HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics II

Tutorial 9

Please prepare the following questions for discussion on Monday 12 March 2007.

Questions 1 to 3 relate to the Semi-Markov model shown in Figure 1.

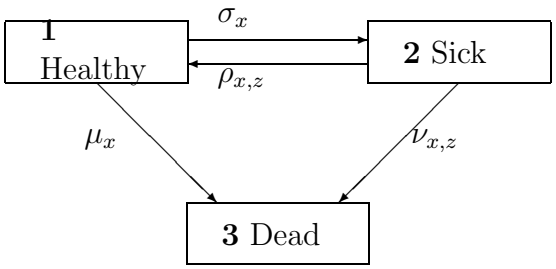

Figure 1: Semi-Markov model..

1. Explain why the formula

$$\frac{d}{dw}{}_{w,t}p_x^{12} = \begin{cases} {}_{t-w}p_x^{11} \cdot \sigma_{x+t-w} \cdot {}_wp_{x+t-w}^{\overline{22}} & \text{for} \quad 0 \le w < t \\ 0 & \text{for} \quad w > t \end{cases}$$

   is correct and describe how you could obtain numerical values for ${}_{w,t}p_x^{12}$ given the transition intensities.

2. Explain why the following two formulae are correct:

$${}_tp_x^{12} = \int_{u=0}^{t} {}_up_x^{11} \cdot \sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{\overline{22}} du.$$

$${}_tp_x^{12} = \int_{u=0}^{t} {}_up_x^{\overline{11}} \cdot \sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{22} du.$$

3. (a) Explain why the following formulae are correct:

$${}_{d,t}p_{x,z}^{22} = \int_{\max(t-d,0)}^{t} {}_up_{x,z}^{21} \cdot \sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{\overline{22}} du \quad \text{for } d < t+z.$$

$$ {}_{d,t}p^{22}_{x,z} = {}_{t}p^{\overline{22}}_{x,z} + \int_0^t {}_{u}p^{21}_{x,z} \cdot \sigma_{x+u} \cdot {}_{t-u}p^{\overline{22}}_{x+u} du \quad \text{for } d \geq t+z. $$

(b) Standard sickness policies issued by an insurance company provide a benefit payable continuously at rate B *p.a.* during periods of sickness of duration greater than d. Premiums are payable at constant rate are are waived while benefit is being paid. All premiums and benefits cease at age 65, or on earlier death.

A life aged x wishes to purchase one of these policies now. The life is currently sick, and has been sick for duration $z(> d)$. The insurance company agrees to issue the policy now on condition that for the duration of the current sickness no benefit will be payable and premiums will be payable, no matter how long this sickness lasts. Derive a formula (in terms of integrals of probabilities and discount factors) from which the premium rate P *p.a.* could be calculated.

4. An insurer sells combined death and sickness policies to healthy lives aged 35. The policies, which are for a term of 30 years, pay a lump sum of £20,000 immediately on death, with an additional £10,000 if the deceased is sick at the time of death. There is also a benefit of £3,000 per annum payable continuously to sick policyholders. There is no deferred period before benefits are payable. Annual premiums of £500 are payable continuously by healthy policyholders.

The mortality and sickness of the policyholders are described by the multiple state model in Figure 2, in which the forces of transition depend on age.

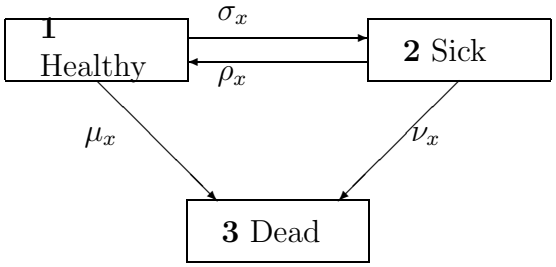

Figure 2: The multiple state model.

${}_{t}p^{gh}_{x}$ is defined as the probability that a life aged x who is in state g ($g = 1,\ 2$ or 3) is in state h at age $x+t$ ($t \geq 0$ and $h = 1,\ 2$ or 3). The force of interest is δ.

