Heriot-Watt University & The University of Edinburgh

M.Sc. in Financial Mathematics

Derivative Pricing and Financial Modelling

Tutorial 6

1. UK government coupon paying bonds are called gilts. What names are given to
   (a) UK money-market bonds
   (b) UK zero-coupon bonds
   (c) US coupon bonds (two types: so distinguish between the two)
   (d) US money-market bonds
   (e) US zero-coupon bonds

2. Prove that the gross redemption yield (yield to maturity) is uniquely defined for a fixed-interest bond under which all of the future cashflows are positive.

3. (*) One consequence of an arbitrage-free bond market is that the instantaneous risk-free rate, $r(T)$, must be non-negative for all T.
   (a) Why must the forward-rate curve, $f(t, t+s)$, also be non-negative?
   (b) What are the consequences for the form of $P(t, T)$?

4. Suppose that the UK government issues two bonds, Treasury 8% 2010-2014 and Treasury 8% 2010, with earliest redemption dates which coincide. Explain which bond will have the highest price?

5. Suppose the government issues two bonds, Treasury 8% 2010 and Conversion 8% 2010. The bonds mature on the same date. On 1 January 2006 (not a coupon payment date) holders of Conversion 8% 2010 will be able to convert their stock into $8\frac{3}{4}$% 2017 on a one for one basis. Explain which of Treasury 8% 2010 and Conversion 8% 2010 will have the higher price?

6. (*) Suppose that the spot rates (continuously compounding) for terms 1, 2 and 3 to maturity are:

T	1	2	3
$R(0,T)$	6%	6.5%	7%

   (a) Find the values of the forward rates $F(0, 1, 2)$, $F(0, 1, 3)$ and $F(0, 2, 3)$.

(b) Assuming that coupons are payable annually in arrears, find the par yields for terms 1, 2 and 3 years.

7. (*) Suppose that $f(0,s) = 0.08$ for all $s > 0$.

   We have available for investment three zero-coupon bonds maturing at times 5, 10 and 15.

   At time 1 the forward-rate curve will be $f(1,s) = f(0,s) + \epsilon$ where $\epsilon = +0.02$ with probability 0.5 and $\epsilon = -0.02$ with probability 0.5.

   Construct the arbitrage which will take advantage of this parallel forward-rate curve shift.

8. (*) In a particular 1-period bond-pricing model 4 bonds are available which mature at times 1, 2, 3 and 4. Their prices at time 0 are 0.9, 0.81, 0.729 and 0.684 respectively. At time 1 there will be one of three outcomes ω_1, ω_2 and ω_3. The prices of the outstanding bonds for each outcome are given in the following table.

	ω_1	ω_2	ω_3
$P(1,2)$	0.88	0.9	0.92
$P(1,3)$	0.77	0.805	0.86
$P(1,4)$	0.7	0.75	x

   No trading is possible between times 0 and 1.

   (a) Find the value of x which will make this model arbitrage free.

   (b) Is this market complete?

   (c) If instead $x = 0.81$ show how to create an arbitrage opportunity.