F71TT Risk Management Techniques and Tools

Questions

(*) questions are types of questions that could come up in the exam. (**) questions are ones that **could** be developed into challenging, high-level questions in an exam. Questions with no (*) are ones that reinforce your basic knowledge of the subject and will help you answer questions on a wide range of other topics.

Personal Risk Management

1. (**) Recap on personal risk management:

   You are planning to buy a new car for £20,000. The car comes with a standard 3-year warranty. You have been asked if you would like to pay an additional £1,999 to extend the warranty to 6 years.

   What issues should you consider before deciding if you want the extended warranty or not?

2. Think about what other types of conscious and unconscious risk management you do in your personal lives.

   How much of what you do as an individual can be applied (in a more sophisticated way) to risk management in corporations etc.?

Risk management control cycle:

3. (*) In class you have seen a number of different ERM *control cycles*.

   From these discussions, and bear in mind that there is no single correct control cycle, write down what you see as the key or important elements of a control cycle.

4. (*) Consider the business-as-usual part of the control cycle. The risk management strategy of an insurance company was last reviewed in January 2019. It is now January 2020. Describe why the experience of the last 12 months might require a change in the risk management strategy.

   In your answer you might refer to Solvency II as well as other issues.

5. (*) For each of banking and insurance: write down one or more examples of (a) an emerging risk, (b) a new opportunity, (c) a new tool or technique. Examples might be real examples from the past or hypothetical. In each case discuss why each might result in a revision to the company's risk management strategy.

6. (*) The IAA Note on Risk Management mentions that a newly emerging risk might represent a new business opportunity (rather than emerging losses that need to be capped in other cases). Explain why this might be the case.

Operational risk

7. (**) For each of a bank, insurance company, pharamaceutical company, resort hotel chain and specialist bridge building company, give one example of each of the following types of risk (as in Lam, Chapter 14):

   - Process risk
   - People risk
   - System risk
   - Event risk (low frequency, high impact)
   - Business risk

   [(**) extensions ⇒ propose how to manage these operational risks!]

8. (*) Explain why best practice in operational risk management requires

   - inclusion of business risk and reputational risk;
   - inclusion of operational risk in economic capital allocation to business units;
   - includes proper consideration of the dependencies between certain operational risks and, for example, credit and market risk;
   - integration of the insurance function into the operational risk function.

   Give examples where appropriate.

9. (*) Explain why a combination of operational risk controls and insurance might be optimal for a particular company.

10. (**) Why might outsourcing of some processes or activities be considered appropriate as a method of operational risk management?

    How does outsourcing create operational risk? Give examples.

11. Give an example of an operational risk management activity that costs very little to implement in advance of an operational risk event.

12. Give some examples of KPI's or KRI's (key performance/risk indicators) that could be used to monitor operational risks in a financial organisation and in a manufacturing organisation.

Basel II and Solvency II

13. (*) Basel II: Credit risk

    Suppose a bank is using an internal ratings based (IRB) approach to assess credit risk on its banking books. Refer to the McNeil and Crouhy textbooks for this question.

    Write some brief notes on the different methods that might be employed to assess the risks associated with personal loans (e.g. mortgages) versus loans to large companies.

14. (**) Basel II: Operational risk

    Under the Advanced Measurement Approach (AMA), list the 8 business lines and 7 loss event types.

    (Hint: see `https://www.bis.org/basel_framework/standard/OPE.htm`)

    Which combinations of business lines and loss events are likely to give rise to high frequency, low loss events, and which are likely to give rise to low frequency, extreme loss events?

    Discuss how dependencies arise in the 8×7 matrix of operational risk losses.

    A large bank has good quality operational risk data from its own business from the last 10 years. Why is it still necessary to use external data to assess its overall exposure to operational risk?

15. (*) Basel II: operational risk

    What is meant by *systemic risk*?

    Why might systemic risk be increased if all banks use exactly the same advanced internal model to measure market, credit and operational risk?

    Give an example of where over-reliance of the banking industry on one specific model caused systemic risk.

    Why is it potentially better if different banks use different internal models?

16. (**) Basel II, Pillar 3: Everything else being equal, why would a change from Value-at-Risk to the use of expected shortfall encourage shareholders to pay more attention to excessive risk taking?

17. (*) Explain what is meant by the expression *mark-to-market* and *mark-to-model* valuation of assets and liabilities.

Basel II and Solvency II

18. (*) Solvency II, MCV's:

Let $P(t,T)$ be the risk-free price at t for £1 payable at time T.

Let $S(t)$ be the price at t for a non-dividend-paying equity stock.

Let $\tau(t)$ be the temperature in Edinburgh at time t.

Now consider three liabilities, for $T > 0$:

- £K at T, $= L_1$
- £$S(T)$ at T, $= L_2$
- £$\tau(T)$ at T, $= L_3$.

Which can be given a market consistent value (MCV) at time 0, and which need a model plus further assumptions?

Give reasons for your answers based on the idea of hedgeable and non-hedgeable risks.

19. (*) Solvency II: The Cost-Of-Capital approach.

The MCV is equal to the best-estimate value of a liability, plus a Risk Margin (RM). The RM is equal to the cost-of-capital rate times the expected present value of holding the Solvency Capital Requirements: that is, at time t, with a liability of L due at time T

$$RM(t) = c \times \sum_{s=t}^{T-1} P(t, s+1) E[SCR(s)|\mathcal{F}_t]$$

where $SCR(s)$ is the SCR to be held from time s to time $s + 1$, $\mathcal{F}_t$ is the information available at time t, $P(t,T)$ is the price at t for £1 (risk-free) payable at time T, and c is the Cost-of-Capital rate (e.g. 6%).

(a) (This part is not a question!)

Consider the following simple example to understand how the MCV and the SCR feedback into one another.

Let $P(t,T) = 1$ for all t, T (i.e. zero interest rates for simplicity).

$L(1) = \mu + \sigma Z_1$ is a liability due at time 1, with $Z_1 \sim N(0, 1)$ under the real-world measure, and not known until time 1. We wish to calculate the BE, RM, MCV and SCR for this liability at time 0.

