Risk Management
7: Economic Capital, RORAC, Capital Allocation

Reading:

- Sweeting: Chapter 18
- Hull (Risk Management): Chapter 23
- Crouhy: Chapter 17
- IAA: Chapter 7 + Ch 8.1-8.2
- McNeil: Chapter 8, section 8.5 (capital allocation)
- *Further reading:*
  - S&P: Economic Capital Models
  - S&P: Economic Capital Models: Methodology
  - Lam: relevant sections from the Chapter on "Risk Analytics"

This chapter:

- What does it mean to be doing good Economic Capital Modelling?
- How to determine requirements for additional assets to cover unanticipated losses?
- How do you assess the success or otherwise of different lines of business taking risk into account?
- How do I allocate capital to different lines of business to utilise my shareholders' capital in the most efficient way?

S&P Insurance Criteria, p12

Optimizing risk-adjusted results. The capital-budgeting process provides the information to allow management to choose the strategic alternatives that can provide the best return for the scarce capital resources of the company. The use of economic capital modeling as a principal driver of the capital budgeting element in strategic planning is indicative of strong strategic risk management.

Outline

- Unit 7.1: Economic Capital, RORAC
- Unit 7.2: Aggregating and Disaggregating Economic Capital
- Unit 7.3: The Euler Capital Allocation Principle
- Unit 7.4: The Economic Capital Model and Final Comments

Unit 7.1: Economic Capital, RORAC

7.1: Economic Capital, RORAC

What do we mean by economic capital? (EC)

- EC = additional assets put up by the shareholders
  - to support risk taking in the pursuit of profit
  - to cover unanticipated losses up to a specified measure of risk tolerance over a specified time horizon
- Associated quantities:
  OC = Other Capital from other sources
  BE = Best Estimate liabilities
  RC = Risk Capital in excess of BE
- $BE + RC = OC + EC$
- Based on internal economic principles and risk tolerances
- Not usually based on supervisory or accounting req.
  Why? These don't take own risk tolerance into account.
- Economic Capital Modelling aims to optimise the use of the limited economic capital

Basic idea (insurance): Example 1

- Invest risk free at $r = 5\%$ p.a.
- Time 1: Aggregate claims, L, has a Gamma distribution
  - $\Gamma(\alpha, \beta)$: $\alpha = 10$, $\beta = 10/105$
  - $E[L] = \alpha/\beta = 105$
  - $Pr(L > 210) \approx 0.005$
- Now: time 0
  Premiums $= 110$ just paid
  Best estimate: $BE(0) = E[L]/(1 + r) = 100$
- Risk $\Rightarrow$ prudent to hold more than 100 at time 0
- e.g. Hold a total of 200 at time 0
  $= 110$ from the policyholders $\Rightarrow$ other capital
  $+90$ from the shareholders $\Rightarrow$ economic capital

Basic idea (insurance): Example 1 (cont.)

- $BE + RC = OC + EC$:
- $BE + RC$ = sufficient to be 99.5% confident of meeting the liabilities

- $OC = 110$ premiums from policyholders
- $EC = 90$ from shareholders

- Premium $= 110 > E[L]/(1 + r) = 100 \Rightarrow$ rewarded risk

Basic idea (insurance): Example 1 (cont.)

- Time 1 total return to shareholders:

$$R = 200 \times 1.05 - L \quad \Rightarrow \quad E[R] = 105$$

(Also $Pr(R > 0) \approx 0.005$)

- Prospective performance measure:
Return on Risk Adjusted Capital (RORAC)

$$RORAC = \frac{E[R]}{EC} - 1 = \frac{105}{90} - 1 = 0.1667 > 0.05$$

Questions

- Is $EC = 90$ the right level?
  $\Rightarrow$ company risk tolerance
- Is a 16.67% return big enough?
- How does it compare to other opportunities for shareholders? (Efficient use of capital)
- Are shareholders getting a good return given the level of risk?