Express in integral form, using the probabilities and the various forces of transition, the expected present value of the profit to the office from one such policy at its commencement.

5. Describe briefly what is meant by each of the following terms, in each case giving an example to illustrate the points you make:

(a) temporary initial selection;

(b) anti-selection (or adverse selection);
(c) class selection.

6. (a) Discuss the suitability of the crude death rate, the standardised death rate and the standardised mortality ratio for comparing
   (i) the mortality, at different times, of the population of a given country,
   (ii) the mortality, at a certain time, of two different occupational groups in the same population.

   (b) The following table gives a summary of mortality for one of the occupational groups and for the country as a whole.

	Occupation A		Whole country	
Age group	Population	Deaths	Population	Deaths
20–34	15,000	52	960,000	3,100
35–49	12,000	74	1,400,000	7,500
50–64	10,000	109	740,000	7,100
Total	37,000	235	3,100,000	17,700

   Calculate the crude death rate, the standardised death rate and the standardised mortality ration for Occupation A, using the whole country as the standard population.

7. The following table contains abridged mortality data for two areas and also for the total population of the country of which they form a part.

	Population			No of deaths		
Age	Area A	Area B	Country	Area A	Area B	Country
30–39	800	1,200	10,000	2,800	5,400	40,000
40–49	840	1,040	10,000	5,880	9,360	80,000
50–59	800	840	9,000	10,000	12,600	126,000
60–69	720	600	7,000	23,040	22,200	248,000
70–79	640	320	4,000	44,800	28,800	320,000
Total	3,800	4,000	40,000	86,520	78,360	814,000

   Population is in units of 1,000.

   (a) Calculate the crude death rate for each area and for the country.
   (b) Calculate the directly standardised death rate for each area, standardising on the country's population.
   (c) Calculate the standardised mortality ratios for each area, using the country's data as the standard.

8. The following data have been extracted from an investigation into the mortality experience in two regions of a country, and in the whole country.

Age	Region 1 ${}^1E_x^c$	Region 2 ${}^2E_x^c$	Country ${}^sE_x^c$	Country ${}^s\mu_x$
0–19	220	180	2,300	0.00106
20–39	140	220	2,520	0.00160
40–59	110	260	2,600	0.01100
60+	100	360	2,800	0.05100
All	570	1,020	10,220	0.01744

${}^1E_x^c$ and ${}^2E_x^c$ are the central exposed to risk (equivalent to mid-year population) over each age range shown, for Region 1 and Region 2 respectively, in units of 1,000. ${}^sE_x^c$ and ${}^s\mu_x$ are respectively the central exposed to risk, in units of 1,000, and the observed rate of mortality for the whole country over each age range.

The observed numbers of deaths in Region 1 and Region 2 were 9,800 and 19,620 respectively.

(a) Calculate the crude death rate and standardised mortality ratio for each region.
(b) Comment on your results in part (a).
(c) Based on your calculations, which region do you think experiences heavier mortality.

9. A first year actuarial student has stated that he does not believe that 'social class' has any influence on mortality in the modern age. Discuss this statement, explaining why you agree or disagree with the student.

10. You have been asked to comment on the following table of standardised mortality ratios (SMR's) for males which have been calculated for a large industrialised country.

Occupation	SMR
Teachers	62
Economists, accountants, statisticians	69
Judges, barristers, solicitors	77
Ministers of religion	78
Foreman, paper processing	83
Construction workers	133
Chemical plant workers	161
General labourers	195

In particular, discuss factors which would explain the differences between occupations.

11. The number of people sick with a new disease is expected to increase according to the logistic model. The initial number sick is 100,000 and it is believed that the number sick with the disease will never exceed 250,000. The exponential parameter for growth is 5% per annum.

Calculate the number of people who will be sick after exactly 10 years.

12. You have been asked to project the population of pupils in state-controlled primary education (ages 5 to 12) in Edinburgh and the Lothians at the start of 2010. State what data you would require to carry out this projection and what assumptions you would have to make.