With zero interest rates, $BE(0) = \mu$.

Now let $A(t)$ be the assets at time t. These are all invested in risk-free bonds, so that $A(1) = A(0)$. We require to find $A(0)$ such that $Pr(A(1) > L(1)) = \alpha = 0.995$. We then have,

$$A(0) = BE(0) + RM(0) + SCR(0) = \mu + RM(0) + SCR(0)$$

The requirement $Pr(A(1) > L(1)) = \alpha = 0.995$ tells us that $A(1) = A(0) = \mu + \sigma q_\alpha$, where q_α is the α quantile of the standard normal distribution.

Finally, the CoC approach tells us that

$$RM(0) = c \times P(0,1) \times E[SCR(0)|\mathcal{F}_0] = c.SCR(0).$$

Hence we have:

$$A(0) = \mu + \sigma q_\alpha = \mu + c.SCR(0) + SCR(0)$$

from which we infer that

$$SCR(0) = \frac{1}{1+c}\sigma q_\alpha, \quad \text{and} \quad RM(0) = \frac{c}{1+c}\sigma q_\alpha.$$

(b) Now consider a two-step problem: $L(2) = \mu + \sigma_1 Z_1 + \sigma_2 Z_2$ is a liability payable at time 2. Z_1, Z_2 are i.i.d. standard normal random variables. Z_1 is known at time 1, Z_2 is known at time 2.

 i. Show that $BE(1) = \mu + \sigma_1 Z_1$, and $BE(0) = \mu$.
 ii. Show that $SCR(1) = \sigma_2 q_\alpha/(1+c)$, and $RM(1) = c\sigma_2 q_\alpha/(1+c)$.
 iii. Hence or otherwise, find expressions for $SCR(0)$ and $RM(0)$.

Warning: in contrast to the above example, in practice, $SCR(t)$ will depend on the state of the world at time t, so the calculation will be significantly more complex!

20. (*) In Solvency II, explain briefly the difference between the SCR and the MCR.

21. Discuss in what ways the Own Risk and Solvency Assessment goes beyond the requirements of Solvency II Pillar 1.

22. Why is it important (pillar 2) that an insurer uses the same internal model to calculate the Pillar 1 capital requirements and for day to day use in capital modelling and risk management decision making.

Corporate Governance:

Note: you will be examined on the underlying principles of ERM and corporate governance and on how these might be applied under a variety of scenarios. You will not be assessed on memorised details of the contents of the various documents in the required reading.

23. (**) List all of the stakeholders in a life insurance company that sells (a) basic life insurance contracts, (b) equity-linked savings contracts with an element of life insurance, (c) pensions.

    Identify who the main stakeholders are, and, for these, discuss their ERM priorities.

    Describe some conflicts that might arise over ERM policy between different groups of stakeholders. How can these conflicts be mitigated through good corporate governance?

24. Risk Dashboard and Daily Reports

    What is the danger of placing too much emphasis on the top 10 risks?

25. Use Google Images to search for "Risk Dashboard".

    From the results, select 3: discuss and explain the individual features of the dashboard, and what you think is good or bad about each dashboard. In each case, how much expertise and experience of the dashboard do you need to get maximum benefit.

    Summarise what do you see as typical elements of a risk dashboard. Is anything missing that you might have expected to see?

26. (*) Write some brief notes on the UK pensions mis-selling scandal (1988-1994). Explain why this was a *compliance* issue.

27. Why is it important to get diversity of backgrounds amongst the membership of a business's Risk Committee and Risk Function?

28. (**) Risk committees: the Vision learning materials web page has links to Lloyds Bank and Standard Life.

    Read through the Lloyds Bank and Standard Life terms of reference (ToR) for their Risk Committees, and also the IAA Practice Note (Chapter 2 and Appendix 4).

    Write down and discuss the key elements of the ToR's. Why do you think they are important?

    Do the terms of reference make clear which types of risk the Committee should address and which should be left out for consideration elsewhere (e.g. the Audit Committee)?

Which elements of the two documents are the same and which are different?

Do you see differences that reflect the different natures of a bank and a life insurer?

29. (*) Consider the different stakeholders in an ERM context for an insurance company. *Compliance* might be considered as being a constraint for some stakeholders and being beneficial to other stakeholders.

    Which stakeholders are the beneficiaries and why?

30. (**) A motor insurer sells its policies through individual branches and has the following bonus structure in place:

    - Targets have been set for individual branches to meet for numbers of new insurance policies sold.

    - Branch managers get a 50%-of-salary bonus if their branch exceeds its target.

    - Other employees in the branch are on a well-defined salary scale with no bonuses. Each year they will move one point up the scale unless they reach the top of their current grade or are promoted.

    Discuss the advantages and disadvantages of this pay structure.

31. (**) A hedge fund has a bonus structure that pays a bonus of 100% of basic salary if the return on their fund exceeds 10% in a given year.

    Discuss the advantages and disadvantages of this bonus structure.

    How might it be improved?

32. (**) Heat Maps

    For earthquake risk, risk management activities might only be able to move the position of the risk in a heat map down (i.e. less impact), but not to the left (reduced likelihood). Explain why this is.

    For a particular form of operational risk, an objective has been set which, if successful, will move the position of the risk in the heat map both down and to the left. Explain how this might happen.

    Why might a risk management action result in the risk in a heat map moving to the left and <u>up</u>? Give an example.

ERM Frameworks and Assessment

33. (*) Quoting from the S&P document (handout, page 4):

    Although the views of company ERM quality have fixed definitions, the exact practices that will be viewed as excellent, strong, adequate, and weak would vary according to the risks of the company. For example, a company with high concentrations of coverage with a single-event probable maximum loss that is a high percentage of surplus would be expected to have a diversified reinsurance program to be considered to have strong ERM, while another company without such concentrations might not need the same sort of reinsurance for its ERM to be considered strong.

    (a) Explain briefly what reinsurance involves.
    (b) Explain why the first company's reinsurance programme should be *diversified*?

