Risk Management
9: Market Risk Measurement and Management II

Reading:

- Lam: Chapter 13
- Sweeting: Chapter 16, sections 16.1-16.2.
- Crouhy: Chapter 7
- Hull (3rd Edition) Chapter 7

Outline

- Unit 9.1: More on Statistical Modelling and Applications
- Unit 9.2: Risk Management Using the Greeks
- Unit 9.3: Other Market Risk Management Topics:

  basis risk; scenario and stress tests etc.

Unit 9.1: More on Statistical Modelling and Applications

Statistical Modelling

Building on the last chapter:

- Market risk $\Rightarrow$ lots of high frequency data for liquid assets over long periods of time
- A good quality internal model will incorporate:
  - Multi-period or continuous time
  - Stochastic volatility (e.g. GARCH(1,1) model)
  - Fat-tailed innovations, $Z(t)$ (e.g. fatter tails than $N(0,1)$)
  - Skewness in the innovations
  - Time-varying mean return
  - An accurate dependency structure (e.g. using copulas) between different market risks
    or some other mechanism for aggregating market risk over distinct risks
  - Use of extreme value theory for tail risks

Statistical Modelling (cont.)

- Uses of stochastic internal models:
  1. model the distribution of surplus over time
     $\Rightarrow$ Value-at-Risk; Expected Shortfall
  2. Medium and long-term risk measurement and management
     (contrast with the approach to short-term risk management in Unit 9.2)
- Stochastic models are often complemented through the use of scenarios and stress tests
  (helpful for more extreme risks where a stochastic model might be less accurate)
- Here: selected mini-topics rather than a full discussion

Aggregation

A good approach:

- Full multivariate model covering all market risks
- Dependencies modelled using copulas
- Fat-tailed, skewed distributions if appropriate

Alternatives:

- Historical simulation
  $\Rightarrow$ multivariate daily outcome drawn at random from historical sample period
- The Mean-Variance approach (McNeil et al., 2.2.3):
  very basic; assumes linear relationships

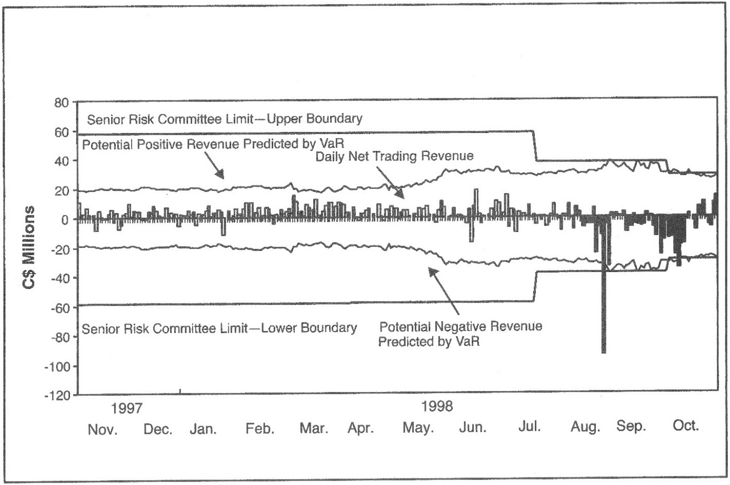
Banking: Monitoring of Market Risk

- 1-day and 10-day Value-at-risk for market risk is very important
- Limits set with reference to Board Risk Tolerance
- At least daily monitoring

Example: Monitoring a Bank's VaR

Source: Crouhy et al., page 245 (hard copy), Figure 7-2B

Net Daily Trading Revenues during 1998 versus One-Day VaR at the 99 Percent Confidence Level.



Risk Committee Limits

- Usually constant
- Changes from time to time in response to perceived dangers
- Individual VaR limits consistent with Group VaR limits

Daily estimated VaR

- Changes daily in response to portfolio mix and market volatility
- Monitor exceedances:
  - 99% VaR $\Rightarrow$ 1 in 100 should exceed
  - If too many or too few $\Rightarrow$ bad model!!!!
- VaR close to Senior Risk Committee limits
  $\Rightarrow$ URGENT action required

Summary

- Understand what the key statistical features are in a good quality stochastic internal model for market risk
- Know what the alternatives are as approximations to a full stochastic model
- Be aware of how an internal model is used to determine VaR and other risk measurement statistics

Unit 9.2: Risk Management Using the Greeks

The Greeks

What are "the Greeks"? The Greeks (e.g. Delta, Gamma) are quantities that measure the sensitivity of an asset or portfolio value to changes in underlying market variables.

