New Directions
in the Modelling of Longevity Risk

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Plan

- Genealogy
- New directions in modelling
- Numerical illustrations

Development of New Models

- Many new stochastic mortality models since Lee-Carter
- Are they fit for purpose?
- Are they robust?

GENEALOGY: 1st GENERATION MODELS

Currie/Richards (M4)
2-D P-splines
2002, ...

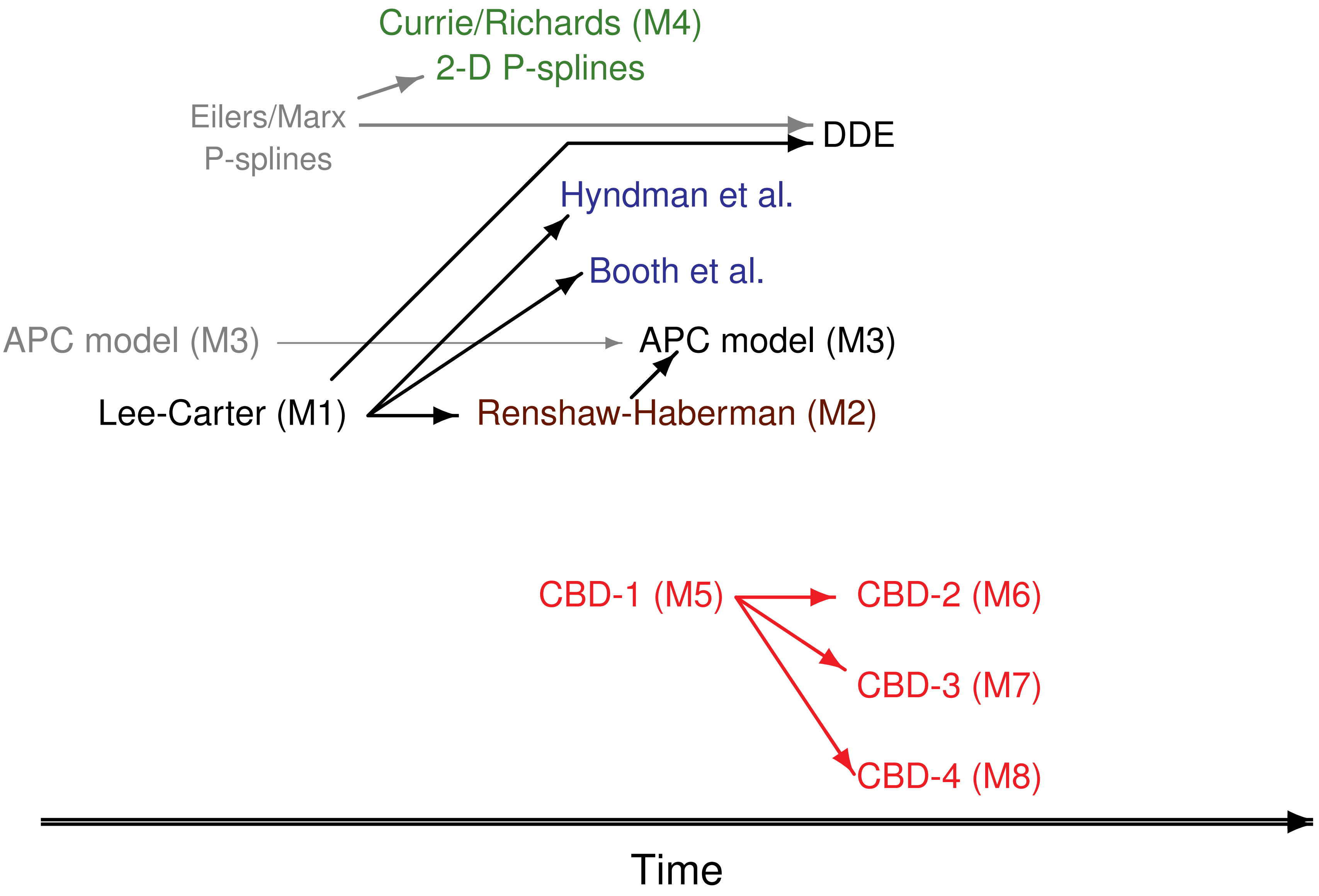
Eilers/Marx
P-splines