34. (*) Refer back to the slide on the S&P 6 indicators of Risk Control.

    For each of the 6 indicators, discuss where each fits into the control cycle.

35. The Sarbanes-Oxley Act (2002) (USA) (see, for example, Crouhy et al. (2006/2014), or other internet resources):

    (a) What events led to the proposal of this law in the USA?
    (b) Does the law deal with risk management in general or some specific alternative issues?
    (c) Do you think of the Sarbanes-Oxley Act as being a compliance issue or a value-creation issue for companies?
    (d) Describe the key *principles* that underpin the Act.
    (e) What elements of ERM does the Act contribute to?

36. (**) Why might staff turnover (the percentage of staff who leave each year) be used as a key performance indicator for monitoring the culture within an organisation?

37. (**) A company has had its ERM practices evaluated by S&P.

    The following statements appear in the final report:

    - *A small number of technical specialists understand the basis of the risk measures and risk-management programs and have not been able to communicate that understanding to management.*
    - *Insurer avoids mention of unpleasant experiences. Management response to a loss situation is often to make sure that it will not repeat that very specific mistake.*

- *The ERM function is staffed by knowledgeable and experienced professionals. However, there are insufficient staff generally to manage the overall workload and specifically too few staff to develop and maintain the company's internal model.*

What rating would you associate with each of these statements?

If you are the CRO, what actions would you take in reaction to these statements?

Risk Appetite

38. Use the internet or other resources to investigate what information is available publicly on risk appetite and risk tolerance in specific banks and insurance companies.

    Are public statements typically quite explicit or are they quite vague?

39. (*) Consider the IRM article on "Risk appetite and tolerance" (class handout and also PDF on Vision).

    Explain how the way that the article distinguishes between the risk universe, risk tolerance and risk appetite links into slide 11 of the Risk Appetite Slides.

40. (**) At the start of 2019, an insurer had a credit rating of AA. At the same time it had a risk appetite statement that it aimed to have a minimum rating of AA at the end of 2019 with probability at least 0.95.

    During 2019 adverse claims experience meant that the financial position deteriorated and the rating changed from AA to A.

    Should the insurer review its stated risk appetite?

41. (*) Explain what is meant by an "unrewarded risk"?

    Give an example of a statement of risk appetite or risk tolerance that concerns unrewarded risk.

42. (**) An insurer is considering engaging in a new line of business.

    - It will be possible to sell N policies for a premium of P per policy.
    - At the end of the year, each policy will incur a loss of $L_i = 1000 + X + Y_i$ where X is independent of the Y_i, and $Y_1, \ldots, Y_N$ are i.i.d random variables with mean 0 and variance σ_Y^2. X takes the value 0 with probability 0.5 and 200 with probability 0.5.
    - The Y_i are non-traded risks.
    - A financial contract exists in which an investor can pay an amount x now per unit and receive the uncertain payment of X per unit in one year. The current market price (buying and selling) for this contract is $x = 110$.

    (a) Discuss whether or not you think that X is an unrewarded risk. [There are two perspectives: accepting the risk, X in the first place as one component of each L_i; and then selling on the risk X.]

    (b) Explain, with reference to P, why the insurer might engage in this type of business. How might the traded market help? (You might assume that the risk-free rate of interest, r, is zero to help with any mathematical arguments that you might develop.)

43. (**) The over-riding objective for a company is to maximise shareholder value.

    At present a company is rated AAA, and has a risk tolerance statement to maintain the AAA rating with a probability of at least 0.95.

    Why might this risk tolerance statement conflict with the objective to maximise shareholder value?

Economic Capital; RORAC; Capital Allocation

(Questions identified as [HR*x.y* indicates that this comes from Hull's book *Risk Management*, Chapter *x*, question *x.y*.)

44. (**) Consider a stock-market-listed insurer that sells a variety of life insurance contracts.

    The company is developing its economic capital model.

    Describe as many *management decisions* as possible that can be based upon this model. (Hint: use this question to help focus your mind when working through the recommended reading for this chapter.)

45. (*) An insurer operates two lines of business. Line 1 has a RORAC of 15% while line 2 has a RORAC of 11%. The company's overall RORAC is 12.5%.

    What recommendations would you make concerning the company's business operations?

    How would your recommendations differ if the company has no excess risk capacity?

46. (*) A retail bank has £2 billion of retail deposits. This has been used to make £2 billion of loans to corporate customers to be repaid in one year's time.

   - Retail depositors will be paid 3% interest at the end of the year.
   - Corporate loans all attract an interest rate of 7.5% per annum.
   - The gross yield on 1-year government bonds is 5% per annum.
   - The bank has an effective tax rate of 30%.
   - Expected operating costs are £13 million.
   - Expected losses as a result of default on corporate loans is £60 million.
   - The bank has determined that it needs £60 million of shareholder's economic capital.

   (a) Develop an Excel spreadsheet to analyse this problem.
   (b) Calculate the RORAC on this business.
   (c) What would the shareholder's economic capital have to be to achieve a RORAC of 25%?
   (d) What other actions could the bank take to increase its RORAC?
   (e) Now suppose that, instead of the above formulation, the default losses, L, have an Exponential distribution with mean 60.

   Define the Surplus, S, as the total available for return to shareholders before tax is calculated.

       i. What is $Pr(S < 0)$ if the initial economic capital is $EC_0 = 60$?
       ii. How much should EC_0 be to ensure that $Pr(S < 0) = 0.05$?
       iii. With this revised EC_0 what is the (prospective) RORAC assuming default losses equal their expected value?

47. (*)[HR23.1] What is the difference between economic capital and regulatory capital?

48. (*) RORAC can be calculated in two different ways. Compare and contrast these approaches.

49. (**) A retail bank has retail deposits of £5 billion that earn 2% per annum interest.

   - Cash can be invested in 1-year government bonds that earn 3.5% per annum interest.
   - The bank has made loans to two types of corporate costomers:
     - £2.7 billion to low risk borrowers

– £2.1 billion to high risk borrowers

The remainder of the deposits is invested in government bonds, with the government being considered as a third type of corporate borrower.

