is a standard Brownian motion under P_{T_k}. Second, for $i < j$, we have

$$\begin{aligned} dW_{T_j}(t) &= dW_{T_i}(t) - \big(S(t,T_j) - S(t,T_i)\big)dt \\ &= dW_{T_i}(t) - \sum_{k=i+1}^{j} \big(S(t,T_k) - S(t,T_{k-1})\big)dt \\ &= dW_{T_i}(t) + \sum_{k=i+1}^{j} \frac{\tau L_k(t)}{1+\tau L_k(t)} v_k(t)dt \quad \text{(by (9.3.2))}. \end{aligned}$$

It follows that simulating under P_{T_1} we have

$$\begin{aligned} dL_1(t) &= L_1(t)v_1(t)'dW_{T_1}(t) \\ \text{and}\quad dL_j(t) &= L_j(t)v_j(t)'\left(dW_{T_1}(t) + \sum_{k=2}^{j} \frac{\tau L_k(t)}{1+\tau L_k(t)} v_k(t)dt\right) \quad \text{for } j = 2,\dots,M. \end{aligned}$$

9.4 Swap market models

It can be seen from the preceding discussion that the Brace, Gatarek & Musiela (1997) method for pricing swaptions is rather cumbersome and contains a number of awkward approximations and assumptions. Jamshidian (1997) and Musiela & Rutkowski (1997) developed a more direct approach which does not suffer from these complications.

For pricing the swaption described in Section 9.2 we use a different numeraire from before, namely,

$$X(t) = \tau \sum_{k=1}^{M} P(t,T_k),$$

with associated forward measure P_X. Under P_X the prices of all tradeable assets discounted by $X(t)$ are martingales. Now the forward swap rate $K_\tau(t,T_0,T_M)$ equals $\Big(P(t,T_0) - P(t,T_M)\Big)/X(t)$ and $\Big(P(t,T_0) - P(t,T_M)\Big)$ equals the value of a tradeable asset. It follows that $K_\tau(t,T_0,T_M)$ is a martingale under P_X. For notational convenience we will write $K(t) = K_\tau(t,T_0,T_M)$. Suppose that

$$dK(t) = K(t)\gamma(t)dW_X(t)$$

where $W_X(t)$ is a Brownian motion under P_X and $\gamma(t)$ is deterministic.