Heriot-Watt University

M.Sc. in Actuarial Science

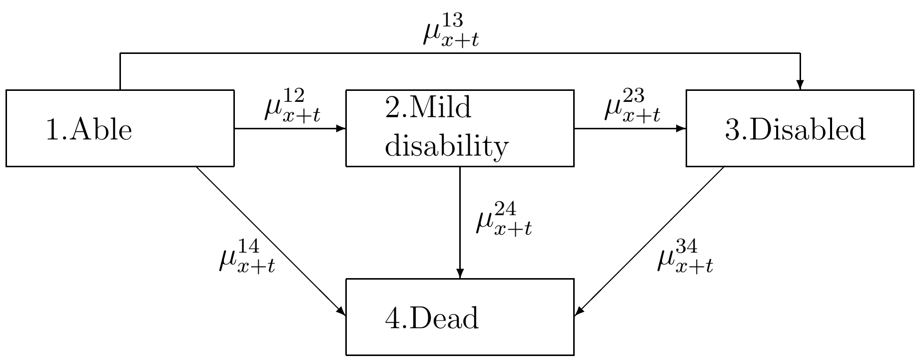
Life Insurance Mathematics I

Tutorial 7

Please prepare the following questions for discussion in the week beginning Monday 26 February 2007.

1. Explain briefly what is meant by the terms 'off-period', 'waiting period' and 'deferred period', as applied to an insurance policy providing benefits during sickness.

2. An insurance company uses the multiple state model shown below to calculate premiums for long-term care contracts:



   (a) Derive Kolmogorov's differential equation for the occupancy probability ${}_tp_x^{\overline{11}}$

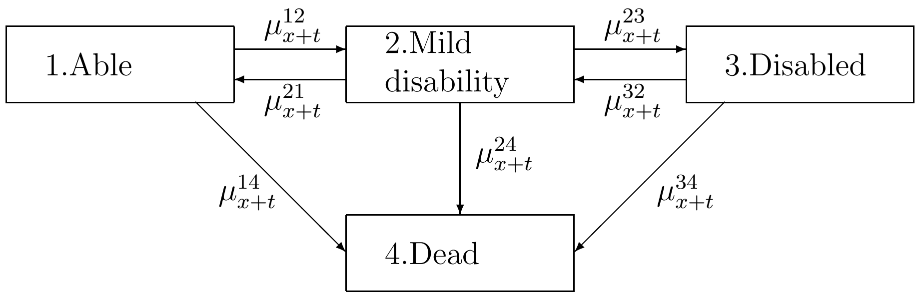
   (b) Hence, show that the following solution satisfies the differential equation derived in part (a) as well as the boundary condition ${}_0p_x^{\overline{11}} = 1$:

$$ {}_tp_x^{\overline{11}} = \exp\left\{ -\int_0^t \mu^{12}_{x+r} + \mu^{13}_{x+r} + \mu^{14}_{x+r} dr \right\} $$

   (c) Hence, or otherwise, write down an expression for ${}_tp_x^{11}$

3. An insurance company uses the multiple state model shown below to calculate premiums for the following long-term care contract:

Premiums:	Payable continuously while in state 1
Benefits:	50% SA per annum payable continuously while in state 2
	100% SA per annum payable continuously while in state 3
Waiver:	Premiums are waived while benefits are payable
Death Benefit:	There are no death benefits



The company has estimated the transition probabilities for the above model as:

$$ {}_tp_x^{11} = \begin{cases} \left[\frac{100-x-t}{100-x}\right]^3 & x+t \le 100 \\ 0 & \text{otherwise} \end{cases} \qquad {}_tp_x^{21} = \begin{cases} \frac{t\,(100-x-t)}{6000} & x+t \le 100 \\ 0 & \text{otherwise} \end{cases} $$

$$ {}_tp_x^{12} = \begin{cases} \frac{t\,(100-x-t)}{4000} & x+t \le 100 \\ 0 & \text{otherwise} \end{cases} \qquad {}_tp_x^{22} = \begin{cases} \left[\frac{100-x-t}{100-x}\right]^4 & x+t \le 100 \\ 0 & \text{otherwise} \end{cases} $$

$$ {}_tp_x^{13} = \begin{cases} \frac{t\,(100-x-t)}{2000} & x+t \le 100 \\ 0 & \text{otherwise} \end{cases} \qquad {}_tp_x^{23} = \begin{cases} \frac{t\,(100-x-t)}{1000} & x+t \le 100 \\ 0 & \text{otherwise} \end{cases} $$

where ${}_tp_x^{ij}$ is the probability that a life aged x in state i will be in state j at age $x+t$.