- Low risk borrowers pay an interest rate of 7% per annum.
  High risk borrowers pay an interest rate of 11% per annum.

- Allocated capital (AC) is 1.05 times lent capital for low-risk borrowers, 1.1 times lent capital for high-risk borrowers, and 1.0001 times lent capital for the government loan.

- Expenses and costs are 1.5% of lent capital plus: 0 for the government loan; £20 M for the low-risk loans; £30 M for the high-risk loans.

- Expected losses are: 0% of capital for the government loan; 2% of capital for the low-risk loans; 5% of capital for the high-risk loans.

- The bank pays 25% tax on gross profits.

(a) Show that the Economic Capital to be put up by the shareholders is £0.02 M for the government loan, £135 M for the low-risk loans, and £210 M for the high-risk loans.

(b) Calculate the gross profit on each of the loan books.
    Calculate the taxes on each, and finally the RORAC on each.

(c) Calculate the RORAC on the total economic capital put up by the shareholders.

(d) How does this investment compare with shareholders simply investing in government bonds?

(e) Based on your results, how should the bank develop its loans business?

50. (**) A company operates two lines of business, $i = 1, 2$. As stand alone units the two lines require risk capital of £100 million and £75 million respectively. These have been calculated using a full internal model for each unit.

The correlation between the risk in the two business units is 0.5.

For both lines of business, other capital is equal to the best estimate liabilities at time 0. This means that the Risk Capital is equal to the Economic Capital at the group level.

(a) Use the Hybrid Approach to calculate the group Risk Capital.

(b) Use the Euler method to allocate the group risk/economic capital to each of the business units.
    Hint: Group $RC = f(x_1 RC_1, x_2 RC_2)$ where, in part (a), $x_1 = x_2 = 1$. Use this representation plus the Euler Method to split $EC = RC$ into $AC_1 + AC_2$.

51. (**) A company operates n lines of business, $i = 1, \ldots, n$ with liabilities of L_i for line i. They company uses a standard deviation risk measure, risk capital, $RC = \rho(L) = 2\sigma_L$ where σ_L is the standard deviation of the group liability $L = L_1 + \ldots + L_n$. Also let $\sigma_{ij} = Cov(L_i, L_j)$ and recall (revision!) that $\sigma_L^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}$.

    Show that, in general, the proportional capital allocation principle gives different results to the covariance capital allocation principle.

F71TT Risk Management Techniques and Tools

F71AP Applied Risk Management

Market risk modelling and measurement

52. Write brief notes on the following graphical diagnostic tools:

    (a) Histograms with fitted density overlaid

    (b) Time series plots

    (c) Autocorrelation functions

    (d) QQ plots with different theoretical distributions

    In each case describe how each graphic can be plotted, and discuss how each can be used to detect evidence for stochastic volatility in a financial time series.

53. Look at the chapter 8 slides, slide 21 "QQ-Geometric plot".

    Explain why clusters of high volatility and clusters of low volatility in the source data results in the convex shape in this QQ plot.

54. (a) Revise what you know about (or learn for the first time!) the runs test for the independence of a sequence of observations $X(1), \ldots, X(n)$.

    State the null hypothesis. Describe how the test statistic is calculated. State how the test statistic is used to assess if the null hypothesis should be rejected.

    (b) From a graphical analysis of a given dataset $X(1), \ldots, X(n)$ it looks like (a) they have a median of zero and (b) little evidence that the sign of $X(t)$ can be used to improve your prediction of the sign of $X(t+1)$. However, there is possible evidence for clustering of large absolute values of $X(t)$ and clusters of small absolute values.

    Show how the runs test can be adapted to draw out this second feature (clusters).

55. Consider the GARCH(1,1) model

$$\delta(t) = \mu + \sigma(t)Z(t) \tag{1}$$
$$\text{where } \sigma(t)^2 = \alpha_0 + (\alpha_1 Z(t-1)^2 + \beta_1)\sigma(t-1)^2 \tag{2}$$

and $Z(1), Z(2), \ldots$ is a sequence of i.i.d. random variables with $E[Z(t)] = 0$ and $Var[Z(t)] = 1$. $\sigma(1)$ has been specified, and $\alpha_0 > 0$, $\alpha_1 > 0$ and $\beta_1 > 0$.

    (a) Suppose that $\alpha_1 + \beta_1 < 1$. By taking expectations of the left and right-hand sides of equation (2) prove that $E[\sigma(t)^2] = \alpha_0/(1 - \alpha_1 - \beta_1)$. Hence show that $Var[\delta(t)] = \alpha_0/(1 - \alpha_1 - \beta_1)$.

(b) By following a similar line of argument, find a formula for $E[(\delta(t) - \mu)^4]$, giving constraints of the GARCH(1,1) parameters for its existence.

Hence find a formula for the unconditional kurtosis of $\delta(t)$.

Is the unconditional kurtosis of $\delta(t)$ higher or lower than the kurtosis of the $Z(t)$?

56. Consider a time series of data $\{X(1), \ldots, X(n)\}$.

It has been proposed that the $X(t)$ are i.i.d. which you set up as your null hypothesis.

(a) You have estimated the autocorrelation function $\rho_1(k) = cor(X(t), X(t + k))$ and have found that the $\rho_1(k)$ are not significantly different from zero for $k = 1, 2, \ldots$.

What can you conclude from this analysis?

Does this prove that the $X(t)$ are i.i.d.?