AIM: To use the Greeks to help reduce exposure to losses resulting from short-term market movements

- π = portfolio of assets e.g. derivatives + underlyings
- $V_\pi(t)$ = value of π at t
- Example: Bank derivatives desk:
  OBJECTIVE: profits come from buy/sell spread, not from profits arising from market risk

Greeks $\Rightarrow$ minimise short term market risk

- Example: Equity derivatives desk
- Inputs: t, $S(t)$, r, σ
- Portfolio of contracts: T_i, payoff$_i(T_i)$ for $i = 1, 2, \ldots$
- $V_\pi(t) \equiv V_\pi(t, S(t), r, \sigma)$

$$\Delta_\pi = \left.\frac{\partial V_\pi}{\partial S}\right|_{(t,S(t),r,\sigma)}$$

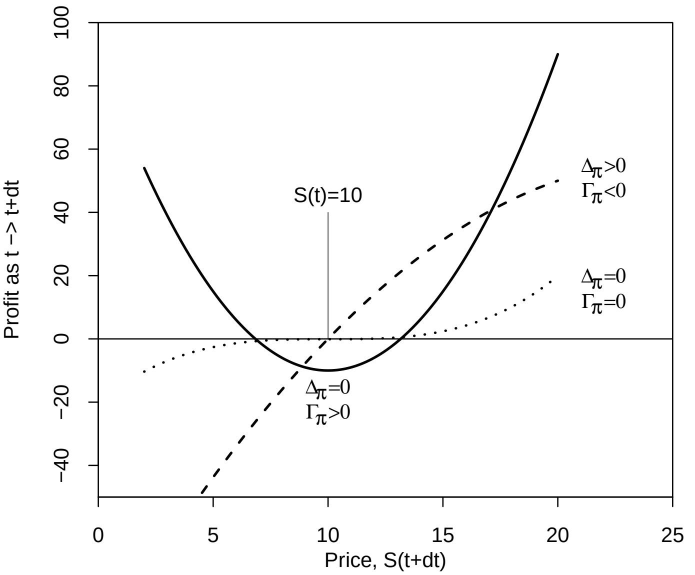
$\Rightarrow$ sensitivity to SMALL changes in $S(t)$

- Delta hedging $\Rightarrow$ keep Δ_π at or close to zero (e.g. Black-Scholes)

Gamma hedging

$$\Gamma_\pi = \left.\frac{\partial^2 V_\pi}{\partial S^2}\right|_{(t,S(t),r,\sigma)}$$

- How much will Δ_π change if $S(t)$ changes?
- Convexity of $V_\pi(t)$ with respect to $S(t)$
- $\Delta_\pi = 0$ and $\Gamma_\pi > 0 \Rightarrow$ downside risk associated with $S(t)$ is limited



Profit as t −> t+dt
100
80
60
40
20
0
−20
−40
0
5
10
15
20
25
Price, S(t+dt)
S(t)=10
Δπ>0
Γπ<0
Δπ=0
Γπ=0
Δπ=0
Γπ>0

Benefits of Gamma hedging

AIM: $\Gamma_\pi \approx 0$ or slightly positive

Assume $\Delta_\pi = 0$

- Small $\Gamma_\pi \Rightarrow$ less rebalancing at $t + dt$ to reset Δ_π to 0
- Limited downside risk
- Deals with some aspects of model risk: e.g. jumps instead of Brownian Motion

[Even if continuous rebalancing is possible with zero transaction costs, jumps $\Rightarrow$ risk of losses. $\Gamma_\pi > 0 \Rightarrow$ losses will be small.]