Lee-Carter (M1)
1992

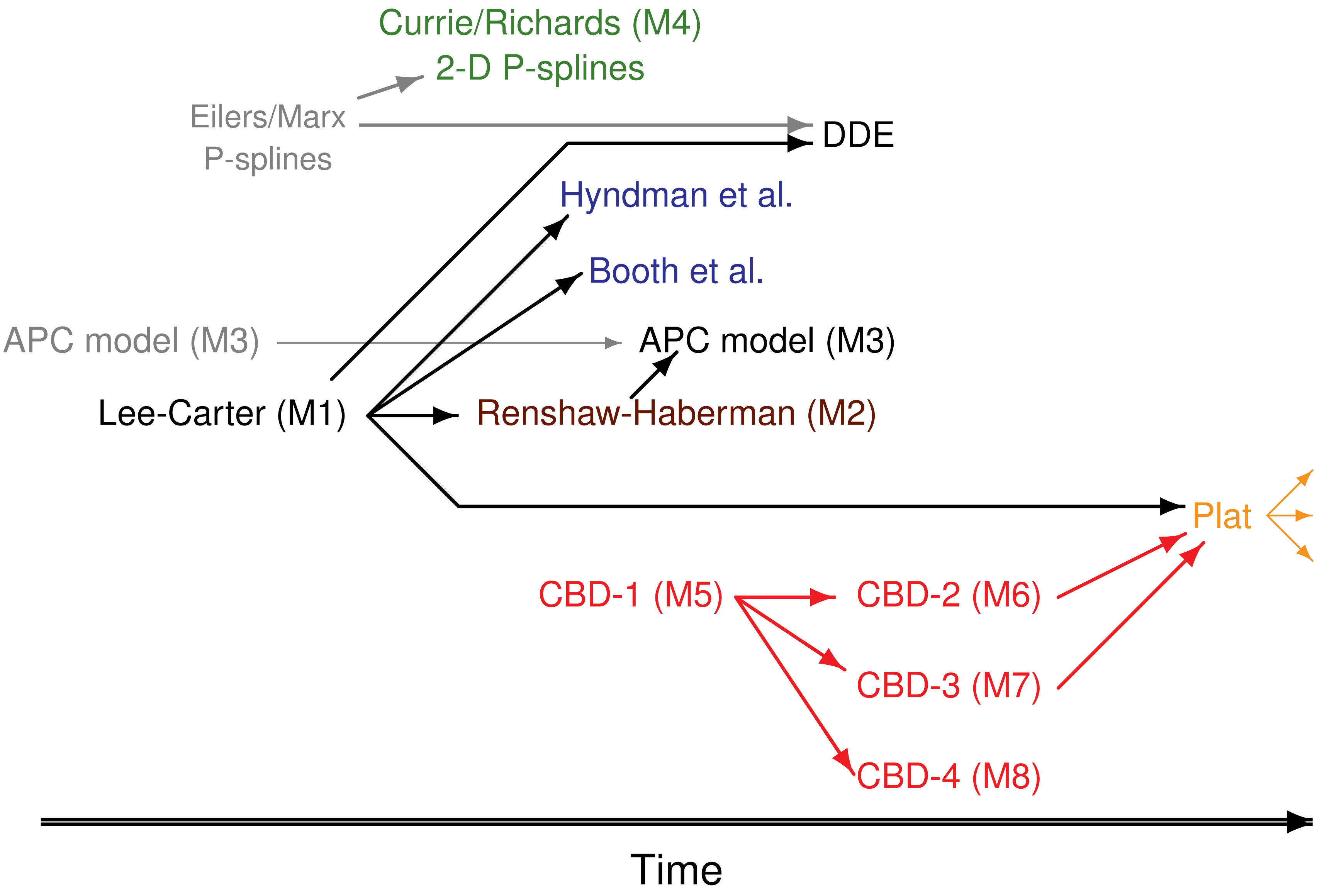
CBD-1 (M5)
2006

Time

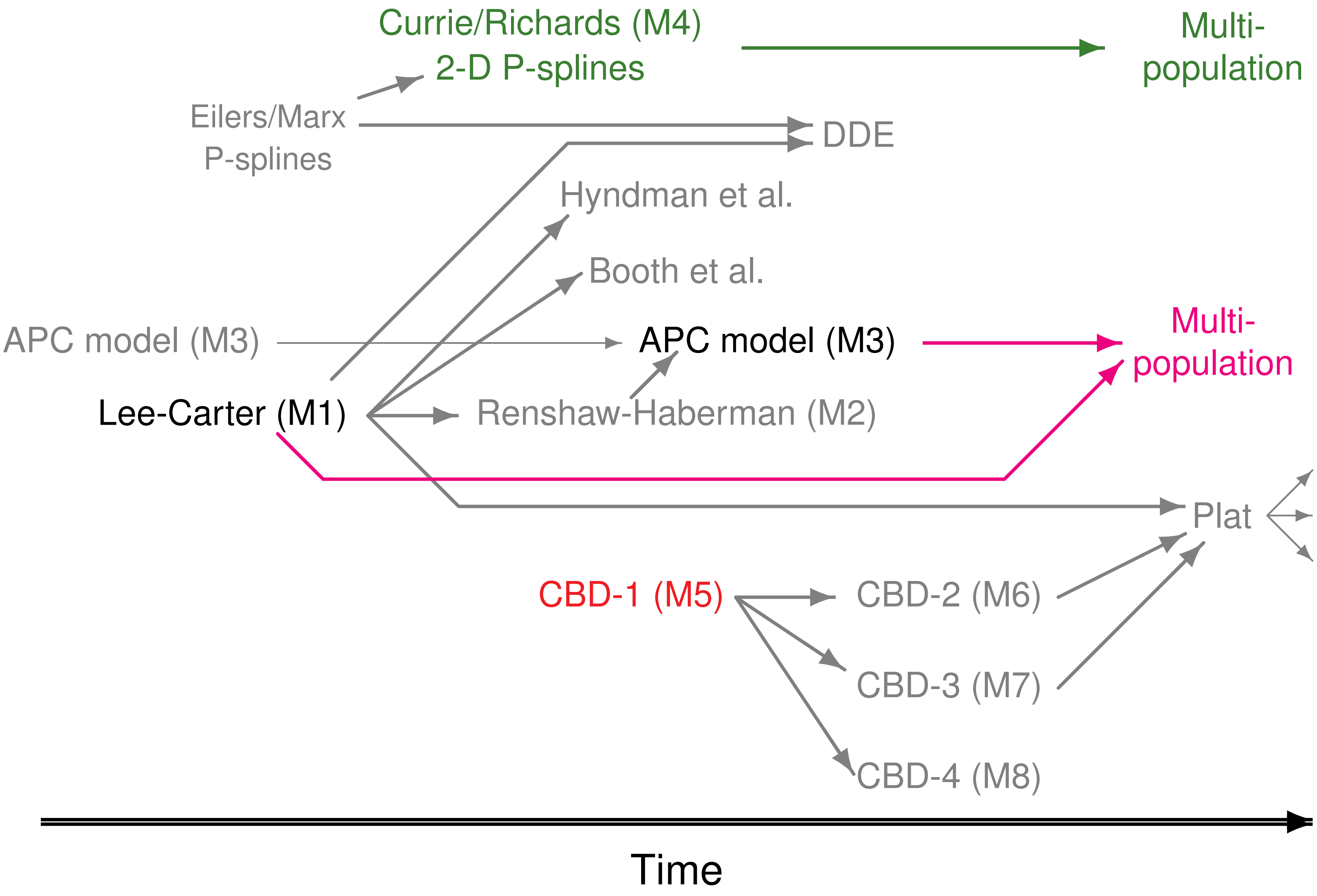
Improvements + more complexity



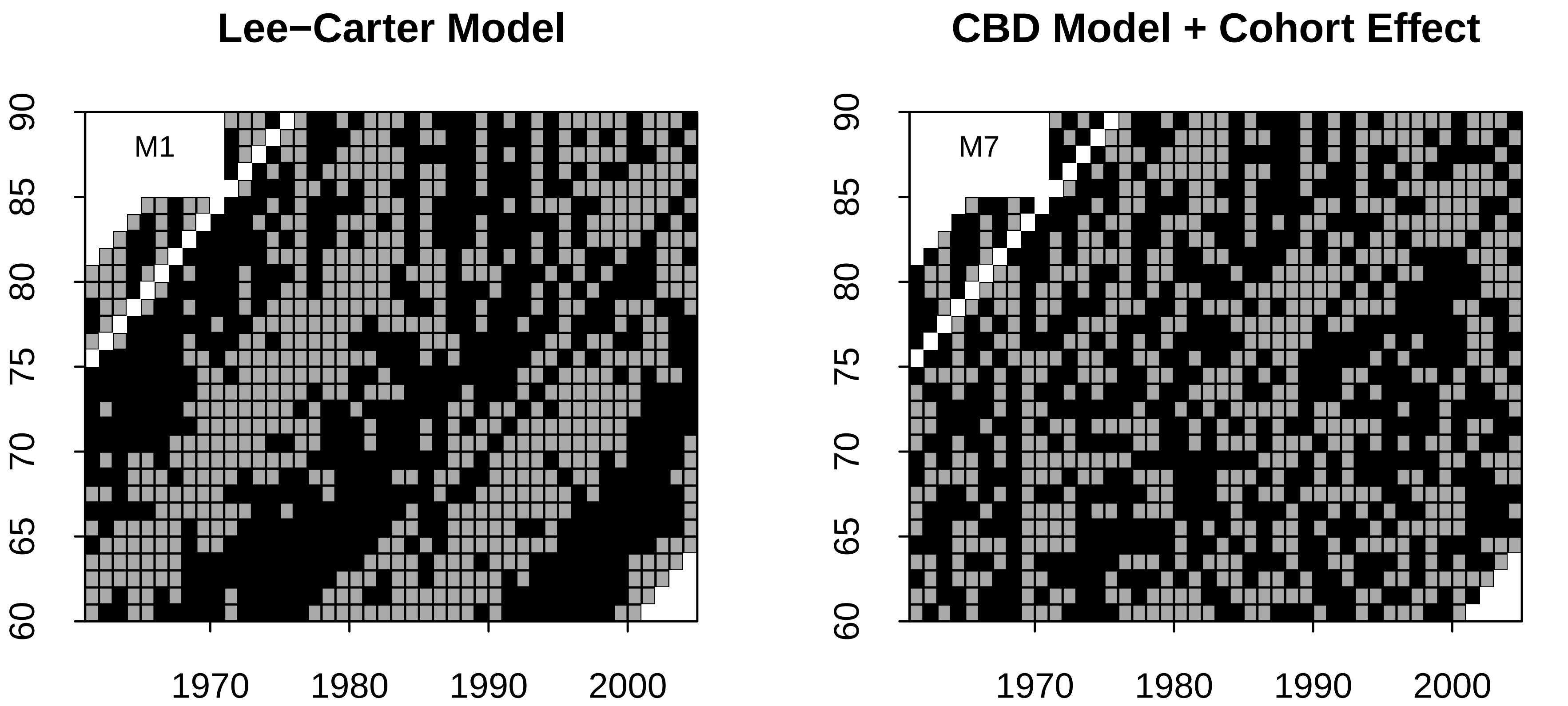
More improvements + even more complexity



Multiple population modelling



Why do we need complexity?



Black $\Rightarrow$ model *over*-estimates $m(x,t)$ death rate

Gray $\Rightarrow$ model *under*-estimates $m(x,t)$ death rate

LC: non-random clusters $+$ errors are too big

Issues on complexity

- Lee-Carter, CBD-1: simple and robust

  BUT underlying assumptions are violated:

  A: Deaths, $D(x,t)$ are cond. Poisson$\Big(m(x,t)E(x,t)\Big)$

  B: Death counts in neighbouring (x,t) cells are independent

- More complexity e.g. CBD-1 $\rightarrow$ CBD-3 $\rightarrow$ Plat ...
  - Underlying assumptions now okay
  - But excessive complexity $\Rightarrow$ less robust forecasts???