Example 2: Crouhy Chapter 17

expanded detail; $ millions

Source		$t = 0$ Use		$t = 1$ Return
s/h \$75 risk cap.	$\longrightarrow$	\$75 in gov. sec.	5% $\longrightarrow$	$75 \times 1.05 = 78.75$
\$1,000 Bank Retail Deposits @ 6%		\$1,000 in loans to corporate customers @ 9% ($\Rightarrow$ default risk)		$1,000 \times 1.09$ Deduct Expected expenses 9 Retail depositors 1,060 Expected corp. defaults 10 30% Tax 4.425 Net Profit: +10.325

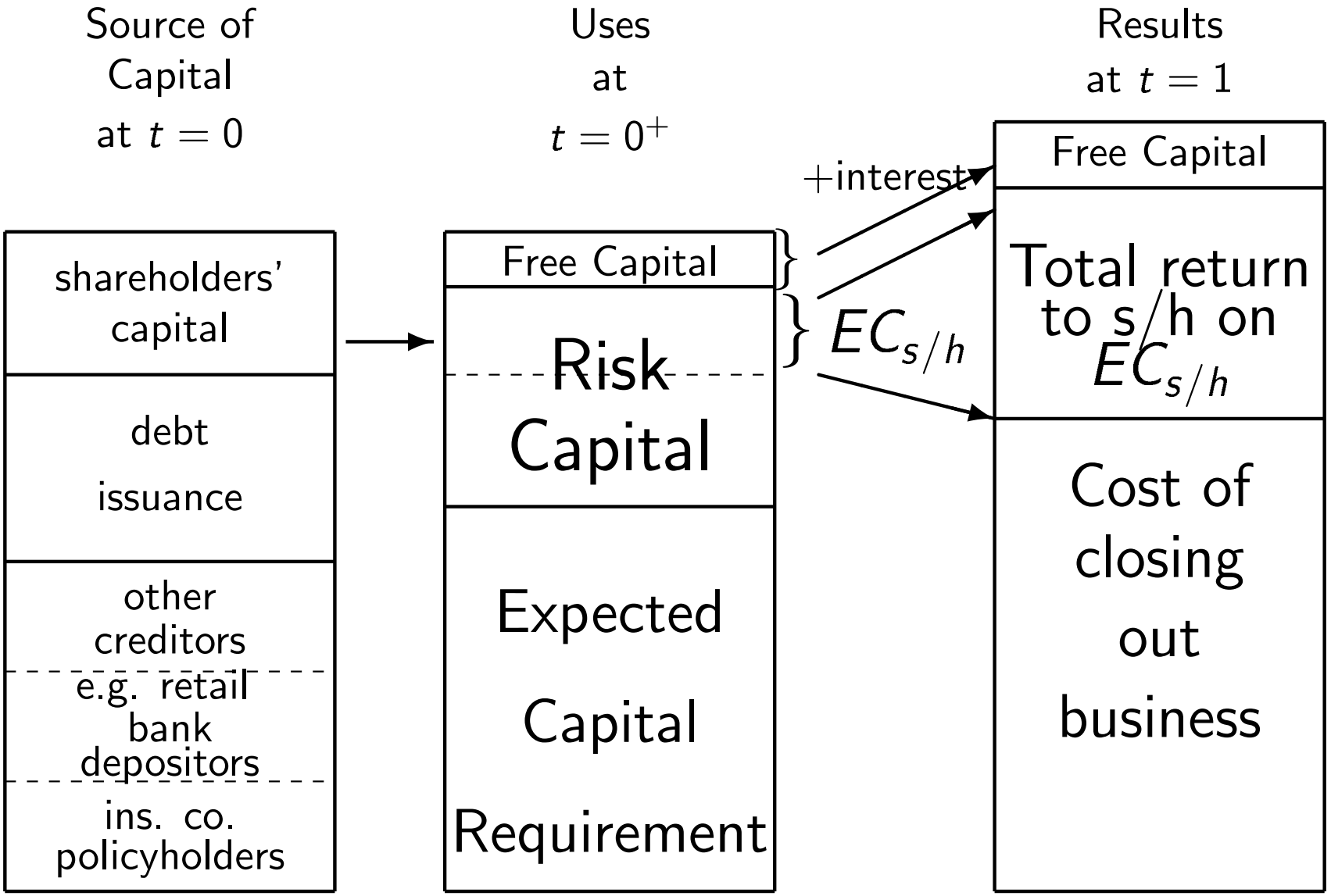
$$\text{Gross profit} = 78.75 + 1090 - 9 - 1060 - 10 \quad - 75 = 14.75$$
$$\text{Tax} = 0.3 \times 14.75 = 4.425$$

