Heriot-Watt University

M.Sc. in Actuarial Science

Life Insurance Mathematics I

Tutorial 4

1. (a) A life insurance company sells a whole life policy product. The premium is calculated by the equivalence principle at the outset. In detail, show that the risk of loss on a portfolio of such policies may be unacceptably high. State two ways of mitigating this risk.

   (b) State (without proof) Lidstone's theorem. How can the results of Lidstone's theorem be used to create risk reserves? Discuss how with-profits policies arise from the creation of risk reserves.

2. A life insurance company sells endowment assurances of term 25 years to persons age 35. The sum assured is £1, payable at the end of the year of death or on maturity. Net premiums and net premium reserves are calculated using A1967–70 Ultimate mortality and 5% interest.

   (a) What is the variance of the present value of the future loss on a single policy at outset? (You are given that $A_{35:\overline{25}|}$ at 10.25% is 0.10046.)

   (b) What loss will be exceeded with probability 5% if the insurer sells (a) 10; (b) 100; or (c) 1,000 contracts? Express your answers as a percentage of one year's total premiums.

   (c) How much additional reserves should the insurer hold to ensure that it will not be bankrupted by a loss in the lower three quartiles, if it sells (a) 10; (b) 100; or (c) 1,000 contracts? Express your answers as a percentage of one year's total premiums.

3. Consider an n-year endowment assurance policy providing a death benefit of S immediately on death of (x) at duration t years ($t < n$) and a survival benefit of E at time n years. Level premiums are payable continuously for n years or until the death of (x), if earlier. Let $V(t)$ denote the net premium reserve for this contract on a given mortality table and at force of interest δ p.a. Let $V^{'}(t)$ denote the net premium reserve on the same mortality table but at a force of interest of $\delta^{'}$ p.a.

   (a) Let π and $\pi^{'}$ be the net annual premiums (payable continuously) at the forces of interest δ and $\delta^{'}$ respectively. Assuming Thiele's differential equations, show that for $0 < t < n$,

$$\frac{d}{dt}\left(V^{'}(t) - V(t)\right) = (\delta^{'} + \mu_{x+t})\,(V^{'}(t) - V(t)) + c_t$$

where

$$c_t = (\pi^{'} - \pi) + (\delta^{'} - \delta)\, V(t).$$

Hence or otherwise show that, for $0 < t < n$,

$$\frac{d}{dt}\Big[G(t)(V^{'}(t) - V(t))\Big] = G(t)\, c_t$$

where

$$G(t) = \exp\left(- \int_0^t (\mu_{x+r} + \delta^{'})dr \right).$$

(b) Use the results of (a) to show that, if $V(t)$ increases as t increases and $\delta^{'} > \delta$, then $V^{'}(t) < V(t)$ at any given duration t years $(0 < t < n)$.