- Dowd et al. (2010a,b): out-of-sample backtesting

  Models that fit *much better* in sample

  are *not obviously better* at out-of-sample forecasting

Issues on complexity

- More complex $\Rightarrow$ More random processes
- More random processes $\Rightarrow$

  MUCH more difficult to model multiple populations

A Possible Way Forward

Single-population models

- Paradigm shift away from *independent* Poisson model

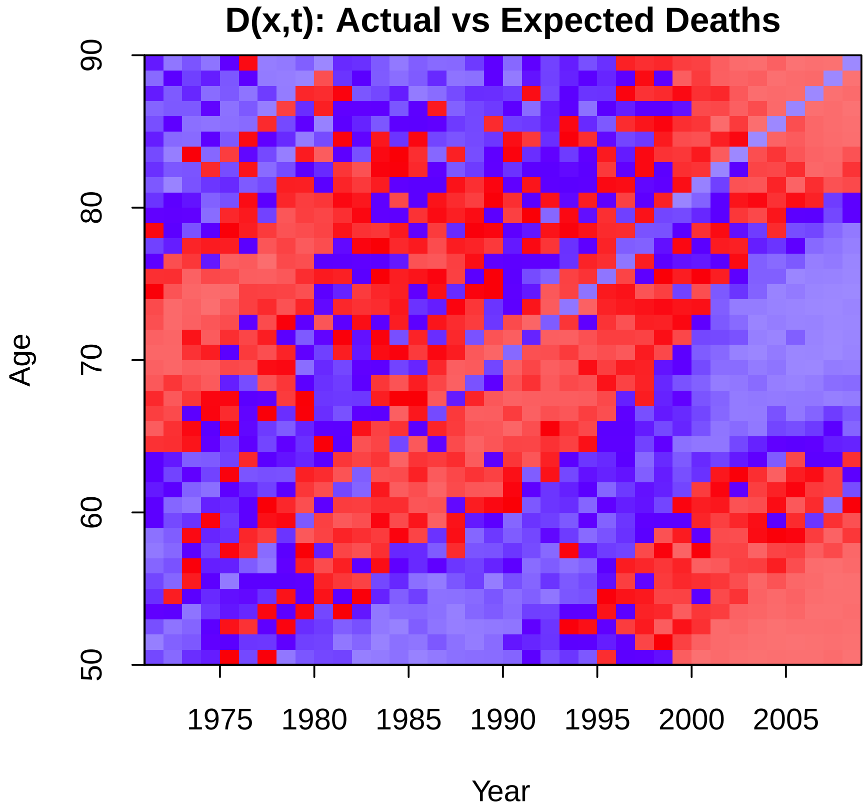
- Focus on small number of key drivers

  $\Rightarrow$ much easier to extend to multi-populations

- Focus on greater robustness of forecasts

Case Study: CBD/Plat Revisited

$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$$



Red $\Rightarrow$ actual deaths $>$ expected deaths

CBD/Plat Revisited: Key Idea: Possible responses

$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$$

Add:

- Cohort effect, $\gamma(t-x)$
- Extra age-period effects
- Do something new ......

Key Idea: CBD/Plat Revisited

Underlying $\log\ m(x,t) =$

- $\beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$: two key drivers

  PLUS

  $R(x,t)$ *Residuals*

- Assume: vector $R(t) \rightarrow R(t+1)$ mean reverting process

  $\Rightarrow$ long term risk depends on two key drivers

Specific Model

$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) + R(t,x)$$

- $\big(\kappa_1(t), \kappa_2(t)\big)$: bivariate random walk
- $R(t) =$ ($n_x \times 1$ vector) VAR(2), reverting to 0

$$R(t) = AR(t-1) + BR(t-2) + Z(t)$$

- $Z(x,t)$ i.i.d. $\sim N(0, \sigma_Z^2)$
- $A = A_1 + A_2$ and $B = -A_2A_1$

VAR matrices A_1 and A_2

$$A_i = \begin{pmatrix} a_i & 0 & 0 & \cdots & & & & \\ c_i & d_i & 0 & 0 & \cdots & & & \\ d_i/2 & c_i & d_i/2 & 0 & 0 & \cdots & & \\ 0 & d_i/2 & c_i & d_i/2 & 0 & 0 & \cdots & \\ 0 & 0 & d_i/2 & c_i & d_i/2 & 0 & 0 & \cdots \\ \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \end{pmatrix}$$

a_i =AR terms for new members;