RORAC: Return on Risk Adjusted Capital

$$
\begin{aligned}
RORAC &= \frac{E[\text{s/h profit}]}{EC_{s/h} \ \text{(i.e. s/h risk capital)}} \\
&= \frac{10.325}{75} \\
&= 0.1377 \quad \text{or } 13.8\%
\end{aligned}
$$

RORAC = risk adjusted expected net profit after tax divided by economic capital

Bigger Picture



Alternatives to RORAC (Sweeting)

- RORAC=r_A
- Hurdle rate = r_H: minimum RORAC for viability
- Economic Income Created:

$$EIC = (r_A - r_H) \times EC$$

Summary

- We have introduced the concepts of
  - EC, economic capital: capital that is put up by the shareholders to support risk taking by the company
  - RORAC: expected return to shareholders adjusted for risk
- Understand the nature of the equation $BE + RC = OC + EC$
- How to calculate RORAC taking account of multiple cashflows and taxes
- Alternatives to RORAC
- EC and RORAC are closely linked but each is important in its own right

Unit 7.2: Aggregating and Disaggregating Economic Capital

7.2: Aggregating and Disaggregating Economic Capital

Group $\leftrightarrow$ business lines/units or risk types $1, \ldots, n$:

- Problem 1 (bottom up):
  - Calculate $EC_1, \ldots, EC_n$ or $RC_1, \ldots, RC_n$
  - How much capital do you need at the group level?
  - E.g., how much benefit is there from diversification
- Problem 2 (top down):
  - EC or RC established at the group level
  - How much capital gets allocated to each business unit to support risk taking within that business unit
    $\Rightarrow EC = \tilde{EC}_1 + \ldots + \tilde{EC}_n$
    where $\tilde{EC}_i$ is economic capital for business unit i with allowance for diversification benefits between business units

Bottom Up Aggregation (Hull)

- RC = group risk capital in excess of the best estimate (expected liability discounted at the risk free rate)
  Also recall: $BE + RC = EC + OC$
- Risk $i \Rightarrow$ **stand alone** risk capital RC_i
  $BE_i + RC_i = EC_i + OC_i$
  EC_i = stand alone economic capital
- How to convert $RC_1, \ldots, RC_n$ into group RC?
- Advanced Measurement Approach: Full multivariate model
  - Individual (non-normal) marginal distributions for each risk
    (e.g. individual models for each business unit and/or risk)
  - Copula to model dependencies between risks

Bottom Up Aggregation (Hull) (cont.)

- Basic Approach: Assume multivariate normality
  Example:
  - Risk Tolerance: Hold enough to be 97.5% confident of meeting liabilities at $t = 1$
  - Liabilities: $L = (L_1, \ldots, L_n)^T \sim$ Multivariate Normal
  - $E(L_i) = \mu_i; \quad Var(L_i) = \sigma_i^2; \quad Cov(L_i, L_j) = \sigma_i \sigma_j \rho_{ij}$
  - $\rho_{ij} =$ correlation between risks i and j
  - $L_G = L_1 + \ldots + L_n \sim N(\mu_G, \sigma_G^2)$

$$\mu_G = \sum_{i=1}^{n} \mu_i, \qquad \sigma_G^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \rho_{ij}$$

  - Assume interest rate, r, is zero
    (otherwise divide liabilities by $(1 + r)$ to discount to time 0)

Bottom Up Aggregation (Hull) (cont.)

- Stand alone:

$$\begin{aligned} BE_i &= \mu_i \\ BE_i + RC_i &= \text{97.5\% quantile of } L_i \\ \Rightarrow RC_i &= 1.96\sigma_i \end{aligned}$$

- Aggregated:

$$\begin{aligned} BE_G &= \mu_G = \mu_1 + \ldots + \mu_n \\ RC_G &= 1.96\sigma_G \end{aligned}$$

$$\text{equivalent to: } RC_G = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} RC_i RC_j \rho_{ij}}$$

Bottom Up Aggregation (Hull) (cont.)

- Hybrid:
  - Individual (non-normal) marginal distributions for each risk $\Rightarrow$ individual RC_i.
  - Then (approximation)

$$RC_G = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} RC_i RC_j \rho_{ij}}$$

(e.g. Solvency II; $RC \equiv SCR$)

- Finally, economic capital from shareholders

$$\begin{aligned} EC_G &= RC_G + \text{best estimate liability} - \text{other capital} \\ &= RC_G + BE_G - OC_G \end{aligned}$$

where $OC_G = OC_1 + \ldots + OC_n$

Capital Allocation (Top Down)

IAA, 7.1:

... the market will allocate capital to the most productive activities and organisations as measured by their ability to provide a return on that capital.