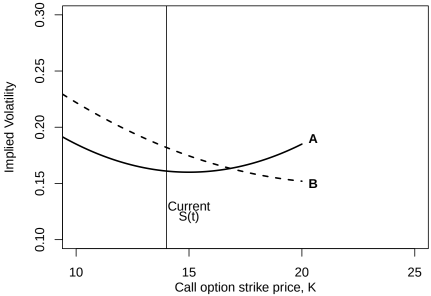
Vega Hedging

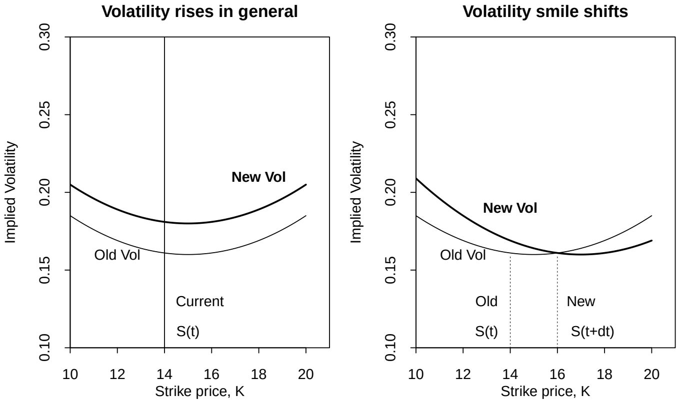
$$\mathcal{V}_\pi = \left.\frac{\partial V_\pi}{\partial \sigma}\right|_{(t,S(t),r,\sigma)}$$

- $\mathcal{V}_\pi \Rightarrow$ sensitivity to changes in the volatility
- σ might change because:
  - (a) limited historical data
  - (b) recalibration of a model that assumes a constant σ (e.g. B-S) using latest market prices
  - (c) volatility is stochastic
- (a), (b) $\Rightarrow$ if $\mathcal{V}_\pi \approx 0$ then we are addressing some elements of parameter and model risk

Warning

- Suppose for $i = 1, \ldots, N$ assets, σ_i =B-S implied volatility $\Rightarrow V_i(t, S(t), r, \sigma_i)$ = market price based on the B-S model
- $\mathcal{V}_i = \partial V_i / \partial \sigma \big|_{\sigma_i}$
- BUT: what is $\mathcal{V}_\pi = \partial V_\pi / \partial \sigma$?
- Volatility smile:





- Left: $\sigma_i \to \sigma_i + \lambda$ for all i
  $\Rightarrow \partial V_\pi / \partial \sigma$ is meaningful
- Right: $\partial V_\pi / \partial \sigma$ is not meaningful

Rho (hedging)

$$\rho_\pi = \left.\frac{\partial V_\pi}{\partial r}\right|_{(t,S(t),r,\sigma)}$$

$\Rightarrow$ sensitivity to changes in the risk-free rate of interest

- r calibrated using spot rate for the same term to maturity as the derivative
- Similar in concept to Vega
  Rho risk not as significant as Vega risk

Practicalities

- Each risk: responsibility for hedging rests with one clearly identified person
  - Δ_π: regular rebalancing to keep $\Delta_\pi \approx 0$
  - $\Gamma_\pi, \mathcal{V}_\pi$: regular monitoring, less frequent rebalancing
- Futures contracts: price is linear in $S(t)$

  $\Rightarrow$ help to keep $\Delta_\pi = 0$ with no impact on $\Gamma_\pi, \mathcal{V}_\pi$
- Risk limits on $\Delta_\pi, \Gamma_\pi, \mathcal{V}_\pi$: Department, trader
- Risk limits on individual underlyings, i: $\Delta_i, \Gamma_i, \mathcal{V}_i$
- Permission required in advance for violations of limits

Greeks: Conclusions

- Hedging and monitoring of the Greeks is an important PART of a good programme of market risk management
- Drawbacks:
    - Cannot add up $\Delta, \Gamma, \mathcal{V}, \rho$
    - Cannot add up Δ's across different risks
      [Previous slides: π implicitly had derivatives all linked to the same underlying $S(t)$.]
    - not able to identify capital at risk

Greeks: Conclusions (cont.)

- Plus points:
  - "cannot add up" $\Rightarrow$ forced to monitor a lot of important quantities
  - maintaining Δ-neutrality and monitoring $\Gamma, \mathcal{V}, \rho$ is MUCH better than not
  - Δ-neutral + low $\Gamma, \mathcal{V}$
    $\Rightarrow$ capital at risk and economic capital is likely to be much lower than would otherwise be the case
  - Good for short-term risk management

Summary

- Know how the Greeks are defined and calculated for individual risks
- Know how to use the Greeks to manage short-term market risks in a portfolio
- Understand the limitations of the Greeks