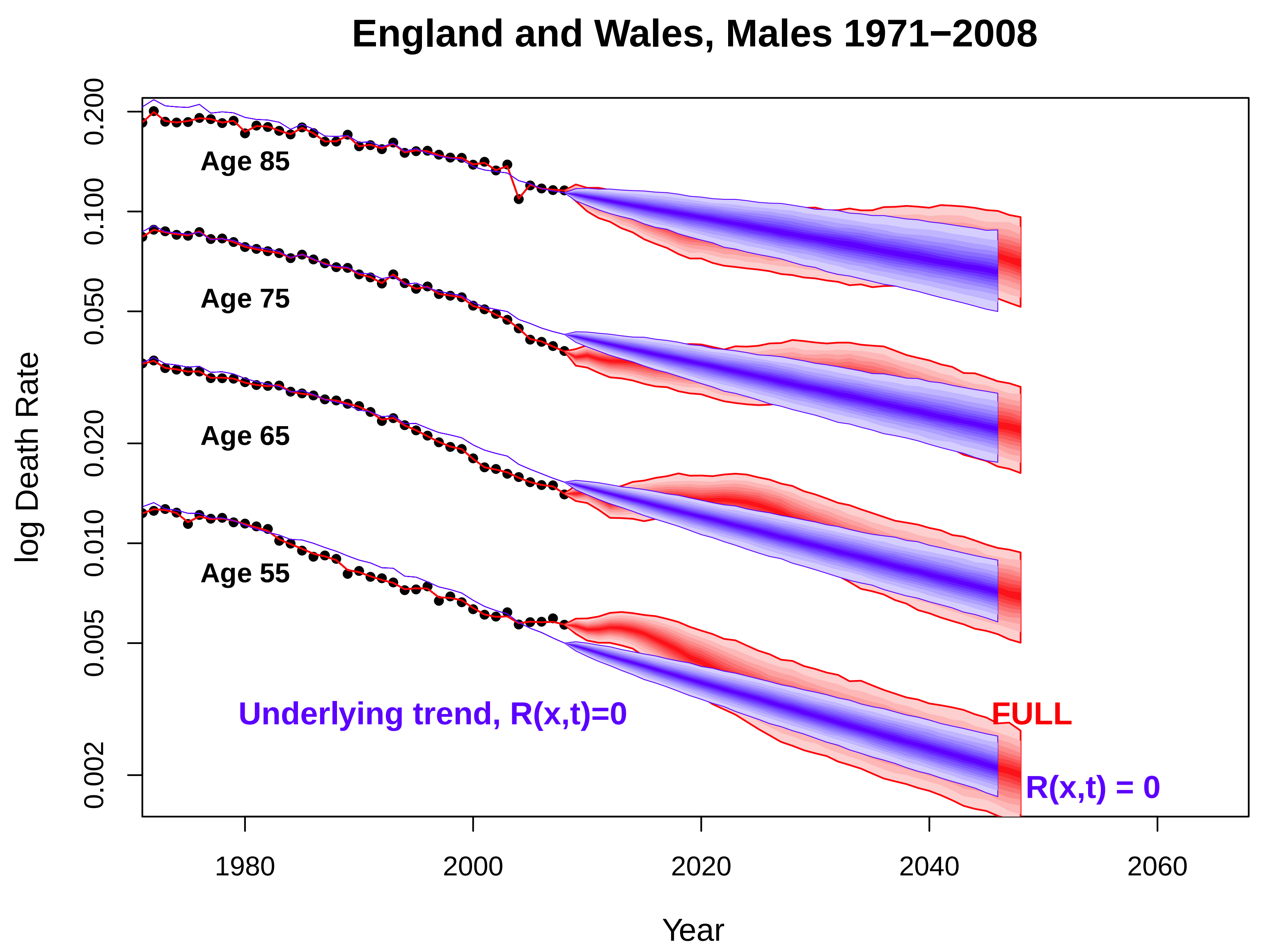
c_i = cohort persistence;

d_i = diffusion coeff.