(b) What other simple diagnostic tests can you look at to investigate whether or not the $X(t)$ are i.i.d.?
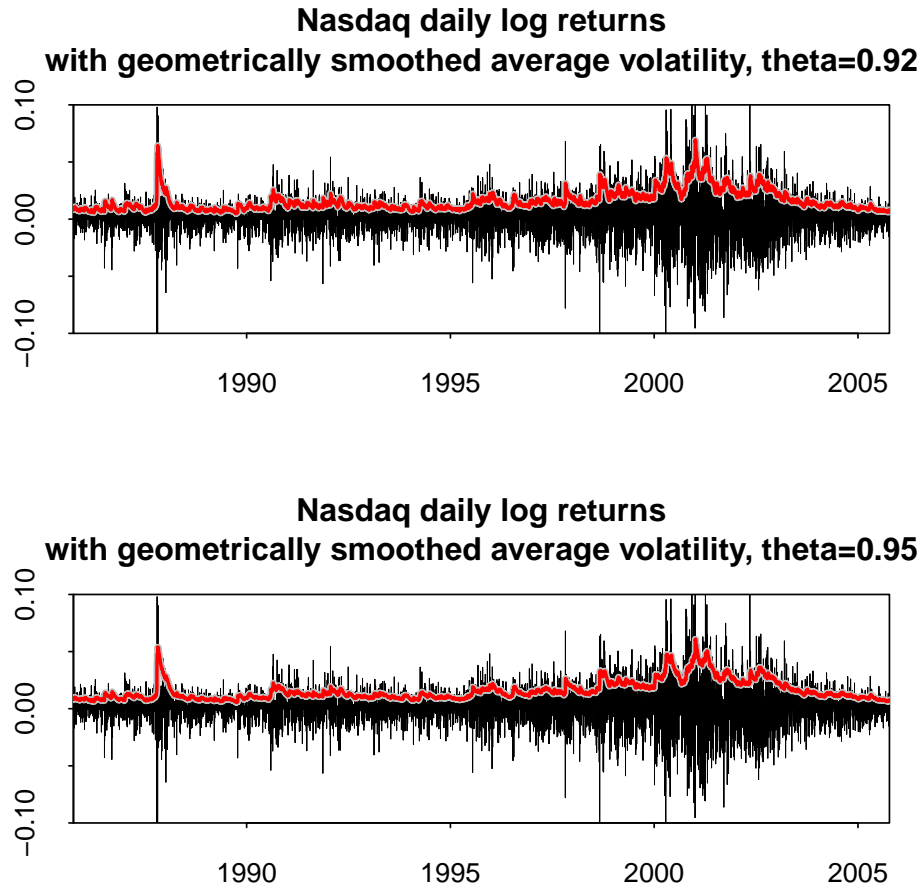
Figure 1: Nasdaq daily log returns with estimated volatility superimposed. Smoothing parameter is $\theta = 0.92$ (top) or $\theta = 0.95$ (bottom).

57. In Figure 1 we have plotted the daily log returns $\delta(t)$ for the Nasdaq index. The estimated squared volatility is

$$v(t) = \sigma(t)^2 = \theta v(t-1) + (1-\theta)(\delta(t-1) - \mu)^2.$$

In the upper and lower plots in Figure 1 we have plotted the estimated $\sigma(t) = \sqrt{v(t)}$ on top of the daily log returns for $\theta = 0.92$ and $\theta = 0.95$.

Based on these two plots, why might you conclude that the parameter θ is not important for forecasting the distribution of $\delta(t)$ over short periods of time?

58. Theoretical QQ plots for positive random variables.

    Let X and Y be continuous random variables distributed on $[0, \infty)$ with continuous and differentiable densities $f(x)$ and $g(x)$ and CDF's $F(x)$ and $G(x)$ respectively.

    Suppose that $f(x) \to f_0 > 0$ and $g(x) \to g_0 > 0$ as $x \to 0$.

    The QQ plot is a plot of $(x, G^{-1}(F(x)))$ for $x > 0$.

    What is the gradient of the QQ plot as $x \to 0$?

59. In Figure 2 we have plotted daily log returns data, $X(t)$, for a stock-market price index over a period of 5000 trading days, along with the estimated GARCH(1,1) (quasi MLE) volatility.

    It has been suggested that the daily log returns are i.i.d..

    (a) Do you agree with this statement?

        Give reasons: why do you agree or disagree?

    (b) What tests could you apply to verify this hypothesis or otherwise?

    (c) Consider the (exceedance) times $T_1 < T_2 < \ldots < T_m$ that $X(t)$ exceeds a threshold equal to its 95% quantile, q_{95}:

    $$\{T_1, T_2, \ldots, T_m\} = \{t : X(t) > q_{95}\}.$$

    The inter-exceedance times are then $S_k = T_{k+1} - T_k$ for $k = 1, \ldots, m-1$.

    The following code in R calculates these times, takes their differences and then does a QQ plot using theoretical geometric quantiles. It produces the QQ plot in Figure 3.

    ```
    # x is the vector of daily log returns
    > nv=1:5000 # full vector of dates
    > q95=quantile(x,0.95)
    > tv=nv[x > q95] # extract the dates on which x exceeds q95
    > sv=diff(tv) # vector of interexceedance times
    > qqgeometric(sv)
    > abline(0,1)
    ```

    What does this QQ plot tells us about the frequency of very small and very large inter-exceedance times?

    (d) The command `abs(x)` will return the absolute values of the vector `x`. How would you amend the R code above to calculate inter-exceedance times where the threshold is that $|X(t)|$ is greater than its 95% quantile?
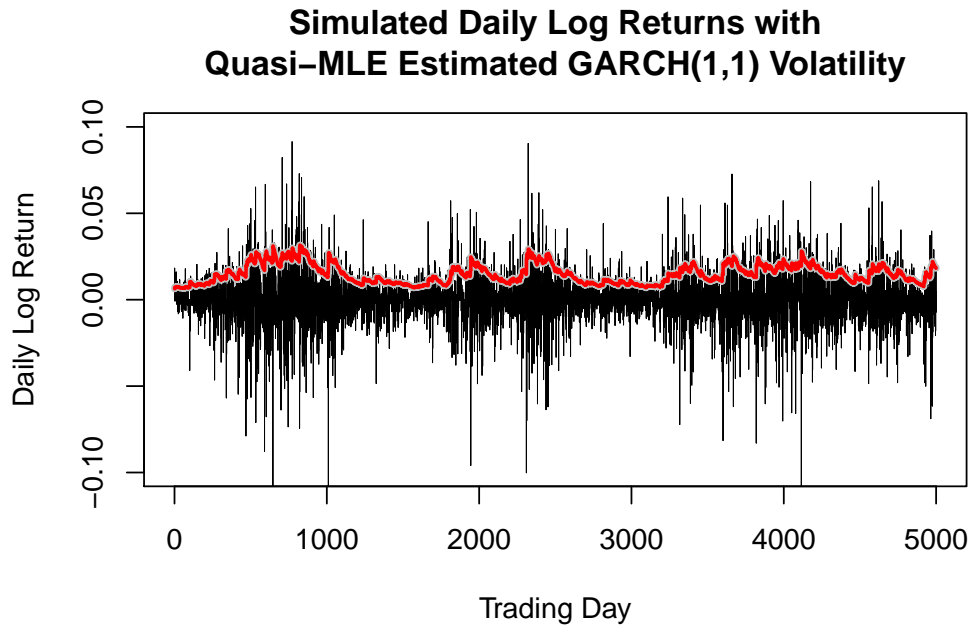
**Simulated Daily Log Returns with
Quasi–MLE Estimated GARCH(1,1) Volatility**