... best economic capital models are able to break up overall capital and return into smaller parts for which individual decisions can be made.

Simplified model

- Risks: $L_1, L_2, \ldots, L_d$; $\quad L = \sum_i L_i$
- $\rho(L)$ = total allocated capital = $EC+$ other capital (OC)
- $AC = \rho(L)$ = total allocated capital
  AIM:
  to subdivide AC into $AC_1, \ldots, AC_d$
  in a meaningful and fair way that
  allows comparison of the contribution
  of each of the risks $L_1, \ldots, L_d$
  to the enterprise's risk-return profile.
- The company decides how to subdivide.

How to use $AC_1, \ldots, AC_d$

- $\tilde{EC}_i = AC_i - OC_i$
  (The difference between stand alone EC_i and $\tilde{EC}_i$ is that $\tilde{EC}_i$ incorporates diversification benefits.)
- $RORAC_i = E[\text{shareholder profit}_i]/\tilde{EC}_i$
- Suppose the risks $(1, \ldots, d)$ have been reordered into $\{i_1, \ldots, i_d\}$ so that
  $RORAC_{i_1} \geq \ldots \geq RORAC_{i_d}$.
  A possible conclusion:
  - *Too much concentration of risk type i_d;*
  - *Too little of risk type i_1*

Examples

- Example 1: Allocate by time of arrival
  - $AC_1 = \rho(L_1)$
  - $AC_2 = \rho(L_1 + L_2) - AC_1$
  - $\vdots$
  - $\sum_{i=1}^{k} AC_i = \rho(\sum_{i=1}^{k} L_i)$

- Examples 2 and 3:
  Sometimes $\rho(L)$ equates to the Risk Capital, RC:
  i.e. the excess required capital over the Best Estimate (or mean)
  (See $\rho^{mean}(L)$ in McNeil et al., Section 8.4.1, equation (8.48))
  $\Rightarrow AC = BE + RC = BE + \rho(L)$
  (Any exam questions will make this clear.)

Examples 2 and 3

Both examples: $\rho(L)$ = required *risk capital* (in excess of the best estimate) either at the group level or on a stand-alone basis

- Example 2: the Proportional Capital Allocation Principle

$$\tilde{RC}_i = \frac{\rho(L_i)}{\sum_{j=1}^{n} \rho(L_j)} \times RC$$

- Example 3: the Covariance Capital Allocation Principle

$$\tilde{RC}_i = \frac{Cov(L_i, L)}{Var(L)} \times RC$$

In both examples: $\tilde{EC}_i = BE_i + \tilde{RC}_i - OC_i$

Summary

- Understand the different approaches to combine standalone economic capital that allows for diversification at the group level
- Begin to understand the concept of top down capital allocation

Unit 7.3: The Euler Capital Allocation Principle

General Formulation

- McNeil, Chapter 8, Section 8.5
- Previously $L = L_1 + \ldots + L_n$
- Now generalise: $L(x) = x_1 L_1 + \ldots + x_d L_d$
- $x = (x_1, \ldots, x_d)^T$ = portfolio weights; $x \in X$
- $r(x) = \rho(L(x))$ = more general capital requirement for portfolio x
- $r(x) = AC_1(x) + \ldots + AC_d(x)$
  where $AC_i(x)$ = risk i Allocated Capital
  $$AC_i(x) = \tilde{EC}_i(x) + OC_i(x)$$

General Formulation (cont.)

- $AC_1(x), \ldots, AC_d(x)$ to be calculated, subject to $\sum_i AC_i(x) = r(x)$
- $\pi : X \to \mathbb{R}^d$

  π is called a per unit capital allocation principle (CAP)

  if, for all $x \in X$, $\sum_i x_i \pi_i(x) = r(x)$.

  Then (definition) $AC_i(x) := x_i \pi_i(x)$

  and $\sum_i AC_i(x) = r(x) = AC(x)$

Example 4: The Euler Capital Allocation Principle

- Assume $\rho(L)$ is a *positive homogeneous function of degree 1.* (Coherent risk measures: Axiom 3)
  $\Rightarrow \rho(kL) = k\rho(L)$ for all $k > 0$ (*)
- Examples:
  1. Expected Shortfall: $ES_\alpha(L)$
  2. Value at Risk: $q_\alpha(L)$
  3. Standard Deviation risk measure:
     $$\rho(L) = \mu_L + \sigma_L \, z \text{ for a constant } z$$

Example 4: The Euler Capital Allocation Principle (cont.)