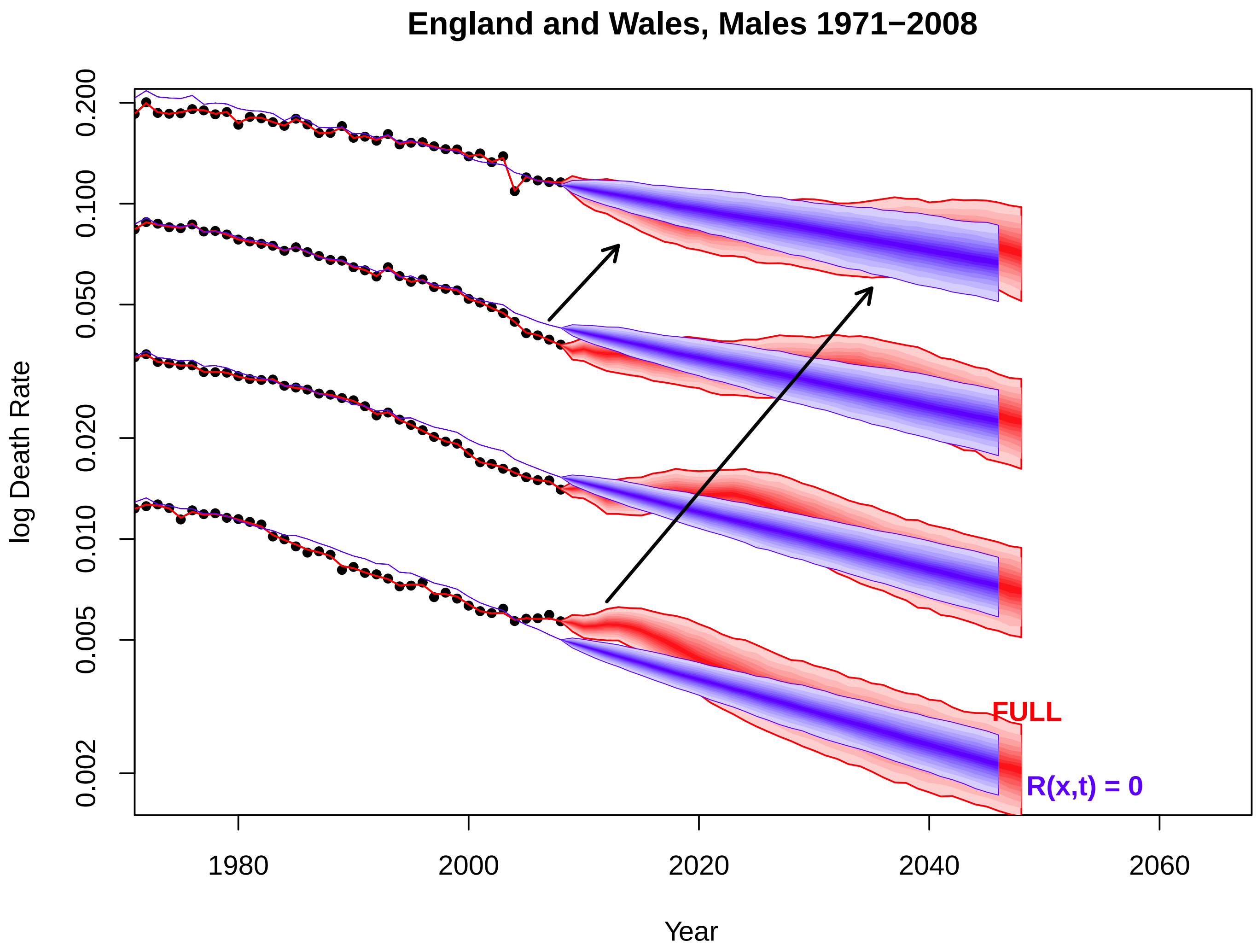
Further details

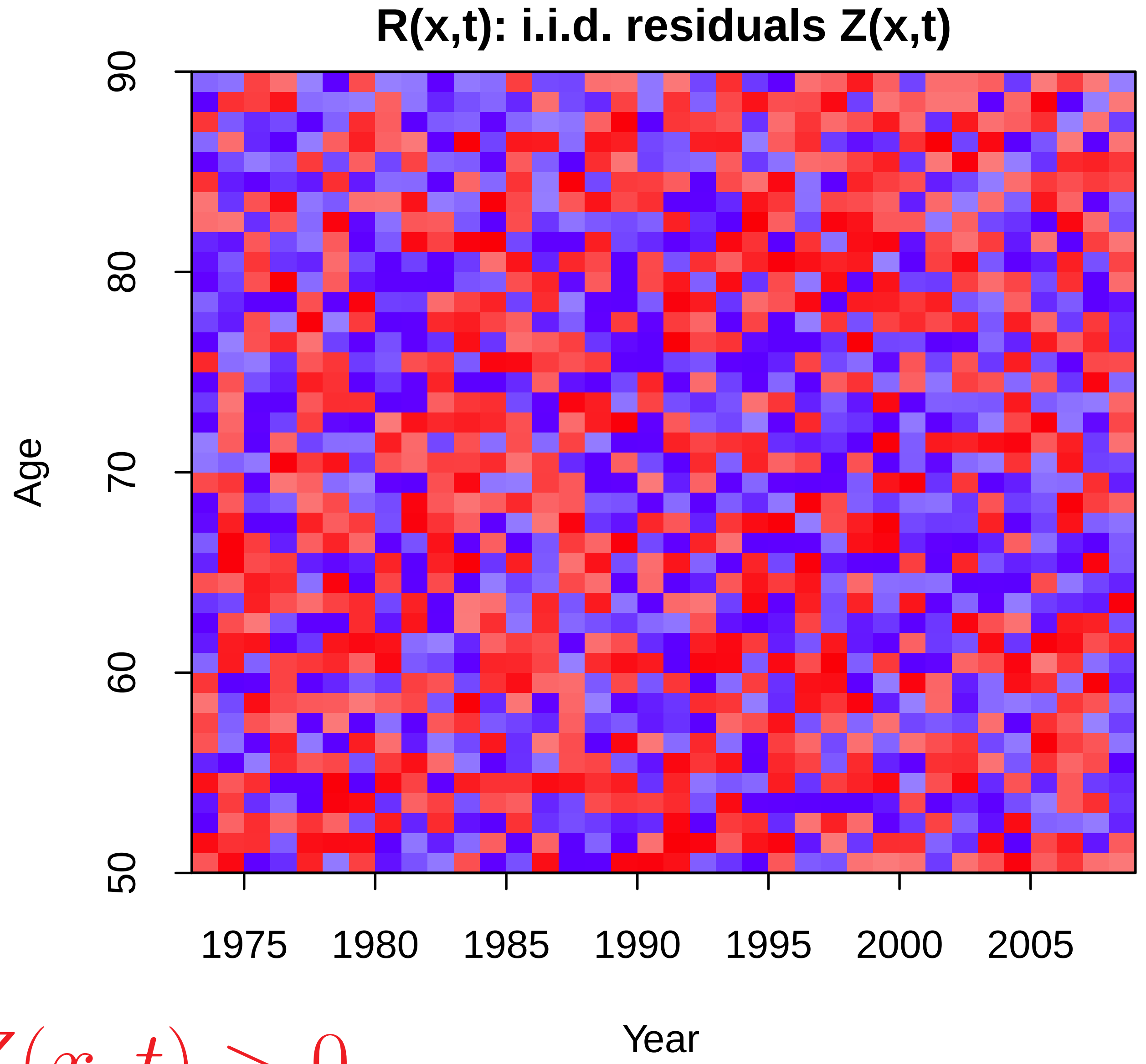
- Deaths: $D(x,t) \sim$ Poisson $(m(x,t)E(x,t))$
- Bayesian approach:

  posterior density $=$ likelihood $\times$ prior
- Upcoming results: mode of posterior density
- Further work: Bayesian parameter uncertainty