(a) Show that the equation of value for the above long-term care policy for a life aged x and with an initial sum assured of rate $\bar{B}$ per annum is:

$$ 0.92\,\bar{P} \int_0^{100-x} {}_tp_x^{11}\,dt = \frac{\bar{B}}{2} \int_0^{100-x} {}_tp_x^{12}\,dt + \bar{B} \int_0^{100-x} {}_tp_x^{13}\,dt $$

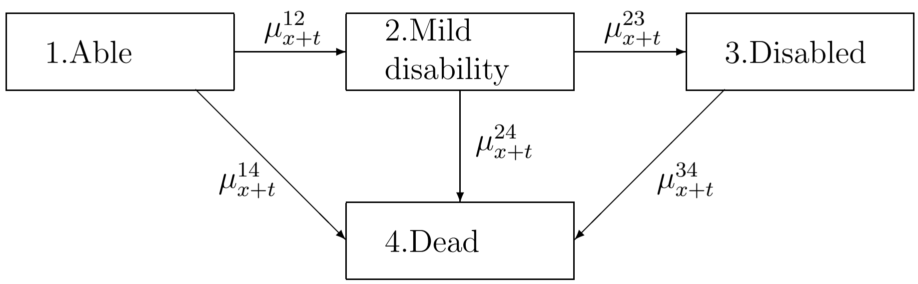
Where $\bar{P}$ is the initial annual premium rate and the company uses the following basis for calculating premiums:

Basis:

Transition probabilities:	as given
Force of interest:	5% per annum continuously
Expenses:	8% of all premiums paid
Indexation:	Premiums and benefits indexed at 5% per annum continuously

(b) Hence, calculate the annual premium rate at outset for a life aged 60 for a contract with an initial sum assured of £10,000 per annum on the above basis.

4. An insurance company uses the multiple state model shown below to calculate premiums for long-term care contracts:



The insurance company uses the following values for the transition intensities, which are independent of age:

$$\mu^{14}_x = 0.01 \quad \mu^{24}_x = 0.02 \quad \mu^{34}_x = 0.04$$
$$\mu^{12}_x = 0.025 \quad \mu^{23}_x = 0.05$$

(a) Derive Kolmogorov's differential equation for the occupancy probability ${}_tp^{12}_x$ is:

$$\frac{d}{dt}{}_tp^{12}_x = {}_tp^{11}_x\,\mu^{12}_{x+t} - {}_tp^{12}_x\,(\mu^{23}_{x+t} + \mu^{24}_{x+t})$$

(b) Show, using Euler's method and the formula above, for a small period of time s:

$${}_sp^{12}_x \approx s\,\mu^{12}_x$$

Hence, in one Euler step, calculate an approximate value of ${}_1p^{12}_x$.

Reminder: Euler's method states that if $\frac{d}{dt}f(t) = g(f(t))$ then for small h:

$$f(t+h) \approx f(t) + h\,g(f(t))$$

(c) By taking one more Euler step derive an expression for ${}_{2s}p^{12}_x$, in terms of the transition intensities and the time step s only, and hence calculate another approximate value of ${}_1p^{12}_x$. You are given:

$${}_tp^{11}_x = \exp\left\{-\int_0^t \mu^{12}_{x+r} + \mu^{14}_{x+r}dr\right\}$$

(d) Calculate ${}_1p^{12}_x$ exactly given that, if the transition intensities are constant, then:

$${}_tp^{12}_x = \frac{\mu^{12}_x}{\mu^{12}_x + \mu^{14}_x - \mu^{23}_x - \mu^{24}_x}\left[\mathrm{e}^{-(\mu^{23}_x+\mu^{24}_x)\,t} - \mathrm{e}^{-(\mu^{12}_x+\mu^{14}_x)\,t}\right]$$

and compare it to your answers for (b) and (c) and comment. How could you improve the accuracy of Euler's method to get a more accurate solution? (No further calculations are required.)

Numerical Solutions: Qu3.(b) 7,246.38 Qu4.(b) 0.025 (c) 0.024346 (d) 0.023723