- Consequence:

$$\begin{aligned} r(kx) &= \rho(L(kx)) = \rho\left(k\sum_i x_i L_i\right) \\ &= k\ \rho\left(\sum_i x_i L_i\right) \\ &= k\ r(x) \quad \text{for all } k > 0 \end{aligned}$$

  so $r(x)$ is positive homogeneous of degree 1.

- Why is positive homogeneity relevant?

Euler's Homogeneous Function Theorem (special case)

Theorem

- Suppose $f(v) : \mathbb{R}^d \to \mathbb{R}$ is homogeneous of degree 1 (i.e. $f(\alpha v) = \alpha f(v)$ for all v)
- Then

$$\sum_{i=1}^{d} v_i \frac{\partial f}{\partial v_i}(v) = f(v)$$

The Euler CAP

- Define $\pi_i(x) = \frac{\partial r}{\partial x_i}(x)$ (per unit of risk)
- and define $AC_i(x) = x_i \pi_i(x)$

Then $$\sum_i AC_i(x) = \sum_i x_i \pi_i(x) = \sum_i x_i \frac{\partial r}{\partial x_i}(x) = r(x)$$

by Euler's Theorem

- Hence: π is a *per unit CAP*

Rationale for the Euler CAP

- Start with $x \longrightarrow r(x)$
- Next: suppose we add a small amount of risk j:

$$\begin{aligned} \tilde{x}_j &= x_j + h \quad \text{for small } h \\ \tilde{x}_i &= x_i \qquad \text{for all } i \neq j \end{aligned}$$

$$\text{Then} \quad r(\tilde{x}) \approx r(x) + h\frac{\partial r}{\partial x_j}(x)$$

- Assume extra $= h \times \frac{\partial r}{\partial x_j}(x)$ all added to $AC_j(x)$

$$\Rightarrow AC_j(\tilde{x}) = (x_j + h)\pi_j(x)$$

Underlying assumption: for small h the $\pi_i(x)$ remain unchanged when $x \rightarrow \tilde{x}$ for all i.

Rationale for the Euler CAP (cont.)

Hence: the Euler CAP allocates capital in proportion to the *marginal* impact of each risk on the current portfolio.

Example 5: Euler CAP + VaR

- $\rho(L) = \alpha$-quantile of $L = q_\alpha(L)$
- If $(L_1, \ldots, L_d)$ has a joint density and $r(x) = \rho(L(x)) = q_\alpha(L(x))$ then (see McNeil 8.5.2) Euler CAP $\Rightarrow$

$$\pi_i(x) = \frac{\partial r}{\partial x_i}(x) = E\left[L_i \mid L(x) = q_\alpha(L(x))\right]$$

- Check:

$$\sum_i x_i \pi_i(x) = E\left[\sum_i x_i L_i \;\middle|\; L(x) = q_\alpha(L(x))\right] = q_\alpha(L(x))$$

RORAC again

- $H_i = E[\text{profit per unit of risk } i]$;
  $P_i = \text{premium per unit}$
- $EC_i(x) = AC_i(x) - OC_i(x)$ (e.g. $OC_i = x_i P_i$)
- $RORAC(x) = \sum_i x_i H_i / \sum_i EC_i(x)$
- $RORAC_i(x) = x_i H_i / EC_i(x)$
- Suppose the risks $(1, \ldots, d)$ have been reordered into $\{i_1, \ldots, i_d\}$ so that
  $RORAC_{i_1}(x) \geq \ldots \geq RORAC_{i_d}(x)$.
  Then $RORAC_{i_1}(x) \geq RORAC(x) \geq RORAC_{i_d}(x)$

Exercise: prove the last point. Hint: the group RORAC can be written as a weighted average of the $RORAC_i$'s.

Desirable Criterion

- Suppose $\hat{x} > 0$ maximises $RORAC(x)$ subject to $EC_{\text{group}} \leq EC_{\text{max}}$ available from shareholders
- Then a desirable criterion for a capital allocation principle is that

$$RORAC_i(\hat{x}) = RORAC(\hat{x}) \quad \text{for all } i$$

- If this is not true then, for some $i \neq j$, $RORAC_i(\hat{x}) > RORAC_j(\hat{x})$ suggesting we can do better with more of risk i and less of risk j.
- The Euler CAP satisfies this criterion.

