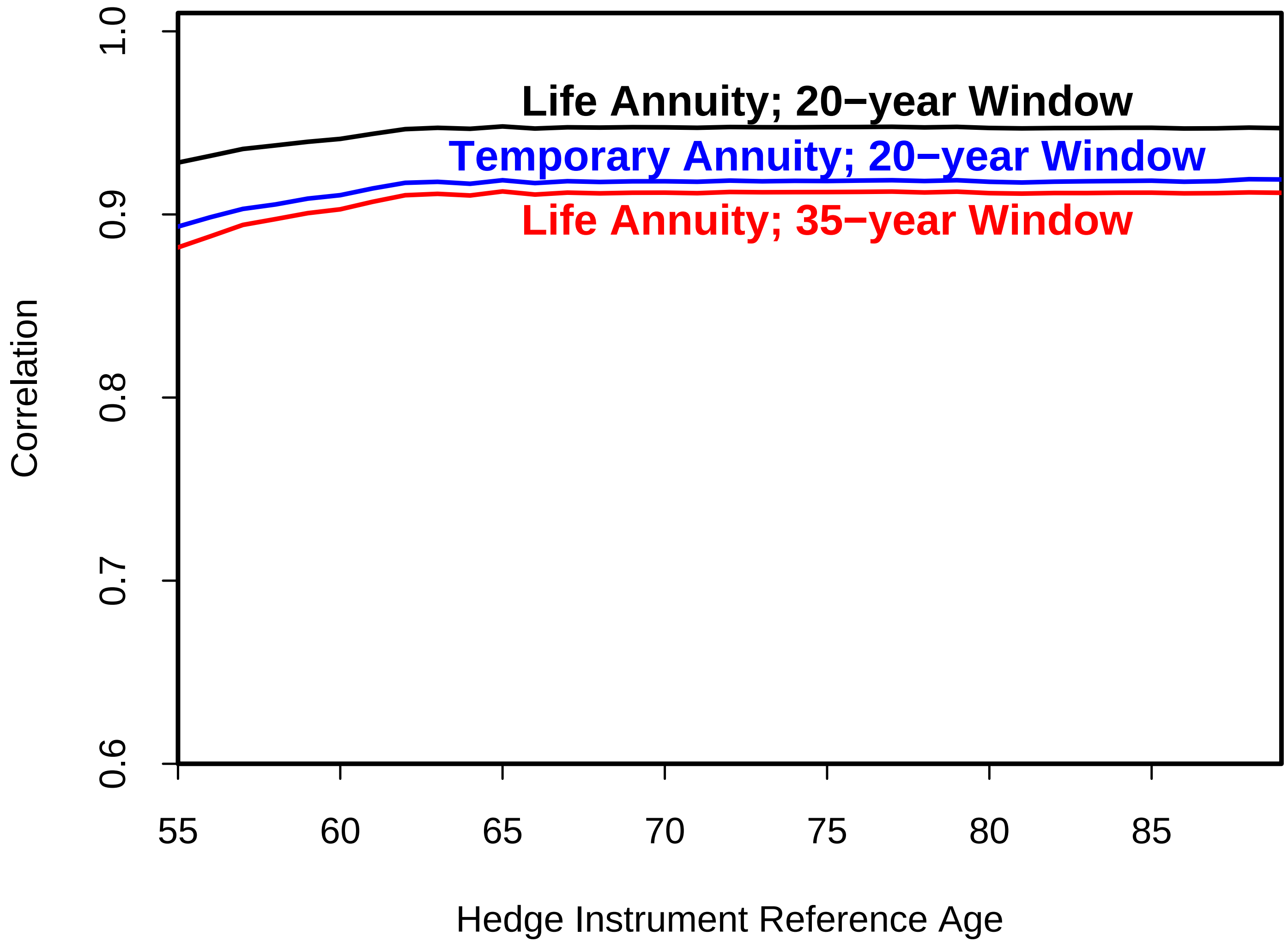
Recalibration risk: simplified example

- Risks: $X_1 = \mu + Z_1, \quad X_2 = \mu + Z_2$
- $Z_1,\ Z_2$ are uncorrelated
- μ known $\Rightarrow cor(X_1, X_2) = 0$
- μ unknown $\Rightarrow cor(X_1, X_2) > 0$