Unit 9.3: Other Market Risk Management Using Derivatives

Market risk management using derivatives on $S(t)$

- Futures
  - traded, liquid, low transaction costs;
  - $\Gamma = \mathcal{V} = 0$
- Call, put options:
  - use to change payoff profile
  - European, American: traded
  - Index options;   single stock options
  - Non-equity options: bonds, commodities
- Exotic options
  - OTC; tailored to your requirements

Basis Risk

Risk associated with imperfect hedging of a liability

e.g. Black Scholes:

- Theory $\Rightarrow$ zero basis risk:
    - continuous rebalancing
    - no transaction costs
    - no jumps in prices

  $\Rightarrow$ replication is possible
- Practice
    - discrete time rebalancing
    - transaction costs
    - hedging asset might not be an exact match for liability risk
    - jumps in prices

Basis Risk (cont.)

- e.g. $S_1(t)$ and $S_2(t)$ are correlated indices
  Trade in $S_2(t)$ to hedge payoff $f(S_1(t))$
  $\Rightarrow$ basis risk
- Hedging instrument is linked to same underlying risk as the liability
  BUT: *payoff profile is not an exact match for the liability*
- Chapter 10 example: hedging interest rate risk:
  - matching $\Rightarrow$ zero basis risk
  - immunisation $\Rightarrow$ basis risk

Market Risk stress tests

- Identify the main market risks that you are exposed to.
- Stress tests $\Rightarrow$ a collection of immediate shocks to key risk factors
- e.g. Interest rates (Chapter 10)
  - Parallel shift in spot rate curve of $\pm$100b.p.'s
  - Twist: $\pm$25b.p.'s at the short end; $\mp$25b.p.'s at the long end
  - Change in curvature: $\pm$10bp's at T_0; $\mp$10bp's at T_1; $\pm$10bp's at T_2 $\quad (T_0 < T_1 < T_2)$

  $\Rightarrow 8 = 2 \times 2 \times 2$ stress tests.
- Equities: index changes by $\pm 10\%$

Market Risk stress tests (cont.)

- Equity volatility: new $\sigma =$ old $\sigma \times 0.8$ or 1.2
  Important for derivatives portfolios, or non-linear market-risk exposures
- FX: USD:GBP changes by $\pm 6\%$
  Extent of FX stress tests depends on detailed exposure to FX
  Constraint: GBP1 $\rightarrow$ USDX $\rightarrow$ EURY $\rightarrow$ GBP1
  $\Rightarrow$ 3 exchange rates but only 2 degrees of freedom
- Δ, Γ, and $\mathcal{V}$ neutral portfolios should do well in stress tests
- Credit risk: spreads widen by 50b.p.'s

Market Risk stress tests (cont.)

- Stress envelopes (Crouhy)
  $\Rightarrow$ combinations of stress events on 2 or more risk factors
- Some combinations of stress tests are more plausible than others:
  e.g. equities fall 10% AND volatility rises by 20%
  e.g. GBP interest rates rise unexpectedly $\Rightarrow$ USD strengthens and credit spreads widen

Scenarios

- More complex collections of stress tests
- Sequences of events over time
- Some based on historical events
- Some “black swans”
- Requires a committee with diverse expertise
- Economically plausible

Results of stress and scenario tests

A: Comfortably survive test

B: Just survive

C: Disaster

Responses to B, C:

- immediate action to reduce or eliminate risk
- no significant action
- prepare contingency plans

Response depends on how many tests result in B or C and on the perceived chance of these events actually happening.

Pros and cons of stress and scenario tests

+ Good way of reducing reliance on stochastic models
+ Simple to implement
+ Highlights dangers that need immediate action or constant monitoring

– Highly subjective
– Difficult to rank in terms of likelihood
– Potential huge number of tests
– Dynamic time element often neglected

Other ways to manage market risk

- Diversify
- Clear guidance on
  - Which investments are acceptable (or not)
  - Limits on exposure to individual market risks
  - Limits of the use of derivatives
  - Limits on portfolio volatility
  - Limits on portfolio Value-at-Risk

Summary

- Demonstrate an understanding of basis risk and how it features in liability hedges in a non-banking context
- Be able to propose stress and scenario tests for market risk to complement the results of stochastic modelling
- Understand the pros and cons of stress and scenario tests
- Be able to analyse the results of stress and scenario tests and develop a risk management strategy that responds to the results of these tests