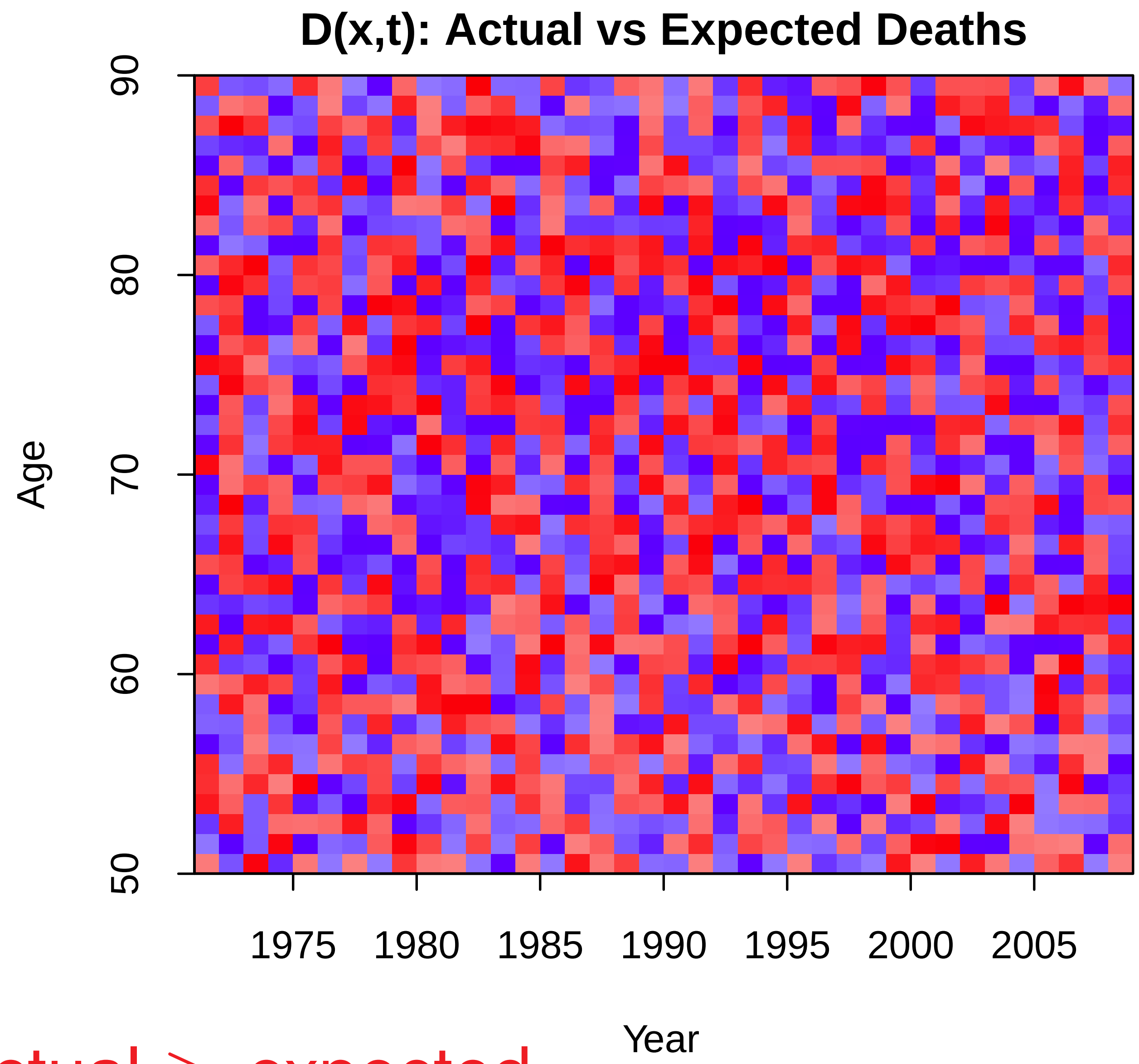

England and Wales, Males 1971−2008
log Death Rate
0.200
0.100
0.050
0.020
0.010
0.005
0.002
Age 85
Age 75
Age 65
Age 55
Underlying trend, R(x,t)=0
FULL
R(x,t) = 0
1980
2000
2020
2040
2060
Year