Summary

- We have introduced the Euler CAP as a way to allocate capital between business units
- The allocation problem is generalised by replacing business unit i liabilities, L_i by $x_i L_i$
- $r(x) = \rho(\sum_i x_i L_i)$
- $AC_i(x) = x_i \frac{\partial r}{\partial x_i}(x)$
- At the optimal $\hat{x}$, $RORAC_i(\hat{x}) = RORAC(\hat{x})$ for all i
- If $r(x)$ is convex, then the optimisation problem is well posed (i.e. there is a unique maximum) [See McNeil et al. for convex risk measures.]

Unit 7.4: The Economic Capital Model and Final Comments

The Economic Capital Model

ECM Aim (IAA 7.1): "...best economic capital models are able to break up overall capital and return into smaller parts for which individual decisions can be made"

ECM Aim (S&P): use economic capital allocation to help optimise RORAC/EIC/other objective

The Economic Capital Model (cont.)

ECM $\Rightarrow$ combination of elements:

- Dynamic modelling:
  - stochastic
  - plus scenarios and stress tests
  - dynamic management actions
  - rigorous and justifiable dependency structure between key risks
  - integrated asset and liability models
  - model and parameter risk
- Risk profile, tolerance, capacity
  RT $\Rightarrow$ links to
  - Regulatory capital
  - Ratings agency capital
  - Internal shareholders' economic capital
  - Capital allocation framework

The Economic Capital Model

- Control cycle
- Relationship with capital management team: $\Rightarrow$ capital allocation; $RORAC_i$ etc.
- Model $\Rightarrow$ sub-units and adequate modelling of all significant risks
- Integration of risk tolerances and risk limits:
  - business units
  - risk factors
- Iterative process (e.g. optimising $RORAC_i$'s, assessing different risk management activities and actions, ...)

The Economic Capital Model (cont.)

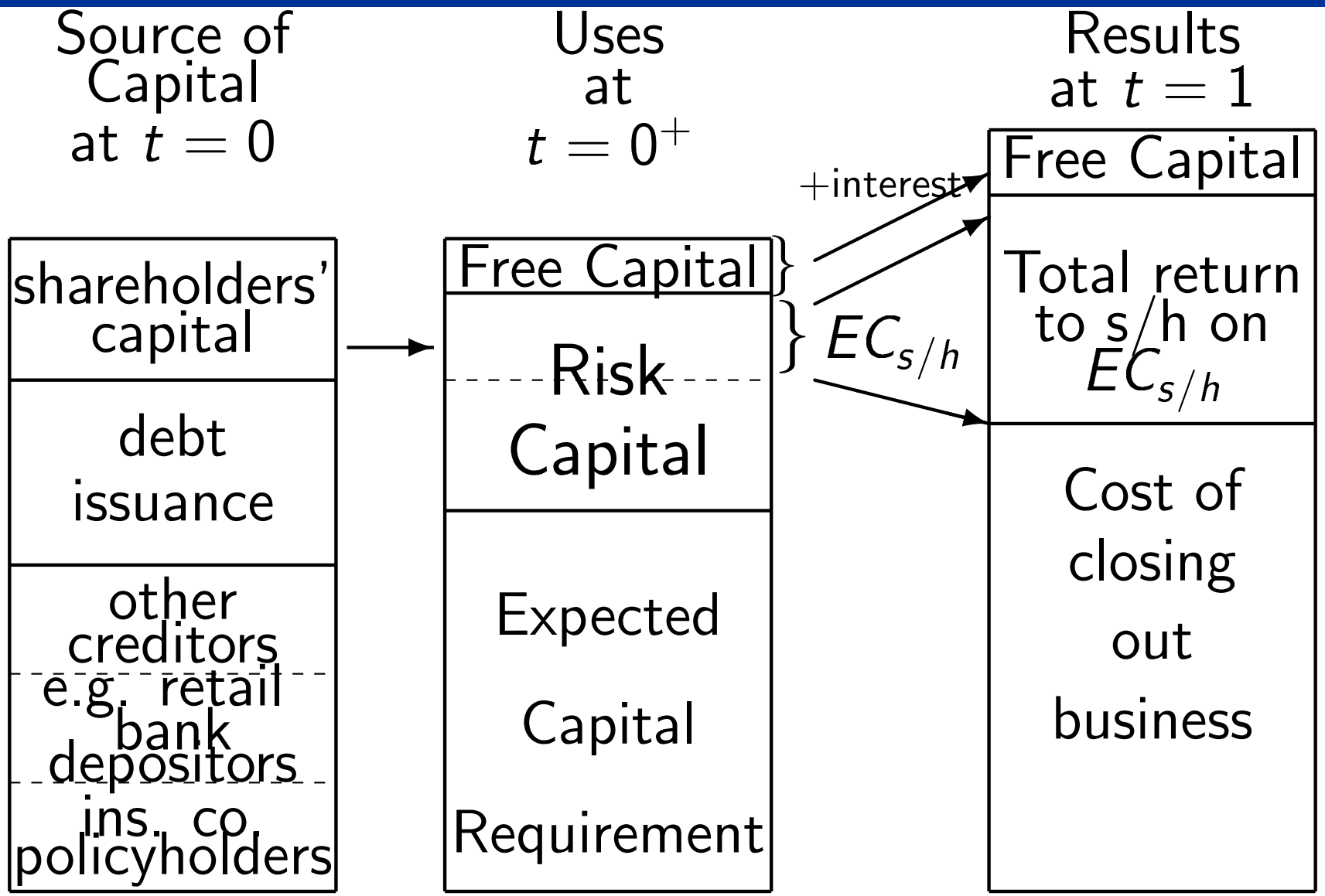
- Improved understanding of balance sheet dynamics
- Frictional and liquidity constraints on business volumes

