HERIOT-WATT UNIVERSITY

M.SC. IN ACTUARIAL SCIENCE

Life Insurance Mathematics I

Tutorial 7

1. Calculate the following on the basis of PMA92/PFA92 mortality and interest of 4% (in each case the first age is of a male life and the second age is of a female life):

   (a) $\ddot{a}_{70|70}$

   (b) $\ddot{a}_{65|64}$

   (c) $\ddot{a}^{(12)}_{65|64}$ (Note: payment commences at the end of the month of (65)'s death, not the end of the year of (65)'s death.)

2. A life office sells an annuity to a man aged 65, for a single premium of £P. The payments are £10,000 per year, payable monthly in advance, and on his death a reduced widow's pension of £5,000 per year will be paid to his wife, now aged 61, provided she is then alive. The payments are guaranteed (at the initial level) for the first 5 years. Initial expenses are £100, and there is an administrative cost of £5 associated with each monthly payment. Both lives are subject to the mortality of the PMA92/PFA92 ultimate table, and the interest basis is 4% per annum. Calculate the single premium P.

3. (a) Express ${}_nq^2_{xy}$ in terms of single life probabilities and contingent probabilities referring to the first death.

   (b) Suppose $\mu_x = (90 - x)^{-1}$. Evaluate ${}_{10}q^2_{30:40}$.

4. Evaluate the following functions on A1967–70 ultimate at 4% per annum interest:

   (a) $\bar{A}^{\,1}_{40:40}$

   (b) $A_{\overline{40:40}}$

   (c) $A^{\,2}_{40:40}$

   (d) $\bar{A}_{\overline{40:40:\overline{10}|}}$.

5. Define the following symbols in words, and give an expression for each of them in terms of an integral:

   (a) ${}_{\infty}q^1_{xy}$

   (b) $\bar{A}^2_{xy}$

   (c) $\bar{A}^1_{xy:\overline{n}|}$.

6. A husband and wife, aged 70 and 65 respectively, effect a policy under which the benefits are

   (a) a reversionary annuity of £10,000 p.a. payable annually in advance throughout the lifetime of the surviving spouse after the death of the first, and

   (b) a lump sum of £5,000 payable immediately on the second death.

   Level premiums are payable annually in advance until the first death. Calculate the annual premium on the following basis

   Males' mortality: PMA92 ultimate

   Females' mortality: PFA92 ultimate

   Interest: 4% p.a.

   Expenses: None.

   Ignore the possibility of divorce.

7. (Excel exercise): Estimate, by repeated application of the trapezium rule or any other suitable rule for approximate integration, the single premium for a temporary contingent assurance of £50,000 payable immediately on the death of Mrs. Smith (aged 50), provided that this event occurs within 10 years and that her husband (aged 55) is alive at the date of her death. Mrs. Smith is subject to the mortality of the AF92 ultimate table and Mr. Smith is subject to the mortality of the AM92 ultimate table. An interest rate of 7.5% p.a. is to be used.

   [Hint: Use the Excel spreadsheet `amf92_mu.xls` as a starting point.]