Figure 2: Simulated daily log returns with estimated GARCH(1,1) (using quasi-MLE) volatility superimposed.
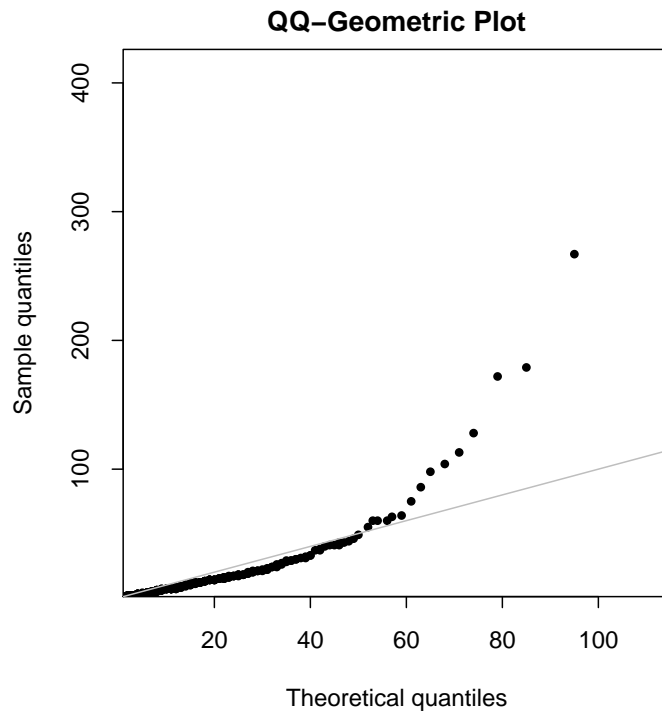
QQ–Geometric Plot

Figure 3: QQ plot for the times between exceedances by the daily log returns of the 95% quantile.

60. Consider Method 2 in the out-of-sample backtesting component of model validation (see the study note on Model Validation on Vision, Section 1.1). This produces a sequence of processed out-of-sample observations $U(1), \ldots, U(T)$.

   (a) Assuming that the model you are re-fitting to the historical data on a daily basis is correct (i.e. the null hypothesis), what distributional properties should the $U(1), \ldots, U(T)$ have?

   (b) Describe some graphical diagnostic tests and formal hypothesis tests that you might use to verify the properties outlined in (a).

   (c) As a graphical diagnostic, what should a scatterplot of $U(t)$ versus $U(t+1)$ for $t = 1, \ldots, T - 1$ look like?

   (d) For a financial time series and GARCH model in particular, why should out-of-sample backtesting involve a relatively long run of data (i.e. several years)?

   (e) Consider the following graphical diagnostic:

   - Calculate the out of sample $U(t)$ as above in (a) along with the corresponding estimated, one-day-ahead $\sigma(t)$'s.
   - Let $R(t)$ be the rank of $\sigma(t)$ out of $\sigma(1), \ldots, \sigma(T)$. (So $\sigma(t)$ is the $R(t)^{th}$ largest out of the T estimated volatilities.)
   - Do a scatterplot of $R(t)$ versus $U(t)$.

   i. If the model you have used is accurate, what should this scatterplot look like?

   ii. In the standard GARCH(1,1) model it is assumed that the $Z(t)$ are *identically* distributed (as well as independent). Suppose, instead, it is suspected that the distribution of $Z(t)$ depends on the value of $\sigma(t)$. (For example, $Z(t)$ still always has a t distribution with mean 0 and variance 1, but where the degrees of freedom depend on $\sigma(t)$.) What would this impact on the scatterplot of $R(t)$ versus $U(t)$?

61. Why is it important to present clearly to users of a model, a model's assumptions and limitations?

Market risk management

Questions with [Hx.y] refer to exercises in Hull's textbook, 6th edition, chapter x, question x.y.

62. # [H15.2] What does it mean to assert that the Delta of a call option is 0.7? How can a short position in 1,000 options be made Delta neutral when the Delta of each option is 0.7?

63. # [H15.5] What is meant by the Gamma of an option position? What are the risks in the situation where the Gamma of a position is large and negative, and the Delta is zero.

64. (*) What are the Delta, Vega and Gamma of a long position of 2 futures contracts on silver with 3 months to maturity? The current spot price for silver is $35,000 per 5,000 ounces, and the risk free rate of interest (non-stochastic) is 5% per annum continuously compounding.

65. (**) A derivatives desk has the following positions on the FTSE-100 index:

Name	Type	Position	Option Delta(*)	Option Gamma(*)	Option Vega(*)
A	Call	-5,000	0.50	2.7	1.80
B	Call	2,000	0.85	0.6	0.20
C	Put	-1,000	-0.50	1.3	0.75
D	Put	-500	-0.50	4.3	0.20

((*) Quoted Deltas, Gammas and Vegas are for one option.)

In addition the manager can invest in the index itself and in cash.

(a) Calculate the total Delta, Gamma, and Vega for the portfolio.

(b) Show how to make the portfolio Delta neutral by investing in the FTSE-100 index.