ECM Applications

- Risk profile (detail + summaries)
- RORAC calculations, Economic Value Added
- Capital allocation
- Capital budgeting
  $\Rightarrow$ allocating scarce capital to profitable lines of business
- Optimisation (e.g. RORAC)
- Mergers and acquisitions
- Product pricing (e.g. supply/demand $\Rightarrow$ volume)
- ALM effectiveness

More detail: IAA Ch7 (+ S&P Economic Capital Models Methodology)

Recap


Source of Capital at $t = 0$
Uses at $t = 0^+$
Results at $t = 1$
shareholders' capital
debt issuance
other creditors
e.g. retail bank depositors
ins. co policyholders
Free Capital
Risk Capital
Expected Capital Requirement
+interest
$EC_{s/h}$
Free Capital
Total return to s/h on $EC_{s/h}$
Cost of closing out business

Final words: Optimisation

- Don't forget the unallocated *Free Capital*
- Recall: alternative to RORAC, r_A

$$EIC = (r_A - r_H)EC$$

where EC = economic capital allocated to risky activities.
Maximise $RORAC \not\Longrightarrow$ maximise EIC

- Alternative: Calculate Group RORAC across all business lines
  PLUS return on free capital
- Remember: multiperiod $\Rightarrow$ free capital is necessary
- Example: Apple – \$ billions in share buybacks to reduce cash surplus

Crouhy, Chapter 17: 4 reasons for using EC/RORAC/CA

(CA = capital allocation)

1. EC $\Rightarrow$ adequate funds to absorb risk
   - unlock potential for profit
   - sources of funds: shareholders + loans/debt (leverage)
     - shareholders
     - Bank $\Rightarrow$ leverage through retail deposits
     - Insurer $\Rightarrow$ leverage through premiums
     - others $\Rightarrow$ leverage through debt issuance
2. Example (risk tolerance):
   EC $\Rightarrow$ capital to ensure target credit rating at $T + 1$
   Good rating $\Rightarrow$
   - ability to do business with net creditors on favourable terms
   - higher volumes of business
   - unlocks further potential future profits

Crouhy, Chapter 17: 4 reasons for using EC/RORAC/CA (cont.)

3. Typically: *my business is opaque to outsiders*
   Therefore: Strong and obvious EC + ERM $\Rightarrow$ signal to external stakeholders concerning my financial integrity
   $\Rightarrow$ S&P evaluation: a good rating helps
4. Strong use of EC technology $\Rightarrow$
   optimal use of available shareholders capital
   KEY reason for doing ERM: *value maximisation*

Summary

- What does Economic Capital Modelling entail?
- Don't forget the unallocated free capital
- What are the benefits of economic capital modelling, and calculating economic capital, risk adjusted performance (e.g. RORAC), and economic capital allocation?