$a_2(T,65)$ vs $a_1(T,x)$: Impact of full PU + Poisson



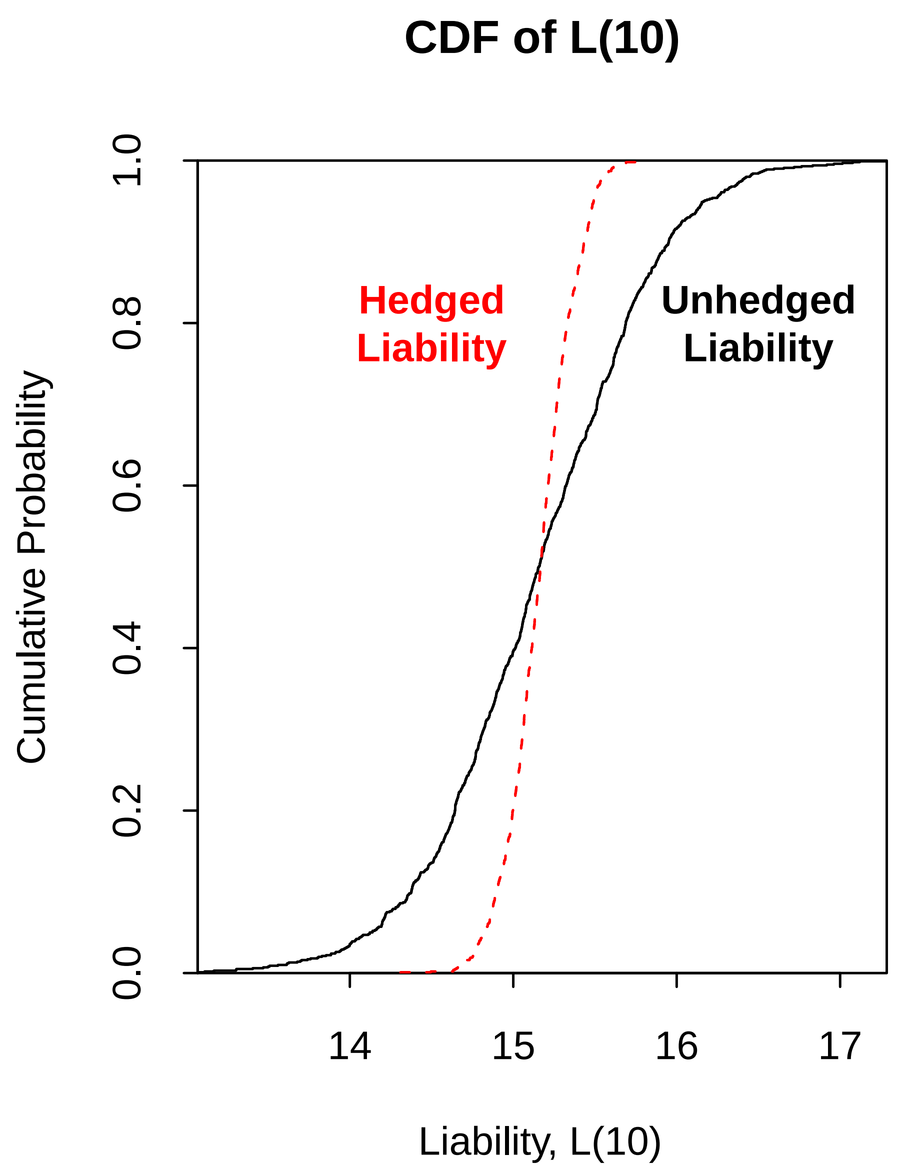
Impact of (a) Calibration window, (b) Term of annuity