(c) The desk manager wishes to make her position Delta and Gamma neutral. Show how this can be done by adjusting the holdings in derivative D and in the index itself.

Calculate the revised Vega for this portfolio.

(d) What would the manager need to do to make the portfolio Delta, Gamma and Vega neutral?

(Numbers are not required here.)

(e) Describe (without doing further calculations) some stress tests that could be carried out to further assess the risk on the portfolio.

66. (*) A derivatives desk has the following positions on the S&P-500 index:

Name	Type	Position	Option Delta(*)	Option Gamma(*)	Option Vega(*)
A	Call	-4,000	0.40	2.3	1.40
B	Put	-2,000	-0.75	1.5	1.50
C	Call	1,000	0.75	0.5	0.80
D	Put	700	-0.20	3.5	1.35

((*) Quoted Deltas, Gammas and Vegas are for one option.)

In addition the manager can invest in the index itself and in cash.

Towards the end of the trading day the portfolio manager needs to ensure that the portfolio is Delta, Gamma and Vega neutral.

Show how this can be achieved by altering the number of units of options C and D and through investment in the index itself.

How would your answer differ if you were investing in futures on the S&P-500 index instead of the index itself?

67. (*) Consider the Market Risk Slides, slide 19.

(a) A portfolio of derivatives linked to a single underlying, $S(t)$, has value $V_\pi(t)$. The portfolio has n derivative positions of various types with a variety of strike prices and maturities. Derivative i has implied volatility, σ_i.

Explain how the implied volatility is calculated.

Suppose now that all implied volatilities are going to have a constant λ added (Slide 19, left-hand plot), so that σ_i is replaced by $\sigma_i + \lambda$ for all i.

Explain how the portfolio Vega might be calculated for this type of shift in implied volatility.

(b) Discuss how Vega hedging might be adapted to deal with tilts in the volatility smile as in the right hand plot in slide 19.

68. (**) An insurer has three lines of business with losses on each denoted by L_1, L_2 and L_3.

(a) Each line has had its own risk assessment and we have:

$$S.D.(L_1) = 100, \quad S.D.(L_2) = 400, \quad \text{and} \quad S.D.(L_3) = 900.$$

The regulator has stated that

$$cor(L_1, L_2) = 0.8, \quad cor(L_1, L_3) = 0.2, \quad \text{and} \quad cor(L_2, L_3) = 0.5.$$

Calculate the standard deviation (under the regulations) of $L = L_1 + L_2 + L_3$.

(b) Discuss relevant issues in how to set the correlations. For example: what if your assets and liabilities are quite unlike other insurers; what if losses on your three lines have a non-Gaussian dependency structure?

69. (**) A manufacturer of pipes is exposed to market risk associated with the market price of copper. The quantity of copper pipes to be manufactured varies from month to month in response to demand for their product.

   (a) Discuss how the manufacturer might manage their exposure to this risk over the next 12 months through commodities derivatives exchange.

   (b) Explain why the manufacturer might have to accept some degree of basis risk.

Interest rate risk management

\# Questions with [Hx.y] refer to exercises in Hull's textbook, 6th edition, chapter x, question x.y.

70. Give two descriptions of what *duration* means in relation to a sequence of liability cashflows.

71. Explain in your own words how delta hedging differs from duration matching.

72. (*) Consider the HJM model on slide 31 of the Slides (Interest Rate Risk Management)

$$df(t,T) \;=\; \alpha(t,T)dt + \sum_{i=1}^{m} \sigma_i(t,T)dW_i(t)$$

where the $W_i(t)$ are independent standard Brownian motions under the risk-neutral measure Q, and compare this with simple risk management model on slide 28,

$$
\begin{aligned}
t &\;\rightarrow\; t + \Delta t \\
R(t,T) &\;\rightarrow\; R(t,T) + \sum_{i=1}^{m} x_i g_i(T-t) \\
x_i &\quad \text{uncertain, } i = 1, \ldots, m: \quad x = (x_1, \ldots, x_m)' \\
g_i(T-t) &\quad \text{known}
\end{aligned}
$$

   (a) Identify the key differences between the two approaches.

   (b) Now suppose that $m = 1$, so we just have a single factor HJM model. For a general volatility function, $\sigma(t,T)$ (i.e. one factor), identify from Hull's textbook the required condition on the drift, $\alpha(t,T)$, for the HJM model to be arbitrage free.

73. (*) A trader has identified that a 10-year government bond seems to be underpriced. As a consequence, he has taken up a leveraged position that is long in the 10-year bond and short in 5-year and 15-year bonds.

   What sort of stress tests would you suggest to assess the riskiness of the position being taken?

74. (**) Outline how an interest-rate futures contract works.

    Outline how an interest rate swap works.

    An insurer holds a portfolio of fixed interest bonds that has the same market value as a portfolio of fixed liability cashflows. The insurer anticipates that the spot rate curve, $R(t, T)$, will be subject to changes in level, slope and curvature proportion to the functions $g_1(T - t)$, $g_2(T - t)$ and $g_3(T - t)$ respectively. Discuss how interest rate derivatives can be used to delta hedge its current position.

75. (**) What is the maximum maturity in the UK for government issued fixed-interest bonds (gilts).

    An annuity provider has fixed liability cashflows that run as far out as 70 years from now. The company has a very low appetite for risk. Discuss the key risk management issues for this company.

    Propose a risk management strategy for dealing with the key risks.

Other risk management approaches

76. Describe as many approaches as possible that reduce the level of risk that a company is exposed to without transferring it. [For example, avoid writing high risk insurance products.]

    Your examples can be drawn from insurance, banking, investment and non-financial enterprises.

Other questions

77. A growing multinational hotel chain has had its risk management policies and processes reviewed by an external consultancy. The final report includes the following statement:

    The company's monitoring of major financial risks is usually consistent and of good quality, but its monitoring of project risks is intermittent and of variable quality.

    What S&P ERM classification (i.e. weak, adequate, strong or excellent) would you give based on this statement? Give reasons.