Cohort-type effects





Red $\Rightarrow Z(x,t) > 0$



Red $\Rightarrow$ actual $>$ expected

Comparison with related models

$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x})$$

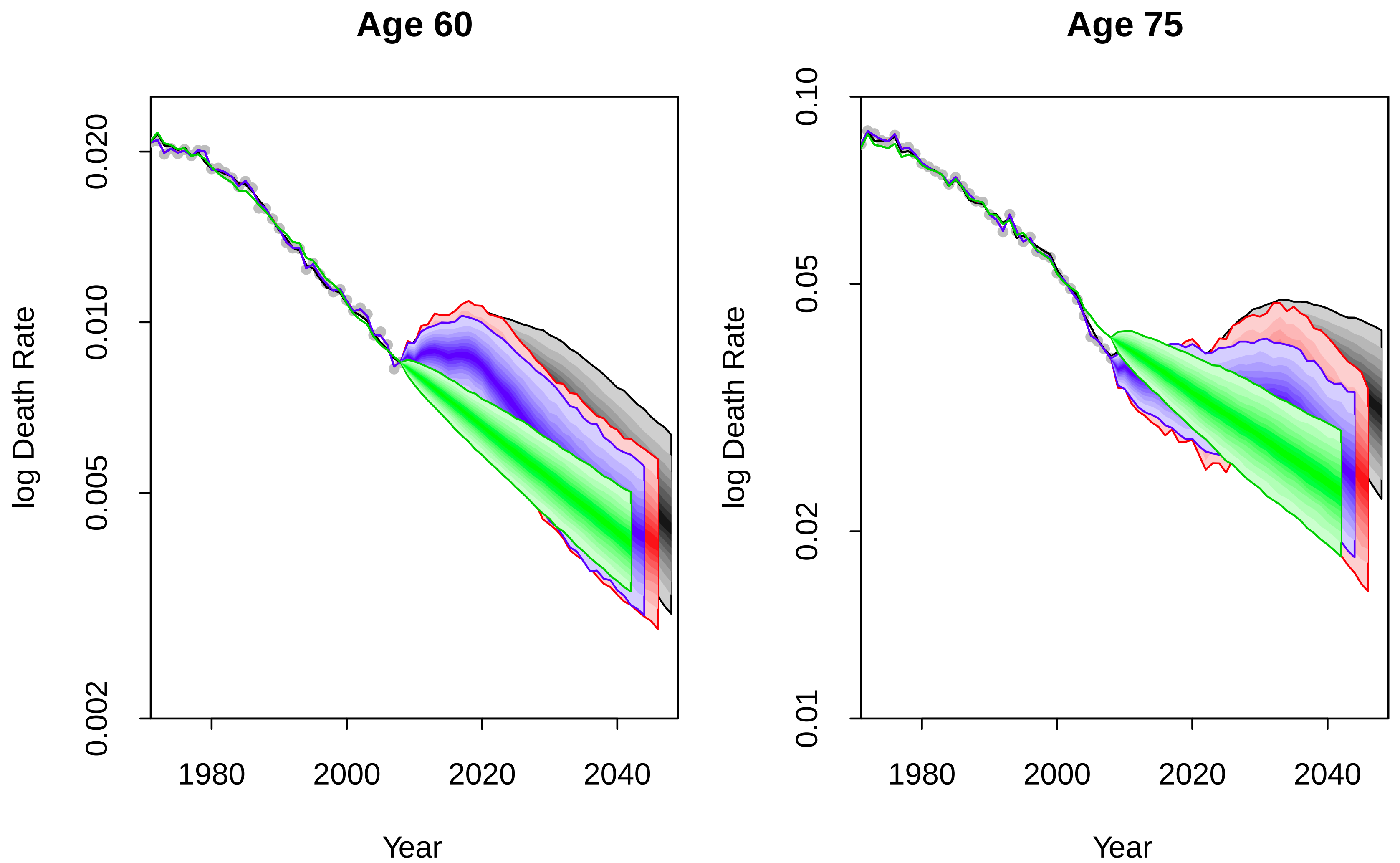
$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t((x - \bar{x}) + \gamma(t - x)$$

$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t)(x - \bar{x}) + R(x,t)$$

$R(t) = AR(t-1) + BR(t-2) + Z(t)$ (A, B as specified earlier)

$$\log m(x,t) = \beta(x) + \kappa_1(t) + \kappa_2(t((x - \bar{x}) + R(x,t)$$

$R(t) = AR(t-1) + BR(t-2) + Z(t)$ (simplified A, B)



Age 60
log Death Rate
0.002
0.005
0.010
0.020
1980
2000
2020
2040
Year
Age 75
log Death Rate
0.01
0.02
0.05
0.10
1980
2000
2020
2040
Year

Conclusions: Model Comparisons

- Long term underlying trends ($\kappa(t)$) are reasonably consistent
- Model risk more evident in the mean reverting $R(x,t)$

Further work

- Bayesian parameter uncertainty
- Multiple populations: focus on underlying $\kappa(t)$

  $\Rightarrow$ less complexity

Multipopulations

Borrow from multifactor asset models: e.g.

- Asset i return: $R_i = \alpha_i + \beta_{i1} F_1 + \beta_{i2} F_2 + \epsilon_i$
- $F_1,\ F_2$ are systematic risk factors
- $\epsilon_i =$ idiosyncratic risks

Multipopulations

Mortality – version 1:

- Population, P, specific $\kappa_i^{(P)}(t)$ correlated
- $R^{(P)}(x,t)$: assume independent

Mortality – version 2:

- All populations have the same $\kappa_i(t)$
- $R^{(P)}(x,t)$: assume independent
- Greater role for $R(x,t)$ as country specific effect

Questions

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Other models for $R(x,t)$

1. $R(x,t) = \phi R(x-1, t-1) + Z_R(x,t)$

2. $R(x,t) = \phi R(x-1, t-1) +$ diffusion $+ Z_R(x,t)$

3. Smooth underlying period effects, $\kappa_1(t), \kappa_2(t)$

   plus annual shocks

   e.g. $R(1), R(2), \ldots$ are i.i.d. vectors, correlated

   across ages