Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 8

HOMEWORK 3 - please prepare answers to Question 1 for submission in the week beginning Monday 5 March 2007

The other questions will be discussed in tutorials that week.

1. Consider the Markov Model for sickness with three states and transition intensities as shown in Figure 1.

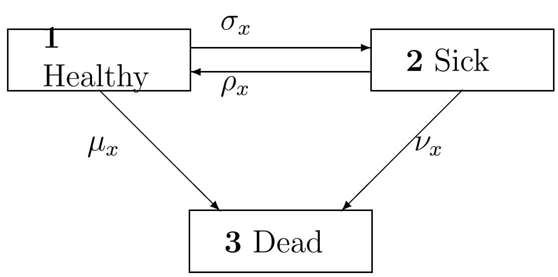

Figure 1: Three state Markov model.

(a) Derive a differential equation for ${}_tp_x^{13}$ and explain briefly how you would solve it numerically.

(b) Assume that all the transition intensities are equal to some number μ.

(i) Derive a second order ordinary differential equation for ${}_tp_x^{11}$ and verify that it is satisfied by

$${}_tp_x^{11} = 0.5 \times (e^{-\mu t} + e^{-3\mu t}).$$

(ii) Show that for $0 \leq d < t$

$${}_{d,t}p_{30}^{12} = 0.5 \times \left(e^{-\mu t} - e^{-3\mu t} - e^{-\mu(t+d)} + e^{-\mu(3t-d)}\right).$$

(iii) Hence, or otherwise, show that for $0 \leq d < t$,

$${}_tp_{30}^{12} - {}_{d,t}p_{30}^{12} = 0.5 \times \left(e^{-\mu(t+d)} - e^{-\mu(3t-d)}\right).$$

(iv) Given that $\mu = 0.01$ and that the force of interest, δ, is 0.05 per annum, calculate the expected present value of a benefit payable continuously at the rate of £10,000 per annum to a life aged 30 until age 60 during any sickness which has lasted more than 3 months.

The next questions relate to the C.M.I.B.'s semi-Markov model shown in Figure 2.

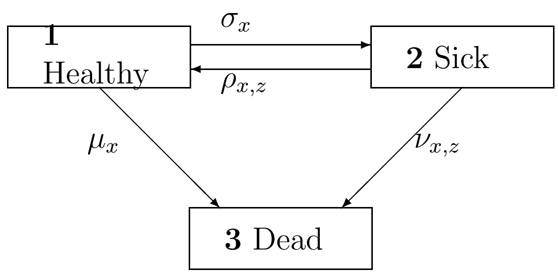

Figure 2: The C.M.I.B.'s semi-Markov model.

2. Explain why the following formula is correct:

$$ {}_tp^{21}_{x,z} = \int_{u=0}^{t} {}_up^{\overline{22}}_{x,z} \cdot \rho_{x+u,z+u} \cdot {}_{t-u}p^{11}_{x+u} du. $$

3. Show that

$$ \frac{d}{dt} {}_tp^{13}_x = {}_tp^{11}_x \mu_{x+t} + \int_{u=0}^{t} {}_up^{11}_x \cdot \sigma_{x+u} \cdot {}_{t-u}p^{\overline{22}}_{x+u} \cdot \nu_{x+t,t-u} du $$

and describe how you could obtain numerical values for ${}_tp^{13}_x$, given the transition intensities.

4. Derive a differential equation for ${}_tp^{23}_{x,z}$.