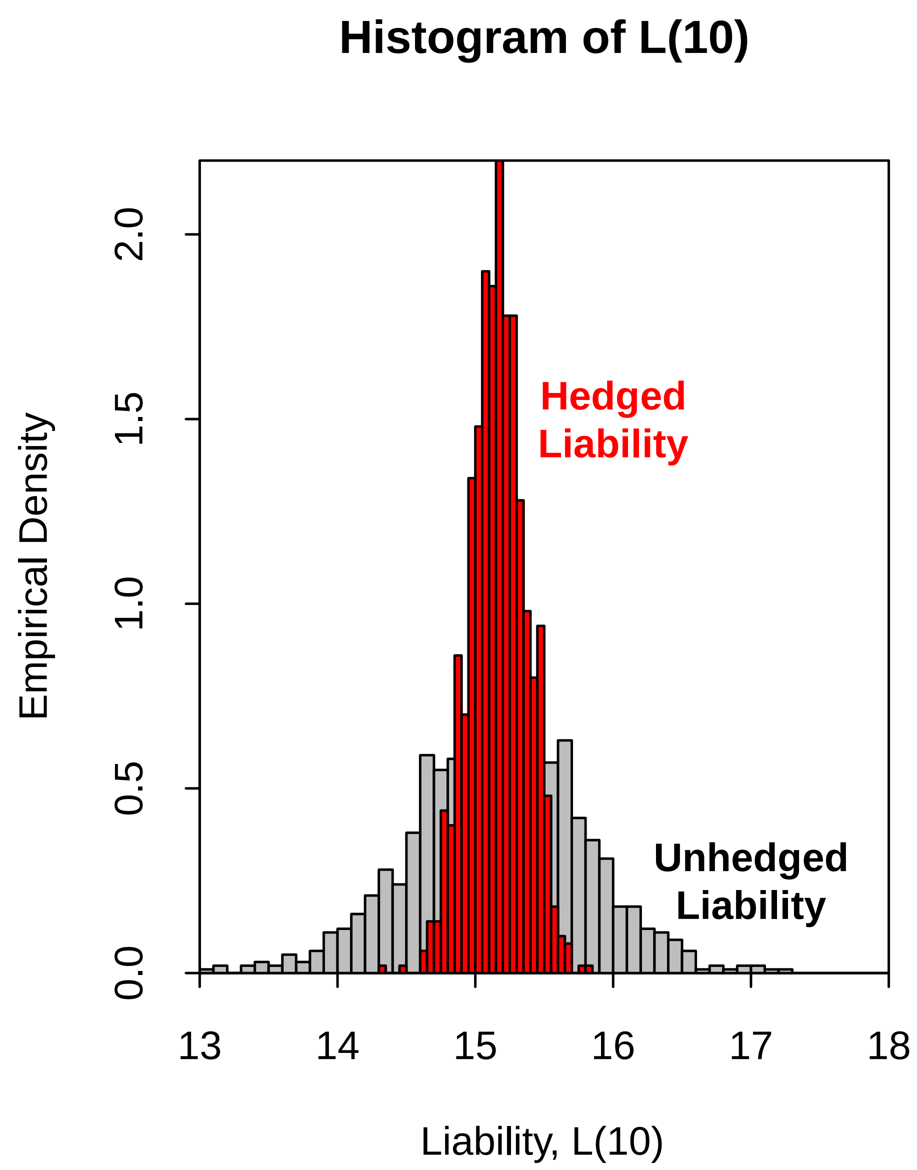


Hedge Effectiveness Example

- $L(10) = a_2(10, 65)$;
  $H(10) = a_1(10, 65) - \hat{a}_1^{\text{fxd}}(0, 10, 65)$
- Risk metric 1: variance of liability
- Risk metric 2: 95% Value-at-Risk in excess of median
- $h^* = -0.846$

Risk metric	Unhedged		Hedged	Hedge Effectiveness	
Variance:	0.4039	$\searrow$	0.0409	0.90	$= \rho^2$
95% VaR:	1.0072	$\searrow$	0.3235	0.68	$\approx 1 - \sqrt{1 - \rho^2}$


CDF of L(10)
Hedged Liability
Unhedged Liability
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
14
15
16
17
Liability, L(10)


Histogram of L(10)
Hedged Liability
Unhedged Liability
Empirical Density
0.0
0.5
1.0
1.5
2.0
13
14
15
16
17
18
Liability, L(10)

Value hedging: basis risk

- Population basis risk (total basis risk UP)
- Latent state variable estimation uncertainty (UP or DOWN)
- Recalibration risk (μ_1) (DOWN)
- Recalibration window (UP or DOWN)
- Duration of annuity (UP or DOWN)
- 2006-2015 Poisson deaths risk (UP)
- Sub-optimal choice of hedging instrument (UP)
- Sub-optimal # units of hedging instrument (UP)
- Additional hedging instruments (DOWN)

Further comments $+$ work

- Robustness of optimal hedge ratios
  - Impact of sub-optimal allocation
  - Sensitivity to PC/PU etc.
- Vega hedging;

  Use of more than one hedging instrument
- Use of more recent EW data
- Models with more complex correlation structure

Questions

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