    If you are the CRO, what actions (if any) would you take in reaction to this statement?

More challenging questions or revision

78. Advice on exam technique:

    - At the start of the exam assess approximately how much time you have for each question and try to stick to that schedule. Build in some time at the end to go back to any question that you feel might benefit from a bit more attention, or to act as a buffer in case you overrun on a specific question.

    - Often in ERM exams there will be more than one right answer. Some will be more right than others so you might get good credit but not full marks e.g. for a proposed risk mitigation strategy if it is adequate but could clearly be better. However, do not spend time agonising over whether or not there might be a better solution than the one that you have spent the last 10 minutes writing down. MOVE ON and only return to this question if you have time at the end.

    - Use the number of marks available as a guide to how much you should write.

79. Consider a multinational shipping company that transports a variety of manu-factured goods around the world. It is considering expanding its fleet of cargo ships to meet increasing demand for its services.

    Discuss in detail the major categories of risk that this company is exposed to.

    What should be the key elements of the company's operational risk strategy.

    Why is it important to ensure consistency as much as possible throughout the organisation?

    Why will it be necessary for some inconsistencies to arise?

80. The Advanced Measurement Approach under Basel II deals with 8 business lines and 7 operational risk categories:

	Business Line
1	Corporate finance
2	Trading and sales
3	Retail banking
4	Commercial banking
5	Payment and settlement
6	Agency services
7	Asset management
8	Retail brokerage

	Event types
1	Internal fraud
2	External fraud
3	Employment practices and workplace safety
4	Clients, products and business practices
5	Damage to physical assets
6	Business disruption and system failures
7	Execution, delivery and process management

(a) Use the internet or other sources to give brief descriptions of each of the 8 business lines.

(b) Give brief descriptions of what is meant by each of the 7 event types.

81. .[IFA, Sept 2011]

The central risk team of a global insurance company is preparing for the introduction of a new risk based regulatory regime. The Chief Risk Officer for the insurance company has responsibility for preparing a set of new risk policies.

Outline the areas that should be covered in each risk policy. [4]

82. .[IFA, Sept 2011]

Investment Bank A is listed on the national stock exchange. It has grown rapidly in recent years and has operations around the world. It uses outsourcing arrangements to provide a large part of its services.

It has just appointed a new external director, who has commented that although the bank has implemented a risk management framework, it does not appear to have a very strong risk management culture.

(i) Discuss this comment. [4]

(ii) Propose ways in which the management of the bank could seek to improve the risk management culture. [5]

Life Insurer B is a domestic, unlisted life insurance company that sells retirement savings products. Investment Bank A is looking to acquire Life Insurer B in order to boost the scale of its investment management operations.

(iii) Outline how Lam's seven key lessons learned can assist the Board when integrating the risk management frameworks of the two businesses. [7]

[Lam's 7 key lesson's learned are:

- 1: Know your business
- 2: Establish checks and balances
- 3: Set limits and boundaries
- 4: Keep your eye on the cash

- 5: Use the right yardstick
- 6: Pay for the performance you want
- 7: Balance the ying and the yang]

[Total 16]

83. .[IFA, Sept 2011]

Over the last few years the price of aviation fuel has generally been increasing, as has the volatility of the price. In response, UK based Snooze Air plc purchased futures contracts listed on the New York Mercantile Exchange relating to the company's forecasted usage of aviation fuel over the following 18 months.

(i) Describe why the company has purchased futures contracts. [2]

(ii) Describe the issues that might arise in relation to this mitigation strategy. [4]

[Total 6]

84. .[IFA, Sept 2011]

ABC is a life insurance company. It sells two product types, namely immediate annuities and term assurances, both of which are backed by portfolios of government bonds.

The two products are sold through different distribution channels. Recognising that the key risk exposures are quite different for each product (longevity and mortality respectively), ABC also has separate administration and back office functions for each product.

ABC has recently begun developing an economic capital model. It believes that these two products to some extent provide a natural hedge and it would like to reflect this in the model.

(i) Explain the role of diversification in measuring economic capital requirements. [2]

ABC has calculated the economic capital requirement for annuities and term assurances on a standalone basis to be £5.5 million and £2.25 million respectively. ABC has estimated that the correlation between the two blocks of business is 0.25.

(ii) Estimate the combined capital requirement and diversification benefit for the business, using a simple correlation matrix approach. [4]

(iii) Suggest an alternative modelling approach that could be used to recognise the natural hedge between the two product lines. [1]

The Business Development Manager has suggested that a third product line would help to increase the relative size of the diversification benefit available

and has suggested selling a savings product with a guaranteed investment return.

(iv) Discuss this proposal. [5]

[Total 12]

85. .[IFA, Sept 2011]

Traditionally, most companies state that their primary objective is to increase profits from year to year. A well-known economist has recently said that, in most cases, a company's stakeholders would be better served if the company's primary objective was to operate at the limit of its stated risk tolerance. Discuss this comment. [12]

86. *This next question is a Do-It-Yourself (DIY) question. Work in groups to develop some new scenarios (see, e.g., question 87) and then use this scenario to answer the following question.*

Make a detailed proposal for a firm's risk appetite and risk tolerances in a given scenario.

Ensure that quantitative statements are internally consistent.

87. Some years ago a large insurance company set up a UK-based subsidiary that only sells annuity products to people aged 60 and over. The market consistent value of the subsidiary's liabilities is £150 million. The subsidiary has outsourced its investments to a specialist fund manager. The fund manager has been given an investment mandate to invest in investment grade corporate bonds (AAA to BBB rated) with a specified range for each credit class (e.g. 30-40% should be invested in AAA bonds). If individual bonds are rerated and the portfolio strays outside its permitted limits then the fund manager has one week to restore the portfolio to a position within permitted bounds.

Discuss all of the risks to which this subsidiary is exposed.

Propose how the subsidiary might manage these risks including any amendments to the strategy described above.

88. Other DIY questions:

Many of the ERM 2 questions this semester have been marked (**).

In groups discuss how these questions might be developed further into more challenging exam questions, and discuss in groups what might constitute a strong answer!