Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 8

1. Explain briefly what is meant by the terms 'off-period', 'waiting period' and 'deferred period', as applied to an insurance policy providing benefits during sickness.

2. Consider the Markov model for sickness with three states and transition intensities as shown in Figure 1.

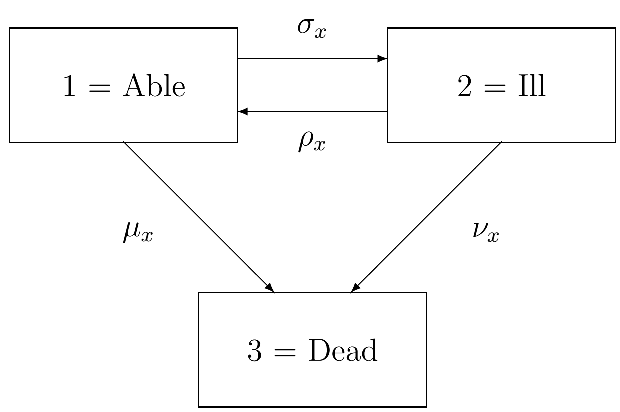

Figure 1: A Markov model of illness and death.

(a) Derive a differential equation for ${}_tp_x^{13}$ and explain briefly how you would solve it numerically.

(b) Assume that all the transition intensities are equal to some constant μ.

(1) Derive a second order ordinary differential equation for ${}_tp_x^{11}$ and verify that it is satisfied by

$${}_tp_x^{11} = 0.5 \times (e^{-\mu t} + e^{-3\mu t}).$$

(2) Show that for $0 \leq d < t$

$${}_{d,t}p_{30}^{12} = 0.5 \times \left(e^{-\mu t} - e^{-3\mu t} - e^{-\mu(t+d)} + e^{-\mu(3t-d)}\right).$$

(3) Hence, or otherwise, show that for $0 \leq d < t$,

$${}_tp_{30}^{12} - {}_{d,t}p_{30}^{12} = 0.5 \times \left(e^{-\mu(t+d)} - e^{-\mu(3t-d)}\right).$$

(4) Given that $\mu = 0.01$ and that the force of interest, δ, is 0.05 per annum, calculate the expected present value of a benefit payable continuously at the rate of £10,000 per annum to a life aged 30 until age 60 during any sickness which has lasted more than 3 months.

[The next questions relate to the C.M.I.B.'s semi-Markov model in Figure 2.]

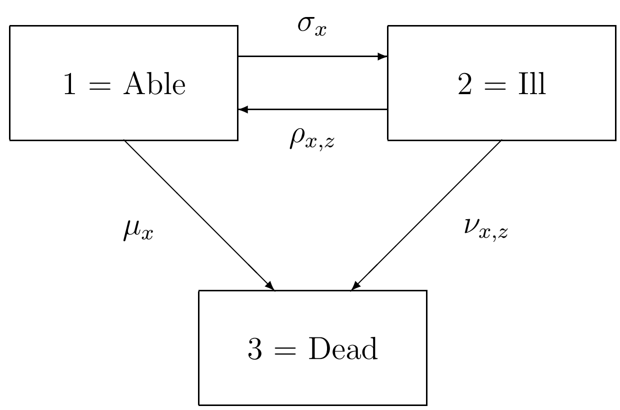

Figure 2: The C.M.I.B.'s semi-Markov model of illness and death. Age is denoted x and duration of the current spell of illness is denoted z.

3. Explain why the following is correct:

$$ {}_tp^{21}_{x,z} = \int_0^t {}_up^{\overline{22}}_{x,z} \cdot \rho_{x+u,z+u} \cdot {}_{t-u}p^{11}_{x+u} du. $$

4. Show that:

$$ \frac{d}{dt} {}_tp^{13}_x = {}_tp^{11}_x \mu_{x+t} + \int_0^t {}_up^{11}_x \cdot \sigma_{x+u} \cdot {}_{t-u}p^{\overline{22}}_{x+u} \cdot \nu_{x+t,t-u} du $$

and describe how you could obtain numerical values for ${}_tp^{13}_x$, given the transition intensities.

5. Derive a differential equation for ${}_tp^{23}_{x,z}$.