Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 1 Solutions

1.

$$
\begin{aligned}
L &= 100,000v^{K_x+1} - P_x\ddot{a}_{\overline{K_x+1}|} = 100,000v^{K_x+1} - P_x\left[\frac{1-v^{K_x+1}}{d}\right] \\
&= v^{K_x+1}\left[100,000 + \frac{P_x}{d}\right] - \frac{P_x}{d}.
\end{aligned}
$$

We have, for ${}^2A_x$ denoting the value of A_x calculated at rate of interest i^2+2i,

$$
S.D.[L] = \sqrt{\mathrm{Var}[L]} = \left[100,000 + \frac{P_x}{d}\right]\sqrt{\mathrm{Var}[v^{K_x+1}]} = \left[100,000 + \frac{P_x}{d}\right]\sqrt{({}^2A_x - A_x^2)}
$$

2. (a) Let $\bar{P}_x = \bar{A}_x/\bar{a}_x$ be the net premium. Then:

$$
\begin{aligned}
V(t) &= \bar{A}_{x+t} - \bar{P}_x\,\bar{a}_{x+t} \\
&= \bar{A}_{x+t} - \frac{\bar{A}_x}{\bar{a}_x}\bar{a}_{x+t} \\
&= 1 - \delta\,\bar{a}_{x+t} - \frac{(1-\delta\,\bar{a}_x)\,\bar{a}_{x+t}}{\bar{a}_x} \\
&= 1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}.
\end{aligned}
$$

(b)

$$\begin{aligned}
\frac{d}{dt}\left(\bar{a}_{x+t}\right) &= \frac{d}{dt}\left(\int_0^\infty v^s\,{}_sp_{x+t}\,ds\right) && (1)\\
&= \int_0^\infty v^s\left(\frac{d}{dt}\,{}_sp_{x+t}\right)ds && (2)\\
&= \int_0^\infty v^s\frac{d}{dt}\exp\left\{-\int_{x+t}^{x+t+s}\mu_r\,dr\right\}ds && (3)\\
&= \int_0^\infty v^s\exp\left\{-\int_{x+t}^{x+t+s}\mu_r\,dr\right\}\frac{d}{dt}\left\{-\int_{x+t}^{x+t+s}\mu_r dr\right\}ds && (4)\\
&= \int_0^\infty v^s\big({}_sp_{x+t}\left(\mu_{x+t}-\mu_{x+t+s}\right)\big)\,ds && (5)\\
&= \mu_{x+t}\int_0^\infty v^s\,{}_sp_{x+t}\,ds - \int_0^\infty v^s\,{}_sp_{x+t}\,\mu_{x+t+s}\,ds && (6)\\
&= \mu_{x+t}\,\bar{a}_{x+t}-\bar{A}_{x+t}. && (7)
\end{aligned}$$

3. (a) Reserves are required to cover the difference between the present value of future liabilities and the present value of future premium income. Reserves are needed for various purposes including to provide surrender values, to demonstrate solvency, to decide bonus rates and to determine the shareholders profits.

   (b) Using the formula ${}_tV_{30:\overline{25}|} = A_{30+t:\overline{25-t}|} - P_{30:\overline{25}|}\cdot\ddot{a}_{30+t:\overline{25-t}|}$, we get

$${}_5V_{30:\overline{25}|}=0.1301\quad {}_{10}V_{30:\overline{25}|}=0.28829\quad {}_{15}V_{30:\overline{25}|}=0.48039\quad {}_{20}V_{30:\overline{25}|}=0.71399$$

   (c) Using $V(t)=A^1_{30+t:\overline{25-t}|}-P^1_{30:\overline{25}|}\,\ddot{a}_{30+t:\overline{25-t}|}$, we get:

$$V(5)=0.00349\quad V(10)=0.00694\quad V(15)=0.00934\quad V(20)=0.00843.$$

4. Since we are given the premium, we can write

$$\begin{aligned}
{}_{15}V^1_{30:\overline{25}|} &= 100,000A^1_{45:\overline{10}|}-0.9(200)\cdot\ddot{a}_{45:\overline{10}|}\\
&= 100,000(0.01946)-180(8.3658)\\
&= 440.20
\end{aligned}$$

5. The bonus declared in year 1 is $0.03(60,000) = 1,800$. The bonus declared in year 2 comprise of $0.045(1,800) = 81$ which is earned on the existing bonus and another $1,800$ earned on the basic benefit. In year three the bonus on the existing bonuses is $0.045\,[2(1,800) + 81] = 165.65$ and another $1,800$ on the basic benefit. Therefore the total bonuses declares to date is,

$$3(1,800) + 81 + 165.66 = 5,646.65$$

(Alternatively, the declared bonuses are $1,800(1.045^2 + 1.045 + 1) = 5,646.65$). The policy value is

$${}_3V_{40:\overline{25}|} = 65,646.65A_{43:\overline{22}|} - 60,000P_{[40]:\overline{25}|} \cdot \ddot{a}_{43:\overline{22}|} = 7,062.81.$$

6. The policy value is:

$$\begin{aligned}
V(15) &= 33,000\left[1.02913v_0|q_{55} + 1.02913^2v^2{}_1|q_{55} + 1.02913^3v^3{}_2|q_{55} + \cdots\right] - 350\ddot{a}_{55} \\
&= 33,000\left[\frac{1.02913}{1.06}{}_0|q_{55} + \left(\frac{1.02913}{1.06}\right)^2{}_1|q_{55} + \left(\frac{1.02913}{1.06}\right)^3{}_2|q_{55} + \cdots\right] - 350\ddot{a}_{55} \\
&= 33,000A_{55} - 350\ddot{a}_{55} = 13,603.75
\end{aligned}$$

since A_{55} is evaluated at 3% and $\ddot{a}_{55}$ is evaluated at 6%.

7. (a) Let P be the annual premium, then

$$P\ddot{a}_{[62]:\overline{3}|} = 2,000A_{[62]:\overline{3}|} + 150 + 20\ddot{a}_{[62]:\overline{3}|} - 20$$

at 6% p.a. interest and A1967–70 select mortality. Therefore $P = 665.07$.

(b) Let P' be the net premium that would be paid at the outset, on the valuation basis (A1967–70 ultimate mortality and 3% interest). From the Tables:

$$P' = 2,000(0.32036) = 640.72$$

.

The policy value at $t = 0$ is 0.
At $t = 1$, the policy value is $2,000\,A_{63:\overline{2}|} - 640.72\,\ddot{a}_{63:\overline{2}|} = £635.61$.
At $t = 2$ the policy value is $2,000\,A_{64:\overline{1}|} - 640.72\,\ddot{a}_{64:\overline{1}|} = £1,301.02$.

8. (a) You will find it helpful to have spreadsheets of commutation functions at several rates of interest, say 3%, 4%, 5% and 6%.

(b) You will find that the answers may be slightly different from those in the solution to Q.3 given above. This is because that solution used the values given on page 58 of the Green Tables, whereas the spreadsheet will use unrounded values.

An Excel workbook showing how this example may be laid out can be found at

`www.ma.hw.ac.uk/~andrea/f79af/worksheets/tut1_q8.xls`

which can also be accessed through the module webpage.

Comment on the results: this shows clearly: (1) that the net premium policy value at outset is zero; (2) that policy values then increase; (3) that for the endowment, the policy values keep increasing up to the sum assured at maturity; and (4) that for the term assurance, the policy values eventually peak and fall to zero at expiry.